APPENDIX B

SOLVING THE INTERMEDIATE GOOD PRICES

The ratio of intermediate good supplies can be viewed as the following equation;

$$\frac{x_{2t}}{x_{1t}} = \frac{\frac{X_{2t}}{N_t}}{\frac{X_{1t}}{N_t}} = \frac{\left(\frac{K_{2t}}{L_{2t}}\frac{L_{2t}}{N_t}\right)^{\alpha_2}\left(\frac{L_{2t}}{N_t}\right)^{1-\alpha_2}}{\left(\frac{K_{1t}}{L_{1t}}\frac{L_{1t}}{N_t}\right)^{\alpha_1}\left(\frac{L_{1t}}{N_t}\right)^{1-\alpha_1}} = \frac{\kappa_{2t}^{\alpha_2} l_{2t}}{\kappa_{1t}^{\alpha_1} l_{1t}}, \qquad \text{(B.1)}$$

where $\kappa_{it} = \frac{K_{it}}{L_{it}}$.

From factor market clearing condition, $k_t = \frac{K_t}{N_t} = \frac{K_{1t}}{L_{1t}}\frac{L_{1t}}{N_t} + \frac{K_{2t}}{L_{2t}}\frac{L_{2t}}{N_t} = \kappa_{1t} l_{1t} + \kappa_{2t} l_{2t}$,

and $l_t = l_{1t} + l_{2t} = 1$.

Then, the FOC of profit maximization of final good equation (3.2.14) after substituting equation (B.1) becomes

$$\frac{\alpha_3}{1-\alpha_3} = \frac{p_{1t}}{p_{2t}}\frac{\kappa_{1t}^{\alpha_1} l_{1t}}{\kappa_{2t}^{\alpha_2}(1-l_{2t})}. \qquad \text{(B.2)}$$

From the first order conditions of profit maximizing problems of intermediate goods, w_t in equation (3.2.3) can be rearranged as

$$w_t = (1-\alpha_1)p_{1t}\left(\frac{K_{1t}}{L_{1t}}\frac{L_{1t}}{N_t}\right)^{\alpha_1}\left(\frac{L_{1t}}{N_t}\right)^{-\alpha_1} = (1-\alpha_2)p_{2t}\left(\frac{K_{2t}}{L_{2t}}\frac{L_{2t}}{N_t}\right)^{\alpha_2}\left(\frac{L_{2t}}{N_t}\right)^{-\alpha_2},$$

$$w_t = (1-\alpha_1)p_{1t}\kappa_{1t}^{\alpha_1} = (1-\alpha_2)p_{2t}\kappa_{2t}^{\alpha_2}. \qquad \text{(B.3)}$$

Rearranging equation (B.3) to be

$$\frac{p_{1t}\kappa_{1t}^{\alpha_1}}{p_{2t}\kappa_{2t}^{\alpha_2}} = \frac{1-\alpha_2}{1-\alpha_1}. \qquad \text{(B.4)}$$

Then, substituting equation (B.4) into (B.2), to get

$$\frac{\alpha_3}{1-\alpha_3} = \frac{1-\alpha_2}{1-\alpha_1}\frac{l_{1t}}{(1-l_{2t})}. \tag{B.5}$$

Rearranging equation (B.5) to get

$$\frac{1}{l_{1t}} - 1 = \frac{1-l_{1t}}{l_{1t}} = \frac{(1-\alpha_3)}{\alpha_3}\frac{(1-\alpha_2)}{(1-\alpha_1)}$$

$$\frac{1}{l_{1t}} = \frac{1-\alpha_2-\alpha_3+\alpha_2\alpha_3}{\alpha_3(1-\alpha_1)} + \frac{\alpha_3-\alpha_1\alpha_3}{\alpha_3(1-\alpha_1)}$$

$$l_{1t} = \frac{\alpha_3(1-\alpha_1)}{1-\alpha_1\alpha_3-(1-\alpha_3)\alpha_2}$$

Let $\gamma = \alpha_1\alpha_3 + \alpha_2(1-\alpha_3)$, thus $l_{1t} = \dfrac{\alpha_3(1-\alpha_1)}{1-\gamma}$, (B.6)

and $l_{2t} = 1 - l_{1t} = \dfrac{1-\alpha_1\alpha_3-\alpha_2(1-\alpha_3)-\alpha_3-\alpha_1\alpha_3}{1-\gamma} = \dfrac{(1-\alpha_2)(1-\alpha_3)}{1-\gamma}$. (B.7)

That is, labor used in each sector can be expressed only in parameters.

From the first order conditions of profit maximizing problems of intermediate goods, r_t in equation (3.2.4) can be rearranged as

$$r_t = \alpha_1 p_{1t}\left(\frac{K_{1t}}{L_{1t}}\frac{L_{1t}}{N_t}\right)^{\alpha_1-1}\left(\frac{L_{1t}}{N_t}\right)^{1-\alpha_1} = \alpha_2 p_{2t}\left(\frac{K_{2t}}{L_{2t}}\frac{L_{2t}}{N_t}\right)^{\alpha_2-1}\left(\frac{L_{2t}}{N_t}\right)^{1-\alpha_2},$$

$$r_t = \alpha_1 p_{1t}\kappa_{1t}^{\alpha_1-1} = \alpha_2 p_{2t}\kappa_{2t}^{\alpha_2-1}. \tag{B.8}$$

Combining equation (B.3) and (B.8) to get wage-rental ratio expressed as the following equation;

$$\frac{w}{r} = \frac{1-\alpha_1}{\alpha_1}\kappa_{1t} = \frac{1-\alpha_2}{\alpha_2}\kappa_{2t}. \tag{B.9}$$

Since rearranging equation (B.9) will receive

$$\kappa_{1t} = \frac{\alpha_1(1-\alpha_1)}{\alpha_2(1-\alpha_2)}\kappa_{2t}, \tag{B.10}$$

then substituting factor market clearing condition of capital, $\kappa_{2t} = \dfrac{k_t - \kappa_{1t}l_{1t}}{1-l_{1t}}$ into,

$$\kappa_{1t} = \frac{\alpha_1(1-\alpha_1)}{\alpha_2(1-\alpha_2)}\frac{k_t}{1-l_{1t}} - \frac{\alpha_1(1-\alpha_1)}{\alpha_2(1-\alpha_2)}\frac{\kappa_{1t}l_{1t}}{1-l_{1t}}$$

$$\kappa_{1t} = \frac{\alpha_1(1-\alpha_2)}{\alpha_2(1-\alpha_1)(1-l_{1t}) + \alpha_1(1-\alpha_2)l_{1t}} k_t \,, \qquad \text{(B.11)}$$

and

$$\kappa_{2t} = \frac{\alpha_2(1-\alpha_1)}{\alpha_2(1-\alpha_1)(1-l_{1t}) + \alpha_1(1-\alpha_2)l_{1t}} k_t \qquad \text{(B.12)}$$

Substituting $l_{1t} = \frac{\alpha_3(1-\alpha_1)}{1-\gamma}$ and $1-l_{1t} = \frac{(1-\alpha_2)(1-\alpha_3)}{1-\gamma}$ into (B.11) and (B.12).

Finally, receiving

$$\kappa_{1t} = \frac{\alpha_1(1-\gamma)}{\gamma(1-\alpha_1)} k_t \,, \qquad \text{(B.13)}$$

and

$$\kappa_{2t} = \frac{\alpha_2(1-\gamma)}{\gamma(1-\alpha_2)} k_t \,. \qquad \text{(B.14)}$$

Since $\alpha_1 < \alpha_2$, hence $\kappa_{1t} < \kappa_{2t}$

Therefore,

$$\frac{x_{2t}}{x_{1t}} = \frac{X_{2t}}{X_{1t}} = \frac{\kappa_{2t}^{\alpha_2} l_{2t}}{\kappa_{1t}^{\alpha_1} l_{1t}} = \left[\frac{\alpha_2(1-\gamma)}{\gamma(1-\alpha_2)}\right]^{\alpha_2} \left[\frac{\gamma(1-\alpha_1)}{\alpha_1(1-\gamma)}\right]^{\alpha_1} \frac{1-\gamma}{\alpha_3(1-\alpha_1)} \frac{(1-\alpha_3)(1-\alpha_2)}{1-\gamma} k_t^{\alpha_2-\alpha_1}$$

$$\frac{x_{2t}}{x_{1t}} = \frac{\alpha_2^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}{\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} \frac{(1-\alpha_3)}{\alpha_3} \left(\frac{1-\gamma}{\gamma}\right)^{\alpha_2-\alpha_1} k_t^{\alpha_2-\alpha_1} \qquad \text{(B.15)}$$

Equation (B.15) is the same as (3.2.18). Substituting this intermediate good supply ratio to the integrated equilibrium prices, equation (3.2.16) and (3.2.17), then the prices of intermediated goods are given by world capital-labor ratio;

$$p_{1t} = \alpha_3 \left[\frac{\alpha_2^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}{\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} \frac{(1-\alpha_3)}{\alpha_3} \left(\frac{1-\gamma}{\gamma}\right)^{\alpha_2-\alpha_1}\right]^{1-\alpha_3} k_t^{(1-\alpha_3)(\alpha_2-\alpha_1)}, \qquad \text{(B.16)}$$

$$p_{2t} = (1\text{-}\alpha_3) \left[\frac{\alpha_2^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}{\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} \frac{(1-\alpha_3)}{\alpha_3} \left(\frac{1-\gamma}{\gamma}\right)^{\alpha_2-\alpha_1}\right]^{-\alpha_3} k_t^{-\alpha_3(\alpha_2-\alpha_1)}, \qquad \text{(B.17)}$$

which are equation (3.2.19) and (3.2.20), respectively.