APPENDIX B

CONSUMER UTILITY MAXIMIZATION

B.1 Consumer Utility Maximization

A representative household maximizes its utility function:

$$Max\ E_t \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right) = E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{\left(C_t - hC_{t-1}\right)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \qquad \text{(B1.1)}$$

subject to a budget constraint

$$P_t C_t + E_t \left(\frac{D_{t+1}}{1+r_t} \right) = D_t + W_t N_t \qquad \text{(B1.2)}$$

First, we calculate the FOC for intratemporal consumption. The Lagrangian function has following form (a Lagrangian multiplier is Λ_t)

$$L\left(C_t, N_t, \Lambda_t\right) = E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{\left(C_t - hC_{t-1}\right)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) + \sum_{t=0}^{\infty} \Lambda_t \beta^t \left(D_t + W_t N_t - P_t C_t - E_t \left(\frac{D_{t+1}}{1+r_t} \right) \right)$$

FOC w.r.t. C_t and N_t respectively

$$\frac{\partial L\left(C_t, N_t, \Lambda_t\right)}{\partial C_t} = \beta^t \left(1-\sigma\right) \frac{\left(C_t - hC_{t-1}\right)^{1-\sigma-1}}{\left(1-\sigma\right)} \left(1\right) - \Lambda_t \beta^t P_t = 0$$

$$\beta^t \left[\left(C_t - hC_{t-1}\right)^{-\sigma} - \Lambda_t P_t \right] = 0$$

$$\frac{\left(C_t - hC_{t-1}\right)^{-\sigma}}{P_t} = \Lambda_t$$

$$\frac{\partial L\left(C_t, N_t, \Lambda_t\right)}{\partial N_t} = \beta^t \left(1-\varphi\right) \frac{N_t^{1+\varphi-1}}{\left(1-\varphi\right)} \left(1\right) + \Lambda_t \beta^t W_t = 0$$

$$\beta^t \left(N_t^{\varphi} + \Lambda_t W_t \right) = 0$$

$$\frac{N_t^{\varphi}}{W_t} = \Lambda_t$$

The FOC is following:

$$\left(C_t - hC_{t-1}\right)^{-\sigma} \frac{W_t}{P_t} = N_t^{\varphi} \tag{B1.3}$$

Log-linearizing both sides (as it is introduced e.g. in Malley (2004) yields:

$$-\sigma \ln\left(C_t - hC_{t-1}\right) + \ln\left(\frac{W_t}{P_t}\right) = \varphi \ln N_t$$

$$-\sigma \ln\left\{C_t - hC_{t-1}\right\} + \ln W_t - \ln P_t = \varphi \ln N_t$$

$$-\sigma d \ln\left\{C_t - hC_{t-1}\right\} + d \ln W_t - d \ln P_t = \varphi d \ln N_t$$

$$-\sigma \frac{1}{C - hC} d\left\{C_t - hC_{t-1}\right\} + \frac{1}{W} dW_t - \frac{1}{P} dP_t = \varphi \frac{1}{N_t} dN_t$$

$$-\frac{\sigma}{C(1-h)}\left\{dC_t - hdC_{t-1}\right\} + \ln\left(\frac{W_t}{W}\right) - \ln\left(\frac{P_t}{P}\right) = \varphi \ln\left(\frac{N_t}{N}\right)$$

$$-\frac{\sigma}{C(1-h)}\left\{C \ln\left(\frac{C_t}{C}\right) - hC \ln\left(\frac{C_{t-1}}{C}\right)\right\} + w_t - p_t = \varphi n_t$$

$$-\frac{\sigma}{C(1-h)} C\left\{\ln\left(\frac{C_t}{C}\right) - h \ln\left(\frac{C_{t-1}}{C}\right)\right\} + w_t - p_t = \varphi n_t$$

$$-\frac{\sigma}{(1-h)}\left\{c_t - hc_{t-1}\right\} + w_t - p_t = \varphi n_t$$

$$\frac{\sigma}{(1-h)}\left\{c_t - hc_{t-1}\right\} + \varphi n_t = w_t - p_t \tag{B1.4}$$

where

$$c_t = \frac{1}{C} dC_t \cong \ln\left(\frac{C_t}{C}\right) = \ln C_t - \ln C, \qquad w_t = \ln\left(\frac{W_t}{W}\right) = \ln W_t - \ln W,$$

$$c_{t-1} = \frac{1}{C} dC_{t-1} \cong \ln\left(\frac{C_{t-1}}{C}\right) = \ln C_{t-1} - \ln C, \qquad p_t = \ln\left(\frac{P_t}{P}\right) = \ln P_t - \ln C$$

The Euler (intertemporal) equation can be solved by the Bellman's equation. To the household's optimizing problem we form a value function.

$$
\begin{aligned}
v(D_t) &= E_t \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \\
&= \beta^0 U(C_t, N_t) + E_t \sum_{t=1}^{\infty} \beta^t U(C_t, N_t) \\
&= U(C_t, N_t) + \beta E_t v(D_{t+1})
\end{aligned}
$$

The Bellman's Equation for this problem is:

$$v(D_t) = \max_{C_t} U(C_t, N_t) + \beta E_t v(D_{t+1})$$

or equivalently for max $U(C_t, N_t) = U\left(\tilde{C}_t, \tilde{N}_t\right)$

$$v(D_t) = U\left(\tilde{C}_t, \tilde{N}_t\right) + \beta E_t v(D_{t+1})$$

We use the budget constraint in form of $D_{t+1} = R_t(D_t + W_t N_t - P_t C_t)$ where $R_t = 1 + r_t$ and plug in Bellman's Equation

$$v(D_t) = U\left(\tilde{C}_t, \tilde{N}_t\right) + \beta E_t v\left(R_t(D_t + W_t N_t - P_t C_t)\right)$$

The calculation of a condition for an optimality is

$$
\begin{aligned}
\frac{\partial v(D_t)}{\partial C_t} &= \frac{\partial U\left(\tilde{C}_t, \tilde{N}_t\right)}{\partial C_t} + \beta E_t \frac{\partial v(D_{t+1})}{\partial C_t} \\
&= \frac{\partial U\left(\tilde{C}_t, \tilde{N}_t\right)}{\partial C_t} + \beta E_t \frac{\partial v(D_{t+1})}{\partial_t D_{t+1}} \frac{\partial\left(R_t(D_t + W_t N_t - P_t C_t)\right)}{\partial C_t} \\
&= \frac{\partial U\left(\tilde{C}_t, \tilde{N}_t\right)}{\partial C_t} - \beta E_t \frac{\partial v(D_{t+1})}{\partial D_{t+1}} R_t P_t = 0
\end{aligned}
$$

Since

$$\frac{\partial U\left(\tilde{C}_t, \tilde{N}_t\right)}{\partial C_t} = \beta E_t\left[\frac{\partial v\left(D_{t+1}\right)}{\partial D_{t+1}} R_t P_t\right]$$

From the result is shown that optimal value of $\tilde{C}_t$ is a function of D_t and it is possible to reformulate the function $v(.)$ in the bellman's equation as

$$v\left(D_t\right) = U\left(\tilde{C}_t\left(D_t\right), \tilde{N}_t\right) + \beta E_t v\left(D_{t+1}\right)$$

Dif $v\left(D_t\right)$ w.r.t. D_t

$$\frac{\partial v\left(D_t\right)}{\partial D_t} = \frac{\partial U\left(\tilde{C}_t\left(D_t\right), \tilde{N}_t\right)}{\partial \tilde{C}_t} \frac{\partial \tilde{C}_t\left(D_t\right)}{\partial D_t} + \beta E_t \frac{\partial v\left(D_{t+1}\right)}{\partial D_{t+1}} \frac{\partial\left(R_t\left(D_t + W_t N_t - P_t \tilde{C}_t\left(D_t\right)\right)\right)}{\partial D_t}$$

$$= \frac{\partial U\left(\tilde{C}_t\left(D_t\right), \tilde{N}_t\right)}{\partial \tilde{C}_t} \frac{\partial \tilde{C}_t\left(D_t\right)}{\partial D_t} + \beta E_t\left[\frac{\partial v\left(D_{t+1}\right)}{\partial D_{t+1}}\left(1 - Pt\frac{\partial \tilde{C}_t}{\partial D_t}\right) R_t\right]$$

Then we plug the optimality condition into FOC

$$\frac{\partial v\left(D_t\right)}{\partial D_t} = \frac{\partial U\left(\tilde{C}_t, \tilde{N}_t\right)}{\partial C_t} \frac{\partial \tilde{C}_t}{\partial D_t} + \beta E_t\left[\frac{\partial v\left(D_{t+1}\right)}{\partial D_{t+1}}\left(1 - Pt\frac{\partial \tilde{C}_t}{\partial D_t}\right) R_t\right]$$

$$= \beta E_t\left[\frac{\partial v\left(D_{t+1}\right)}{\partial D_{t+1}} R_t P_t\right] \frac{\partial \tilde{C}_t}{\partial D_t} + \beta E_t\left[\frac{\partial v\left(D_{t+1}\right)}{\partial D_{t+1}}\left(1 - Pt\frac{\partial \tilde{C}_t}{\partial D_t}\right) R_t\right]$$

$$= \beta E_t\left[\frac{\partial v\left(D_{t+1}\right)}{\partial D_{t+1}} R_t\right]\left(P_t\frac{\partial \tilde{C}_t}{\partial D_t} + 1 - Pt\frac{\partial \tilde{C}_t}{\partial D_t}\right)$$

$$= \beta E_t\left[\frac{\partial v\left(D_{t+1}\right)}{\partial D_{t+1}} R_t\right]$$

We compare the previous equation together with the optimality condition

$$\frac{\partial U\left(\tilde{C}_t,\tilde{N}_t\right)}{\partial C_t}=\beta E_t\left[\frac{\partial v\left(D_{t+1}\right)}{\partial D_{t+1}}R_tP_t\right] \text{ and } \frac{\partial v\left(D_t\right)}{\partial D_t}=\beta E_t\left[\frac{\partial v\left(D_{t+1}\right)}{\partial D_{t+1}}R_t\right]$$

So

$$\frac{\partial U\left(\tilde{C}_t,\tilde{N}_t\right)}{\partial C_t}=\frac{\partial v\left(D_t\right)}{\partial D_t}P_t$$

and for $t+1$

$$\frac{\partial U\left(\tilde{C}_{t+1},\tilde{N}_{t+1}\right)}{\partial C_{t+1}}=\frac{\partial v\left(D_{t+1}\right)}{\partial D_{t+1}}P_{t+1}$$

Now we plug it back to the optimality condition

$$\frac{\partial U\left(\tilde{C}_t,\tilde{N}_t\right)}{\partial C_t}=\beta E_t\left[\frac{\partial v\left(D_{t+1}\right)}{\partial D_{t+1}}R_tP_t\right]$$

$$\frac{\partial U\left(\tilde{C}_t,\tilde{N}_t\right)}{\partial C_t}=\beta E_t\left[\frac{\partial v\left(D_{t+1}\right)}{\partial D_{t+1}}\frac{R_tP_t}{P_{t+1}}\right]$$

Plug the specific utility form

$$\frac{\partial U\left(\tilde{C}_t,\tilde{N}_t\right)}{\partial C_t}=\beta E_t\left[\frac{\partial v\left(D_{t+1}\right)}{\partial D_{t+1}}\frac{R_tP_t}{P_{t+1}}\right]$$

$$\frac{\partial\left(\frac{\left(C_t-hC_{t-1}\right)^{1-\sigma}}{1-\sigma}-\frac{N_t^{1+\varphi}}{1+\varphi}\right)}{\partial C_t}=\beta E_t\frac{\partial\left(\frac{\left(C_{t+1}-hC_t\right)^{1-\sigma}}{1-\sigma}-\frac{N_{t+1}^{1+\varphi}}{1+\varphi}\right)}{\partial C_{t+1}}\frac{R_tP_t}{P_{t+1}}$$

$$\left(1-\sigma\right)\frac{\left(C_t-hC_{t-1}\right)^{1-\sigma-1}}{1-\sigma}=\beta E_t\left(1-\sigma\right)\frac{\left(C_{t+1}-hC_t\right)^{1-\sigma-1}}{1-\sigma}\frac{R_tP_t}{P_{t+1}}$$

$$\left(C_t-hC_{t-1}\right)^{-\sigma}=\beta R_tE_t\left(C_{t+1}-hC_t\right)^{-\sigma}\frac{P_t}{P_{t+1}}$$

$$1=\beta R_tE_t\left\{\left(\frac{C_{t+1}-hC_t}{C_t-hC_{t-1}}\right)^{-\sigma}\frac{P_t}{P_{t+1}}\right\} \tag{B1.5}$$

The equation is the Euler intertemporal equation (5.12). The log-linearinzing of the equation around the steady state is following

$$\log 1=\log\beta+\log R_tE_t\left\{\log\left[\frac{\left(C_{t+1}-hC_t\right)^{-\sigma}}{\left(C_t-hC_{t-1}\right)^{-\sigma}}\right]+\log\left[\frac{P_t}{P_{t+1}}\right]\right\}$$

$$\log 1=\log\beta+\log R_t+E_t\left\{-\sigma\log\left(C_{t+1}-hC_t\right)+\sigma\log\left(C_t-hC_{t-1}\right)+\log P_t-\log P_{t+1}\right\}$$

Taking diff both sides

$$\log 1=\log\beta+\log R_t+E_t\left\{-\sigma\log\left(C_{t+1}-hC_t\right)+\sigma\log\left(C_t-hC_{t-1}\right)+\log P_t-\log P_{t+1}\right\}$$

$$0=0+d\left(\log R_t\right)+E_t\left\{-\sigma d\left[\log\left(C_{t+1}-hC_t\right)\right]+\sigma d\left[\log\left(C_t-hC_{t-1}\right)\right]+d\left[\log P_t\right]-d\left[\log P_{t+1}\right]\right\}$$

$$0=r_t+E_t\left\{-\frac{\sigma}{\left(C-hC\right)}d\left(C_{t+1}-hC_t\right)+\frac{\sigma}{\left(C-hC\right)}d\left(C_t-hC_{t-1}\right)+p_t-p_{t+1}\right\}$$

$$0=r_t+E_t\left\{-\frac{\sigma}{C\left(1-h\right)}\left(dC_{t+1}-hdC_t\right)+\frac{\sigma}{C\left(1-h\right)}\left(dC_t-hdC_{t-1}\right)+p_t-p_{t+1}\right\}$$

$$0=r_t+E_t\left\{-\frac{\sigma}{C\left(1-h\right)}\left(C\ln\left(\frac{C_{t+1}}{C}\right)-hC\ln\left(\frac{C_t}{C}\right)\right)+\frac{\sigma}{C\left(1-h\right)}\left(C\ln\frac{C_t}{C}-hC\ln\frac{C_{t-1}}{C}\right)+p_t-p_{t+1}\right\}$$

$$0=r_t+E_t\left\{-\frac{\sigma}{C\left(1-h\right)}C\left(\ln\left(\frac{C_{t+1}}{C}\right)-h\ln\left(\frac{C_t}{C}\right)\right)+\frac{\sigma}{C\left(1-h\right)}C\left(\ln\frac{C_t}{C}-h\ln\frac{C_{t-1}}{C}\right)+p_t-p_{t+1}\right\}$$

$$0=r_t+E_t\left\{-\frac{\sigma}{\left(1-h\right)}\left(c_{t+1}-hc_t\right)+\frac{\sigma}{\left(1-h\right)}\left(c_t-hc_{t-1}\right)+p_t-p_{t+1}\right\}$$

$$0=r_t+E_t\left\{-\frac{\sigma}{\left(1-h\right)}\left(c_{t+1}-hc_t\right)+\frac{\sigma}{\left(1-h\right)}\left(c_t-hc_{t-1}\right)-\left(p_{t+1}-p_t\right)\right\}$$

$$0 = r_t + E_t\left\{-\frac{\sigma}{(1-h)}(c_{t+1} - hc_t) + \frac{\sigma}{(1-h)}(c_t - hc_{t-1}) - (\pi_{t+1})\right\}$$

$$0 = E_t\left\{-\frac{\sigma}{(1-h)}(c_{t+1} - hc_t) + \frac{\sigma}{(1-h)}(c_t - hc_{t-1}) + r_t - \pi_{t+1}\right\}$$

$$0 = E_t\left\{\frac{(1-h)}{\sigma}(r_t - \pi_{t+1}) - (c_{t+1} - hc_t) + (c_t - hc_{t-1})\right\}$$

$$0 = E_t\left\{-\frac{(1-h)}{\sigma}(r_t - \pi_{t+1}) + (c_{t+1} - hc_t) - (c_t - hc_{t-1})\right\}$$

$$(c_t - hc_{t-1}) = E_t(c_{t+1} - hc_t) - \frac{(1-h)}{\sigma}E_t(r_t - \pi_{t+1}) \qquad \text{(B1.6)}$$

B.2 International Risk Sharing Condition

Conditions connected to a complete international markets and perfect mobility can be expressed as in equation:

$$R_t^* E_t\left(\frac{Z_t}{Z_{t+1}}\right) = R_t \qquad \text{(B2.1)}$$

together with the domestic and foreign Euler equation, respectively

$$1 = \beta R_t E_t\left\{\left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}}\right)^{-\sigma} \frac{P_t}{P_{t+1}}\right\}$$

$$R_t = \frac{1}{\beta} E_t\left\{\left(\frac{C_t - hC_{t-1}}{C_{t+1} - hC_t}\right)^{-\sigma} \frac{P_{t+1}}{P}\right\} \qquad \text{(B2.2)}$$

and

$$1 = \beta R_t^* E_t\left\{\left(\frac{C_{t+1}^* - hC_t^*}{C_t^* - hC_{t-1}^*}\right)^{-\sigma} \frac{P_t^*}{P_{t+1}^*}\right\}$$

$$R_t^* = \frac{1}{\beta} E_t\left\{\left(\frac{C_t^* - hC_{t-1}^*}{C_{t+1}^* - hC_t^*}\right)^{-\sigma} \frac{P_{t+1}^*}{P_t^*}\right\} \qquad \text{(B2.3)}$$

We plug both equations in the international risk condition as:

$$\frac{1}{\beta}E_t\left\{\left(\frac{C_t^*-hC_{t-1}^*}{C_{t+1}^*-hC_t^*}\right)^{-\sigma}\frac{P_{t+1}^*}{P_t^*}\right\}E_t\left(\frac{Z_t}{Z_{t+1}}\right)=\frac{1}{\beta}E_t\left\{\left(\frac{C_t-hC_{t-1}}{C_{t+1}-hC_t}\right)^{-\sigma}\frac{P_{t+1}}{P_t}\right\}$$

$$\beta E_t\left\{\left(\frac{C_{t+1}^*-hC_t^*}{C_t^*-hC_{t-1}^*}\right)^{-\sigma}\frac{P_t^*}{P_{t+1}^*}\right\}E_t\left(\frac{Z_{t+1}}{Z_t}\right)=\beta E_t\left\{\left(\frac{C_{t+1}-hC_t}{C_t-hC_{t-1}}\right)^{-\sigma}\frac{P}{P_{t+1}}\right\}$$

$$\frac{\left(C_t-hC_{t-1}\right)^{-\sigma}}{\left(C_t^*-hC_{t-1}^*\right)^{-\sigma}}\frac{P_t^*}{P_tZ_t}=E_t\left\{\frac{\left(C_{t+1}-hC_t\right)^{-\sigma}}{\left(C_{t+1}^*-hC_t^*\right)^{-\sigma}}\frac{P_{t+1}^*}{P_{t+1}Z_{t+1}}\right\} \tag{B2.4}$$

From the relationship for the real exchange rate $Q_t\equiv\frac{P_tZ_t}{P_t^*}$, we use it to substitute above equation.

$$\frac{\left(C_t-hC_{t-1}\right)^{-\sigma}}{\left(C_t^*-hC_{t-1}^*\right)^{-\sigma}}\frac{1}{Q_t}=E_t\left\{\frac{\left(C_{t+1}-hC_t\right)^{-\sigma}}{\left(C_{t+1}^*-hC_t^*\right)^{-\sigma}}\frac{1}{Q_{t+1}}\right\}$$

$$\left(C_t-hC_{t-1}\right)^{-\sigma}=E_t\left\{\frac{\left(C_{t+1}-hC_t\right)^{-\sigma}}{\left(C_{t+1}^*-hC_t^*\right)^{-\sigma}}\frac{1}{Q_{t+1}}\right\}\left(C_t^*-hC_{t-1}^*\right)^{-\sigma}Q_t$$

$$\left(C_t-hC_{t-1}\right)=E_t\left\{\frac{\left(C_{t+1}-hC_t\right)^{-\sigma}}{\left(C_{t+1}^*-hC_t^*\right)^{-\sigma}}\frac{1}{Q_{t+1}}\right\}^{-\frac{1}{\sigma}}\left(C_t^*-hC_{t-1}^*\right)Q_t^{-\frac{1}{\sigma}}$$

$$\left(C_t-hC_{t-1}\right)=E_t\left\{\frac{C_{t+1}-hC_t}{C_{t+1}^*-hC_t^*}Q_{t+1}^{\frac{1}{\sigma}}\right\}\left(C_t^*-hC_{t-1}^*\right)Q_t^{-\frac{1}{\sigma}}$$

$$\left(C_t-hC_{t-1}\right)=\vartheta\left(C_t^*-hC_{t-1}^*\right)Q_t^{-\frac{1}{\sigma}} \tag{B2.5}$$

This term $\vartheta=E_t\left(\frac{C_{t+1}-hC_t}{C_{t+1}^*-hC_t^*}Q_{t+1}^{\frac{1}{\sigma}}\right)$ indicates that the expected development of the consumptions (a change of domestic consumption to foreign with respect to the real exchange rate) influences the current domestic consumption.

Log-linearizing the equation around the steady state gives:

$$\log\left(C_t - hC_{t-1}\right) = \log\left\{\vartheta\left(C_t^* - hC_{t-1}^*\right)Q_t^{-\frac{1}{\sigma}}\right\}$$

$$\log\left(C_t - hC_{t-1}\right) = \log\vartheta + \log\left(C_t^* - hC_{t-1}^*\right) + \log\left(Q_t^{-\frac{1}{\sigma}}\right)$$

$$\log\left(C_t - hC_{t-1}\right) = 0 + \log\left(C_t^* - hC_{t-1}^*\right) - \frac{1}{\sigma}\log Q_t$$

$$d\log\left(C_t - hC_{t-1}\right) = d\log\left(C_t^* - hC_{t-1}^*\right) - \frac{1}{\sigma}d\log Q_t$$

$$\frac{1}{C - hC}d\left(C_t - hC_{t-1}\right) = \frac{1}{C^* - hC^*}d\left(C_t^* - hC_{t-1}^*\right) - \frac{1}{\sigma Q}dQ_t$$

$$\frac{1}{(1-h)C}\left(dC_t - hdC_{t-1}\right) = \frac{1}{(1-h)C^*}\left(dC_t^* - hdC_{t-1}^*\right) - \frac{1}{\sigma Q}dQ_t$$

$$\frac{1}{(1-h)C}\left(C\ln\frac{C_t}{C} - hC\ln\frac{C_{t-1}}{C}\right) = \frac{1}{(1-h)C^*}\left(C^*\ln\frac{C_t^*}{C^*} - hC^*\ln\frac{C_{t-1}^*}{C^*}\right) - \frac{1}{\sigma Q}Q\ln\frac{Q_t}{Q}$$

$$\frac{1}{(1-h)C}\left(C\ln\frac{C_t}{C} - hC\ln\frac{C_{t-1}}{C}\right) = \frac{1}{(1-h)C^*}\left(C^*\ln\frac{C_t^*}{C^*} - hC^*\ln\frac{C_{t-1}^*}{C^*}\right) - \frac{1}{\sigma Q}Q\ln\frac{Q_t}{Q}$$

$$\frac{1}{(1-h)C}C\left(\ln\frac{C_t}{C} - h\ln\frac{C_{t-1}}{C}\right) = \frac{1}{(1-h)C^*}C^*\left(\ln\frac{C_t^*}{C^*} - h\ln\frac{C_{t-1}^*}{C^*}\right) - \frac{1}{\sigma}\ln\frac{Q_t}{Q}$$

$$\frac{c_t - hc_{t-1}}{1-h} = \frac{c_t^* - hc_{t-1}^*}{1-h} - \frac{1}{\sigma}q_t$$

$$c_t - hc_{t-1} = c_t^* - hc_{t-1}^* - \frac{1-h}{\sigma}q_t \qquad \text{(B2.6)}$$

and with using the relationship $y_t^* = c_t^*$, we get equation

$$c_t - hc_{t-1} = \left(y_t^* - hy_{t-1}^*\right) - \frac{1-h}{\sigma}q_t \qquad \text{(B2.7)}$$

$$c_t = \frac{1}{C}dC_t \cong \ln\left(\frac{C_t}{C}\right) = \ln C_t - \ln C,$$

$$c_{t-1} = \frac{1}{C}dC_{t-1} \cong \ln\left(\frac{C_{t-1}}{C}\right) = \ln C_{t-1} - \ln C,$$

$$q_t = \frac{1}{Q}dQ_t \cong \ln\left(\frac{Q_t}{Q}\right) = \ln Q_t - \ln Q,$$

More precisely, the relationship for the foreign economy takes the form $y_t^* = c_t^* + c_{F,t}$, but the influence of domestic economy is negligible and it is possible to simplify the equation to $y_t^* = c_t^*$,