# APPENDIX E

# COMMON PROBABILITY DISTRIBUTIONS

**Definition 1:** The gamma distributions

The gamma distribution is a two-parameter family of continuous probability distributions. It is that a continuous random variable Y has a *gamma* distribution with mean $\mu > 0$ and degrees of freedom $v > 0$ , denoted by $Y \sim \gamma(\mu, v)$, if its p.d.f is

$$f_\gamma(y|\mu, v) = \begin{cases} c_\gamma^{-1} y^{\frac{v-2}{2}} \exp\left(-\frac{yv}{2\mu}\right) & \text{if } 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

where the integrating constant is given by $c_\gamma = (2\mu / v)^{\frac{v}{2}} \Gamma\left(\frac{v}{2}\right)$ and $\Gamma(a)$ is the gamma function. It is also common to parameterize the gamma in terms of $\alpha = v/2$ and $\beta = 2\mu / v$ , in which case we denote the distribution as $Y \sim G(\alpha, \beta)$. The associated density function under this parameterization is denoted by , where

$$f_G(y|\alpha, \beta) = \begin{cases} c_G^{-1} y^{\alpha-1} \exp(-y/\beta) & \text{if } 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

and $c_G = \beta^\alpha \Gamma(\alpha)$

**Theorem 1:** Mean and variance of the gamma distribution

If $Y \sim G(\alpha, \beta)$ then $E(Y) = \alpha\beta$ and $Var(Y) = \alpha\beta^2$ . If $Y \sim \gamma(\mu, v)$, then $E(Y) = \mu$ and $Var(Y) = 2\mu^2 / v$

Notes: The gamma distribution is very important one in Bayesian econometrics as it usually relates to the error precision. Distribution related to the Gamma include the Chi-squared distribution which is a Gamma distribution with $v = \mu$. It is denot by $Y \sim \chi^2(v)$. The exponential distribution is a Gamma distribution with $v = 2$

**Definition 2:** The inverse gamma distributions

The inverse gamma distribution is a two-parameter family of continuous probability distributions which is the distribution of the *reciprocal* of a variable distributed according to the gamma distribution. If Y has an inversed gamma distribution, then 1/Y has a gamma distribution. It has p.d.f. as

$$Y \sim IG(\alpha,\beta) \Rightarrow p(y) = \left[\Gamma(\alpha)\beta^{\alpha}\right]^{-1} y^{-(\alpha+1)} \exp\left(-1/\left[y\beta\right]\right)$$

**Theorem 2:** Mean and variance of the inverse gamma distribution

If $Y \sim IG(\alpha,\beta)$ then $E(Y) = \left[\beta(\alpha-1)\right]$, for $\alpha > 1$ and the variance is $Var(Y) = \left[\beta^2(\alpha-1)^2(\alpha-2)\right]$ for $\alpha > 2$.

**Definition 3:** The beta distribution

The beta distribution is a family of continuous probability distributions defined on the interval $[0,1]$ parameterized by two positive shape parameters, typically denoted by $\alpha$ and $\beta$. The p.d.f. of beta distribution is

$$\begin{aligned} f_B(y;\alpha,\beta) &= \frac{y^{\alpha-1}(1-y)^{\beta-1}}{\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}\,du} \\ &= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1}(1-y)^{\beta-1} \\ &= \frac{1}{B(\alpha,\beta)} y^{\alpha-1}(1-y)^{\beta-1} \end{aligned}$$

where $\Gamma$ is the gamma function. The beta function, B, appears as a normalization constant to ensure that the total probability integrates to unity.

**Theorem 3:** Mean and variance of the beta distribution

If $Y \sim B(\alpha,\beta)$ then $E(Y) = \dfrac{\alpha}{\alpha+\beta}$ and the variance is $Var(Y) = \dfrac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$.

**Definition 4:** The normal distribution

The normal distribution, also called the Gaussian distribution, is an important family of continuous probability distributions. It is that a continuous random variable Y has a normal distribution with mean $\mu$ and variance $\sigma^2 \geq 0$, denoted by $Y \sim N\left(\mu, \sigma^2\right)$, if its p.d.f is

$$f_N\left(y \middle| \mu, \sigma^2\right) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{y-\mu}{\sigma}\right)^2} \qquad \text{for } -\infty < y < \infty$$

**Theorem 4:** Mean and variance of the normal distribution

If $Y \sim N\left(\mu, \sigma^2\right)$ then $E\left(Y\right) = \mu$ and the variance is $Var\left(Y\right) = \sigma^2$.