APPENDIX C

THE NEW KEYNESIAN PHILLIPS CURVE

C.1 Optimal Price Setting in the Calvo Model

According to the Calvo style every firm resets its price with the probability $1-\theta$ each period in other words it means simply that only a $1-\theta$ portion of firms is able to reoptimise their prices. An individual firm's probability of re-optimizing in any given period is independent on the time elapsed since it last rested its price implied by the time is independent of the time since the last adjustment. The rest of the firms keep the price adjusted by the indexation to the last period inflation.

In the Calvo price setting holds $P_{H,t+k}(j)=\overline{P}_{H,t}(j)$ with probability θ^k for $k=0,1,2,...,$ which represents the likelihood that new price chosen at current period would last for the next k-period ahead, where $\overline{P}_{H,t}$ is the new price set by a firm j adjusting it in period t. Because we suppose that all firms choose the best new price, which is the same one for all firms, we drop the subscript j.

Every firm set its new price $\overline{P}_{H,t}$ in period t to maximize the discounted value of all future profits to reach the most effective behavior during its optimizing. For the $j-$th firms we can write:

$$E_t\sum_{k=0}^{\infty}\theta_H^k\frac{1}{R_{t+k}}\left\{Y_{t+k}(j)\left(\overline{P}_{H,t}(j)-MC_{t+k}^n\right)\right\}$$

The chosen firm's price is the same we can rewrite the previous function into the following form:

$$E_t\sum_{k=0}^{\infty}\theta_H^k\frac{1}{R_{t+k}}\left\{Y_{t+k}\left(\overline{P}_{H,t}-MC_{t+k}^n\right)\right\}$$

Every firm tries to maximize it by setting the new price $\overline{P}_{H,t}$ subject to the sequence of demand constraints (expressed as the demand constraints):

$$Y_{t+k} \le \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \left(C_{H,t+k} + C^*_{H,t+k} \right),$$

where MC^n_{t+k} are nominal marginal costs and the demand constraint for the $i-$th good is

$$Y^d_{t+k}(i) \equiv \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} Y_{t+k} = \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \left(C_{H,t+k} + C^*_{H,t+k} \right),$$

The first order condition (FOC) of the optimizing behavior is calculated with using the Lagrangian function.

$$\max E_t \sum_{k=0}^{\infty} \theta_H^k \frac{1}{R_{t+k}} \left\{ Y^d_{t+k}(i) \left(\overline{P}_{H,t} - MC^n_{t+k} \right) \right\} \quad \text{(C1.1)}$$

$$s.t. \qquad Y^d_{t+k}(i) \equiv \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} Y_{t+k} \quad \text{(C1.2)}$$

The Lagrangian has the following form:

$$L\left(\overline{P}_{H,t}\right) = E_t \sum_{k=0}^{\infty} \theta_H^k \frac{1}{R_{t+k}} \left\{ \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} Y_{t+k} \left(\overline{P}_{H,t} - MC^n_{t+k} \right) \right\}$$

$$L\left(\overline{P}_{H,t}\right) = E_t \sum_{k=0}^{\infty} \theta_H^k \frac{1}{R_{t+k}} \left\{ \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} Y_{t+k} \overline{P}_{H,t} - \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} Y_{t+k} MC^n_{t+k} \right\}$$

$$L\left(\overline{P}_{H,t}\right) = \sum_{k=0}^{\infty} \theta_H^k E_t \frac{1}{R_{t+k}} \left\{ \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon+1} Y_{t+k} P_{H,t+k} - \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} Y_{t+k} MC^n_{t+k} \right\}$$

and the calculation:

$$\frac{\partial L\left(\overline{P}_{H,t}\right)}{\partial \overline{P}_{H,t}} = \sum_{k=0}^{\infty} \theta_H^k E_t \frac{1}{R_{t+k}} \left\{ \left(-\varepsilon+1\right) \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \frac{1}{P_{H,t+k}} Y_{t+k} P_{H,t+k} - \left(-\varepsilon\right) \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon-1} \frac{1}{P_{H,t+k}} Y_{t+k} MC_{t+k}^n \right\}$$

$$= \sum_{k=0}^{\infty} \theta_H^k E_t \frac{1}{R_{t+k}} \left\{ \left(-\varepsilon+1\right) \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} Y_{t+k} + \varepsilon \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-1} \frac{1}{P_{H,t+k}} Y_{t+k} MC_{t+k}^n \right\}$$

$$= \sum_{k=0}^{\infty} \theta_H^k E_t \frac{1}{R_{t+k}} \left\{ \left(-\varepsilon+1\right) Y_{t+k}^d\left(i\right) + \varepsilon \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \frac{P_{H,t+k}}{\overline{P}_{H,t}} \frac{1}{P_{H,t+k}} Y_{t+k} MC_{t+k}^n \right\}$$

$$= \sum_{k=0}^{\infty} \theta_H^k E_t \frac{1}{R_{t+k}} \left\{ \left(-\varepsilon+1\right) Y_{t+k}^d\left(i\right) + \varepsilon \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \frac{1}{\overline{P}_{H,t}} Y_{t+k} MC_{t+k}^n \right\}$$

$$= \sum_{k=0}^{\infty} \theta_H^k E_t \frac{1}{R_{t+k}} \left\{ \left(-\varepsilon+1\right) Y_{t+k}^d\left(i\right) + \varepsilon Y_{t+k}^d\left(i\right) \frac{1}{\overline{P}_{H,t}} MC_{t+k}^n \right\}$$

$$= \sum_{k=0}^{\infty} \theta_H^k E_t \frac{1}{R_{t+k}} Y_{t+k}^d\left(i\right) \left\{ \left(-\varepsilon+1\right) + \varepsilon \frac{1}{\overline{P}_{H,t}} MC_{t+k}^n \right\}$$

The derivative of the Lagrangian function equals zero:

$$\sum_{k=0}^{\infty} \theta_H^k E_t \frac{1}{R_{t+k}} Y_{t+k}^d\left(i\right) \left\{ \left(-\varepsilon+1\right) + \varepsilon \frac{1}{\overline{P}_{H,t}} MC_{t+k}^n \right\} = 0$$

$$\sum_{k=0}^{\infty} \theta_H^k E_t \frac{1}{R_{t+k}} \left\{ Y_{t+k}^d\left(i\right) \left(\overline{P}_{H,t} - \frac{\varepsilon}{\left(\varepsilon-1\right)} MC_{t+k}^n \right) \right\} = 0$$

The last equation is the first order condition for the firm's optimizing problem. We use the Euler equation

$$1 = \beta R_t E_t \left\{ \left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}$$

$$\frac{1}{R_{t+1}} = \beta E_t \left\{ \left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\} \qquad \text{(C1.3)}$$

and for $t+k$:

$$\frac{1}{R_{t+k}} = \beta^k E_t \left\{ \left(\frac{C_{t+k} - hC_{t+k-1}}{C_t - hC_{t-1}} \right)^{-\sigma} \frac{P_t}{P_{t+k}} \right\} \qquad \frac{1}{R_{t+k}} = \beta^k E_t \left\{ \left(\frac{C_{t+k+1} - hC_{t+k}}{C_{t+k} - hC_{t+k-1}} \right)^{-\sigma} \frac{P_{t+k}}{P_{t+k+1}} \right\}$$

$$\frac{1}{R_{t+k}} = \beta^k E_t \left\{ \left(\frac{\tilde{C}_{t+k}}{\tilde{C}_t} \right)^{-\sigma} \frac{P_t}{P_{t+k}} \right\} \qquad \text{where } \tilde{C}_t = C_t - hC_{t-1}$$

Now we rule out R_{t+k} from the FOC

$$0 = E_t \sum_{k=0}^{\infty} \theta_H^k \beta^k \left\{ \left(\frac{\tilde{C}_{t+k}}{\tilde{C}_t} \right)^{-\sigma} \frac{P_t}{P_{t+k}} \right\} \left\{ Y_{t+k}^d (i) \left(\overline{P}_{H,t} - \frac{\varepsilon}{(\varepsilon - 1)} MC_{t+k}^n \right) \right\}$$

$$0 = E_t \sum_{k=0}^{\infty} (\beta \theta_H)^k \left\{ \left(\frac{\tilde{C}_{t+k}}{\tilde{C}_t} \right)^{-\sigma} \frac{P_{t+k}^{-1}}{P_t^{-1}} \right\} \left\{ Y_{t+k}^d (i) \left(\overline{P}_{H,t} - \frac{\varepsilon}{(\varepsilon - 1)} MC_{t+k}^n \right) \right\}$$

$$0 = E_t \sum_{k=0}^{\infty} (\beta \theta_H)^k \left\{ \frac{\tilde{C}_{t+k}^{-\sigma}}{\tilde{C}_t^{-\sigma}} \frac{P_{t+k}^{-1}}{P_t^{-1}} \right\} \left\{ Y_{t+k}^d (i) \left(\overline{P}_{H,t} - \frac{\varepsilon}{(\varepsilon - 1)} MC_{t+k}^n \right) \right\}$$

$$0 = E_t \sum_{k=0}^{\infty} (\beta \theta_H)^k \left\{ \frac{\tilde{C}_{t+k}^{-\sigma}}{\tilde{C}_t^{-\sigma}} \frac{P_{t+k}^{-1}}{P_t^{-1}} \right\} \left\{ Y_{t+k}^d (i) \left(\overline{P}_{H,t} - \frac{\varepsilon}{(\varepsilon - 1)} MC_{t+k}^n \right) \right\}$$

$$0 = \frac{1}{P_t^{-1} \tilde{C}_t^{-\sigma}} E_t \sum_{k=0}^{\infty} (\beta \theta_H)^k \tilde{C}_{t+k}^{-\sigma} P_{t+k}^{-1} \left\{ Y_{t+k}^d (i) \left(\overline{P}_{H,t} - \frac{\varepsilon}{(\varepsilon - 1)} MC_{t+k}^n \right) \right\}$$

$$0 = E_t \sum_{k=0}^{\infty} (\beta \theta_H)^k \left\{ \tilde{C}_{t+k}^{-\sigma} P_{t+k}^{-1} Y_{t+k}^d (i) \left(\overline{P}_{H,t} - \frac{\varepsilon}{(\varepsilon - 1)} MC_{t+k}^n \right) \right\}$$

$$0 = E_t \sum_{k=0}^{\infty} (\beta \theta_H)^k \left\{ \tilde{C}_{t+k}^{-\sigma} Y_{t+k}^d (i) \frac{P_{H,t-1}}{P_{t+k}} \left(\frac{\overline{P}_{H,t}}{P_{H,t-1}} - \frac{\varepsilon}{(\varepsilon - 1)} \frac{MC_{t+k}^n}{P_{H,t-1}} \right) \right\}$$

According to the nominal marginal costs are $MC_t^n = P_{H,t} MC_t$, MC_t are real marginal cost. Therefore,

$$0 = E_t \sum_{k=0}^{\infty} (\beta \theta_H)^k \left\{ \tilde{C}_{t+k}^{-\sigma} Y_{t+k}^d (i) \frac{P_{H,t-1}}{P_{t+k}} \left(\frac{\overline{P}_{H,t}}{P_{H,t-1}} - \frac{\varepsilon}{\varepsilon - 1} \frac{P_{H,t+k}}{P_{H,t-1}} MC_{t+k} \right) \right\}$$

and $\Pi_{t-1,t+k}^H = \dfrac{P_{H,t+k}}{P_{H,t-1}}$

$$0 = E_t \sum_{k=0}^{\infty} (\beta \theta_H)^k \left\{ \tilde{C}_{t+k}^{-\sigma} Y_{t+k}^d (i) \frac{P_{H,t-1}}{P_{t+k}} \left(\frac{\overline{P}_{H,t}}{P_{H,t-1}} - \frac{\varepsilon}{\varepsilon - 1} \Pi_{t-1,t+k}^H MC_{t+k} \right) \right\}$$

Log–linearizing the previous equation around the steady state (zero inflation perfect foresight steady state) gives with a balance trade we obtain:

$$\overline{p}_{H,t} = p_{H,t-1} + \sum_{k=0}^{\infty} (\beta\theta_H)^k E_t(\pi_{H,t+k}) + (1-\beta\theta_H)\sum_{k=0}^{\infty}(\beta\theta_H)^k E_t\left(\log\frac{\varepsilon}{\varepsilon-1}MC_{t+k}\right)$$

for $mc_t = \log\frac{\varepsilon}{\varepsilon-1}MC_t$, where the steady state value is $mc = \log\frac{\varepsilon}{\varepsilon-1}$

We can rewrite the equation into the form of

$$\overline{p}_{H,t} = p_{H,t-1} + E_t\sum_{k=0}^{\infty}(\beta\theta_H)^k\left\{\pi_{H,t+k} + (1-\beta\theta_H)mc_{t+k}\right\}$$

Next,

$$\begin{aligned}\overline{p}_{H,t} &= p_{H,t-1} + \sum_{k=0}^{\infty}(\beta\theta_H)^k E_t\left[\pi_{H,t+k} + (1-\beta\theta_H)mc_t\right] \\ &= p_{H,t-1} + \left[\pi_{H,t} + (1-\beta\theta_H)mc_t\right] + \sum_{k=1}^{\infty}(\beta\theta_H)^k E_t\left[\pi_{H,t+k} + (1-\beta\theta_H)mc_{t+k}\right] \\ &= p_{H,t-1} + \left[\pi_{H,t} + (1-\beta\theta_H)mc_t\right] + (\beta\theta_H)\left\{\sum_{k=0}^{\infty}(\beta\theta_H)^k E_t\left[\pi_{H,t+k+1} + (1-\beta\theta_H)mc_{t+k+1}\right]\right\}\end{aligned}$$

Then we use the log-linearized condition expressed for $t+1$

$$\overline{p}_{H,t} = p_{H,t-1} + \left[\pi_{H,t} + (1-\beta\theta_H)mc_t\right] + (\beta\theta_H)E_t\left\{\overline{p}_{H,t+1} - p_{H,t}\right\}$$

$$\overline{p}_{H,t} - p_{H,t-1} = \pi_{H,t} + (1-\beta\theta_H)mc_t + (\beta\theta_H)E_t\pi_{H,t+1}$$

We use the last equation together with the log-linearized version of the domestic price level (4.33)

$$\begin{aligned}\pi_{H,t} &= (1-\theta_H)\left(\overline{p}_{H,t} - p_{H,t-1}\right) + \theta_H^2\pi_{H,t-1} \\ \pi_{H,t} &= (1-\theta_H)\left[\pi_{H,t} + (1-\beta\theta_H)mc_t + (\beta\theta_H)E_t\pi_{H,t+1}\right] + \theta_H^2\pi_{H,t-1} \\ &= (1-\theta_H)\pi_{H,t} + (1-\theta_H)(1-\beta\theta_H)mc_t + (1-\theta_H)(\beta\theta_H)E_t\pi_{H,t+1} \\ &\quad + \theta_H^2\pi_{H,t-1}\end{aligned}$$

$$\begin{aligned}\pi_{H,t} - (1-\theta_H)\pi_{H,t} &= (1-\theta_H)(1-\beta\theta_H)mc_t + (1-\theta_H)(\beta\theta_H)E_t\pi_{H,t+1} + \theta_H^2\pi_{H,t-1} \\ \theta_H\pi_{H,t} &= (1-\theta_H)(\beta\theta_H)E_t\pi_{H,t+1} + \theta_H^2\pi_{H,t-1} + (1-\theta_H)(1-\beta\theta_H)mc_t \\ \pi_{H,t} &= \beta(1-\theta_H)E_t\pi_{H,t+1} + \theta_H\pi_{H,t-1} + \frac{(1-\theta_H)(1-\beta\theta_H)}{\theta_H}mc_t\end{aligned}$$

$$\pi_{H,t} = \beta(1-\theta_H)E_t\pi_{H,t+1} + \theta_H\pi_{H,t-1} + \lambda_H mc_t \qquad \text{(C1.4)}$$

This equation is the New Keynesian Phillips Curve in terms of domestic goods.

C.2 Aggregate Domestic Price Level

We use a log-linearized version of the aggregate domestic price level for the calculation of the Phillips Curve.

The aggregate domestic price level is expressed in the following form:

$$P_{H,t}=\left[\left(1-\theta_H\right)\overline{P}_{H,t}^{1-\rho}+\theta_H\tilde{P}_{H,t}^{1-\rho}\right]^{\frac{1}{1-\rho}} \qquad \text{and} \qquad \tilde{P}_{H,t}=P_{H,t-1}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\theta_H}$$

$$P_{H,t}=\left\{\left(1-\theta_H\right)\overline{P}_{H,t}^{1-\rho}+\theta_H\left[P_{H,t-1}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\theta_H}\right]^{1-\rho}\right\}^{\frac{1}{1-\rho}}$$

$$P_{H,t}^{1-\rho}=\left(1-\theta_H\right)\overline{P}_{H,t}^{1-\rho}+\theta_H\left[P_{H,t-1}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\theta_H}\right]^{1-\rho}$$

$$-\left(1-\theta_H\right)\overline{P}_{H,t}^{1-\rho}=-P_{H,t}^{1-\rho}+\theta_H\left[P_{H,t-1}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\theta_H}\right]^{1-\rho}$$

$$\left(1-\theta_H\right)\overline{P}_{H,t}^{1-\rho}=P_{H,t}^{1-\rho}-\theta_H\left[P_{H,t-1}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\theta_H}\right]^{1-\rho}$$

and the log-linearizing:

$$\log\left\{\left(1-\theta_H\right)\overline{P}_{H,t}^{1-\rho}\right\}=\log\left\{P_{H,t}^{1-\rho}-\theta_H\left[P_{H,t-1}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\theta_H}\right]^{1-\rho}\right\}$$

$$\log\left(1-\theta_H\right)+\log\overline{P}_{H,t}^{1-\rho}=\frac{\left\{\Delta P_{H,t}^{1-\rho}-\theta_H\Delta\left[P_{H,t-1}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\theta_H}\right]^{1-\rho}\right\}}{P_H^{1-\rho}-\theta_H\left[P_H\left(\frac{P_H}{P_H}\right)^{\theta_H}\right]^{1-\rho}}$$

$$(1-\rho)\log \overline{P}_{H,t} = \frac{\Delta P_{H,t}^{1-\rho}\frac{P_H^{1-\rho}}{P_H^{1-\rho}} - \theta_H \Delta\left[P_{H,t-1}\frac{P_H}{P_H}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\theta_H}\frac{\left(\frac{P_H}{P_H}\right)^{\theta_H}}{\left(\frac{P_H}{P_H}\right)^{\theta_H}}\right]^{1-\rho}}{P_H^{1-\rho}-\theta_H\left[P_H\left(\frac{P_H}{P_H}\right)^{\theta_H}\right]^{1-\rho}}$$

$$(1-\rho)\overline{p}_{H,t} = \frac{P_H^{1-\rho}\left\{\frac{\Delta P_{H,t}^{1-\rho}}{P_H^{1-\rho}} - \theta_H\Delta\left[\frac{P_{H,t-1}}{P_H}\left(\frac{P_{H,t-1}\left(\frac{P_H}{P_H}\right)}{P_{H,t-2}\left(\frac{P_H}{P_H}\right)}\right)^{\theta_H}\right]^{1-\rho}\right\}}{P_H^{1-\rho}\left\{1-\theta_H\left[\left(\frac{P_H}{P_H}\right)^{\theta_H}\right]^{1-\rho}\right\}}$$

$$(1-\rho)\overline{p}_{H,t} = \frac{(1-\rho)\,p_{H,t} - \theta_H\Delta\left[\frac{P_{H,t-1}}{P_H}\left(\frac{\frac{P_{H,t-1}}{P_H}P_H}{\frac{P_{H,t-2}}{P_H}P_H}\right)^{\theta_H}\right]^{1-\rho}}{1-\theta_H\left[(1)^{\theta_H}\right]^{1-\rho}}$$

$$(1-\rho)\overline{p}_{H,t} = \frac{(1-\rho)\,p_{H,t} - \theta_H\Delta\left[\frac{P_{H,t-1}}{P_H}\left(\frac{\frac{P_{H,t-1}}{P_H}}{\frac{P_{H,t-2}}{P_H}}\right)^{\theta_H}\right]^{1-\rho}}{1-\theta_H}$$

$$(1-\theta_H)(1-\rho)\overline{p}_{H,t} = (1-\rho)\,p_{H,t} - \theta_H(1-\rho)\left[p_{H,t-1}+\theta_H\left(p_{H,t-1}-p_{H,t-2}\right)\right]$$

$$(1-\theta_H)(1-\rho)\overline{p}_{H,t} = (1-\rho)\left(p_{H,t}-\theta_H\right)\left[p_{H,t-1}+\theta_H\left(p_{H,t-1}-p_{H,t-2}\right)\right]$$

$$(1-\theta_H)\overline{p}_{H,t} = p_{H,t}-\theta_H\left[p_{H,t-1}+\theta_H\left(\pi_{H,t-1}\right)\right]$$

$$(1-\theta_H)\overline{p}_{H,t} = p_{H,t}-p_{H,t-1}+p_{H,t-1}-\theta_H p_{H,t-1}-\theta_H^2\pi_{H,t-1}$$

$$(1-\theta_H)\overline{p}_{H,t} = \pi_{H,t}+(1-\theta_H)\,p_{H,t-1}-\theta_H^2\pi_{H,t-1}$$

After rearranging, we get:

$$\pi_{H,t} = (1-\theta_H)\left(\overline{p}_{H,t}-p_{H,t-1}\right)+\theta_H^2\pi_{H,t-1} \quad \text{(C2.1)}$$