APPENDIX A

ALLOCATION FUNCTIONS OF EXPENDITURE, DEMAND FUNCTION AND OVERALL CONSUMER PRICE INDEX

This appendix clarifies the calculating of allocation functions of expenditure, demand function and overall consumer price index.

A.1 Allocation Function of Expenditure

Basically, a representative household decides about optimal allocation of expenditures between domestic and foreign goods[1]. It maximizes of total consumption expressed by the equation (5.2).

$$\underset{C_{H,t},C_{F,t},\Lambda_t}{Max} \; C_t \qquad ; \; C_t \equiv \left((1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

Subject to its expenditure constraint

$$P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t}$$

Forming the Lagrangian function (with a multiplier Λ_t), we solve the first order condition (FOC) and take its partial derivatives as following;

$$L_t\left(C_{H,t}, C_{F,t}, \Lambda_t\right) = \left((1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} + \Lambda_t\left(P_t C_t - P_{H,t} C_{H,t} - P_{F,t} C_{F,t} \right)$$

[1] The optimizing problem can be solved in two ways (a dual problem solving). That is the representative household tries to minimize the total expenditure $P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t}$ subject to a possible consumption $C_t \equiv \left((1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$

$$\frac{\partial L_t}{\partial C_{H,t}} = \left(\frac{\eta}{\eta-1}\right)\left((1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}-1} (1-\alpha)^{\frac{1}{\eta}} \left(\frac{\eta-1}{\eta}\right) C_{H,t}^{\frac{\eta-1}{\eta}-1} - \Lambda_t P_{H,t} = 0$$

$$\frac{\partial L_t}{\partial C_{F,t}} = \left(\frac{\eta}{\eta-1}\right)\left((1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}-1} (\alpha)^{\frac{1}{\eta}} \left(\frac{\eta-1}{\eta}\right) C_{F,t}^{\frac{\eta-1}{\eta}-1} - \Lambda_t P_{F,t} = 0$$

Rearranging both above equations, we get

$$\Lambda_t = \frac{1}{P_{H,t}}\left(\frac{\eta}{\eta-1}\right)\left((1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{1}{\eta-1}} (1-\alpha)^{\frac{1}{\eta}} \left(\frac{\eta-1}{\eta}\right) C_{H,t}^{-\frac{1}{\eta}}$$

and

$$\Lambda_t = \frac{1}{P_{F,t}}\left(\frac{\eta}{\eta-1}\right)\left((1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{1}{\eta-1}} (\alpha)^{\frac{1}{\eta}} \left(\frac{\eta-1}{\eta}\right) C_{F,t}^{-\frac{1}{\eta}}$$

Next, we equal both formulas for Λ_t and yield:

$$\frac{1}{P_{F,t}}(\alpha)^{\frac{1}{\eta}} C_{F,t}^{-\frac{1}{\eta}} = \frac{1}{P_{H,t}}(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{-\frac{1}{\eta}}$$

$$\frac{P_{H,t}}{P_{F,t}} = \frac{(\alpha)^{-\frac{1}{\eta}}}{(1-\alpha)^{-\frac{1}{\eta}}} \frac{C_{H,t}^{-\frac{1}{\eta}}}{C_{F,t}^{-\frac{1}{\eta}}}$$

$$\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} = \frac{(\alpha)}{(1-\alpha)} \frac{C_{H,t}}{C_{F,t}}$$

In the above equation, we can formulate the demand function by rearranging the budget constraint $P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t}$ as following:

i) For Domestic Goods

$$\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} = \frac{(\alpha) C_{H,t}}{(1-\alpha) C_{F,t}} \quad \text{and} \quad C_{F,t} = \frac{P_t C_t - P_{H,t} C_{H,t}}{P_{F,t}}$$

$$\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}=\frac{(\alpha)C_{H,t}}{(1-\alpha)\frac{P_tC_t-P_{H,t}C_{H,t}}{P_{F,t}}}$$

$$\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}=\frac{(\alpha)P_{F,t}C_{H,t}}{(1-\alpha)\left(P_tC_t-P_{H,t}C_{H,t}\right)}$$

Multiplying both sides by $(1-\alpha)P_tC_t$, we can derive step by step for the above equation as:

$$\begin{aligned}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}(1-\alpha)P_tC_t &= \frac{(\alpha)P_{F,t}C_{H,t}}{(1-\alpha)\left(P_tC_t-P_{H,t}C_{H,t}\right)}(1-\alpha)P_tC_t\\ &=\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}(1-\alpha)\left(P_{H,t}C_{H,t}+P_{F,t}C_{F,t}\right)\\ &=\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}(1-\alpha)\left(P_{H,t}C_{H,t}\right)+\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}(1-\alpha)\left(P_{F,t}C_{F,t}\right)\\ &=\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}(1-\alpha)\left(P_{H,t}C_{H,t}\right)+\frac{(\alpha)C_{H,t}}{(1-\alpha)C_{F,t}}(1-\alpha)\left(P_{F,t}C_{F,t}\right)\\ &=C_{H,t}\left[(1-\alpha)P_{H,t}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}+(\alpha)P_{F,t}\right]\end{aligned}$$

Therefore,

$$C_{H,t} = \frac{(1-\alpha) P_t C_t \left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}}{(1-\alpha) P_{H,t}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} + (\alpha) P_{F,t}}$$

$$= \frac{(1-\alpha) \frac{P_t C_t}{P_{F,t}} \left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}}{(1-\alpha) \frac{P_{H,t}}{P_{F,t}}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} + \alpha}$$

$$= \frac{(1-\alpha) \frac{P_t C_t}{P_{F,t}} \left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}}{(1-\alpha) \left(\frac{P_{H,t}}{P_{F,t}}\right)^{1-\eta} + \alpha} \qquad \text{(A1.1)}$$

ii) For Foreign (Imported) Goods

$$\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} = \frac{(\alpha) C_{H,t}}{(1-\alpha) C_{F,t}} \qquad \text{and} \qquad C_{H,t} = \frac{P_t C_t - P_{F,t} C_{F,t}}{P_{H,t}}$$

$$\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} = \frac{(\alpha)\left(\frac{P_t C_t - P_{F,t} C_{F,t}}{P_{H,t}}\right)}{(1-\alpha) C_{F,t}}$$

$$\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} = \frac{(\alpha)\left(P_t C_t - P_{F,t} C_{F,t}\right)}{(1-\alpha) P_{H,t} C_{F,t}}$$

$$\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} (1-\alpha) P_{H,t} C_{F,t} = (\alpha)\left(P_t C_t - P_{F,t} C_{F,t}\right)$$

$$\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} (1-\alpha) P_{H,t} C_{F,t} + \alpha P_{F,t} C_{F,t} = \alpha P_t C_t$$

$$\left[\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} (1-\alpha) P_{H,t} + \alpha P_{F,t}\right] C_{F,t} = \alpha P_t C_t$$

$$C_{F,t} = \frac{\alpha P_t C_t}{\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} \left(1-\alpha\right) P_{H,t} + \alpha P_{F,t}}$$

$$C_{F,t} = \frac{\alpha \frac{P_t C_t}{P_{F,t}}}{\left(1-\alpha\right)\left(\frac{P_{H,t}}{P_{F,t}}\right)^{1-\eta} + \alpha} \tag{A1.2}$$

Now, we use the relationship for the overall CES consumer price index (CPI) to substitute into above the equations:

$$P_t \equiv \left\{\left(1-\alpha\right) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right\}^{\frac{1}{1-\eta}}$$

$$P_t^{1-\eta} = \left(1-\alpha\right) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}$$

$$\left(\frac{P_t}{P_{F,t}}\right)^{1-\eta} = \left(1-\alpha\right)\left(\frac{P_{H,t}}{P_{F,t}}\right)^{1-\eta} + \alpha$$

$$\left(1-\alpha\right)\left(\frac{P_{H,t}}{P_{F,t}}\right)^{1-\eta} = \left(\frac{P_t}{P_{F,t}}\right)^{1-\eta} - \alpha$$

$$\left(\frac{P_{H,t}}{P_{F,t}}\right)^{1-\eta} = \frac{1}{\left(1-\alpha\right)}\left[\left(\frac{P_t}{P_{F,t}}\right)^{1-\eta} - \alpha\right] \tag{A1.3}$$

Lastly, we plug the previous equation (A1.3) back to the demand function.

i) For Domestic Consumption Goods

$$C_{H,t}=\frac{(1-\alpha)\frac{P_tC_t}{P_{F,t}}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}}{(1-\alpha)\left(\frac{P_{H,t}}{P_{F,t}}\right)^{1-\eta}+\alpha}$$

$$=\frac{(1-\alpha)\frac{P_tC_t}{P_{F,t}}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}}{(1-\alpha)\frac{1}{(1-\alpha)}\left[\left(\frac{P_t}{P_{F,t}}\right)^{1-\eta}-\alpha\right]+\alpha}$$

$$=\frac{(1-\alpha)\frac{P_tC_t}{P_{F,t}}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}}{\left(\frac{P_t}{P_{F,t}}\right)^{1-\eta}}$$

$$=(1-\alpha)\frac{P_tC_t}{P_{F,t}}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}\left(\frac{P_{F,t}}{P_t}\right)^{1-\eta}$$

$$=(1-\alpha)\frac{P_tC_t}{P_{F,t}}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}\left(\frac{P_{F,t}}{P_t}\right)^{-\eta}\frac{P_{F,t}}{P_t}$$

$$=(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t \qquad \text{(A1.4)}$$

ii) For Foreign (Imported) Consumption Goods

$$C_{F,t} = \frac{\alpha \dfrac{P_t C_t}{P_{F,t}}}{(1-\alpha)\left(\dfrac{P_{H,t}}{P_{F,t}}\right)^{1-\eta} + \alpha}$$

$$= \frac{\alpha \dfrac{P_t C_t}{P_{F,t}}}{(1-\alpha)\dfrac{1}{(1-\alpha)}\left[\left(\dfrac{P_t}{P_{F,t}}\right)^{1-\eta} - \alpha\right] + \alpha}$$

$$= \frac{\alpha \dfrac{P_t C_t}{P_{F,t}}}{\left(\dfrac{P_t}{P_{F,t}}\right)^{1-\eta}}$$

$$= \alpha \frac{P_t C_t}{P_{F,t}} \left(\frac{P_{F,t}}{P_t}\right)^{1-\eta}$$

$$= \alpha \frac{P_t C_t}{P_{F,t}} \left(\frac{P_{F,t}}{P_t}\right)\left(\frac{P_{F,t}}{P_t}\right)^{-\eta}$$

$$= \alpha \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \qquad \text{(A1.5)}$$

Both optimal allocation functions of expenditure between domestic and imported goods correspond to the relationship equation (5.6).

A.2 Demand Function: The Optimizing Behavior of Demand for Domestic and Foreign Goods

A.2.1 Demand for Domestic Goods

A representative household optimizes its behavior. What the way to do is trying to minimize its expenditure for consumption of the domestic goods.

$$\underset{C_{H,t}(i),\Lambda_t}{Min} \int_0^1 P_{H,t}(i) C_{H,t}(i)\, di$$

Subject to

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

The Lagrangian function is in the following form (Λ_t is a Lagrangian multiplier):

$$L_t\left(C_{H,t}(i), \Lambda_t\right) = \int_0^1 P_{H,t}(i) C_{H,t}(i)\, di + \Lambda_t \left\{ C_{H,t} - \left(\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \right\} \qquad \text{(A2.1)}$$

Next, we solve the first order condition (FOC) with respect to $C_{H,t}(i)$ for each domestic consumption goods as

$$\frac{\partial L_t\left(C_{H,t}(i), \Lambda_t\right)}{\partial C_{H,t}(i)} = P_{H,t}(i) - \Lambda_t \left\{ \left(\frac{\varepsilon}{\varepsilon-1} \right) \left(\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}-1} \left(\frac{\varepsilon-1}{\varepsilon} \right) C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}-1} \right\} = 0$$

$$= P_{H,t}(i) - \Lambda_t \left(\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{1}{\varepsilon-1}} C_{H,t}(i)^{-\frac{1}{\varepsilon}} = 0$$

$$P_{H,t}(i) = \Lambda_t \left(\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{1}{\varepsilon-1}} C_{H,t}(i)^{-\frac{1}{\varepsilon}} \qquad \text{(A2.2)}$$

After we get FOC, taking the term of $C_{H,t}(i)$ multiply both side of the above equation:

$$P_{H,t}(i) C_{H,t}(i) = \Lambda_t \left(\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{1}{\varepsilon-1}} C_{H,t}(i)^{-\frac{1}{\varepsilon}} C_{H,t}(i)$$

$$P_{H,t}(i)C_{H,t}(i)=\Lambda_t\left(\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}}\,di\right)^{\frac{1}{\varepsilon-1}}C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} \qquad \text{(A2.3)}$$

Then both sides of last equation are integrated and the constraint $C_{H,t}=\left(\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}}\,di\right)^{\frac{\varepsilon}{\varepsilon-1}}$ is used:

$$\int_0^1 P_{H,t}(i)C_{H,t}(i)di=\int_0^1\Lambda_t\left(\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}}\,di\right)^{\frac{1}{\varepsilon-1}}C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}}di$$

$$\int_0^1 P_{H,t}(i)C_{H,t}(i)di=\int_0^1\Lambda_t C_{H,t}^{\frac{1}{\varepsilon}}C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}}di\ ;\ \left[C_{H,t}\right]^{\frac{1}{\varepsilon}}=\left[\left(\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}}\,di\right)^{\frac{\varepsilon}{\varepsilon-1}}\right]^{\frac{1}{\varepsilon}}$$

where

$$P_{H,t}C_{H,t}=\Lambda_t C_{H,t}^{\frac{1}{\varepsilon}}\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}}di$$

$$P_{H,t}C_{H,t}=\Lambda_t C_{H,t}^{\frac{1}{\varepsilon}}C_{H,t}^{\frac{\varepsilon-1}{\varepsilon}} \qquad ;\ \left[C_{H,t}\right]^{\frac{\varepsilon-1}{\varepsilon}}=\left[\left(\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}}\,di\right)^{\frac{\varepsilon}{\varepsilon-1}}\right]^{\frac{\varepsilon-1}{\varepsilon}}$$

$$P_{H,t}C_{H,t}=\Lambda_t C_{H,t}$$

$$P_{H,t}=\Lambda_t \qquad \text{(A2.4)}$$

We can see that the multiplier is just identical to the domestic price index $\left(\Lambda_t=P_{H,t}\right)$. Finally, we plug it back to the FOC (eq.A2.3):

$$P_{H,t}(i)C_{H,t}(i)=\Lambda_t\left(\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}}\,di\right)^{\frac{1}{\varepsilon-1}}C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}}$$

$$P_{H,t}(i)C_{H,t}(i)=P_{H,t}\left(\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}}\,di\right)^{\frac{1}{\varepsilon-1}}C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}}$$

$$P_{H,t}(i)C_{H,t}(i)=P_{H,t}C_{H,t}^{\frac{1}{\varepsilon}}C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}}$$

$$\frac{P_{H,t}(i)}{P_{H,t}}C_{H,t}^{-\frac{1}{\varepsilon}}=C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}}C_{H,t}(i)^{-1}$$

$$\frac{P_{H,t}(i)}{P_{H,t}}C_{H,t}^{-\frac{1}{\varepsilon}} = C_{H,t}(i)^{-\frac{1}{\varepsilon}}$$

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} \qquad \text{(A2.5)}$$

This equation is the representative household's demand function for domestic produced consumption goods. It is expressed the relationship as the equation (5.9)

A.2.2 Demand for Foreign (Imported) Goods

The whole procedure of finding optimizing behavior of demand for foreign goods is similar for the demand function for the foreign goods. As we shown above, it is as;

$$\underset{C_{F,t}(i),\Lambda_t}{Min} \int_0^1 P_{F,t}(i)C_{F,t}(i)\,di$$

Subject to

$$C_{F,t} = \left(\int_0^1 C_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}}\,di\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

Constructing the Lagrangian function and solving the first order condition (FOC) with respect to $C_{F,t}(i)$, the result is finally following:

$$C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t} \qquad \text{(A2.6)}$$

A.3 Overall Consumer Price Index (CPI)

As we have shown how to derive optimal allocation of expenditures between domestic and foreign good, the representative household has following optimal allocation function (5.6):

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \quad \text{and} \quad C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{A3.1}$$

and the composite consumption index of household consists of the domestically and foreign produced goods defined as:

$$C_t \equiv \left((1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{A3.2}$$

In order to yield the overall Consumer Price Index (CPI), we have to combine these three equations together as following:

$$C_t \equiv \left((1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$$

$$= \left\{(1-\alpha)^{\frac{1}{\eta}}\left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t\right]^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}\left[\alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t\right]^{\frac{\eta-1}{\eta}}\right\}^{\frac{\eta}{\eta-1}}$$

$$= \left\{(1-\alpha)^{\frac{1}{\eta}}(1-\alpha)^{\frac{\eta-1}{\eta}}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta\frac{\eta-1}{\eta}} C_t^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}\alpha^{\frac{\eta-1}{\eta}}\left(\frac{P_{F,t}}{P_t}\right)^{-\eta\frac{\eta-1}{\eta}} C_t^{\frac{\eta-1}{\eta}}\right\}^{\frac{\eta}{\eta-1}}$$

$$C_t^{\frac{\eta-1}{\eta}} \equiv (1-\alpha)^{\frac{1}{\eta}}(1-\alpha)^{\frac{\eta-1}{\eta}}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta\frac{\eta-1}{\eta}} C_t^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}\alpha^{\frac{\eta-1}{\eta}}\left(\frac{P_{F,t}}{P_t}\right)^{-\eta\frac{\eta-1}{\eta}} C_t^{\frac{\eta-1}{\eta}}$$

$$= (1-\alpha)^{\frac{1}{\eta}+\frac{\eta-1}{\eta}}\left(\frac{P_{H,t}}{P_t}\right)^{-(\eta-1)} C_t^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}+\frac{\eta-1}{\eta}}\left(\frac{P_{F,t}}{P_t}\right)^{-(\eta-1)} C_t^{\frac{\eta-1}{\eta}}$$

$$= (1-\alpha)^{\frac{\eta}{\eta}}\left(\frac{P_{H,t}}{P_t}\right)^{1-\eta} C_t^{\frac{\eta-1}{\eta}} + \alpha^{\frac{\eta}{\eta}}\left(\frac{P_{F,t}}{P_t}\right)^{1-\eta} C_t^{\frac{\eta-1}{\eta}}$$

$$C_t^{\frac{\eta-1}{\eta}} \equiv (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{1-\eta} C_t^{\frac{\eta-1}{\eta}} + \alpha\left(\frac{P_{F,t}}{P_t}\right)^{1-\eta} C_t^{\frac{\eta-1}{\eta}}$$

$$1=(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{1-\eta}+\alpha\left(\frac{P_{F,t}}{P_t}\right)^{1-\eta}$$

$$1=\frac{1}{P_t^{1-\eta}}\left[(1-\alpha)P_{H,t}^{1-\eta}+\alpha P_{F,t}^{1-\eta}\right]$$

$$P_t^{1-\eta}\equiv(1-\alpha)P_{H,t}^{1-\eta}+\alpha P_{F,t}^{1-\eta}$$

$$P_t\equiv\left\{(1-\alpha)P_{H,t}^{1-\eta}+\alpha P_{F,t}^{1-\eta}\right\}^{\frac{1}{1-\eta}} \qquad \text{(A3.3)}$$

Let's move to how we can derive the log-linearizing the Consumer Price Index around the steady state:

$$P_t\equiv\left\{(1-\alpha)P_{H,t}^{1-\eta}+\alpha P_{F,t}^{1-\eta}\right\}^{\frac{1}{1-\eta}}$$

$$P_t^{1-\eta}\equiv(1-\alpha)P_{H,t}^{1-\eta}+\alpha P_{F,t}^{1-\eta}$$

$$\frac{P_t^{1-\eta}}{P_{H,t}^{1-\eta}}\equiv(1-\alpha)+\alpha\frac{P_{F,t}^{1-\eta}}{P_{H,t}^{1-\eta}}$$

$$\left(\frac{P_t}{P_{H,t}}\right)^{1-\eta}\equiv(1-\alpha)+\alpha\left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\eta}$$

Using the Taylor approximation,[2] it can be possible to write:

$$\left(\frac{P}{P_H}\right)^{1-\eta}e^{(1-\eta)(p_t-p_{H,t})}\equiv(1-\alpha)+\alpha\left(\frac{P_F}{P_H}\right)^{1-\eta}e^{(1-\eta)(p_{F,t}-p_{H,t})}$$

where $p_t=\ln\left(\frac{P_t}{P}\right)$, $p_{H,t}=\ln\left(\frac{P_{H,t}}{P_H}\right)$ and $p_{F,t}=\ln\left(\frac{P_{F,t}}{P_F}\right)$

$$\left(\frac{P}{P_H}\right)^{1-\eta}\left[1+(1-\eta)(p_t-p_{H,t})\right]\equiv(1-\alpha)+\alpha\left(\frac{P_F}{P_H}\right)^{1-\eta}\left[1+(1-\eta)(p_{F,t}-p_{H,t})\right]$$

$$\left(\frac{P}{P_H}\right)^{1-\eta}+\left(\frac{P}{P_H}\right)^{1-\eta}(1-\eta)(p_t-p_{H,t})\equiv(1-\alpha)+\alpha\left(\frac{P_F}{P_H}\right)^{1-\eta}+\alpha\left(\frac{P_F}{P_H}\right)^{1-\eta}(1-\eta)(p_{F,t}-p_{H,t})$$

[2] For more detail see e.g. Malley (2004).

Because $\left(\frac{P_t}{P_{H,t}}\right)^{1-\eta} \equiv (1-\alpha)+\alpha\left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\eta}$ as it can be seen in the former calculation, in steady state holds $\left(\frac{P}{P_{H,}}\right)^{1-\eta} \equiv (1-\alpha)+\alpha\left(\frac{P_{F,}}{P_{H,}}\right)^{1-\eta}$ too. Subsequently it is possible to use it substituting for the calculation and continue.

$$\left[(1-\alpha)+\alpha\left(\frac{P_{F,}}{P_{H,}}\right)^{1-\eta}\right]+\left(\frac{P}{P_H}\right)^{1-\eta}(1-\eta)\left(p_t-p_{H,t}\right) \equiv (1-\alpha)+\alpha\left(\frac{P_F}{P_H}\right)^{1-\eta}$$
$$+\alpha\left(\frac{P_F}{P_H}\right)^{1-\eta}(1-\eta)\left(p_{F,t}-p_{H,t}\right)$$

$$\left(\frac{P}{P_H}\right)^{1-\eta}(1-\eta)\left(p_t-p_{H,t}\right)=\alpha\left(\frac{P_F}{P_H}\right)^{1-\eta}(1-\eta)\left(p_{F,t}-p_{H,t}\right)$$
$$P^{1-\eta}\left(p_t-p_{H,t}\right)=\alpha P_F^{\,1-\eta}\left(p_{F,t}-p_{H,t}\right)$$
$$\left(p_t-p_{H,t}\right)=\alpha\left(p_{F,t}-p_{H,t}\right)$$
$$p_t=\alpha p_{F,t}-\alpha p_{H,t}+p_{H,t}$$
$$p_t=(1-\alpha)p_{H,t}+\alpha p_{F,t} \qquad \text{(A3.4)}$$

The last equation is a linearized version of the overall CPI used for the derivation of a connection between term of trade and inflation.

Since the assumption that we have in the term of calculation, it is in terms of the steady state holds $P=P_H=P_F$ or equivalently and we assume the condition $\pi=\pi_H=\pi_F$. In the steady state the development of the overall, domestic and foreign inflation is the same. It is logically consistent with equation (5.2)