CHAPTER 2 THEORIES

The algorithm for estimating fractal dimension based on area perimeter method can be developed from Sierpinski triangle

2.1 Sierpinski Triangle

An example of algorithm for estimating fractal dimension based on area perimeter method for Sierpinski triangle is presented in this section.

The construction of Sierpinski triangle is shown in Figure 2.1.

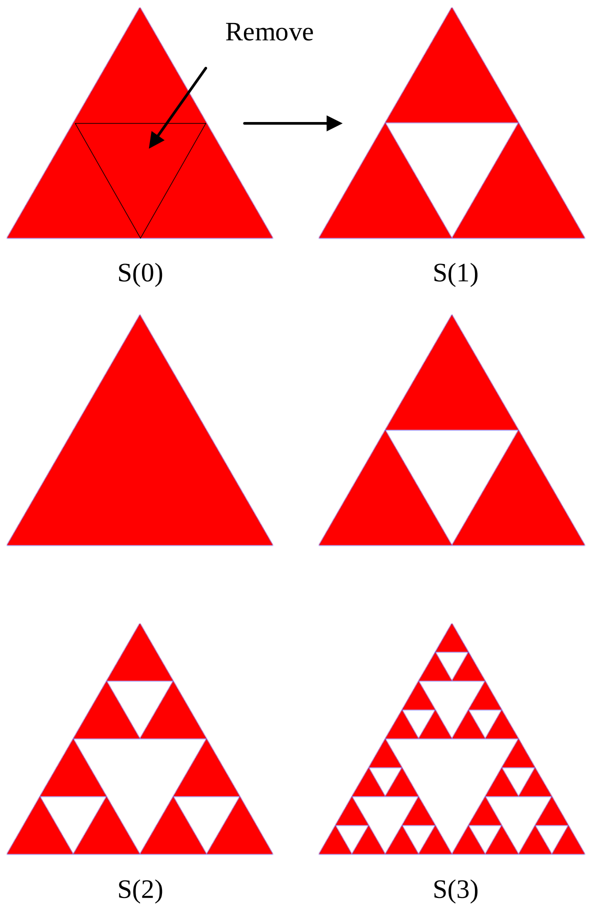


Figure 2.1 Construction steps of the Sierpinski triangle (Riddle, 2006).

The steps to construct the Sierpinski triangle can be summarized as follows,

1. Take an equilateral triangle.
2. Create four triangles out of that one by connecting the center of each side.
3. Cut out the middle triangle.
4. Repeat the steps.

2.2 Fractal Objects: Iterative Construction of Sierpinski Triangle

The dimension of the Sierpinski triangle can be calculated from Hausdorff measure which is based on the Hausdorff dimension $H_\delta{}^d$ concept (Kenneth, 1990).

Let F be a subset of $\Re^n, S$ and δ are non-negative real values.

$$H_\delta^d(F) = \text{int}\{\sum \|U_i\|^d \text{ such that} \{U_i\} \text{is a } \delta\text{- cover of } F\} \tag{2.1}$$

where $\{U_i\}$ is a δ -cover of F if $F \subset \text{U}_{i=1}^{\infty} U_i$, being $0 < \|U_i\| \leq \delta$.

The Hausdorff d-measure, H^s is given by

$$H^d(F) = \lim_{\delta \to 0} H_\delta^d(F) \tag{2.2}$$

An interesting and important characteristic of this measure is that $H_\delta^d(F)$ is always 0 for any $d < d_H$ and ∞ for any $d > d_H$. The real value d_H is the so-called Hausdor dimension of F.

That is

$$d_H(X) = \inf\left\{d \left| H^d(X) = 0\right.\right\} = \sup\left\{d \left| H^d(x) = \infty\right.\right\} \tag{2.3}$$

From the above discussion,

$$H^d(d) = \begin{cases} \infty \, if \; d < \dim_H F \\ 0 \, if \; d < \dim_H F \end{cases} \tag{2.4}$$

At the point $d = \dim_H F,$ the value of $H^d\left(F\right)$ may be zero or infinity (here d = D, dimension),

$$0 < H^d(F) < \infty \tag{2.5}$$

So Sierpinski triangle S is defined to be the intersection $\bigcap_{i=0}^{\infty} S_{n,}$ where S_0 is a equilateral triangle, and $S_i\,(i \geq 1)$ is obtained by removing all points inside the medial triangles of each “basic triangle” of S_{i-1} show in Figure 2.1. If it is assumed that $0 < H^d(S) < \infty$ at the critical point

$$d = \dim_H S \tag{2.6}$$

there is a simple method to find $\dim_H S,$ note that $S,$ can split into

$$S = S_1 \cup S_2 \cup S_3 \tag{2.7}$$

where S_1, S_2, S_3 are geometrically similar to S.

Then

$$H^d(S) = H^d(S_1) + H^d(S_2) + H^d(S_3) \quad (2.8)$$

$$= \frac{1}{2^d} H^d(S_1) + \frac{1}{2^d} H^d(S_2) + \frac{1}{2^d} H^d(S_3) \quad (2.9)$$

$$= \frac{3}{2} H^d(S) \quad (2.10)$$

Since $0 < H^d(S) < \infty$, Thus $\frac{3}{2^d} = 1$ and hence $d = \frac{\log 3}{\log 2}$.

2.3 Linear Regression Analysis

Linear regression attempts to model the relationship between two variables by fitting a linear equation to observed data. One variable is considered to be an explanatory variable, and the other is considered to be a dependent variable. A linear regression line has an equation of the form $Y = a + bX$, where X is the explanatory variable and Y is the dependent variable. The slope of the line is b, and a is the intercept (the value of y when $x = 0$) (Yale University, 2014).

2.4 Area–Perimeter Method

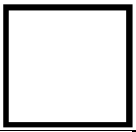
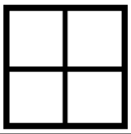
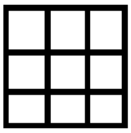
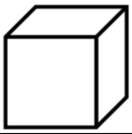
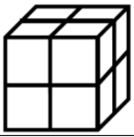
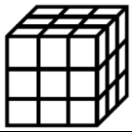
The relation between the number of Perimeter (P), Area (A) for the line, square, and cube is shown in Table 2.1.

From Table 2.1, the fractal power law equation relating the area to perimeter is given by (National Research Ethics Service, 2008)

$$A = (kP)^D \quad (2.11)$$

where A is area
P is some measure of perimeter
k is a constant that is dependent upon the measure used for P
D is fractal dimension

Table 2.1 Relation between the number of Perimeter (P) and Area (A).

Dimension	Object	Reduction Factor (r)	Perimeter	Area	Relation between P and A
$D = 1$ line		1	$1r_1$	$1r_1$	$P = A$
		1/2	$2r_2$	$2r_2$	$P = A$
		1/3	$3r_3$	$3r_3$	$P = A$
		$1/n$	$P = nr_n$	$A = nr_n$	$P = A$
$D = 2$ square		1	$4r_1 = 4nr_1$	$1(1)^2 = 1(r_1)^2$	$P = 4(1r_1)^2$ $P = 4\sqrt{A}$
		1/2	$12r_1 = 6nr_2$	$4(1/2)^2 = 4(r_2)^2$	$P = 6(2r_2)^2$ $P = 6\sqrt{A}$
		1/3	$24r_1 = 8nr_3$	$9(1/3)^2 = 9(r_3)^2$	$P = 8(3r_3)^2$ $P = 8\sqrt{A}$
		$1/n$	$P = (2(n-1)+4)\ nr_n$	$A = n^2(1/n)^2 = (nr_n)^2$ $\sqrt{A} = nr_n$	$P = (2(n-1)+4)\sqrt{A}$ $P = C\sqrt{A} = CA^{1/2}$ $A = (kP)^2$, $k = 1/C$ $A = (kP)^D$, $D = 2$
$D = 3$ cube		1	$(6)(1^2)(1)^2 = 6n^2(r_1)^2$	$(1)(1)(1)^3 = n^2n(r_1)^3$	$P = 6nr_n\sqrt[3]{A}$
		1/2	$(6)(2^2)(1/2)^2 = 6n^2(r_2)^2$	$(4)(2)(1/2)^3 = n^2n(r_2)^3$	$P = 6nr_n\sqrt[3]{A}$
		1/3	$(6)(3^2)(1/3)^2 = 6n^2(r_3)^2$	$(9)(3)(1/3)^3 = n^2n(r_3)^3$	$P = 6nr_n\sqrt[3]{A}$
		$1/n$	$P = 6\ n^2(r_n)^2$ $P = 6(nr_n)^2$	$A = (nr_n)^3$ $\sqrt[3]{A} = nr_n$	$P = 6nr_n\sqrt[3]{A}$ $P = C\sqrt[3]{A} = CA^{1/3}$ $A = (kP)^3$, $k = 1/C$ $A = (kP)^D$, $D = 3$

Taking the logarithm of Eq. (2.11)

$$\log(A) = D\log\left(P\right) + D\log(k) \tag{2.12}$$

$$\log(A) = D\log\left(P\right) + \text{constant} \tag{2.13}$$

So equation (2.13) is the general form to find fractal dimension with the area-perimeter method.

If log(*A*) and log(*P*) are plotted as a log-log plot and linear regression is determined. Then *D* is the slope of the regression line as shown in Figure 2.6.

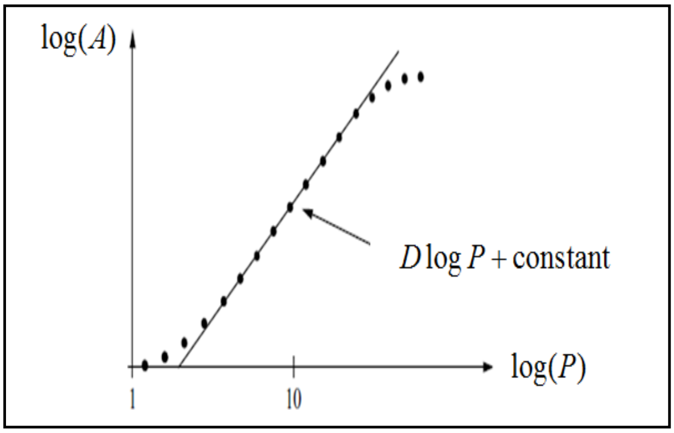


Figure 2.2 Graph of log(*P*) versus log(*A*) (Gruiz, 2006).

The curve log(*P*) versus log(*A*) deviates from a straight line of slope *D* both for resolution approaching unity and for very small resolution. The respective reasons are that for coarse resolution no power-law behaviour can be expected, and that on very fine scales new effects set in and the system behaves differently (Gruiz, 2006).

2.5 Satellite Images

Satellite Image is a tool for measuring data for monitoring short-time atmospheric phenomena including the cloud motion and drift of weather systems. It is also used for monitoring of climate changes based on the accumulation of the global data over a long period (Nanjing University of Information Science & Technology, 2006).

2.5.1 Infrared (IR) Images

IR imagery derived from emissions by the Earth and its atmosphere at thermal-infrared wavelengths (10-12 µm), Conventional IR imagery is derived from terrestrial radiation emitted in the 10-12µm region, Warm temperatures (0-30^{o}C) generally mean land or sea without cloud cover. Most of the radiation reaching the satellite originates from the Earth's surface or from the clouds, and is largely unmodified by the atmosphere (Nanjing University of Information Science & Technology, 2006).

2.5.2 Geostationary Meteorological Satellite - 5 (GMS-5)

For geostationary meteorological satellite-5, calibration gives temperature and reflectance in relation to their observed levels and a correspondence (calibration) tables are provided. Therefore, the temperature and reflectance at the observed point can be known by knowing their gray levels and referring to the calibration table (Ito, 2000). In this study data from GMS-5 in the infrared-1 (IR1) channel (10.5-11.5 µm) are used. The wavelength the GMS-5 are presented in Table 2.3.

Table 2.2 Sensor characteristics of GMS-5 (Ito, 2000).

	Visible	Infrared
Wavelength characteristics (µm)	0.55 - 0.90	10.5 - 11.5 (Infrared1) 11.5 - 12.5 (Infrared 2) 6.5 - 7.0 (Water vapor)
Resolution	1.25 km	5 km
Gray scale of image	64	256