Sunanta Srisopha 2012: The $\mathrm{Q}_\alpha-$ Convolution of Arithmetic Functions.
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Let $\mathcal{A}$ denote the set of arithmetic functions. Let $\alpha \in \mathcal{A}$ be such that $\alpha(n) \neq 0$ for all $n \in \mathbb{N}$, for $f, g \in \mathcal{A}$, we define the $\mathrm{Q}_\alpha-$ convolution as

$$(f \diamond g)(n) = \sum_{xy=n} \frac{\alpha(n)}{\alpha(x)\alpha(y)} f(x)g(y).$$

In this thesis, we establish some properties of the $\mathrm{Q}_\alpha-$ convolution $\diamond$, connections between the Dirichlet convolution $*$ and $\mathrm{Q}_\alpha-$ convolution $\diamond$, characterizations of completely multiplicative functions under the $\mathrm{Q}_\alpha-$ convolution and the algebraic independence of arithmetic funtions under the $\mathrm{Q}_\alpha-$ convolution.

Let g^{*k} denote the convolution power $g * \cdots * g$ with k factor $g \in \mathcal{A}$. Consider the polynomial convolution equation of the form

$$Tg = a_d * g^{*d} + a_{d-1} * g^{*(d-1)} + \cdots + a_1 * g + a_0 = 0 \tag{1}$$

with fixed coefficients $a_d, a_{d-1}, \ldots, a_1, a_0 \in \mathcal{A}$ and $a_d \neq 0$.

In 2007, H. Glöckner, L. G. Lucht and $\tilde{\mathrm{S}}$. Porubský gave a condition which is necessary for existence of solutions $g \in \mathcal{A}$ to equation (1) as follows: if z_0 is a simple zero of the polynomial

$$f(z) = a_d(1)z^d + a_{d-1}(1)z^{d-1} + \cdots + a_1(1)z + a_0(1),$$

then there exists a uniquely determined solution $g \in \mathcal{A}$ to the polynomial convolution equation $Tg = 0$ satisfying $g(1) = z_0$. We investigate the solvability of polynomial convolution equation $Tg = 0$ where $f(z)$ has no simple zero and of polynomial $\mathrm{Q}_\alpha-$ convolution equation

$$T_\alpha g = a_d \diamond g^{\diamond d} + a_{d-1} \diamond g^{\diamond(d-1)} + \cdots + a_1 \diamond g + a_0 = 0.$$

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