APPENDIX A

SOLVING THE FACTOR MARKET CLEARING CONDITIONS

From factor market clearing conditions, since w^j and r^j are known, then solving for x_{1t}^j and x_{2t}^j.

$$\begin{bmatrix} \left(\dfrac{\alpha_1}{1-\alpha_1}\dfrac{w_t^j}{r_t^j}\right)^{-\alpha_1} & \left(\dfrac{\alpha_2}{1-\alpha_2}\dfrac{w_t^j}{r_t^j}\right)^{-\alpha_2} \\ \left(\dfrac{\alpha_1}{1-\alpha_1}\dfrac{w_t^j}{r_t^j}\right)^{1-\alpha_1} & \left(\dfrac{\alpha_2}{1-\alpha_2}\dfrac{w_t^j}{r_t^j}\right)^{1-\alpha_2} \end{bmatrix} \begin{bmatrix} x_{1t}^j \\ x_{2t}^j \end{bmatrix} = \begin{bmatrix} 1 \\ k_t \end{bmatrix} \qquad \text{(A.1)}$$

Substituting $\dfrac{w}{r}$ into the linear system (A.1), where

$$\begin{aligned} \frac{w}{r} &= \frac{1-\alpha_1}{\alpha_1} A\left(\frac{p_{1t}}{p_{2t}}\right)^{\frac{1}{\alpha_2-\alpha_1}} = \frac{1-\alpha_2}{\alpha_2} B\left(\frac{p_{1t}}{p_{2t}}\right)^{\frac{1}{\alpha_2-\alpha_1}} \\ &= \left(\frac{\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}{\alpha_2^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}\frac{p_{1t}}{p_{2t}}\right)^{\frac{1}{\alpha_2-\alpha_1}} \end{aligned} \qquad \text{(A.2)}$$

Then, the linear system becomes

$$\begin{bmatrix} A^{-\alpha_1}(p_{1t}/p_{2t})^{\frac{\alpha_1}{\alpha_1-\alpha_2}} & B^{-\alpha_2}(p_{1t}/p_{2t})^{\frac{\alpha_2}{\alpha_1-\alpha_2}} \\ A^{1-\alpha_1}(p_{1t}/p_{2t})^{\frac{\alpha_1-1}{\alpha_1-\alpha_2}} & B^{1-\alpha_2}(p_{1t}/p_{2t})^{\frac{\alpha_2-1}{\alpha_1-\alpha_2}} \end{bmatrix} \begin{bmatrix} x_{1t}^j \\ x_{2t}^j \end{bmatrix} = \begin{bmatrix} 1 \\ k_t \end{bmatrix} \qquad \text{(A.3)}$$

let $\Lambda = \begin{bmatrix} A^{-\alpha_1}(p_{2t}/p_{1t})^{\frac{-\alpha_1}{\alpha_1-\alpha_2}} & B^{-\alpha_2}(p_{2t}/p_{1t})^{\frac{-\alpha_2}{\alpha_1-\alpha_2}} \\ A^{1-\alpha_1}(p_{2t}/p_{1t})^{\frac{1-\alpha_1}{\alpha_1-\alpha_2}} & B^{1-\alpha_2}(p_{2t}/p_{1t})^{\frac{1-\alpha_2}{\alpha_1-\alpha_2}} \end{bmatrix}$,

then $|\Lambda| = (B-A)A^{-\alpha_1}B^{-\alpha_2}(p_{2t}/p_{1t})^{\frac{1-\alpha_1-\alpha_2}{\alpha_1-\alpha_2}}$.

By Cramer's rule, $\left[x_{it}^j\right]=\Lambda^{-1}\left[1,k_t^j\right]$; that is,

$$\begin{bmatrix} x_{1t}^j \\ x_{2t}^j \end{bmatrix}=\frac{1}{(B-A)A^{-\alpha_1}B^{-\alpha_2}(p_{2t}/p_{1t})^{\frac{1-\alpha_1-\alpha_2}{\alpha_1-\alpha_2}}}\begin{bmatrix} B^{1-\alpha_2}(p_{2t}/p_{1t})^{\frac{1-\alpha_2}{\alpha_1-\alpha_2}} & -B^{-\alpha_2}(p_{2t}/p_{1t})^{\frac{-\alpha_2}{\alpha_1-\alpha_2}} \\ -A^{1-\alpha_1}(p_{2t}/p_{1t})^{\frac{1-\alpha_1}{\alpha_1-\alpha_2}} & A^{-\alpha_1}(p_{2t}/p_{1t})^{\frac{-\alpha_1}{\alpha_1-\alpha_2}} \end{bmatrix}\begin{bmatrix} 1 \\ k_t \end{bmatrix}$$

Thus,

$$x_{1t}^j=\frac{1}{(B-A)A^{-\alpha_1}B^{-\alpha_2}(p_{2t}/p_{1t})^{\frac{1-\alpha_1-\alpha_2}{\alpha_1-\alpha_2}}}\left[B^{1-\alpha_2}(p_{2t}/p_{1t})^{\frac{1-\alpha_2}{\alpha_1-\alpha_2}}-B^{-\alpha_2}(p_{2t}/p_{1t})^{\frac{-\alpha_2}{\alpha_1-\alpha_2}}k_t^j\right]$$

$$=\frac{A^{\alpha_1}B}{B-A}(p_{2t}/p_{1t})^{\frac{1-\alpha_2-1+\alpha_1+\alpha_2}{\alpha_1-\alpha_2}}-\frac{A^{\alpha_1}}{B-A}k_t^j(p_{2t}/p_{1t})^{\frac{-\alpha_2-1+\alpha_1+\alpha_2}{\alpha_1-\alpha_2}},$$

$$=\frac{A^{\alpha_1}}{B-A}\left[B-k_t^j(p_{2t}/p_{1t})^{\frac{1}{\alpha_2-\alpha_1}}\right](p_{2t}/p_{1t})^{\frac{\alpha_1}{\alpha_1-\alpha_2}} \tag{A.4}$$

and

$$x_{1t}^j=\frac{1}{(B-A)A^{-\alpha_1}B^{-\alpha_2}(p_{2t}/p_{1t})^{\frac{1-\alpha_1-\alpha_2}{\alpha_1-\alpha_2}}}\left[-A^{1-\alpha_1}(p_{2t}/p_{1t})^{\frac{1-\alpha_1}{\alpha_1-\alpha_2}}+A^{-\alpha_1}(p_{2t}/p_{1t})^{\frac{-\alpha_1}{\alpha_1-\alpha_2}}k_t^j\right]$$

$$=\frac{B^{\alpha_2}}{B-A}k_t^j(p_{2t}/p_{1t})^{\frac{-\alpha_1-1+\alpha_1+\alpha_2}{\alpha_1-\alpha_2}}-\frac{AB^{\alpha_2}}{B-A}(p_{2t}/p_{1t})^{\frac{1-\alpha_1-1+\alpha_1+\alpha_2}{\alpha_1-\alpha_2}}.$$

$$=\frac{B^{\alpha_2}}{B-A}\left[k_t^j(p_{2t}/p_{1t})^{\frac{1}{\alpha_2-\alpha_1}}-A\right](p_{2t}/p_{1t})^{\frac{\alpha_2}{\alpha_1-\alpha_2}} \tag{A.5}$$

Substituting x_{1t}^j and x_{2t}^j into equation (3.1.21) and (3.1.22), then all factor demands are determined by intermediate good price ratio and each economy's capital-labor ratio as the following equations.

$$
\begin{aligned}
l_{1t}^{j} &= A^{-\alpha_1}(p_{2t}/p_{1t})^{\frac{-\alpha_1}{\alpha_1-\alpha_2}}x_{1t}^{j}\\
&= A^{-\alpha_1}(p_{2t}/p_{1t})^{\frac{-\alpha_1}{\alpha_1-\alpha_2}}\frac{A^{\alpha_1}}{B-A}\left[B-k_t^{j}(p_{2t}/p_{1t})^{\frac{1}{\alpha_2-\alpha_1}}\right](p_{2t}/p_{1t})^{\frac{\alpha_1}{\alpha_1-\alpha_2}}\\
&= \frac{1}{B-A}\left[B-k_t^{j}(p_{2t}/p_{1t})^{\frac{1}{\alpha_2-\alpha_1}}\right] \qquad \text{(A.6)}
\end{aligned}
$$

$$
\begin{aligned}
l_{2t}^{j} &= B^{-\alpha_2}(p_{2t}/p_{1t})^{\frac{-\alpha_2}{\alpha_1-\alpha_2}}x_{2t}^{j}\\
&= B^{-\alpha_2}(p_{2t}/p_{1t})^{\frac{-\alpha_2}{\alpha_1-\alpha_2}}\frac{B^{\alpha_2}}{B-A}\left[k_t^{j}(p_{2t}/p_{1t})^{\frac{1}{\alpha_2-\alpha_1}}-A\right](p_{2t}/p_{1t})^{\frac{\alpha_2}{\alpha_1-\alpha_2}}\\
&= \frac{1}{B-A}\left[k_t^{j}(p_{2t}/p_{1t})^{\frac{1}{\alpha_2-\alpha_1}}-A\right] \qquad \text{(A.7)}
\end{aligned}
$$

$$
\begin{aligned}
k_{1t}^{j} &= A^{1-\alpha_1}(p_{2t}/p_{1t})^{\frac{1-\alpha_1}{\alpha_1-\alpha_2}}x_{1t}^{j}\\
&= A^{1-\alpha_1}(p_{2t}/p_{1t})^{\frac{1-\alpha_1}{\alpha_1-\alpha_2}}\frac{A^{\alpha_1}}{B-A}\left[B-k_t^{j}(p_{2t}/p_{1t})^{\frac{1}{\alpha_2-\alpha_1}}\right](p_{2t}/p_{1t})^{\frac{\alpha_1}{\alpha_1-\alpha_2}}\\
&= \frac{A}{B-A}\left[B(p_{2t}/p_{1t})^{\frac{1}{\alpha_1-\alpha_2}}-k_t^{j}\right] \qquad \text{(A.8)}
\end{aligned}
$$

$$
\begin{aligned}
k_{2t}^{j} &= B^{1-\alpha_2}(p_{2t}/p_{1t})^{\frac{1-\alpha_2}{\alpha_1-\alpha_2}}x_{2t}^{j}\\
&= B^{1-\alpha_2}(p_{2t}/p_{1t})^{\frac{1-\alpha_2}{\alpha_1-\alpha_2}}\frac{B^{\alpha_2}}{B-A}\left[k_t^{j}(p_{2t}/p_{1t})^{\frac{1}{\alpha_2-\alpha_1}}-A\right](p_{2t}/p_{1t})^{\frac{\alpha_2}{\alpha_1-\alpha_2}}\\
&= \frac{B}{B-A}\left[k_t^{j}-A(p_{2t}/p_{1t})^{\frac{1}{\alpha_1-\alpha_2}}\right] \qquad \text{(A.9)}
\end{aligned}
$$