CHAPTER 2 THEORIES

2.1 The Weather Research and Forecasting (WRF) Model

The Weather Research and Forecasting model (WRF) is a numerical weather prediction and atmospheric simulation system model designed by the National Center for Atmospheric Research (NCAR) (Skamarock *et al.* 2008). This model is capable of creating simulation using real time data, or idealized atmospheric simulations. The WRF-ARW contains with vertical coordinate, horizontal grid, initial conditions, mapping to sphere, full set of physical parameterization options, and etc. The WRF-ARW can applications across scales ranging from meters to thousands of kilometers include: idealized simulations, parameterization research, and data assimilation research, forecast research, real-time, regional climate research, coupled-model application, and teaching. The principal components of the WRF system are depicted in Figure 1.1. The WRF Software Framework (WSF) provides the infrastructure that accommodates the dynamics solvers, physics packages that interface with the solvers, programs for initialization, the Advanced Research WRF (ARW) solver developed primarily at NCAR, and the Nonhydrostatic Mesoscale Model (NMM) solver developed at National Centers for Environmental Prediction (NCEP). Community support for the former is provided by the Mesoscale and Microscale Meteorology (MMM) Division of NCAR (Skamarock *et al.* 2008).

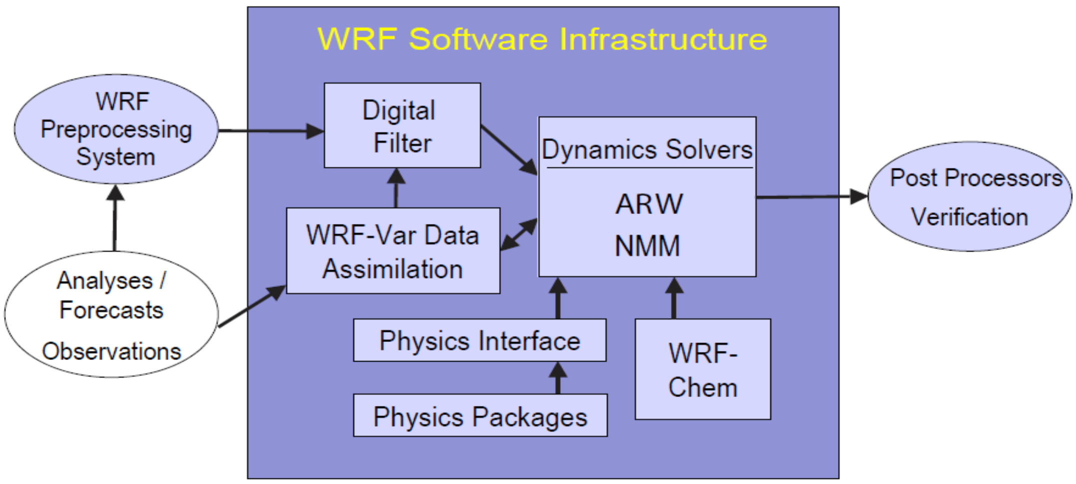

Figure 2.1 WRF system components (Skamarock *et.al*, 2008)

2.2 The WRF Modeling System Program Component

The framework of the WRF-ARW model follows that of the WRF flowchart in Figure 2.2. The first, the WRF Preprocessing System (WPS): used preparing about observation data and primarily for real-data simulation, such as defining simulation research domain (Map projections), to interpolate terrestrial from USGS 24 category and MODIS 20 category land dataset to simulation research domain and the result from interpolate terrestrial is terrain height, land-use, and soil types, to degribbing and interpolate meteorological data such as the Global Forecast System (GFS), and the NCEP Final Operational Global Analysis (FNL) to simulation research domain. The final analysis is processed into gridded simulate by the WRF-ARW dynamics solver. The dynamic solver uses third order Runge-Kutta schemes to solve time integration of simulation. Also during this stage, the full physics parameterization schemes such as atmospheric, surface radiation, cumulus convection and planetary boundary later are applied to the analysis between time integration. After the WRF-ARW dynamics solver has achieved simulation, the data can then be output to meteorological packages such as the Grid Analysis and Display System (GRADS), NCAR Command Language (NCL) and the Read/Interpolate/Plot (RIP4) for create graphic, data analysis and data assimilation.

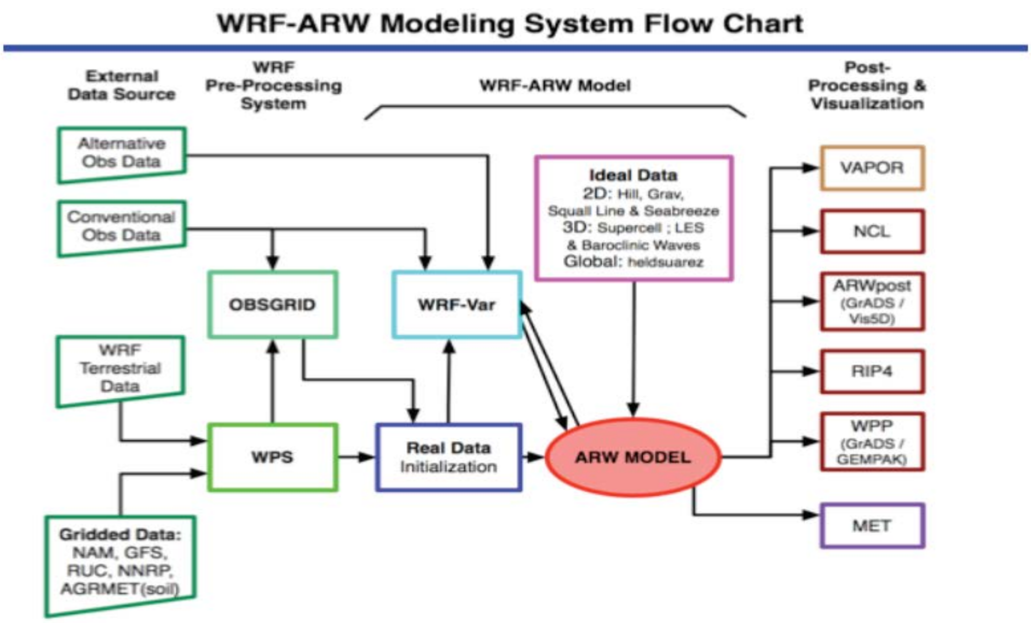

Figure 2.2 WRF flow chart (Skamarock *et.al*, 2011)

2.3 Governing Equation of the WRF Model

2.3.1 Vertical Coordinate and Variables of the WRF Model

The WRF equations are formulated using a terrain-following hydrostatic-pressure vertical coordinate denote by η and defined by

$$\eta = \frac{(p_h - p_{ht})}{\mu}, \tag{2.1}$$

while $\mu = p_{hs} - p_{ht}$ the mass per unit area with the column in the model domain

where p_h is the hydrostatic pressure,

p_{ht} is the hydrostatic pressure at the top of the model,

p_{hs} is the hydrostatic pressure at the model surface.

The coordinate definition (2.1) is the traditional σ coordinate, η varies from a value of 1 at the surface to 0 at the top boundary of the model domain (Figure 2.3).

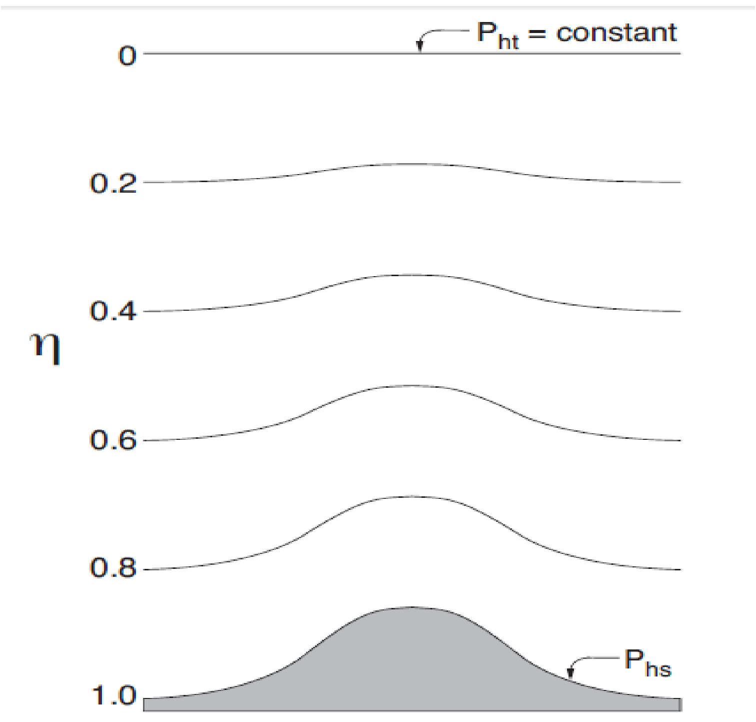

Figure 2.3 η coordinate of WRF model (Skamarock et.al, 2008)

2.3.2 WRF Flux-Form Euler Equations

The flux-form Euler equations can be written as

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial \eta}-\frac{\partial p\phi_\eta}{\partial x}+\frac{\partial p\phi_x}{\partial \eta}=F_u, \tag{2.2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial \eta}-\frac{\partial p\phi_\eta}{\partial y}+\frac{\partial p\phi_y}{\partial \eta}=F_v, \tag{2.3}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial \eta}-g\left(\frac{\partial p}{\partial \eta}-\mu\right)=F_w, \tag{2.4}$$

$$\frac{\partial \Theta}{\partial t}+\left(u\frac{\partial \Theta}{\partial x}+v\frac{\partial \Theta}{\partial y}+w\frac{\partial \Theta}{\partial \eta}\right)=F_\Theta, \tag{2.5}$$

$$\frac{\partial \mu}{\partial t}+\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial \eta}\right)=0, \tag{2.6}$$

$$\frac{\partial \phi}{\partial t}+\frac{1}{\mu}\left(u\frac{\partial \phi}{\partial x}+v\frac{\partial \phi}{\partial y}+w\frac{\partial \phi}{\partial \eta}-gw\right)=0, \tag{2.7}$$

and the equation of state

$$p=p_0\left(\frac{R_d\theta}{p_0\alpha}\right)^\gamma, \tag{2.8}$$

where

u	is the horizontal velocity in x direction,
v	is the horizontal velocity in y direction,
w	is the vertical velocity in η direction,
$x, y,$ and η	are the east, north, and upward direction, respectively,
t	is time,
μ	is hydrostatic pressure difference between surface and top of the model,
Θ	is the potential temperature,

ϕ	is the geopotential,
p	is the pressure,
ρ	is the density,
α	is the inverse density,
$\gamma = \frac{c_p}{c_v} = 1.4$	is the ratio of the heat capacities for dry air,
$R_d = \left(\frac{2}{7}\right) c_p$	is the gas constant for dry air,
p_0	is a reference sea-level pressure value,
F_u	is forcing terms for u,
F_v	is forcing terms for v,
F_w	is forcing terms for w,
F_Θ	is forcing terms for Θ.

2.3.3 WRF Inclusion of Moisture Equation

When including moisture, the vertical coordinate from (2.1) can defined by

$$\eta = \frac{(p_{dh} - p_{dht})}{\mu_d}, \tag{2.9}$$

while μ_d the mass of the dry air in the column,

where p_{dh} is the hydrostatic pressure,

p_{dht} is the hydrostatic pressure at the top of the model.

The moist Euler equations can be written as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial \eta} + \mu_d \alpha \frac{\partial p}{\partial x} + \left(\frac{\alpha}{\alpha_d}\right)\frac{\partial p}{\partial \eta}\frac{\partial \phi}{\partial x} = F_u, \tag{2.10}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial \eta} + \mu_d \alpha \frac{\partial p}{\partial y} + \left(\frac{\alpha}{\alpha_d}\right)\frac{\partial p}{\partial \eta}\frac{\partial \phi}{\partial y} = F_v, \tag{2.11}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial \eta} - g\left(\left(\frac{\alpha}{\alpha_d}\right)\frac{\partial p}{\partial \eta} - \mu_d\right) = F_w, \tag{2.12}$$

$$\frac{\partial \Theta}{\partial t} + \left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial \eta} \right) = F_{\Theta}, \tag{2.13}$$

$$\frac{\partial \mu_d}{\partial t} + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial \eta} \right) = 0, \tag{2.14}$$

$$\frac{\partial \phi}{\partial t} + \frac{1}{\mu_d}\left(u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y} + w\frac{\partial \phi}{\partial \eta} - gw \right) = 0, \tag{2.15}$$

$$\frac{\partial q_m}{\partial t} + \left(u\frac{\partial q_m}{\partial x} + v\frac{\partial q_m}{\partial y} + w\frac{\partial q_m}{\partial \eta} \right) = F_{q_m}, \tag{2.16}$$

and the equation of state

$$p = p_0\left(\frac{R_d \theta_m}{p_0 \alpha_d} \right)^{\gamma}, \tag{2.17}$$

where

u	is the horizontal velocity in x direction,
v	is the horizontal velocity in y direction,
w	is the vertical velocity in η direction,
$x, y,$ and η	are the east, north, and upward direction, respectively,
t	is time,
μ_d	is the dry hydrostatic pressure difference between surface and top of the model,
Θ	is the potential temperature,
ϕ	is the geopotential,
p	is the pressure,
ρ	is the density,
α	is the inverse density,
$\gamma = \frac{c_p}{c_v} = 1.4$	is the ratio of the heat capacities for dry air,

$R_d = \left(\frac{2}{7}\right) c_p$	is the gas constant for dry air,
q_m	is gain and loss of water through phase changes,
p_0	is a reference sea-level pressure,
F_u	is forcing terms for u,
F_v	is forcing terms for v,
F_w	is forcing terms for w,
F_Θ	is forcing terms for Θ.

2.3.4 WRF Map Projections Equation

In the WRF's computational space, Δx and Δy are constants. Orthogonal projections to the sphere require that the physical distances between grid points in the projection vary with position on the grid. To transform the governing equations, map scale factors m_x and m_y are defined as the ratio of the distance in computational space to the corresponding distance on the earth's surface:

$$(m_x, m_y) = \frac{(\Delta x, \Delta y)}{\text{distance on the earth}}, \tag{2.18}$$

Using these redefined momentum variables, the governing equations, including map factors and rotational terms, can be written as

$$\frac{\partial u}{\partial t} + m_x\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] + w\frac{\partial u}{\partial \eta} + \left(\frac{m_x}{m_y}\right)\left[\mu_d \alpha \frac{\partial p}{\partial x} + \left(\frac{\alpha}{\alpha_d}\right)\frac{\partial p}{\partial \eta}\frac{\partial \phi}{\partial x}\right] = F_u, \tag{2.19}$$

$$\frac{\partial v}{\partial t} + m_y\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] + \left(\frac{m_y}{m_x}\right) w\frac{\partial v}{\partial \eta} + \left(\frac{m_y}{m_x}\right)\left[\mu_d \alpha \frac{\partial p}{\partial y} + \left(\frac{\alpha}{\alpha_d}\right)\frac{\partial p}{\partial \eta}\frac{\partial \phi}{\partial y}\right] = F_v, \tag{2.20}$$

$$\frac{\partial w}{\partial t}+\left(\frac{m_x m_y}{m_y}\right)\left[u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}\right]+w\frac{\partial w}{\partial \eta}-\frac{1}{m_y}g\left(\left(\frac{\alpha}{\alpha_d}\right)\frac{\partial p}{\partial \eta}-\mu_d\right)=F_w, \tag{2.21}$$

$$\frac{\partial \Theta}{\partial t}+m_x m_y\left[u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}\right]+m_y\left(w\frac{\partial \theta}{\partial \eta}\right)=F_\Theta, \tag{2.22}$$

$$\frac{\partial \mu_d}{\partial t}+m_x m_y\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right]+m_y\frac{\partial w}{\partial \eta}=0, \tag{2.23}$$

$$\frac{\partial \phi}{\partial t}+\frac{1}{\mu_d}\left[m_x m_y\left(u\frac{\partial \phi}{\partial x}+v\frac{\partial \phi}{\partial y}\right)+m_y w\frac{\partial \phi}{\partial \eta}-m_y g w\right]=0, \tag{2.24}$$

$$\frac{\partial q_m}{\partial t}+m_x m_y\left[u\frac{\partial q_m}{\partial x}+v\frac{\partial q_m}{\partial y}\right]+m_y w\frac{\partial q_m}{\partial \eta}=F_{q_m}, \tag{2.25}$$

and the equation of state

$$p=p_0\left(\frac{R_d\theta_m}{p_0\alpha_d}\right)^\gamma, \tag{2.26}$$

where

u	is the horizontal velocity in x direction,
v	is the horizontal velocity in y direction,
w	is the vertical velocity in η direction,
$x, y,$ and η	are the east, north, and upward direction, respectively,
t	is time,
μ_d	is the dry hydrostatic pressure difference between surface and top of the model,
m_x and m_y	are defined as the ratio of the distance in computational space to the corresponding distance on the earth's surface,
Θ	is the potential temperature,
ϕ	is the geopotential,
p	is the pressure,
ρ	is the density,
α	is the inverse density,

$\gamma = \dfrac{c_p}{c_v} = 1.4$	is the ratio of the heat capacities for dry air,
$R_d = \left(\dfrac{2}{7}\right) c_p$	is the gas constant for dry air,
q_m	is gain and loss of water through phase changes,
p_0	is a reference sea-level pressure,
F_u	is forcing terms for u,
F_v	is forcing terms for v,
F_w	is forcing terms for w,
F_Θ	is forcing terms for Θ.

2.3.5 WRF Perturbation Form of the Governing Equation

The hydrostatically balanced portion of the pressure gradients in the reference sounding can be removed without approximation to the equations using these perturbation variables.

The momentum equations (2.19) – (2.21) are written as

$$\frac{\partial u}{\partial t} + m_x\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] + w\frac{\partial u}{\partial \eta} + \left(\frac{m_x}{m_y}\right)\left(\frac{\alpha}{\alpha_d}\right)\bullet\left[\mu_d\left(\frac{\partial \phi'}{\partial x} + \alpha_d\frac{\partial p'}{\partial x} + \alpha_d'\frac{\partial \overline{p}}{\partial x}\right) + \frac{\partial \phi}{\partial x}\left(\frac{\partial p'}{\partial \eta} - \mu_d'\right)\right] = F_u, \tag{2.27}$$

$$\frac{\partial v}{\partial t} + m_y\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] + \left(\frac{m_y}{m_x}\right)w\frac{\partial v}{\partial \eta} + \left(\frac{m_y}{m_x}\right)\left(\frac{\alpha}{\alpha_d}\right)\bullet\left[\mu_d\left(\frac{\partial \phi'}{\partial y} + \alpha_d\frac{\partial p'}{\partial y} + \alpha_d'\frac{\partial \overline{p}}{\partial y}\right) + \frac{\partial \phi}{\partial y}\left(\frac{\partial p'}{\partial \eta} - \mu_d'\right)\right] = F_v, \tag{2.28}$$

$$\frac{\partial w}{\partial t} + \left(\frac{m_x m_y}{m_y}\right)\left[u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y}\right] + w\frac{\partial w}{\partial \eta} - \frac{1}{m_y}g\left(\frac{\alpha}{\alpha_d}\right)\left(\frac{\partial p'}{\partial \eta} - \overline{\mu}_d(q_v + q_c + q_r)\right) + \frac{1}{m_y}\mu_d' g = F_w, \tag{2.29}$$

and the mass conservation equation (2.23) and geopotential equation (2.24) become

$$\frac{\partial \mu_d'}{\partial t} + m_x m_y \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right] + m_y \frac{\partial w}{\partial \eta} = 0, \tag{2.30}$$

$$\frac{\partial \phi'}{\partial t} + \frac{1}{\mu_d} \left[m_x m_y \left(u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} \right) + m_y w \frac{\partial \phi}{\partial \eta} - m_y g w \right] = 0, \tag{2.31}$$

Remaining unchanged are the conservation equations for the potential temperature and scalars

$$\frac{\partial \Theta}{\partial t} + m_x m_y \left[u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} \right] + m_y \left(w \frac{\partial \theta}{\partial \eta} \right) = F_\Theta , \tag{2.32}$$

$$\frac{\partial Q_m}{\partial t} + m_x m_y \left[u \frac{\partial q_m}{\partial x} + v \frac{\partial q_m}{\partial y} \right] + m_y w \frac{\partial q_m}{\partial \eta} = F_{Q_m} , \tag{2.33}$$

where

u	is the horizontal velocity in x direction,
v	is the horizontal velocity in y direction,
w	is the vertical velocity in η direction,
$x, y,$ and η	are the east, north, and upward direction, respectively,
t	is time,
μ_d	is the dry hydrostatic pressure difference between surface and top of the model,
m_x and m_y	are defined as the ratio of the distance in computational space to the corresponding distance on the earth's surface,
Θ	is the potential temperature,
ϕ	is the geopotential,
p	is the pressure,
ρ	is the density,
α	is the inverse density,
$\gamma = \frac{c_p}{c_v} = 1.4$	is the ratio of the heat capacities for dry air,

$R_d = \left(\frac{2}{7}\right) c_p$ is the gas constant for dry air,

q_m is gain and loss of water through phase changes,

q_v is the mixing ratios for water vapor,

q_c is the mixing ratios for cloud,

q_r is the mixing ratios for rain,

p_0 is a reference sea-level pressure,

F_u is forcing terms for u,

F_v is forcing terms for v,

F_w is forcing terms for w,

F_Θ is forcing terms for Θ.

2.3.6 Model Discretization

The Runge-Kutta 3 order scheme, integrates a set of ordinary differential equations using a predictor-corrector formulation. Defining the prognostic variables in the WRF solver as $\Phi = (U, V, W, \Theta, \phi', \mu', q_m)$ and the model equations as $\Phi_t = R(\Phi)$, the Runge-Kutta 3 order integration takes the form of 3 order to advance a solution $\Phi(t)$ to $\Phi(t + \Delta t)$:

$$\Phi^* = \Phi^t + \frac{\Delta t}{3} R\left(\Phi^t\right), \tag{2.34}$$

$$\Phi^{**} = \Phi^t + \frac{\Delta t}{2} R\left(\Phi^*\right), \tag{2.35}$$

$$\Phi^{t+\Delta t} = \Phi^t + \Delta t R\left(\Phi^{**}\right), \tag{2.36}$$

where Δt is the time step for the low-frequency modes (the model time step). In equations (2.34) – (2.36), superscripts denote time levels. This scheme is not a true Runge-Kutta scheme per se because, while it is third-order accurate for linear equations, it is only second-order accurate for nonlinear equations. With respect to the WRF equations, the time derivatives Φ_t are the partial time derivatives (the leftmost terms) in equations (2.34) – (2.36), and $R(\Phi)$ are the remaining terms in equations(2.34) – (2.36).

2.4 The Climate of Thailand

Thailand is located in the tropical area between latitudes $5^\circ\ 37'\ N$ to $20^\circ\ 27'\ N$ longitudes $97^\circ\ 22'\ E$ to $105^\circ\ 37'\ E$. The total area is 513,115 square kilometers or around 200,000 square miles, as show in Figure 2.4. According to the climate pattern and meteorological conditions Thailand may be divided into 5 parts i.e. Northern, Northeastern, Central, Eastern and Southern Parts. The boundaries of Thailand with near areas are Myanmar and Laos in Northern part of Thailand. The Eastern part of Thailand is near Laos, Cambodia and the Gulf of Thailand. The Southern part of Thailand is near Malaysia and the western part of Thailand is near Myanmar and the Andaman Sea. The climate of Thailand is under the influence of monsoon winds of seasonal character i.e. southwest monsoon and northeast monsoon. The southwest monsoon which starts in May brings a stream of warm moist air from the Indian Ocean towards Thailand causing abundant rain over the country, especially the windward side of the mountains. The northeast monsoon which starts in October brings the cold and dry air from the anticyclone in china mainland over major parts of Thailand, especially the Northern and Northeastern Parts which is higher latitude areas. In the southern Part, this monsoon causes mild weather and abundant rain along the eastern coast of the part (Thai Meteorological Department, 2013).

2.5 The Season of Thailand

The meteorological point of view the climate of Thailand may be divided into three seasons. The first season is summer or pre-monsoon season to begin middle of February to middle of May. This is the transitional period from the northeast monsoon to southwest monsoons. The weather becomes warmer, especially in upper Thailand and April is the hottest month. The second season is rainy or southwest monsoon season to begin middle of May to middle of October. The southwest monsoon prevails over Thailand and abundant rain occurs over the country. The wettest period of the year is during August to September. The exception is found in the Southern Thailand East Coast where abundant rain remains until the end of the year that is the beginning period of the northeast monsoon and November is the wettest month. The third season is winter or northeast monsoon season to begin middle of October to middle of February. This mild period of the year with quite cold in December and January in upper Thailand but there is a great amount of rainfall in Southern Thailand East Coast especially during October to November (Thai Meteorological Department, 2013).

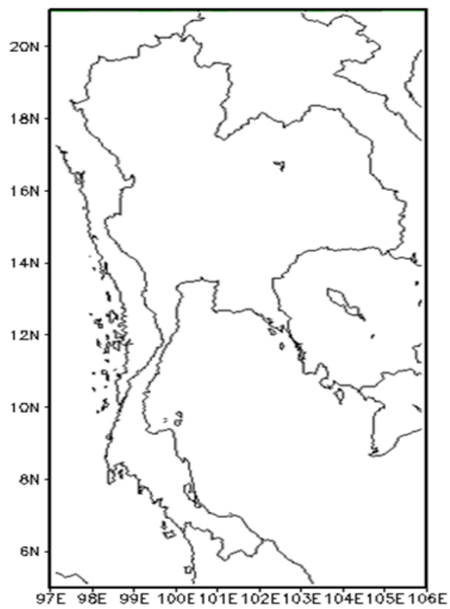

Figure 2.4 Thailand is located in the tropical area between latitudes $5^\circ\ 37' N$ to $20^\circ\ 27' N$ longitudes $97^\circ\ 22' E$ to $105^\circ\ 37' E$

2.6 Physical Parameterizations of WRF Model

Physical process in the WRF Model shown in the Figure 2.5 are microphysics (resolves water vapor, cloud, ice), cumulus parameterization (resolve cloud process), surface layer model (calculates friction velocities and exchange coefficients that enable the calculation of surface heat and moisture fluxes) and radiation (provide atmospheric heating due to radiative flux divergence and surface downward long wave and shortwave radiation for the ground heat budget) (Skamarock,2008).

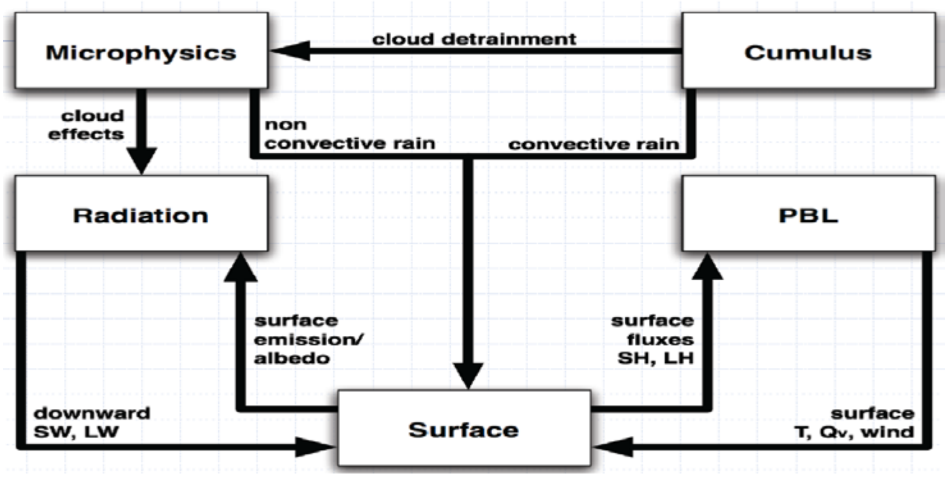

Figure.2.5 Physical process in the WRF Model (Skamarock *et.al*, 2008)

2.7 Microphysical Particles and Processes

From (Warner, 2011), to follow summary is provided for the reader who has not had the benefit of a course in cloud microphysics. The particle types and the microphysical processes that undergo are important in the context of the generation of precipitation in its various forms, and thus they should be parameterized in some way in atmospheric models. The particle types that are involved in microphysical processes are listed below.

Cloud droplets: These are liquid drops, with a typical radius of 10 μm, that form through the condensation of water vapor in the presence of a cloud-condensation nucleus (CCN, small particles that have an affinity for water).

Rain drops: Cloud droplets can grow to rain drops through the accretion mechanism described below, or rain drops can result from the melting of snow crystals. Rain drop radii range from 100 to 1000 μm.

Ice crystals: Water droplets freeze in the presence of an ice nucleus (IN, similar to a CCN) at temperatures below the normal freezing point. Larger droplets freeze at higher temperatures.

Aggregates of ice crystals, snow flakes: These are clusters of ice crystals formed when ice crystals with different terminal velocities collide and coalesce. Snow flakes are formed by this process.

Rimed ice particles*:* These form when ice crystals collide and coalesce with cloud droplets at temperatures below freezing. If the features of the ice crystal can be distinguished, it is called a rimed ice particle.

Graupel particle: If the crystal features of a rimed ice particle are not recognizable, it is called a graupel particle. Graupel also results from the instantaneous freezing of rain drops, when the sub-freezing drops collide with ice crystals.

Hail stones: As graupel particles fall through the cloud of sub-freezing liquid, they grow by riming. Hail stones result from cases of extreme riming.

Condensation: Liquid droplets form when water saturation is exceeded at temperatures from −40 oC to above freezing. The condensation takes place on CCN that are natural or anthropogenic, typically submicrometer-sized, particles.

Accretion: In the warm-cloud process, droplets with different masses have different terminal velocities, and the resulting collisions between droplets can result in coalescence and droplet growth. As a droplet grows, so does its vertical velocity relative to the smaller cloud droplets, thus increasing the rate of collisions.

Evaporation*:* Cloud droplets and rain drops evaporate.

Ice and snow aggregation*:* When ice crystals and snowflakes collide and coalesce, it is called aggregation.

Accretion by frozen particles*:* Snow, graupel, or hail collect other solid or liquid particles as they fall.

Vapor deposition: The saturation vapor pressure with respect to liquid water is higher than the saturation vapor pressure with respect to ice. Thus, if a cloud that contains both droplets and ice crystals is saturated with respect to water, it is supersaturated with respect to ice. As the ice crystals grow by vapor deposition, the air becomes subsaturated with respect to the liquid surface, and cloud droplets evaporate. This process is called the Bergeron–Findeisen mechanism.

Melting: As snowflakes fall into the lower troposphere, below the freezing level, they may melt and form rain drops. Similarly, hail and graupel begin to melt as they fall below the freezing level.

Freezing*:* Water droplets freeze in the presence of IN, riming involves the freezing of water droplets that collide with ice crystals, and rain drops can freeze to from graupel. Figure 2.6 illustrates the microphysical processes that must be represented in some form in a model in order to predict the types of precipitation shown at the bottom of the figure.

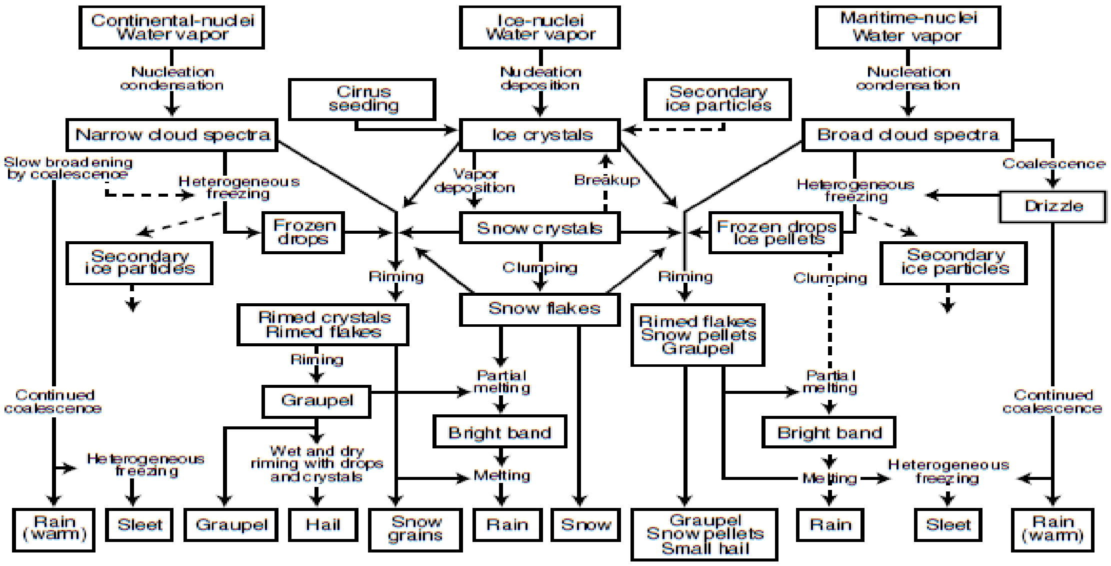

Figure 2.6 Illustrates the microphysical processes (Warner, 2011)

2.7.1 Microphysics Parameterizations

Microphysical parameterizations aim to represent, as thoroughly as possible, the processes described in the previous section. The parameterizations are divided into two categories, based on how the size distributions of particle types are represented. In *bin models*, the particle size spectrum is divided into intervals, and the particle concentrations are predicted for each interval, or bin. Changes for each bin can result from conversions between particle types, and from the increase or decrease of particle sizes. This requires a predictive equation for each particle type and size bin, which must be solved at each grid point. Thus, the use of bin models is very computationally intensive and is presently limited to research activities, and not operational weather and climate prediction. In contrast, *bulk microphysical parameterizations* assume a prescribed analytic form for the size spectrum of each particle type – e.g., exponential (Kessler 1969). To illustrate an example of how bulk microphysical parameterizations are represented in a single-moment model, the following are predictive equations for the specific humidity of five different forms of water: water vapor (q_v), cloud water (q_{cw}), cloud ice (q_{ci}), snow (q_s), and rain (q_r). Where there are vertical derivatives in the mass of a species, there will be a contribution to the tendency in proportion to the terminal velocity. The rest of the terms on the right (*F*) represent various sources and sinks associated with conversions from one type of particle to another (Warner, 2011).

$$\frac{\partial q_v}{\partial t} = -u_i \frac{\partial q_v}{\partial x_i} - \frac{1}{\rho_0}\frac{\partial}{\partial x_i}\rho_0 \overline{u_i' q_v'} - F_{deps} - F_{depci} - F_{evapr} - F_{vcondtocw}, \tag{2.37}$$

$$\frac{\partial q_{cw}}{\partial t} = -u_i \frac{\partial q_{cw}}{\partial x_i} - \frac{1}{\rho_0}\frac{\partial}{\partial x_i}\rho_0 \overline{u_i' q_{cw}'} + F_{vcondtocw} - F_{freezcw} - F_{cwtor} - F_{acccwbyr} - F_{acccwbys}, \tag{2.38}$$

$$\frac{\partial q_{ci}}{\partial t} = -u_i \frac{\partial q_{ci}}{\partial x_i} - \frac{1}{\rho_0}\frac{\partial}{\partial x_i}\rho_0 \overline{u_i' q_{ci}'} + F_{freezcw} + F_{depci} - F_{citos} - F_{acccibys}, \tag{2.39}$$

$$\frac{\partial q_s}{\partial t} = -u_i \frac{\partial q_s}{\partial x_i} - \frac{1}{\rho_0}\frac{\partial}{\partial x_i}\rho_0 \overline{u_i' q_s'} - V_{Ts}\frac{\partial q_s}{\partial z} + F_{citos} + F_{acccibys} + F_{acccwbys} + F_{deps} - F_{smelttor}, \tag{2.40}$$

$$\frac{\partial q_r}{\partial t} = -u_i \frac{\partial q_r}{\partial x_i} - \frac{1}{\rho_0}\frac{\partial}{\partial x_i}\rho_0 \overline{u_i' q_r'} - V_{Tr}\frac{\partial q_r}{\partial z} - F_{evapr} + F_{acccwbys} + F_{cwtor} + F_{smelttor}, \tag{2.41}$$

where

q_v	is the mixing ratios for water vapor,
q_{cw}	is the mixing ratios for cloud water,
q_{ci}	is the mixing ratios for cloud ice,
q_s	is the mixing ratios for snow,
q_r	is the mixing ratios for rain,
F_{evapr}	is evaporation of rain drops,
$F_{acccwbyr}$	is accretion of cloud-water droplets by rain drops,
F_{cwtor}	is growth of cloud-water droplets to rain drops by cold-cloud (Bergeron–Findeisen) process,
F_{cwtor}	is melting of snow to produce rain drops,
F_{citos}	is growth of cloud ice to snow,
$F_{acccibys}$	is accretion of cloud ice by snow,

$F_{acccwbys}$	is accretion of cloud water by snow,
F_{deps}	is growth of snow by vapor deposition,
$F_{freezcw}$	is freezing of cloud water to produce cloud ice,
F_{depci}	is growth of cloud ice by vapor deposition,
$F_{vcondtocw}$	is condensation of vapor to form cloud-water droplets.

It is within these "*F*" terms that the parameterizations of microphysical processes are represented.

2.7.2 Microphysics Parameterizations of WRF Model

Microphysics includes explicitly resolved water vapor, cloud, and precipitation processes. The model is general enough to accommodate any number of mass mixing-ratio variables, and other quantities such as number concentrations. Table 2.1 shows a summary of the options indicating the number of moisture variables, and whether ice-phase and mixed-phase processes are included. Mixed-phase processes are those that result from the interaction of ice and water particles, such as riming that produces graupel or hail. As a general rule, for grid sizes less than 10 km, where updrafts may be resolved, mixed-phase schemes should be used, particularly in convective or icing situations. For coarser grids the added expense of these schemes is not worth it because riming is not likely to be well resolved.

Table 2.1 Microphysics Option

mp_physics	Scheme	Cores	Mass Variable	Number Variable
1	Kessler	ARW	q_c q_r	-
2	Purdue Lin	ARW(Chem)	q_c q_r q_i q_s q_g	-
3	WSM3	ARW	q_c q_r	-
4	WSM5	ARW/NMM	q_c q_r q_i q_s	-
5	Eta	ARW/NMM	q_c q_r q_i q_s	-
6	WSM6	ARW/NMM	q_c q_r q_i q_s q_g	-
8	Thompson	ARW/NMM	q_c q_r q_i q_s q_g	N_i N_r
9	Milbrandt	ARW	q_c q_r q_i q_s q_g q_h	N_c N_r N_i N_s N_g N_h

Table 2.1 Microphysics Option (Cont.)

mp_physics	Scheme	Cores	Mass Variable	Number Variable
10	Morrison	ARW(Chem)	$q_c\ q_r\ q_i\ q_s\ q_g$	$N_r\ N_i\ N_s\ N_g$
13	SBU-YLin	ARW	$q_c\ q_r\ q_i\ q_s$	-
14	WDM5	ARW	$q_c\ q_r\ q_i\ q_s$	$Nn(CCNnumber)\ N_c\ N_r$
16	WDM6	ARW	$q_c\ q_r\ q_i\ q_s\ q_g$	$Nn(CCNnumber)\ N_c\ N_r$
17	NSSL	ARW	$q_c\ q_r\ q_i\ q_s\ q_g\ q_h$	$N_c\ N_r\ N_i\ N_s\ N_g\ N_h$
18	NSSL+CCN	ARW	$q_c\ q_r\ q_i\ q_s\ q_g\ q_h$	$N_c\ N_r\ N_i\ N_s\ N_g\ N_h$

2.8 Statistical Comparison Methods

Once the WRF-ARW model grid point values were interpolated to each surface observation location in the horizontal and vertical direction, the data were compared by simulation accuracy. The accuracy of each simulation was calculated using Correlation, Mean Error, and Mean Absolute. This technique can be done for any scalar quantity. The Correlation is defined by the following equation:

$$r = \frac{n\sum_{i=1}^{n} x_i \cdot y_i - \left(\sum_{i=1}^{n} x_i\right)\cdot\left(\sum_{i=1}^{n} x_i\right)}{\sqrt{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}\cdot\sqrt{n\sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i\right)^2}},$$

where

r is Correlation Coefficient,

x is defined as the simulation value,

y is the observation value,

n is defined as the number of i pairs of observations and simulation values.

The value of r is such that $-1 \le r \le +1$. The $+$ and $-$ signs are used for positive linear correlations and negative linear correlations, respectively. Positive correlation: If x and y have a strong positive linear correlation, r is close to $+1$. A r value of exactly $+1$ indicates a perfect positive fit. Positive values indicate a relationship between x and y variables such that as values for x increases, values for y also increase, in the same direction. Negative correlation: If x and y have a strong negative linear correlation, r is close to -1. An r value of exactly -1 indicates a perfect negative fit. Negative values indicate a relationship between x and y such that as values for x increase, values y decrease, in the opposite direction (MathBits.com, 2013).

The mean error is defined by the following equation:

$$ME = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - y_i\right),$$

where

ME is mean error,

x is defined as the simulation value,

y is the observation value,

n is defined as the number of i pairs of observations and simulation values.

Mean error allows both positive and negative errors to be used in the average. As a result, mean error is also known as bias. if mean error = 0, there is no bias, if mean error > 0, simulation, on average, are too high and if mean error < 0, simulation, on average, are too low (Wilks, D.S., 2006)

The mean absolute error is defined by the following equation:

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|x_i - y_i\right|,$$

where

MAE is mean absolute error,

x is defined as the simulation value,

y is the observation value,

n is defined as the number of i pairs of observations and simulation values.

Each forecast-observation pair gives an error value. This measure sums the absolute values of these errors and divides by the number of forecasts to give an average error. MAE = 0 for a perfect forecast (Wilks, D.S., 2006).