

CHAPTER 1 INTRODUCTION

1.1 Rational/Problem

Heavy rain with thunder is common in Thailand. Thunderstorm clouds are sometimes gathered in cluster. This cloud cluster can produce very heavy rain, very strong wind and hail. The characteristic of this cluster in satellite image should be able to identify by fractal dimension.

1.2 Cloud

Cloud is caused by aggregation or clumping of water vapor, eventually condense and fall as rain. Clouds are divided into two types, horizontal layer and upward growth (convective) (Wikipedia, 2013). The large clouds that are formed in the vertical are called cumulonimbus which can cause wind gusts, lightning, torrent rain and severe air turbulence, and possible hail falls. Clouds have irregular shapes and surface which can not be represented by regular geometric pattern. Clouds have self-similarity characteristic which is the most important property of fractals. Self similar can be found in nature, for examples coastlines, trees and mountains.

1.3 Thunderstorm

A thunderstorm is a storm with lightning and thunder. It is produced by a cumulonimbus cloud, usually producing gusty winds, heavy rain and sometimes hail (Wicker, 2013).

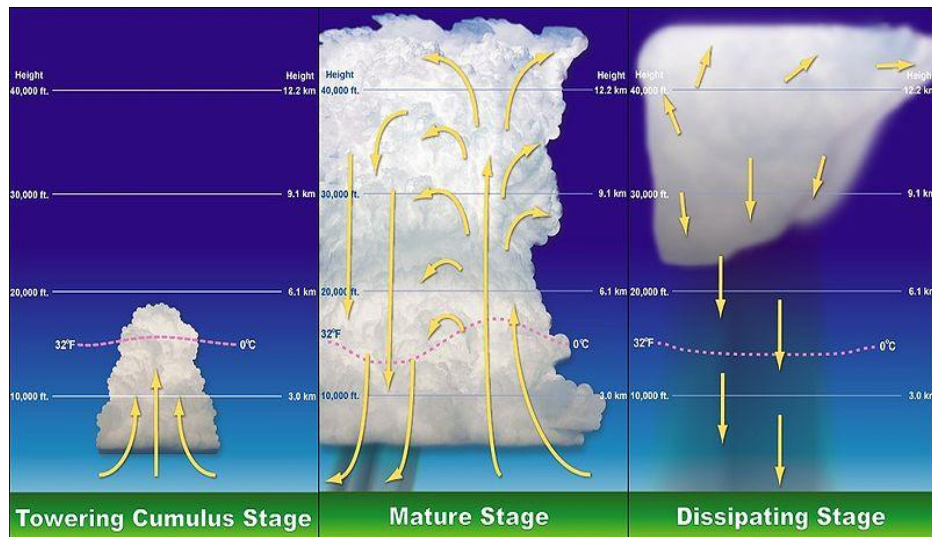


Figure 1.1 Thunderstorm formation (Wikipedia, 2013).

Thunderstorms occur on a regular basis according to season and can occur throughout the year in the vicinity of the equator because the weather is hot. Thunderstorms can be seen in Thailand mostly in the period from April to May.

1.4 Fractal

Fractal is a geometric pattern that has irregular shapes and surfaces which characterize the self similarity (Sripana, 2006). Many objects in nature are fractals. For examples, cauliflowers, ferns, tree and cloud are fractals because of the self similarity characteristic (Figure2.1).



Figure 1.2 Many objects in nature are fractals (Mandelbrot, 1990).

Clouds have complicated and irregular shapes. Thus, analyzing and characterizing the shapes of clouds using fractal dimension methods has a number of advantages for metrological and atmospheric studies.

1.4.1 Regular Dimension

Regular dimension describes dimension of regular shape, which is integer such as circles, triangles and squares.

Dimension is the number of coordinates required to locate a point in a space. Figure 1.3 shows relation between the scale factor r and the number of pieces, N , covering the original structure in the dimension D .

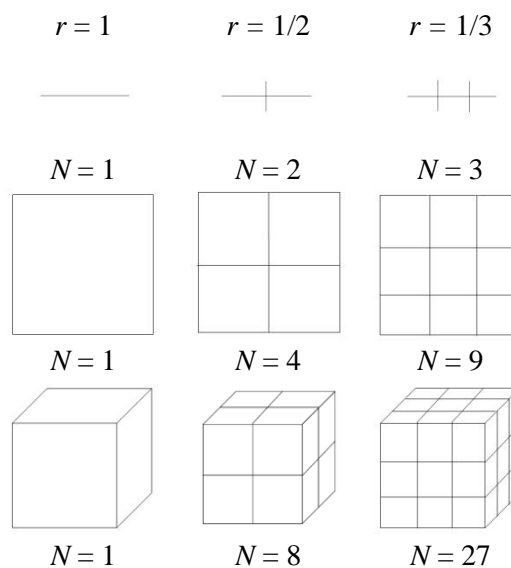


Figure 1.3 Scaling of a line, square, and cube (Florindo and Bruno, 2011).

From Table 1.1, for the line, square, and cube there is a power law relation between the number of pieces N and the reduction factor r .

Table 1.1 Relation between the number of pieces (N) and the reduction factor (r) (Peigen, 2004).

Object	Number of Pieces (N)	Reduction Factor (r)
line	2	1/2
line	3	1/3
square	$4=2^2$	1/2
square	$9=3^2$	1/3
cube	$8=2^3$	1/2
cube	$27=3^3$	1/3

This is the power law (Peigen, 2004)

$$N = \frac{1}{r^D} \quad (1.1)$$

where $D = 1$ for the line, $D = 2$ for the square, and $D = 3$ for the cube. D is the dimension.

1.4.2 Fractal Dimension

Fractal dimension is a measure of irregular shapes with non-integer dimension. Fractal dimension can describe geometric pattern that has irregular shapes.

Examples of irregular shapes are shown in Figure 1.4.

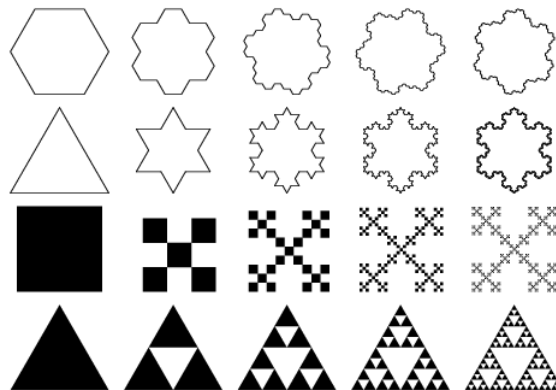


Figure 1.4 Geometric pattern that has irregular shapes (Mathworld, 2012).

From Eq. 1.1, take logarithm

$$\log N = D \log \frac{1}{r} \quad (1.2)$$

$$D = \frac{\log N}{\log \frac{1}{r}} \quad (1.3)$$

Given a self-similar structure, there is a relation between the reduction factor r and the number of pieces N into which the structure can be divided (Peigen, 2004). Fractal dimension has non integer values between $0 < D < 1$ for line, $1 < D < 2$ for area, $2 < D < 3$ for volume. An example of geometric pattern that has irregular shape of mathematical structures is Sierpinski triangle as shown in Figure 1.5.

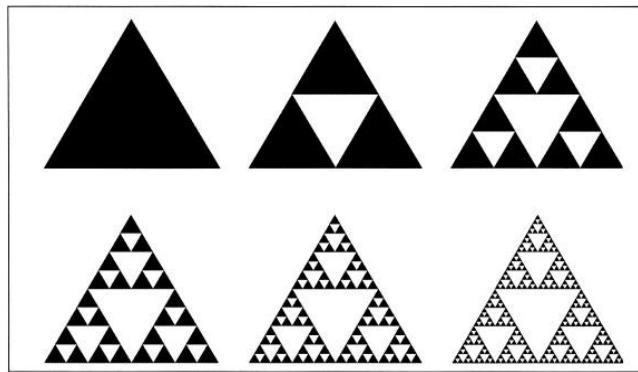


Figure 1.5 The basic construction steps of the Sierpinski triangle (Peigen, 2004).

1.4.3 Calculation of Fractal Dimension

There are several methods to measure fractal dimension, however only five methods are presented in this research.

1.4.3.1 Box-Counting Method

This method approximates fractal dimension by counting the number of boxes that cover the fractal. For different size of the boxes r and the number of the boxes that cover the fractal curve $N(r)$, determine the smallest number of boxes of side length r needed to cover the shape (Sutherland, 2002), as shown in Figure 1.6.

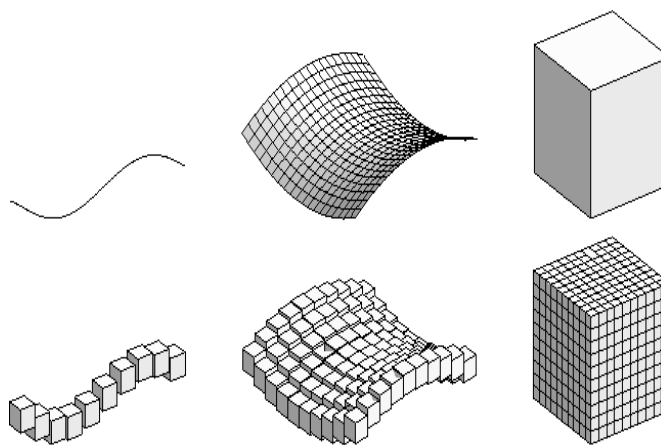


Figure 1.6 Covering a curve, a surface, and a solid cube with cubes of diameter r (Santiago, 2002).

The box-counting dimension is defined as

$$N_r(r) = \frac{1}{r^D}$$

For any $r > 0$, if there is a number D such that

$$N_r(r) \propto \frac{1}{r^D} \text{ as } r \rightarrow 0.$$

by power law (Santiago, 2002)

$$N_r(r) = k \left(\frac{1}{r^D} \right) \quad (1.4)$$

$$\lim_{r \rightarrow 0} \frac{N_r(F)}{\frac{1}{r^D}} = k \quad (1.5)$$

$$\lim_{r \rightarrow 0} (\log N_r(r) + D \log r) = k \quad (1.6)$$

Then

$$D = \lim_{r \rightarrow 0} \frac{\log k - \log N_r(r)}{\log r} \quad (1.7)$$

$$= \lim_{r \rightarrow 0} \frac{\log N_r(r)}{\log(1/r)} \quad (1.8)$$

So

$$D_B = \lim_{r \rightarrow 0} \frac{\log N(r)}{\log \frac{1}{r}} \quad (1.9)$$

where N is the number of the boxes that cover the fractal curve.
 r is the size of the boxes.
 D_B is the fractal dimension by box-counting method.
 k is constant.

The fractal dimension is given by the slope of $\log N(r)$ versus $\log(1/r)$.

1.4.3.2 ε – Blanket Method

In ε –Blanket method (Novianto et al., 2003), all points in dimensional space at distance ε from the surface are considered. Covering the surface with a blanket of thickness 2ε . (Figure 1.7), the estimated surface area is the volume of the blanket divided by 2ε . For different ε , the blanket area can be iteratively estimated as follows. Cover an image surface $g(i, j)$ by using a blanket with top u_ε and bottom b_ε surfaces. Initially, given the gray level function $g(i, j) = u_0(i, j) = b_0(i, j)$.

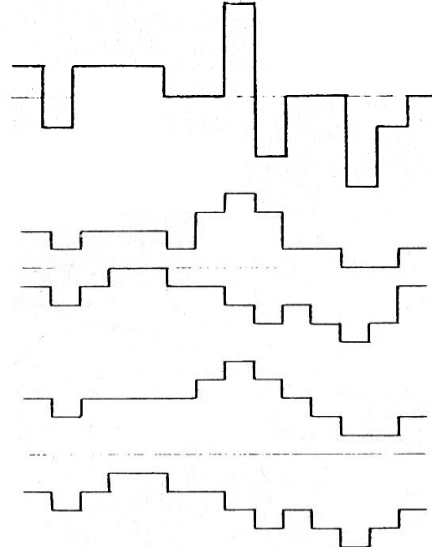


Figure 1.7 Area covers a surface of the substrate thickness of two pieces (Peleg et al., 1984).

For $\varepsilon = 1, 2, 3, \dots$, the blanket surfaces are defined as follows

$$u_{\varepsilon}(i, j) = \max \left\{ u_{\varepsilon-1}(i, j) + 1, \max_{|(m,n)-(i,j)| \leq 1} u_{\varepsilon-1}(m, n) \right\} \quad (1.10)$$

$$b_{\varepsilon}(i, j) = \max \left\{ b_{\varepsilon-1}(i, j) - 1, \max_{|(m,n)-(i,j)| \leq 1} b_{\varepsilon-1}(m, n) \right\} \quad (1.11)$$

where $g(i, j) = u_0(i, j) = b_0(i, j)$.

If ε is the number of blanket, the area of the blanket is $A(\varepsilon)$ computed by

$$A(\varepsilon) = \frac{\sum_{i,j} (u_{\varepsilon}(i, j) - b_{\varepsilon}(i, j))}{2\varepsilon} \quad (1.12)$$

The area of fractal surface behaves according to the expression

$$A(\varepsilon) = F\varepsilon^{2-D} \quad (1.13)$$

where F is a constant and D is the fractal dimension of the surface.

The fractal dimension value may be estimated from the linear fit of $\log A(\varepsilon)$ against $\log \varepsilon$ with the blanket's scale range from 1 to ε , and the slope should be equal to $2-D$ (Novianto, 2003).

1.4.3.3 Bouligand-Minkowski Method

Bouligand-Minkowski method (Backes, 2008) is applied by constructing circles with radius r centered at each point of the fractal. The radius varies at each step of the

method and the set of points pertaining to the union of the circles of radius r composes the dilation area A .

Consider $A \in \mathbb{R}^2$ the shape under analysis, the dilation of A , $A(r)$ is defined as the set of points in \mathbb{R}^2 such that distance from A is smaller than or equal to r

$$A(r) = \{x \in \mathbb{R}^2 \mid \exists y \in A : |x - y| \leq r\} \quad (1.14)$$

This dilation can also be defined as

$$A(r) = \bigcup_{x \in A} B_r(x) \quad (1.15)$$

where $B_r(x)$ is a disc of radius r . The influence area, $A(r)$ and the radius, r , follows the relation

$$A(r) = \mu r^{2-D} \quad (1.16)$$

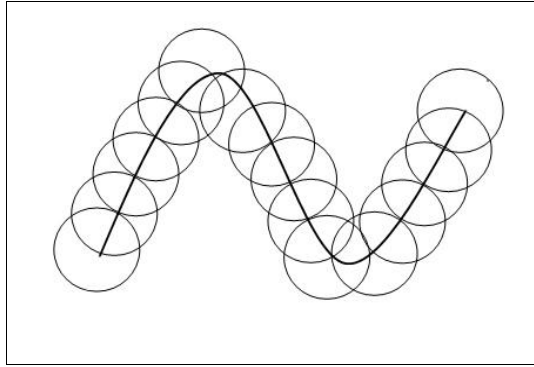


Figure 1.8 Dilation of a shape with a disc of radius r (Backes, 2008).

Figure 1.8 shows dilation shape using a disc of radius r , small modifications on the shape produce modifications in the computed influence area. So, the fractal dimension can be estimated as

$$D = 2 - \lim_{r \rightarrow 0} \frac{\log A(r)}{\log r} \quad (1.17)$$

Through line regression of log-log curve $A(r)$ it is possible to calculate a line with α slope, where $D = 2 - \alpha$ is the fractal dimension of the shape using the Bouligand-Minkowski method.

1.4.3.4 Compass Method

Compass method (Hotar, 2007) is a measurement of the dividing line (roughness profile) by different size of a ruler

$$L_i(r_i) = N_i(r_i)r_i \quad (1.18)$$

where L_i is a length in i -step of the measurement.

r_i is a ruler size.
 N_i is a number of steps needed.

For the measurement length that is given by a power law

$$N(r_i) = \text{const.} r_i^{-D_R} \quad (1.19)$$

if the fractal dimension is larger than the topological fractal dimension, the measured length increases as the ruler size is reduced using Equations (1.18) and (1.19)

$$L_i(r_i) = N_i(r_i)r_i = \text{const.} r_i^{-D_R} r_i = \text{const.} r_i^{-D_R} \quad (1.20)$$

where D_R is the compass dimension.

Logarithmic dependence between $\log_2 N(r_i)$ and $\log_2 r_i$ is called the Richardson-Mandelbrot plot. The compass dimension is then determined from slope s of the regression line

$$D_R = 1 - s = 1 - \frac{\Delta \log_2 L(r)}{\Delta \log_2 r} \quad (1.21)$$

Although the typical dependence consists of three-part slope, only central part (the central slope) is important for the compass dimension computing.

1.4.3.5 Area–Perimeter Method

The area-perimeter method (Sezer et al., 2008) is known as the easiest application of fractal geometry. In this process, Mandelbrot's proposal of "linear ratio of extents" can be used (Mandelbrot, 1990). The length formula of an irregular curve having fractal structure has the form

$$c = \frac{P^{1/D_R}}{A^{1/2}} \quad (1.22)$$

where c is a constant value for the fractal shape possessing similarity.
 P is the perimeter of the grain.
 A is the area of the grain.
 D_R is the average roughness fractal dimension of the grains.

Taking the logarithm of the two sides in Eq. (1.22), the following expression can be obtained

$$\frac{1}{D_R} \log P = \log(c\sqrt{A}) \quad (1.23)$$

Making the necessary arrangements

$$\log P = \frac{D_R}{2} (\log c^2 + \log A) \quad (1.24)$$

Finally, the following equation is obtained

$$\log P = k + \frac{D_R}{2} (\log A) \quad (1.25)$$

In Eq. (1.25), the slope of the best fit line to area-perimeter plot is equal to $\frac{2}{D_R}$.

So the fractal dimension is given by the slope of D_R .

1.5 Literature Review

Lovejoy (1982) studies the statistically self-similar cloud and the structure ranging from 1 to $1.2 \times 10^6 \text{ km}^2$ with a fractal dimension of about $4/3$ using the area perimeter relation. He analyzes data from radar images of tropical rain areas and from infrared (IR) satellite images of Indian Ocean clouds at the resolutions of 1 and 4.8 km.

Davies (1984) explains that clouds with complex geometry possess radiative properties which are quite different from those of clouds with simple geometry. They may affect the estimation of fractal dimensions of cloud in satellite images.

Rys and Waldvogel (1986) use the area-perimeter method with radar analysis of hail clouds and find that the value of the fractal dimension for mid latitude hail cloud changes abruptly at a perimeter of 0.5 km ($2 \sim 3 \text{ km}^2$ in area). For larger perimeters, $D_p^{(L)} = 1.36$, and for smaller ones, $D_p^{(S)} = 1.0$. They conclude that horizontal winds and severe convective storms contribute to the smoothing of the cloud surface, which is described by a lower fractal dimension.

Cahalan and Joseph (1989) use the area-perimeter method with measurement of the cloud from satellite. The dimension are 1.55 for clouds larger than 0.25 km^2 and 1.33 for smaller fair weather clouds above coastal waters.

Beyer et al. (1994) use the box counting method with ground observation and find that 1.37 and 1.88 are the fractal dimensions of the upper and lower sections, respectively, for cumulus cloud cover.

Gotoh and Fuji (1998) present a study of cloud shape from the area-perimeter relation for cumulus surface structure to differentiate the size of the cumulus cloud. Results from calculated fractal dimension of the cloud by the area-perimeter method are fractal dimension for large clouds is 1.677 and for small clouds is 1.364.

National Research Ethics Service (2008) explains fractal as objects characterized by fractal geometry, shapes remain unchanged as the scale of observation is progressively refined. "The structure of every piece holds the key to the whole structure" and increasingly accurate measurements no longer converge to the true extent of the object, since the object (e.g. a curve) does not converge to a single rectifiable length or area. Instead, the process of repeatedly zooming in to observed increasing detail will generate infinite series with common dimension D . Fractal patterns are generated by iterative processes. Simple rules for pattern generation can generate rich, complex structures. Since the same pattern generation rule is applied at all scales, pattern at different scales is self-similar. Self-similarity may also be referred to as scale invariance. Self-similarity

means that spatial objects or time series phenomena reveal an underlying simple form that repeats itself at different scales of observation. The level of variation (i.e. shape) at all scales can be described by a single parameter (fractal dimension).

Madhushani and Sonnadara (2012) use the area-perimeter relationship and the box-counting method to calculate fractal dimensions of cloud shapes from satellite images and present relationship between temperature and fractal dimensions of high level clouds. Results show that fractal dimension decreased linearly with decreasing temperature by 0.04 for every 10 celsius change. The average fractal dimension of clouds 7 km above ground level is 1.5 ± 0.1 .

The review mentioned above can be summarized in Table 1.2.

Table 1.2 Fractal dimension of cloud.

Method	Gotoh and Fuji (1998)	Cahalan and Joseph (1989)	Beyer et al. (1994)	Rys and Waldvogel (1986)
	Area-perimeter by satellite measurement	Area-perimeter by satellite measurement	Box counting by ground observation	Area-perimeter by radar analysis of hail clouds
Fractal dimension for large clouds	1.677	1.550	1.876	1.360
Fractal dimension for small clouds	1.364	1.330	1.310	1.000

The fractal dimension calculated by the 5 methods can be compared as shown in Table 1.3.

Table 1.3 Comparison of the fractal dimension methods.

Methods	Computational speed	Error	Concept
1. Box-counting	Very fast	Largest	Count the number of boxes that cover the area.
2. ε -blanket	Fast	Small	Measuring surface area.
3. Bouligand-Minkowski	Fast	Medium	Sphere with radius r centered at each point.
4. Compass	Slow	Medium	Measurement of the dividing line.
5. Area-perimeter	Very fast	Small	Increase the size of the scale for the distance between grid points.

From the 5 methods mentioned in Section 1.3, fractal dimension can be calculated in a variety of ways. However the area-perimeter is fast and easy to apply with small error. Thus, in this research the area-perimeter method is used to identify fractal dimension of severe thunderstorm cloud over Thailand.

1.6 Objectives

To determine fractal dimension of severe thunderstorm clouds over Thailand in satellite images using a modified area-perimeter method.