CHAPTER 5

THE MODEL: THE SPECIFICATION AND PROBLEM SOLVING

In this chapter we introduce a New Keynesian Model of a small open economy. The New Keynesian Dynamic stochastic General Equilibrium model is derived from microeconomic foundation. Most of equations are derived as a solution of economic agent's optimization problems.

In doing so, we lay out of derivation of key structural equations implied by the model proposed by Gali and Monacelli (2005), Monacelli (2005), Liu (2005), Adolfson et al. (2005) and Lubik and Schorfheide (2005). The model's dynamics are filled out by allowing habit formation as in Justiniano and Preston (2004).

The model is a small open economy model of the Thai economy and its structure is closely related to the New Open Economy Macroeconomic (NOEM) approach which consists of representative agents whom are households, firms, a central monetary authority and a foreign sector. Representative households and firms optimize its behavior from present to future $\left(\text{for } t = 0,1,2,\ldots,\infty\right)$. For representative firms, there are two groups firms: representative firms producing some goods in the domestic economy for either domestic consumption or export and representative firms importing goods from abroad for the domestic consumption.

In the following text, we use this notation that all domestic goods and activity take the subscript H, while goods imported from foreign country are denoted by F and variables with a superscript (*) hold for the foreign economy.

5.1 The Representative Household

In this part, we introduce basic principles connected with the behavior of a representative household and its connection to a foreign economy. The first part is a optimizing problem of the representative household. The second part is shown the linkage of inflation, the real exchange rate and terms of trade. The final part is the part of international financial market.

5.1.1 Optimization

The domestic small open economy is populated by a representative household who supplies a labor service to production sector and seek to maximize

$$E_t \sum_{t=0}^{\infty} \beta^t \left[U(C_t) - V(N_t) \right] \quad (5.1)$$

where $U(C_t) = \frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma}$ and $V(N_t) = \frac{N_t^{1-\varphi}}{1-\varphi}$

$(\text{for } t = 0,1,2,\ldots,\infty)$.

For the utility function as a constant relative risk aversion (CRRA)[1], E_t expresses an expected value of the utility function, β $(0 < \beta < 1)$ is a discount factor, C_t is the consumption, h $(0 < h < 1)$ is an external habit taken as exogenous by the household which made the utility function is also non-separable of preferences over time[2], σ $(\sigma > 0)$ is the inverse elasticity of intertemporal substitution, N_t denote hours of labor and φ $(\varphi > 0)$ is the inverse elasticity of labor supply.

The household can consume both of domestic produced consumption goods $(C_{H,t})$ and the imported foreign consumption goods $(C_{F,t})$ is a composite consumption index of foreign and domestically produced goods defined by:

$$C_t \equiv \left((1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \quad (5.2)$$

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[1] Given a utility function; with $U' > 0$ and $U' \leq 0$, the coefficient of relative risk aversion is defined as $CRRA = -C\frac{U''(C)}{U'(C)}$ measuring how risk averse the household is. The higher σ, the higher is the risk aversion of the household. For more details see Guner (2006) and Krüger (2005)

[2] See more details for definition and implication in Chapter 3 section 3.2

with $\eta\ (\eta > 0)$ is the elasticity of substitution between home and foreign goods and $(0 \le \alpha \le 1)$ is the import ratio. We suppose a continuum of domestic and foreign products indexed by (i). Such indices are given by the following CES aggregators of the quantities consumed of each type of goods as

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{and} \quad C_{F,t} \equiv \left(\int_0^1 C_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \qquad (5.3)$$

for $t = 0,1,2,...,\infty$, where $\varepsilon\ (\varepsilon > 0)$ is the elasticity between different types of goods for consumption in domestic economy. Without loss of generality, it can assume ε is the same for both countries (as it is shown in Gali and Monacelli (2005)).

The maximization (5.1) is subject to a sequence of intertemporal budget constraints of the form:

$$\int_0^1 P_{H,t}(i)C_{H,t}(i)di + \int_0^1 P_{F,t}(i)C_{F,t}(i)di + E_t\left(\frac{D_{t+1}}{1+r_t}\right) = D_t + W_t N_t \qquad (5.4)$$

for $t = 0,1,2,...$ where $P_{H,t}(i)$ and $P_{F,t}(i)$ denote the domestic prices of domestic and foreign goods (i) respectively. D_{t+1} is nominal pay-off on a portfolio held at $t-1$, r_t is nominal interest rate and W_t is nominal wage[3].

The total consumption expenditure between domestic and foreign goods can be written as

$$P_{H,t}C_{H,t} = \int_0^1 P_{H,t}(i)C_{H,t}(i)di \quad \text{and} \quad P_{F,t}C_{F,t} = \int_0^1 P_{F,t}(i)C_{F,t}(i)di$$

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[3] If we take the foreign economy as a group of foreign economies, it is possible to write more general budget constraint as following;

$$\int_0^1 P_{H,t}(i)C_{H,t}(i)di + \int_0^1\int_0^1 P_{Fj,t}(i)C_{F,t}(i)di\ dj + E_t\left(\frac{D_{t+1}}{1+r_t}\right) \le D_t + W_t N_t$$

where $P_{Fj,t}(i)$ is the price of the i-th imported good from the j-th foreign country (expressed in the domestic currency) and $C_{Fj,t}(i)$ is the i-th good produced aboard in the j-th foreign economy. To simplify the equation we assume:

$$C_{F,t}(i) = \left(\int_0^1 C_{Fj,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{and} \quad P_{F,t}(i) = \left(\int_0^1 P_{Fj,t}(i)^{1-\varepsilon} dj \right)^{1-\varepsilon}$$

It is explained by Gali and Monacelli (2005).

and so, $$P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t} \tag{5.5}$$

The representative household aims to allocation optimally its expenditure for the total consumption goods. Because, the results of this optimizing behavior are following set of optimality condition for all t, then optimal allocations of expenditure between domestic and imported goods are[4]

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \quad \text{and} \quad C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{5.6}$$

where the price index of home produced goods and the import price index are given by

$$P_{H,t} \equiv \left(\int_0^1 P_{H,t}(i)^{1-\varepsilon}\, di\right)^{\frac{1}{1-\varepsilon}} \quad \text{and} \quad P_{F,t} \equiv \left(\int_0^1 P_{F,t}(i)^{1-\varepsilon}\, di\right)^{\frac{1}{1-\varepsilon}} \tag{5.7}$$

Then we can derive the overall consumption price index P_t (CPI) as [5]

$$P_t \equiv \left\{(1-\alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right\}^{\frac{1}{1-\eta}} \tag{5.8}$$

Since we know that how much the household spend on domestic and imported consumption, nest decision connected with the optimal allocation is to choose for each category of consumption goods (the i-th good with these two groups of production). The result of this optimizing behavior is the following demand function[6]:

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} \quad \text{and} \quad C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t} \tag{5.9}$$

for all $t = 0,1,2,...$ where $P_{H,t}(i)$ and $P_{F,t}(i)$ are prices of the domestic and imported goods, respectively.

Accordingly, the total consumption expenditure is given by $P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t}$. Using this relationship, the intertemporal budget constraint can be rewritten as:

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[4] See appendix A (A.1)

[5] See appendix A (A.3)

[6] See appendix A (A.2)

$$P_tC_t + E_t\left(\frac{D_{t+1}}{1+r_t}\right) = D_t + W_tN_t \quad (5.10)$$

Now we can take this budget constraint together with the utility function (5.1) and solve the household's optimization problem

$$Max\ E_t\sum_{t=0}^{\infty}\beta^t\left[\frac{\left(C_t - hC_{t-1}\right)^{1-\sigma}}{1-\sigma} - \frac{N_t^{\ 1-\varphi}}{1-\varphi}\right]$$

Subject to

$$P_tC_t + E_t\left(\frac{D_{t+1}}{1+r_t}\right) = D_t + W_tN_t\ ]$$

Solving this problem yields the following set of first order conditions (FOCs), the result are following:

$$\left(C_t - hC_{t-1}\right)^{-\sigma}\frac{W_t}{P_t} = N_t^{\varphi} \quad (5.11)$$

$$\beta R_tE_t\left\{\left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}}\right)^{-\sigma}\frac{P_t}{P_{t+1}}\right\} = 1 \quad (5.12)$$

for all $t = 0,1,2,...$ where $R_t = 1 + r_t$. Equation (5.11) is intra-temporal consumption which balances marginal utility of consumption to marginal value of labor and equation (5.12) expresses inter-temporal Euler equation. Log-linearizing these two equation yields for all t[7]:

$$w_t - p_t = \varphi n_t + \frac{\sigma}{(1-h)}\left(c_t - hc_{t-1}\right) \quad (5.13)$$

$$\left(c_t - hc_{t-1}\right) = E_t\left(c_{t+1} - hc_t\right) - \frac{(1-h)}{\sigma}E_t\left(r_t - \pi_{t+1}\right) \quad (5.14)$$

where $\pi_t = p_t - p_{t-1}$

According to the model, we assume behavior of a foreign household is similar to the behavior of the domestic household that means as we get the same

[7] See more detail in derivation in appendix B

optimality conditions. Variables with a superscript * are valid for the foreign economy[8].

5.2 Linkage of Inflation, The Real Exchange Rate and Terms of Trade

In this research, we maintain to assumption that the law of one price (LOP) holds for the export sector, but incomplete pass-through of nominal exchange rate change to the domestic currency price of imported goods is allowed. The behind this assumption is that Thailand is a price taker with little bargaining power in international markets. For its export bundle, prices are determined exogenously in the rest of the world. On the import side, competition in the world market is assumed to bring import prices equal to marginal cost at the wholesale level, but rigidities arising from inefficient distribution networks and monopolistic retailers allow domestic import prices to deviate from the world price.

The definition of terms of trade (TOT) can be defined as $S_t = \frac{P_{F,t}}{P_{H,t}}$ express a relationship between the aggregate price of exports and imports, or in the other words, the price of foreign goods per unit of home goods. In logs form for all t is

$$s_t = p_{F,t} - p_{H,t} \tag{5.15}$$

Note that, an increase in s_t is equivalent to an increase in competitiveness for the domestic economy because foreign prices $(p_{F,t})$ increase or home prices fall $(p_{H,t})$. Log-linearizing the CPI formula around the steady state yields the following relationship between aggregate price and the terms of trade.

$$p_t \equiv (1-\alpha)\, p_{H,t} + \alpha\, p_{F,t} \tag{5.16}$$

$$= p_{H,t} - \alpha\, p_{H,t} + \alpha\, p_{F,t} = p_{H,t} + \alpha\left(p_{F,t} - p_{H,t}\right)$$

$$p_t = p_{H,t} + \alpha s_t \tag{5.17}$$

Taking the first difference of eq. (5.16), we get

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t \tag{5.18}$$

[8] In the context, it means $C_t^* = C_{F,t}^*, P_t^* = P_{F,t}^*$

From the definition of overall inflation

$$\pi_t \equiv (1-\alpha)\pi_{H,t} + \alpha\pi_{F,t} \tag{5.19}$$

It will identify the linkage of domestic inflation and the terms of trade (TOT) by using above equations combining them together. We get $\pi_{H,t} + \alpha\Delta s_t = (1-\alpha)\pi_{H,t} + \alpha\pi_{F,t}$ and rearrange it as

$$\Delta s_t = \frac{1}{\alpha}(\pi_t - \pi_{H,t}) = \pi_{F,t} - \pi_{H,t} \tag{5.20}$$

The equation (5.20) shows that the difference between foreign and domestic inflation is proportional to the change in TOT or according to (5.18), the difference between overall and domestic inflation is proportional to the change in TOT - the higher the import ratio, α , the smaller the change in the terms of trade

We define Z_t [9] as a nominal exchange rate expressed in terms of foreign currency per a domestic currency. As we state the law of one price, it suppose that not hold strictly, that is, incomplete exchange rate pass-through especially for imports. Law of one price can be shown in the following form:

$$\Psi_t = \frac{P_t^*}{Z_t P_{F,t}} \qquad \text{for all } t = 0,1,2,... \tag{5.21}$$

If LOP holds, i.e. if $\Psi_t = 1,$ the foreign price index equals the import price index expressed in foreign currency $Z_t P_{F,t} = P_t^*$ But if it is not hold, we can say that a law of one price gap is a difference between the foreign world price and the domestic price of import.

We take log in equation (5.21) and substitute it into equation (5.15)

$$\psi_t = p_t^* - z_t - p_{F,t} \qquad \text{or} \qquad p_{F,t} = p_t^* - z_t - \psi_t$$

and terms of trade, therefore, is

$$s_t = p_t^* - z_t - \psi_t - p_{H,t} \tag{5.22}$$

The terms of ψ_t can be understood as a difference between the world price p_t^* and the domestic price expressed in foreign currency $z_t + p_{F,t}$

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[9] An increase of Z_t is an appreciation of domestic currency.

Next, the real exchange rate is $Q_t = \frac{Z_t P_t}{P_t^*}$ for all $t = 0,1,2,...$ or in logs form as following

$$q_t = z_t + p_t - p_t^* \tag{5.23}$$

The relationship between terms of trade and real exchange rate is important to formulate the competitive price index in terms of exchange rate in real conditions, that is, substituting p_t^* from equation (5.22) into the real exchange rate equation

$$\begin{aligned} q_t &= z_t + p_t - \left(s_t + z_t + \psi_t + p_{H,t}\right) \\ &= z_t + p_t - s_t - z_t - \psi_t - p_{H,t} \\ &= p_t - p_{H,t} - s_t - \psi_t \end{aligned}$$

and using equation (5.17) $p_t = p_{H,t} + \alpha s_t$ plugs in to above equation, so

$$\begin{aligned} q_t &= p_{H,t} + \alpha s_t - p_{H,t} - s_t - \psi_t \\ &= \alpha s_t - s_t - \psi_t \\ q_t &= -\left(1-\alpha\right) s_t - \psi_t \end{aligned}$$

Therefore the law of one price gap is

$$\psi_t = -\left[\left(1-\alpha\right) s_t + q_t\right] \tag{5.24}$$

The law of one price gap is an inversely proportionate to the real exchange rate and the degree of international competitiveness for the domestic economy.

5.3 International Financial Market

In this study, we assume that international financial markets are complete together with perfect capital mobility to simplify the model. There are two consequences – the international risk sharing and the uncovered interest parity.

The first one is under international risk sharing. A price of similar bonds must be same in the domestic and foreign economy – expressed as a rate of return in terms of nominal interest rate:

$$1+r_t=\left(1+r_t^*\right)E_t\left(\frac{Z_t}{Z_{t+1}}\right)$$

$$R_t=R_t^*E_t\left(\frac{Z_t}{Z_{t+1}}\right) \qquad (5.25)$$

for all $t=0,1,2,\ldots$ where $R_t=1+r_t$ and $R_t=1+r_t^*$ are gross nominal domestic and foreign interest rates respectively. From equation (5.25), we can use it plug into the intertemporal optimality condition for domestic and foreign household's optimization respectively, the Euler equation (5.14)[10] becomes,

$$\beta E_t\left\{\frac{\left(C_{t+1}-hC_t\right)^{-\sigma}}{\left(C_t-hC_{t-1}\right)^{-\sigma}}\frac{P_t}{P_{t+1}}\right\}=E_t\left(\frac{Z_{t+1}}{Z_t}\right)\beta E_t\left\{\frac{\left(C_{t+1}^*-hC_{t+1}^*\right)^{-\sigma}}{\left(C_t^*-hC_{t-1}^*\right)^{-\sigma}}\frac{P_t}{P_{t+1}}\right\}$$

for all $t=0,1,2,\ldots$, we assume that the habit formation parameter and discount factor for a representative domestic and foreign are identical.

Solving above equation and combining its and term of trade, the result is

$$C_t-hC_{t-1}=\vartheta\left(C_t^*-hC_{t-1}^*\right)Q_t^{-\frac{1}{\sigma}}$$

where ϑ is a constant depending on initial assets positions. Log-linearizing of the equation around steady-state obtains:

$$c_t-hc_{t-1}=\left(c_t^*-hc_{t-1}^*\right)-\frac{1-h}{\sigma}q_t \qquad (5.26)$$

$$c_t-hc_{t-1}=\left(y_t^*-hy_{t-1}^*\right)-\frac{1-h}{\sigma}q_t \qquad (5.27)$$

for all $t=0,1,2,\ldots$,

The condition for parallel optimizing of domestic and foreign household (5.26) supposes relationship $y_t^*=c_t^*$ for all t. More detailed derivation of the previous equations is included in appendix A

The second one is under uncovered interest rate parity. We can connect with another important as following:

$$R_t=R_t^*E_t\left(\frac{Z_t}{Z_{t+1}}\right)$$

[10] How to derive see appendix B

Log-linearizing around steady state yields:

$$\log R_t = \log R_t^* + E_t\left(\log Z_t - \log Z_{t+1}\right)$$
$$r_t = r_t^* + E_t\left(z_t - z_{t+1}\right)$$
$$r_t = r_t^* - E_t\left(z_{t+1} - z_t\right)$$
$$r_t^* - r_t = E_t \Delta z_{t+1}$$

Then we take the first difference equation (5.23) for all $t+1$

$$\Delta q_t = \Delta z_t + \Delta p_t - \Delta p_t^* = \Delta z_t + \pi_t - \pi_t^*$$
$$E_t \Delta q_{t+1} = E_t \Delta z_{t+1} + E_t\left(\pi_{t+1} - \pi_{t+1}^*\right)$$
$$E_t \Delta z_{t+1} = E_t \Delta q_{t+1} - E_t\left(\pi_{t+1} - \pi_{t+1}^*\right)$$

We use both equations $r_t^* - r_t = E_t \Delta z_{t+1}$ and $E_t \Delta z_{t+1} = E_t \Delta q_{t+1} - E_t\left(\pi_{t+1} - \pi_{t+1}^*\right)$ to combine together, give

$$E_t \Delta q_{t+1} - E_t\left(\pi_{t+1} - \pi_{t+1}^*\right) = r_t^* - r_t$$
$$E_t \Delta q_{t+1} = r_t^* - r_t + E_t\left(\pi_{t+1} - \pi_{t+1}^*\right)$$
$$E_t \Delta q_{t+1} = \left(r_t^* - E_t \pi_{t+1}^*\right) - \left(r_t - E_t \pi_{t+1}\right) \qquad (5.28)$$

The equation (5.28) is shown that the familiar relationship between the expected change in real exchange rate and the real interest rate differential[11].

5.4 <u>The Representative Firms</u>

This part will introduce basic characteristic connected with the behavior of a representative firm. In the first subsection there is a description of production possibilities. The second is explained price setting behavior toward though a New Keynesian Phillips Curve (NK-Phillips Curve).

[11] The interest rate differential is calculated as the foreign real interest rate reduced by the domestic real interest rate which can interpret as the motive of international activity.

5.4.1 Production Technology

The aggregate output is described by the constant elasticity of substitution (CES) function:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\delta-1}{\delta}} \, di \right]^{\frac{\delta}{\delta-1}} \tag{5.29}$$

for all $t = 0,1,2,\ldots$, where δ is the elasticity between different types of goods $Y_t(i)$.

Given there is a continuum of monopolistically firms producing differentiated good $Y_t(i)$ with a linear technology represented by the production function:

$$Y_t(i) = A_t N_t(i) \tag{5.30}$$

for the $i-$th firm, for $t = 0,1,2,\ldots$ where $a_t = \log A_t$, describing the technological progress – the firms specific productivity index by the following AR (1) process for all t.

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \tag{5.31}$$

We take log-linear approximation of the aggregate production function is

$$y_t = a_t + n_t \tag{5.32}$$

Next, we can calculate the real total cost as a product of a real wage $\frac{W_t}{P_{H,t}}$ multiplying by a total number of used hours of labor $N_t = \frac{Y_t}{A_t}$ for $t = 0,1,2,\ldots$

$$TC_t = \frac{W_t}{P_{H,t}} \frac{Y_t}{A_t}$$

and we derive marginal costs and log-linearizing them:

$$MC_t = \frac{\partial TC_t}{\partial Y_t} = \frac{\partial \left(\frac{W_t}{P_{H,t}} \frac{Y_t}{A_t} \right)}{\partial Y_t}$$

$$MC_t = \frac{W_t}{P_{H,t}} \frac{1}{A_t}$$

$$\log MC_t = \log \frac{W_t}{P_{H,t}} + \log \frac{1}{A_t}$$

$$mc_t = w_t - p_{H,t} - a_t \qquad (5.33)$$

Equation (5.33) is shown that Marginal cost is increasing in the term of nominal wages and decreasing with domestic prices and technology improvements.

5.4.2 Price Setting Behavior

In order to every representative firms sets a price of its production to maximize its profit, a New Keynesian model is emphasizing on the price stickiness. In the point of view we introduce stickiness in terms of price setting for domestic firms following Calvo's set-up of random price signals[12].

According to the Calvo price setting, in any period t, only $1-\theta_H$ probability of domestic firms $\left(0 \leq \theta_H \leq 1\right)$ are able to reset the prices of production to optimize the behavior. The rest of the probability of representative firms $\left(\theta_H\right)$ is not able to do this.

Let $\overline{P}_{H,t}$ denote the price level that optimizing firms set each period, and $\tilde{P}_{H,t}$ is the price level of firms that only adjust their prices by indexing. Therefore the aggregate domestic price level is a result of setting new prices of the fraction of firms, which are able to optimize $\left(1-\theta_H\right)$, and the remaining fraction, which is not able to optimize prices $\left(\theta_H\right)$ as following

$$P_{H,t} = \left[\left(1-\theta_H\right)\overline{P}_{H,t}^{\,1-\rho} + \theta_H \tilde{P}_{H,t}^{\,1-\rho}\right]^{\frac{1}{1-\rho}}$$

Firms, which are not able to reset their prices optimally, are assumed to adjust their price $\tilde{P}_{H,t}$ by indexing it to the last period's inflation. It can be described according to the relation:

$$\tilde{P}_{H,t} = P_{H,t-1}\left(1+\Pi_{H,t-1}\right)^{\theta_H} = P_{H,t-1}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\theta_H}$$

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[12] For an example of Calvo's price setting is in Chapter 3

where $\left(1+\Pi_{H,t-1}\right)=\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)$, $\Pi_{H,t-1}=\frac{P_{H,t-1}-P_{H,t-2}}{P_{H,t-2}}$ for every period. The new non-optiminized price of $\left(\theta_H\right)$ fraction of firms is created as an adjustment of the last period price $P_{H,t-1}$ with respect to the last period inflation $\Pi_{H,t-1}$.

Log-linearizing its equation results:

$$\hat{p}_{H,t} = p_{H,t-1} + \theta_H \pi_{H,t-1} \quad (5.34)$$

plug previous equation into $P_{H,t}$ so,

$$P_{H,t} = \left\{ \left(1-\theta_H\right) \overline{P}_{H,t}^{1-\rho} + \theta_H \left[P_{H,t-1} \left(\frac{P_{H,t-1}}{P_{H,t-2}} \right)^{\theta_H} \right]^{1-\rho} \right\}^{\frac{1}{1-\rho}} \quad (5.35)$$

the log-linearizing of the equation (5.35) yields[13]:

$$\pi_{H,t} = \left(1-\theta_H\right)\left(\overline{p}_{H,t} - p_{H,t-1}\right) + \theta_H^2 \pi_{H,t-1} \quad (5.36)$$

When firms set a new price, $\overline{P}_{H,t}$, in period t, optimizing firms will seek to maximize the current value of its dividend stream as following:

$$\max_{\overline{P}_{H,t}} E_t \sum_{k=0}^{\infty} \theta_H^k \frac{1}{R_{t+k}} \left\{ Y_{t+k} \left(\overline{P}_{H,t} - MC_{t+k}^n \right) \right\} \quad (5.37)$$

Subject to the current demand constraint:

$$Y_{t+k} \le \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \left(C_{H,t+k} + C_{H,t+k}^* \right) \quad (5.38)$$

where $\frac{\theta_H^k}{R_{t+k}}$ is the effective stochastic discount rate and MC_{t+k}^n are the nominal marginal costs[14]. Solving this problem, we obtain the decision rule for

$$\overline{p}_{H,t} = p_{H,t-1} + E_t \sum_{k=0}^{\infty} \left(\beta\theta_H\right)^k \left\{ \pi_{H,t+k} + \left(1-\beta\theta_H\right) mc_{t+k} \right\} \quad (5.39)$$

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[13] See Appendix C

[14] Detailed description and analysis of the marginal costs are contained in appendix C

that is, firms set their prices according to the future discounted sum of inflation and deviations of real marginal cost from its steady state. We can rewrite equation (5.39) as:

$$\overline{p}_{H,t} = (1-\beta\theta_H)\sum_{k=0}^{\infty}(\beta\theta_H)^k E_t mc_{t+k}^n \tag{5.40}$$

where the real marginal costs are $mc_t = mc_{t+k}^n - p_{H,t+k}$. According to equation (5.40) firms set the price as a markup over a weighted average of expected future marginal costs.

From getting an equation (5.40), plug it into equation (5.36) following the condition of price setting between reoptimized and non-reoptimized price of firms. The result of the optimal price setting of domestic firms, therefore, is a rule of the development of the domestic inflation, so called the New Keynesian Phillips Curve (NKPC).

$$\pi_{H,t} = \beta(1-\beta\theta_H)E_t\pi_{H,t+1} + \theta_H\pi_{H,t-1} + \lambda_H mc_t \tag{5.41}$$

where $\lambda_H = \frac{(1-\beta\theta_H)(1-\theta_H)}{\theta_H}$, for all $t = 0,1,2,...$

The New Keynesian Phillips Curve (NKPC) in this study is a domestic inflation dynamics that is not only backward-looking but the forward-looking as well. On the one hand, if no firm is able to optimize the new prices $(\theta_H \to 1)$, the NKPC is purely backward-looking with adaptive expectations[15]. On the other hand, if all firms in the economy had a chance for reoptimizing of their prices, $(\theta_H \to 0)$, the domestic inflation would be forward-looking and disinflationary policy would be fully costless. However, the domestic inflation is always influenced by the marginal costs of firms, not only in both extreme cases.

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[15] In a case of $\theta_H = 1$, it makes $\lambda_H = 0$ and the NKPC has following form: $\pi_{H,t} = \pi_{H,t-1}$

5.4.3 Incomplete Pass-Through and Import Pricing

In this part we use a setup featuring incomplete exchange rate pass-through in terms of Importers, which induces deviations from the law of one price. We assume that the domestic market is populated by local retailers who import differentiated goods for which the law of one price holds for all import, but the distribution of the goods channels by monopolistic retailers are keeping domestic import prices over the marginal cost, which is they increase their prices after importing. Hence the law of one price for the final buyers does not hold.

We use a similar way of Calvo price setting for domestic importers to find the import inflation. The result will in line of the domestic inflation. That is the fraction θ_F $(0 \le \theta_F \le 1)$ of importers can not reoptimize their prices every period. The rest of the firms $(1-\theta_F)$ set the new price of imports as:

$$\overline{p}_{F,t} = p_{F,t-1} + E_t \sum_{k=0}^{\infty} (\beta\theta_F)^k \left\{ \pi_{F,t+k} + (1-\beta\theta_F)\psi_{t+k} \right\} \qquad (5.42)$$

for all $t = 0,1,2,...$ It is similar to equation (5.39) – the new price depends on the last period price and future path of import inflation and the law of one price gap ψ_t as well.

A positive law of one price gap implies a difference between the foreign economy price and domestic import price. It is a mark–up over the import price. The law of one price gap is a factor for an incomplete import pass-through and provides an influence of the foreign economy prices to the domestic aggregate price level.

The result of this behavior is the Phillips Curve of the import inflation similar to equation (5.41) for all t:

$$\pi_{F,t} = \beta(1-\beta\theta_F) E_t \pi_{F,t+1} + \theta_F \pi_{F,t-1} + \lambda_F \psi_t \qquad (5.43)$$

where $\lambda_F = \dfrac{(1-\beta\theta_F)(1-\theta_F)}{\theta_F}$

By the definition of overall consumer price index (CPI) is

$$P_t \equiv \left\{ (1-\alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right\}^{\frac{1}{1-\eta}}$$

We take the first difference of a log-linear definition of CPI, then it is

$$\pi_t = (1-\alpha)\pi_{H,t} + \alpha\pi_{F,t} \tag{5.44}$$

Taking this definition together with equations (5.41) and (5.43) completes the specification of inflation dynamics for the small open economy. The firms' decisions to smooth prices make the prices sticky that gives rise to nominal rigidities. There are some costs of inflation in case of no price optimization or it means that not valid for the fully flexible prices because there is no deviation of the marginal costs and the law of one price gap.

5.5 The Central Monetary Authority

A domestic central monetary authority is the third agent in the model. The central bank implements monetary policy. Its basic aim is to stabilize both inflation and output. The Taylor rule tells the central bank how to change the interest rate if there is an output gap or a deviation of inflation from the target inflation. The rule is expressed e.g. by Woodford (2001). The central bank increases nominal interest rate in case of a positive output and/or inflation gap especially in case of the inflation targeting regime. The central bank monetary policy development can be approximated by a causal relation of the modified Taylor rule (in a gap form) and the inflation targeting is obtained in the relation implicitly. It is in a development of the inflation gap(π_t) , π_t is a deviation of the consumer price inflation from its target:

$$r_t = \rho_r r_{t-1} + (1-\rho_r)(\phi_1\pi_t + \phi_2 y_t), \tag{5.45}$$

for all t, where ρ_r $(0 \le \rho_r \le 1)$ is the degree of interest rate smoothing (backward–looking parameter for the interest rate gap), ϕ_1 and ϕ_2 $(\phi_1, \phi_2 \ge 0)$ are the relative weights on inflation gap and growth rate of output of the economy gap (output gap). The higher the value of the degree of interest rate smoothing, the lower the influence of inflation and output on the interest rate. The extreme situation $(\rho_r \to 1)$ means that the central bank is only backward–looking and sets the current value of the interest rate only according to its last value. It is not interested in a development of the inflation nor in output. On the other hand, in the opposite situation $(\rho_r \to 0)$, the central bank is devoted only to the basic economic goals of specific rate of inflation and growth rate of output.

5.6 The Foreign Economy

We introduce a foreign economy in the simplest way. Although it is a simplification, it allows us to establish some basic relationships between domestic and foreign economy.

The foreign sector is assumed to be exogenous to the small open economy. It is described by two equations. The first one is connected to the output of the foreign economy:

$$y_t^* = \lambda_1 y_{t-1}^* + \varepsilon_t^{y^*}, \tag{5.46}$$

for $t = 0,1,2,...$ The development of the foreign output y_t^* is described by AR(1) process for $0 < \lambda_1 < 1$ and the production shock $\varepsilon_t^{y^*}$.

The second equation describes behavior of the foreign real interest rate:

$$r_t^* - E_t \pi_{t+1}^* = \rho_{r*} \left(r_{t-1}^* - \pi_t^* \right) + \varepsilon_t^{r*}, \tag{5.47}$$

for all t, where ρ_{r*} $\left(0 < \lambda_1 < 1\right)$ is a parameter of an AR(1) process. The short run real interest rate is expressed in terms of nominal interest rate and inflation. This expression is useful especially for the interpretation because the foreign inflation influences the domestic inflation through the prices of imported goods.

5.7 Equilibrium

To complete the model it is necessary to establish two conditions of the equilibrium. The first condition goes out from the goods market. A goods market–clearing condition expresses a basic fact that the domestic output depends on the foreign output. It is described in part 5.7.1. The inflation dynamics with respect to a development of marginal costs of domestic firms is contained in part 5.7.2. There is a derivation of marginal costs and an introduction of basic relations between marginal costs and variables, which influence the costs. Lastly, we summarize all linearized system equations in part 5.7.3.

5.7.1 Aggregate Demand and Output Determination

The equilibrium on goods market for the domestic economy needs a logical condition that domestic product (Y_t) amounts to the domestic consumption $(C_{H,t})$ and foreign consumption of the home produced goods $(C^*_{H,t})$ [16].

We know that according to the equation (5.9) there is the demand function for the *i*-th product and the same relationship holds for the foreign demand for the *i*-th domestic product for all *t*:

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} \quad \text{and} \quad C^*_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C^*_{H,t} \qquad (5.48)$$

In the calculation we also use the optimal allocation function of the household for the domestic produced consumption goods

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t$$

Then it is necessary to find the optimal allocation function of the foreign household for the imported product. This amount of the domestic consumption $(C_{H,t})$ from the domestic production depends on:

i) The amount of the total domestic consumption (C_t).

ii) The import ratio $(1-\alpha)$

iii) The elasticity of substitution between domestic and foreign consumption goods (η)

iv) and the relative price of the good that is purchased to the aggregate domestic price level $\left(\frac{P_{H,t}}{P_t}\right)$

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[16] We can think that the foreign consumption of home produced goods $(C^*_{H,t})$ like a export of the domestic economy (X_t) or $C^*_{H,t} = X_t$

According to the identical behavior of optimal allocation, it is possible to derivate the foreign consumption from the foreign production $\left(C_{H,t}^{*}\right)$ as following

$$C_{H,t}^{*} = \alpha \left(\frac{Z_t P_{H,t}}{P_t^{*}} \right)^{-\eta} C_t^{*} \tag{5.49}$$

The foreign productions $\left(C_{H,t}^{*}\right)$ are influenced by:

i) The amount of the total consumption in the bigger economy C_t^{*}
ii) The import ration of the bigger economy α .
iii) The elasticity of substitution between domestic and foreign consumption goods (η)
iv) and the relative price of domestic good that is purchased (in this case the purchased good is from the small open economy and expressed in the price of the domestic currency in the bigger economy) to the aggregate price level in the bigger economy $\left(\frac{Z_t P_{H,t}}{P_t^{*}} \right)$

The goods market-cleaning condition holds for the *i*-th domestic product and can be expressed in the following form.

$$Y_t(i) = C_{H,t}(i) + C_{H,t}^{*}(i)$$

for $t = 0,1,2,...$ and now we plug both equations (5.9) to the previous formula:

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t} + \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t}^{*}$$

and then we use the equation (5.6) and (5.49) to eliminate $C_{H,t}$ and $C_{H,t}^{*}$:

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} \alpha \left(\frac{Z_t P_{H,t}}{P_t^{*}} \right)^{-\eta} C_t^{*}$$

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} \left((1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \left(\frac{Z_t P_{H,t}}{P_t^{*}} \right)^{-\eta} C_t^{*} \right)$$

$$Y_t(i)^{\frac{\delta-1}{\delta}} = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon\left(\frac{\delta-1}{\delta}\right)} \left((1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \left(\frac{Z_t P_{H,t}}{P_t^{*}} \right)^{-\eta} C_t^{*} \right)^{\frac{\delta-1}{\delta}}$$

Substituting the equation (5.29) for the aggregate output $Y_t = \left[\int_0^1 Y_t(i)^{\frac{\delta-1}{\delta}} di \right]^{\frac{\delta}{\delta-1}}$ into the previous result yields:

$$Y_t = \left[\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon\left(\frac{\delta-1}{\delta}\right)} \left((1-\alpha)\left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \left(\frac{Z_t P_{H,t}}{P_t^*} \right)^{-\eta} C_t^* \right)^{\frac{\delta-1}{\delta}} di \right]^{\frac{\delta}{\delta-1}}$$

$$Y_t^{\frac{\delta}{\delta-1}} = \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon\left(\frac{\delta-1}{\delta}\right)} \left((1-\alpha)\left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \left(\frac{Z_t P_{H,t}}{P_t^*} \right)^{-\eta} C_t^* \right)^{\frac{\delta-1}{\delta}} di$$

$$Y_t = \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} \left((1-\alpha)\left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \left(\frac{Z_t P_{H,t}}{P_t^*} \right)^{-\eta} C_t^* \right) di$$

$$Y_t = \left(\frac{1}{P_{H,t}} \right)^{-\varepsilon} \left((1-\alpha)\left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \left(\frac{Z_t P_{H,t}}{P_t^*} \right)^{-\eta} C_t^* \right) . \int_0^1 P_{H,t}(i)^{-\varepsilon} di$$

$$Y_t = \left(\frac{1}{P_{H,t}} \right)^{-\varepsilon} \left((1-\alpha)\left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \left(\frac{Z_t P_{H,t}}{P_t^*} \right)^{-\eta} C_t^* \right) P_{H,t}^{-\varepsilon}$$

$$Y_t = \left(\frac{1}{P_{H,t}} \right)^{-\varepsilon} \left((1-\alpha)\left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \left(\frac{Z_t P_{H,t}}{P_t^*} \right)^{-\eta} C_t^* \right) P_{H,t}^{-\varepsilon}$$

$$Y_t = (1-\alpha)\left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \left(\frac{Z_t P_{H,t}}{P_t^*} \right)^{-\eta} C_t^*$$

$$Y_t = C_{H,t} + C_{H,t}^* \tag{5.50}$$

In the last step we used optimal allocation functions (5.6) and (5.49). The total differential of the first order condition (FOC) yields for all *t:*

$$Y y_t = C_H c_{H,t} + C_H^* c_{H,t}^*$$

$$y_t = \frac{C_H}{Y} c_{H,t} + \frac{C_H^*}{Y} c_{H,t}^*$$

$$y_t = \frac{C_H}{C} c_{H,t} + \frac{C_H^*}{C} c_{H,t}^*$$

$$y_t = (1-\alpha) c_{H,t} + \alpha c_{H,t}^* \tag{5.51}$$

The results are intuitive. Both equations (5.50) and (5.51) give the similar explanation. In equation (5.50), the aggregate domestic output is divided into the domestic and the foreign consumption. According to the expression (5.51), the increase in aggregate output is divided between the increase in domestic and foreign consumption with respect to the import ratio (some part is consumed at home and the rest is exported). An equation (5.51) describes the previous equation in a growing form.

Log-linearizing optimal allocation functions for the domestic economy (5.6) and $p_t = p_{H,t} + \alpha s_t$ from the equation gives for all *t*:

$$c_{H,t} = -\eta\left(p_{H,t} - p_t\right) + c_t \qquad p_{H,t} - p_t = -\alpha s_t$$

$$c_{H,t} = \eta\alpha s_t + c_t \tag{5.52}$$

It is evident that an improvement in terms of trade for the domestic economy (s_t or domestic competitiveness on the foreign market increases) enables to the domestic representative households to augment its consumption and substitute out the foreign produced goods for a given level of consumption. The magnitude depends on the possibility of substitution between domestic and foreign goods η and the import ratio α .

The allocation function for the foreign economy is also log-linearized and simultaneously is used log-linearized version of law of one price:

$$c^*_{H,t} = -\eta\left(z_t - p^*_t + p_{H,t}\right) + c^*_t \qquad \psi_t = p^*_t - z_t - p_{F,t} \rightarrow z_t - p^*_t = -p_{F,t} - \psi_t$$

$$c^*_{H,t} = -\eta\left(p_{H,t} - p_{F,t} - \psi_t\right) + c^*_t$$

and together with log-linearized version of terms of trade

$$c^*_{H,t} = -\eta\left(p_{H,t} - p_{F,t} - \psi_t\right) + c^*_t \qquad p_{F,t} - p_{H,t} = s_t$$

$$c^*_{H,t} = -\eta\left(-s_t - \psi_t\right) + c^*_t$$

$$c^*_{H,t} = \eta\left(s_t + \psi_t\right) + c^*_t \tag{5.53}$$

for $t = 0,1,2,\ldots$

The explanation of the previous equation is similar. An increase in s_t causes higher consumption of goods produced in the small open economy for foreigners accompanied by a decrease of domestic goods for consumption.

Plugging (5.52) and (5.53) in (5.51) the equation has the following form:

$$y_t = (1-\alpha)(\eta\alpha s_t + c_t) + \alpha\left[\eta(s_t + \psi_t) + c_t^*\right]$$

$$y_t = \alpha\eta s_t + c_t - \alpha^2\eta s_t - \alpha c_t + \alpha\eta s_t + \alpha\eta\psi_t + c_t^*$$

$$y_t = (2-\alpha)\alpha\eta s_t + (1-\alpha)c_t + \alpha\eta\psi_t + \alpha y_t^* \qquad (5.54)$$

for all $t = 0,1,2,...$ The last equation is the goods market-clearing condition for the small open economy. In case of closed economy $(\alpha = 0)$, we have condition of this form $(y_t = c_t)$.

5.7.2 <u>The Supply Side: Marginal Cost and Inflation Dynamics</u>

The equation (5.41) is a result of behavior of domestic firms with respect to the Calvo style pricing. This domestic New Keynesian Phillips Curve shows the development of domestic inflation (inflation dynamics). We have derived this form:

$$\pi_{H,t} = \beta(1-\beta\theta_H)E_t\pi_{H,t+1} + \theta_H\pi_{H,t-1} + \lambda_H mc_t$$

for $t = 0,1,2,...$

The evolution of the current inflation depends on last period inflation (by indexation of some firms) and discounted value of expected inflation for the next period (by optimizing behavior of the rest of the firms). The real marginal costs are the third important factor. They stem from the production possibilities (expressed as a CES production function) of monopolistic firms. A symmetric equilibrium assumes:

$$mc_t = w_t - p_{H,t} - a_t$$

$$mc_t = (w_t - p_t) + (p_t - p_{H,t}) - a_t$$

Now we employ the log–linearized FOC of household's optimizing expressed as the intratemporal consumption (5.13) and the adjusted formula for the terms of trade (5.17) to plug them into the equation for marginal costs.

$$w_t - p_t = \varphi n_t + \frac{\sigma}{1-h}\left(c_t - hc_{t-1}\right) \qquad p_t = p_{H,t} + \alpha s_t \rightarrow p_t - p_{H,t} = \alpha s_t$$

$$mc_t = \varphi n_t + \frac{\sigma}{1-h}\left(c_t - hc_{t-1}\right) + \alpha s_t - a_t$$

From the domestic New Keynesian Phillips Curve is shown that it is not only depending on both of forward-looking and backward-looking decision but also the marginal cost:

The last task is to substitute out the term n_t with using log–linear version of the firms' production function (5.32) $y_t = a_t + n_t \rightarrow n_t = y_t - a_t$ in this way:

$$mc_t = \varphi\left(y_t - a_t\right) + \frac{\sigma}{1-h}\left(c_t - hc_{t-1}\right) + \alpha s_t - a_t$$

$$mc_t = \varphi\left(y_t - a_t\right) + \frac{\sigma}{1-h}\left(c_t - hc_{t-1}\right) + \alpha s_t - a_t$$

$$mc_t = \frac{\sigma}{1-h}\left(c_t - hc_{t-1}\right) + \varphi y_t + \alpha s_t - \left(1+\varphi\right) a_t \qquad (5.55)$$

for $t = 0,1,2,...$

The marginal costs are positively related to the domestic output and terms of trade and inversely related to the level of technological progress (the firm specific productivity index).

5.7.3 Conclusion of the System of the Model

In this section we summarize all the key equations derived in previous chapters. For the purpose of the empirical analysis, a log-linear approximation around the steady state is employed. The log-linearized model consist of 14 equations that are rearranged and complete by exogenous domestic and foreign shocks. The system is following:

$$\psi_t = -\left[\left(1-\alpha\right) s_t + q_t\right] \qquad (5.23)$$

$$\Delta s_t = \pi_{F,t} - \pi_{H,t} + \varepsilon_t^s \qquad (5.20)$$

$$E_t \Delta q_{t+1} = \left(r_t^* - E_t \pi_{t+1}^*\right) - \left(r_t - E_t \pi_{t+1}\right) + \varepsilon_t^q \qquad (5.28)$$

$$\pi_t = \left(1-\alpha\right) \pi_{H,t} + \alpha \pi_{F,t} \qquad (5.19)$$

$$\pi_{F,t} = \beta\left(1-\beta\theta_F\right)E_t\pi_{F,t+1} + \theta_F\pi_{F,t-1} + \lambda_F\psi_t + \varepsilon_t^{\pi_F} \tag{5.43}$$

$$\pi_{H,t} = \beta\left(1-\beta\theta_H\right)E_t\pi_{H,t+1} + \theta_H\pi_{H,t-1} + \lambda_H mc_t + \varepsilon_t^{\pi_H} \tag{5.41}$$

$$mc_t = \frac{\sigma}{1-h}\left(c_t - hc_{t-1}\right) + \varphi y_t + \alpha s_t - \left(1+\varphi\right)a_t \tag{5.55}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \tag{5.31}$$

$$\left(c_t - hc_{t-1}\right) = E_t\left(c_{t+1} - hc_t\right) - \frac{\left(1-h\right)}{\sigma}E_t\left(r_t - \pi_{t+1}\right) \tag{5.14}$$

$$c_t - hc_{t-1} = \left(y_t^* - hy_{t-1}^*\right) - \frac{1-h}{\sigma}q_t \tag{5.27}$$

$$y_t = \left(2-\alpha\right)\alpha\eta s_t + \left(1-\alpha\right)c_t + \alpha\eta\psi_t + \alpha y_t^* \tag{5.54}$$

$$r_t = \rho_r r_{t-1} + \left(1-\rho_r\right)\left(\phi_1\pi_t + \phi_2 y_t\right) + \varepsilon^{r_t} \tag{5.45}$$

$$y_t^* = \lambda_1 y_{t-1}^* + \varepsilon_t^{y^*} \tag{5.46}$$

$$r_t^* - E_t\pi_{t+1}^* = \rho_{r*}\left(r_{t-1}^* - \pi_t^*\right) + \varepsilon_t^{r*} \tag{5.47}$$

for $t = 0,1,2,...$

The linearized model consists of 11 equations for endogenous and 3 equations for exogenous processes – equation (5.31), (5.46), (5.47). There are eight shocks: $\varepsilon_t^s, \varepsilon_t^q, \varepsilon_t^{\pi_F}, \varepsilon_t^{\pi_H}, \varepsilon_t^a, \varepsilon_t^r, \varepsilon_t^{y^*}, \varepsilon_t^{r*}$.

- Equation (5.23) – law of one price (LOP) gap,
- Equation (5.20) – terms of trade with a measurement error ε_t^s
- Equation (5.28) – uncovered interest parity (UIP) with a risk premium shock ε_t^q
- Equation (5.19) – overall inflation,
- Equation (5.43) – the New Keynesian Phillips Curve (NKPC) for import inflation with a foreign inflation shock $\varepsilon_t^{\pi_F}$
- Equation (5.41) – the New Keynesian Phillips Curve (NKPC) for domestic inflation with a domestic inflation shock $\varepsilon_t^{\pi_H}$
- Equation (5.55) – Firm’s marginal costs,

• Equation (5.31) – AR(1) process for a technological progress with an innovation ε_t^a

• Equation (5.14) – consumption Euler equation,

• Equation (5.27) – international risk sharing condition,

• Equation (5.54) – goods market–clearing condition,

• Equation (5.45) – modified Taylor rule with a monetary shock ε_t^r

• Equation (5.46) – exogenous AR(1) process for the foreign economy output with an innovation $\varepsilon_t^{y^*}$

• Equation (5.47) – exogenous AR(1) process for the foreign economy short- run real interest rate with a shock ε_t^{r*}

The vector x_t is the endogenous state vector, y_t is the endogenous vector of unobservable variables and z_t is the exogenous stochastic process

$$x_t = \{y_t, q_t, r_t, \pi_t, \pi_{F,t}, s_t, c_t, r_t^*, y_t^*, \pi_{H,t}\}$$

$$y_t = \{\psi_t, mc_t\}$$

$$z_t = \{a_t, \varepsilon_t^s, \varepsilon_t^q, \varepsilon_t^{\pi_H}, \varepsilon_t^{\pi_F}, \varepsilon_t^r, \varepsilon_t^{y^*}, \varepsilon_t^{r^*}\}$$

where r_t^* express foreign real interest rate instead of foreign nominal interest rate as it was used so far.

Table 5.1

Parameters of the Linearized Model

Parameter	**Equation**	**Interpretation of the Parameter**	**Restriction**
α	5.23, 5.19, 5.55, 5.54	Import Ratio	$(0:1)$
β	5.43, 5.41	Discount factor	$(0:1)$
h	5.55, 5.14, 5.27	Habit formation parameter in consumption	$(0:1)$
σ	5.55, 5.14, 5.27	Inverse elasticity of intertemporal substitution	$(0:\infty)$
η	5.54	Elasticity of substitution between home and foreign goods	$(0:\infty)$
φ	5.55	Inverse elasticity of labor supply	$(0:\infty)$

Table 5.1

Parameters of the Linearized Model (Continued)

Parameter	**Equation**	**Interpretation of the Parameter**	**Restriction**
θ_H	5.41	Fraction of non-optimizing firms	$(0:1)$
θ_F	5.43	Fraction of non-optimizing importers	$(0:1)$
ϕ_1	5.45	Elasticity of interest rate to inflation	$(0:\infty)$
ϕ_2	5.45	Elasticity of interest rate to output	$(0:\infty)$
ρ_r	5.45	Backward-looking parameter for interest rate	$(0:1)$
ρ_r^*	5.47	Foreign real interest rate inertia parameter	$(0:1)$
ρ_a	5.31	Inertia of technology development	$(0:1)$
λ_1	5.46	Foreign output inertia parameter	$(0:1)$