CHAPTER 4

ANALYSIS

A main discussion that would be presented in this chapter involves an explanation of why a rationally forward-looking household with higher initial wisdom tends to concentrate more on improving their wisdom by dedicating their resources to the religion practice. This also affects the consumption of the secular commodity in such a way that the derived demand for its input of both periods is lower relative to the case of lower initial wisdom.

To begin with, section 4.1 regards the effect of initial wisdom on consumption. The consumption behavior based on equations (3.34-3.47) of the previous chapter would be analyzed. It will be proved that an agent with more initial wisdom may consume less of the optimal level of purchased input of both periods for the case that their initial wisdom level does not exceed a critical value of ω_0. Moreover, the upper and the lower bound of market input consumption, including with the reason why an agent is not capable of consuming the first-period market inputs lower than their lower bound will be stated and explained. Next, for the case that initial wisdom is greater than the critical value ω_0, why an agent chooses to maintain his consumption levels would be reasoned. At the end of this section, economic reasons will be stated to support 1) how the amount of all choice variables of the model changes according to a rise in initial wisdom and 2) how the technology in capital accumulation affects the consumption of the life-time purchased input.

Section 4.2 presents comparative static analysis of the effect of change in wage rate on consumption.

4.1 The Effect of Initial Wisdom on Consumption

According to Proposition 3.1, for $P_1 \in (0, \omega_0)$ [1], it can be shown that the amount of 1st-period market input x_1 is monotonic decreasing in initial wisdom P_1 by taking first order derivatives of x_1 with respect to P_1 and it can be obtained that

$$\frac{\partial x_1}{\partial P_1} = \frac{2wa}{\gamma_3}\left(-\frac{\gamma_1}{P_1^2} + \frac{\left(\frac{\gamma_1^2}{P_1^3} + \left(\gamma_1\gamma_2 - \frac{\gamma_3}{2} \right) \cdot \frac{1}{P_1^2} \right)}{\sqrt{\left(\frac{\gamma_1}{P_1} + \gamma_2 \right)^2 - \frac{\gamma_3}{P_1}}} \right), \text{ for } \quad P_1 \in (0, \omega_0). \qquad (4.1)$$

According to equation[2] (4.1), it can be proved by inequality[3] that $\partial x_1 / \partial P_1$ is always negative. Therefore, within the domain of $(0, \omega_0)$, x_1 is monotonic decreasing in P_1. Due to the fact that x_1 is monotonic decreasing in P_1 for $P_1 \in (0, \omega_0)$, it is implied that the maximum value of x_1 according to equation (3.34) occurs as x_1 approaches 0^+ and its value[4] is wa/γ_1. Moreover, the minimum value of x_1 (within the interval $(0, \omega_0]$) is $(\beta aw) / (a + \beta(a+1))$ as $P_1 = \omega_0$.

Moreover, since the amount of purchased market input of period 2 x_2 equals to x_1 as shown in equation (3.31)[5], it can be implied that the amount of the 2nd-period market inputs x_2 fall with P_1. Therefore it can be concluded that how the amount of

---

[1] $\omega_0 = \left(2\gamma_1\gamma_2\left((1-\beta)/(1+\beta)\right) + \gamma_3\right) / \gamma_2^2 \left(1 - \left((1-\beta)/(1+\beta)\right)^2\right)$

[2] $\gamma_1 = a+1$, $\gamma_2 = \beta(1-a)$, and $\gamma_3 = \left(4\beta(1-a)(a + \beta(a+1))\right) / (1+\beta)$

[3] See the inequality proof in Appendix A.1

[4]

$$\lim_{P_1 \to 0} x_1 = \lim_{P_1 \to 0} \frac{2wa}{\gamma_3}\left(\frac{\gamma_1}{P_1} + \gamma_2 - \sqrt{\left(\frac{\gamma_1}{P_1} + \gamma_2 \right)^2 - \frac{\gamma_3}{P_1}} \right) = \lim_{P_1 \to 0} \frac{2wa}{\gamma_3}\left(\frac{\gamma_3}{\gamma_1 + \gamma_2 P_1 + \sqrt{(\gamma_1 + \gamma_2 P_1)^2 - \gamma_3 P}} \right) = \frac{wa}{\gamma_1}$$

[5] By letting $\beta = 1/(1+r)$, and $b = 1-b$

life-time purchased market input x responds to a unit of change in P_1 can be demonstrated as, for $P_1 \in (0, \omega_0)$,

$$\frac{\partial x}{\partial P_1} = \frac{\partial x_1}{\partial P_1} + \frac{1}{1+r} \cdot \frac{\partial x_2}{\partial P_1} = \frac{2(1+\beta)wa}{\gamma_3} \left(-\frac{\gamma_1}{P_1^2} + \frac{\left(\frac{\gamma_1^2}{P_1^3} + \left(\gamma_1 \gamma_2 - \frac{\gamma_3}{2} \right) \cdot \frac{1}{P_1^2} \right)}{\sqrt{\left(\frac{\gamma_1}{P_1} + \gamma_2 \right)^2 - \frac{\gamma_3}{P_1}}} \right), \tag{4.2}$$

The level of initial wisdom of an agent determines how much they consume market goods and/or service for all of their life time[6]. The equation (4.2) means that an additional unit of increase in initial wisdom P_1 of an agent leads to a decrease of the amount of their life-time purchased market inputs x at present value by the term on the RHS.

However, the dependence between x_1 and P_1 holds for only when $P_1 \in (0, \omega_0]$. For any P_1 that exceeds the critical value ω_0, x_1 may no longer relate to P_1 as shown in equation (3.35). According to equation (3.35), it can be seen that the amount of x_1 is not influenced by P_1 and constant as $P_1 > \omega_0$. Similarly, based on equation (3.46), the 2nd-period consumption of x_2 is independent of P_1 and holds as $P_1 > \omega_0$. While other things being equal, it is implied that any two agents with initial wisdom of $\omega_0 + \varepsilon_1$ and $\omega_0 + \varepsilon_2$ chooses to consume the same amount of life-time market inputs x[7].

---

[6] The amount of life-time x at present value for $P_1 \in (0, \omega_0]$ equals

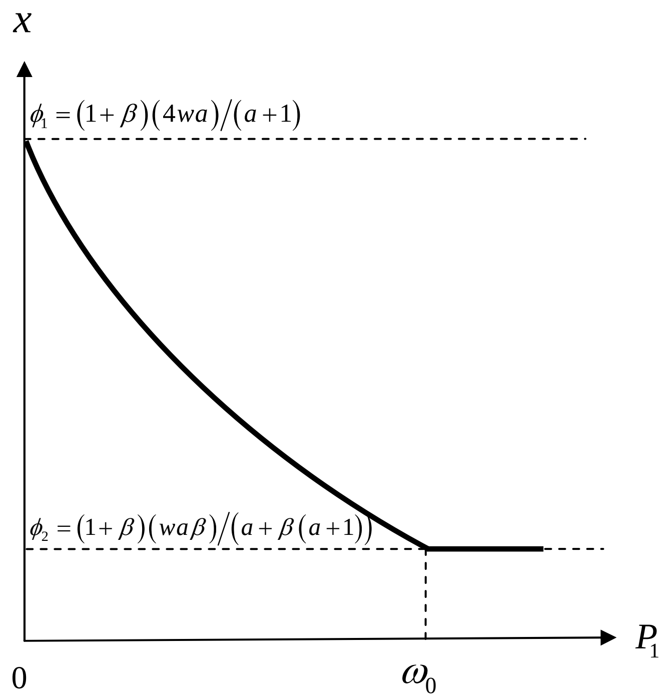
$$x = x_1 + \frac{1}{1+r} \cdot x_2 = \frac{2(1+\beta)wa}{\gamma_3} \left(\frac{\gamma_1}{P_1} + \gamma_2 - \sqrt{\left(\frac{\gamma_1}{P_1} + \gamma_2 \right)^2 - \frac{\gamma_3}{P_1}} \right), \text{ where } \beta = \frac{1}{1+r}$$

[7] which is shown at present value as $x = x_1 + \frac{1}{1+r} \cdot x_2 = \frac{\beta(1+\beta)aw}{a+\beta(a+1)}$, for $P_1 > \omega_0$

The following figure 4.1 shows a rough sketch of the relationships between the amount of purchased x at present value and that of initial wisdom capital P_1. The thick line demonstrates points of the mapping between x_1 and P_1. The lowest and the highest value of P_1 that maintains the negative trend is 0 and ω_0. At given β, a ,and w, the maximum present value of x occurs at $P_1 = 0$, which equals $\phi_1 = (1+\beta)(4wa)/(a+1)$. At $P_1 = \omega_0$, x reaches its minimum level of $\phi_2 = (1+\beta)(wa\beta)/(a+\beta(a+1))$ and this value is held constant although P_1 is greater than ω_0.

Figure 4.1

The Relationships between the Amount of Purchased x at Present Value and that of Initial Wisdom Capital P_1



Within the interval of dependence, it can be seen in equation 4.1 that the amount of x_1 turns to fall with initial wisdom capital. This is due to an incentive to future wisdom investment.

At period 1, an agent weighs costs and benefits and makes a choice on the amount of purchased input x_1, the amount of religion time t_{R_1}, and the amount of savings b_2. The cost of x_1 is the market price which is normalized to unity and its benefits are in terms of higher current marginal utility[8] which equals $a(1-b)/x_1$. For b_2, its cost is the price of bonds which is one and its marginal benefits are the present value of marginal utility of the future. The cost of t_{R_1} is the opportunity cost of working time, w, and the benefits are in forms of higher in both current and future utility. Focused on the marginal benefits that are derived from future utility, it equals $\beta(1-a)(1-b)P_1$ which means that more initial wisdom P_1 boosts the future marginal benefits of the investment in t_{R_1} [9]. This leads to higher in investment incentive in the religion time t_{R_1} and consequently encourages an agent to allocate more time at the present to religion (Equation 3.36 shows the positive relationships between t_{R_1} and P_1.)[10] and less to working activity, leading to a reduction in the current use of market input x_1.

However, the effect of the decrease in secular input x_1 is outweighed by that in lowering income which results from a drop in working hours. Therefore, it leads to a reduction in the amount of savings b_2. The equation 3.39 demonstrates the negative trend between b_2 and P_1 [11]. A reason to support a reduction of x_2 with P_1 as

---

[8] Based on the FOC of equation 3.27

[9] Based on the FOC of equation 3.26

[10] $$t_{R_1} = 1+\beta-\frac{(1+\beta)}{2\beta(1-a)}\cdot\left(\frac{\gamma_1}{P_1}+\gamma_2-\sqrt{\left(\frac{\gamma_1}{P_1}+\gamma_2\right)^2-\frac{\gamma_3}{P_1}}\right)$$

[11] $$b_2 = \frac{2w(a+1)\beta}{\gamma_3}\left(\frac{\gamma_1}{P_1}+\gamma_2-\sqrt{\left(\frac{\gamma_1}{P_1}+\gamma_2\right)^2-\frac{\gamma_3}{P_1}}\right)-w\beta$$

demonstrated in equation 3.37[12] comes from the small return from savings b_2. Moreover, the inadequate returns forces a household to spend more working hours. Therefore, the amount of time left for religion practice at the terminal period is lower with P_1 as shown in equation 3.38[13].

The role of technology in accumulating wisdom capital over periods is significant in such a way that the speed of technology will determine the relationships between P_1 and t_{R_1} at first. Moreover, this may relate to the amount of other choice variables such as x_1 and b_2. In this study, an exponential specification of the form $P_2 = \exp\left(P_1 \cdot t_{R_1}\right)$ is assumed as a transition function of wisdom capital. This reflects the increasing returns to scale and results in the FOC with respect to t_{R_1} of the form $b/t_{R,1} + \beta(1-a)(1-b)P_1 \;=\; \lambda w$ (equation 3.36). This implies the positive relationships between P_1 and t_{R_1}; that is, more initial wisdom P_1 will induce higher t_{R_1}. On the other hand, if the constant returns, for example $P_2 = P_1 + t_{R,1}$,was assumed, the FOC would become $b/t_{R,1} + \beta(1-a)(1-b)/\left(t_{R,1} + P_1\right) \;=\; \lambda w$ which would relate P_1 to t_{R_1} in the opposite direction. Apart for the speed of technology, in the art of selecting the technology, the functional form of capital transition should be concerned. Since the utility specification is the log-Cobb Douglas, the interaction between P_1 and t_{R_1} would vanish if some kinds of the transition function that wisdom variable is isolated from religion time variable after plugging the form into the utility function, for example, $P_2 = P_1 \cdot t_{R,1}^{\lambda}$, where $0 < \lambda < 1$, are adopted.

For the case of $P_1 \le \omega_0$, it can be concluded that each agent endows with different levels of initial religious human capital P_1. At the first period, an agent with

---

[12] $$x_2 \;=\; \frac{2wa}{\gamma_3}\left(\frac{\gamma_1}{P_1} + \gamma_2 - \sqrt{\left(\frac{\gamma_1}{P_1} + \gamma_2\right)^2 - \frac{\gamma_3}{P_1}}\right)$$

[13] $$t_{R,2} \;=\; \frac{2w\beta}{\gamma_3}\left(\frac{\gamma_1}{P_1} + \gamma_2 - \sqrt{\left(\frac{\gamma_1}{P_1} + \gamma_2\right)^2 - \frac{\gamma_3}{P_1}}\right)$$

higher initial wisdom capital may consume less of first-period secular purchased goods and/or service x_1 since they choose to devote more time to religion practice partly with an eye to boost more of the future wisdom capital. The positive relationships between the level of initial endowed wisdom and that of the religion practice are determined by the speed of technology. The small return of bonds lowers the amount of second-period purchased goods or service x_2. Therefore, the agent with higher wisdom capital would consume less of life-time secular input. Although less inputs, an agent with higher wisdom gain more of optimal life-time utility derived from two commodities. To prove this, the Envelope theorem is applied to the value function of equation 3.23 and 3.24 and it would be obtained as

$$V_1'(P_1) = (1-a)(1-b)\left(\frac{1}{P_1}+\beta t_{R_1}\right) \quad \text{for } P_1 \le \omega_0 , \qquad (4.3)$$

which is always positive. Therefore, equation 4.3 implies a rise of optimal life-time utility with initial wisdom for $P_1 \le \omega_0$.

For the case of $P_1 > \omega_0$, it is inevitably true that a rise of initial wisdom P_1 contributes to higher future marginal benefit of the investment in t_{R_1} in terms of higher future marginal utility, which is $\beta(1-a)(1-b)P_1$ [14]. Nevertheless, this does not raise the optimal religion time of period 1 since an agent is incapable of spending the religion time more than one due to the time constraint. Therefore, an agent would decide to hold their religion time at its peak, which is one, no matter how their initial wisdom is. Moreover, the costs and benefits of other choice variables (x_1, x_2, and b_2) are independent of P_1. Consequently, this is why the decision-making by an agent is not altered by $P_1 > \omega_0$ no matter how the technology is. However, initial

[14] according to equation 3.40

wisdom P_1 including with the technology takes part in shifting the levels of life-time utility by $(1-a)(1-b)(1/P_1+\beta P_1)$ [15] as $P_1>\omega_0$.

4.2 The Effect of Wage on Consumption

For the case of $P_1\le\omega_0$, with the first-period budget constraint of the form

$$x_1+wt_{R_1}+b_2=w, \qquad (4.4)$$

a rise of wage rate generates wealth and price effect and shifts the amount of x_1 [16]. Nevertheless, despite the wealth effect, t_{R_1} is insensitive to any change in wage as shown in equation 3.36. This is due to the fact that the rising wage also implies higher indirect price of t_R that consequently inhibits the effect of increasing wealth. Moreover, the increasing wage rate with unchanged working hours leads to a higher in the amount of earnings. The rise of earnings outweighs that of x_1 purchased. Therefore, the amount for b_2 is also increased[17]. For the terminal period, according to the budget constraint,

$$x_2+wt_{R_2} = w+b_2(1+r), \qquad (4.5)$$

the shift of wage, together with the higher b_2 that transferred from the wage change in the first period, generates wealth and price effect that boosts the amount of the market

---

[15] By the Envelope theorem, it can be shown that $V'_1(P_1)=(1-a)(1-b)(1/P_1+\beta P_1)$ for any values of the choice variables and for the technology of $P_2=\exp\left(P_1\cdot t_{R_1}\right)$.

[16] x_1 increases by $\dfrac{\partial x_1(P_1)}{\partial w} = \dfrac{2a}{\gamma_3}\left(\dfrac{\gamma_1}{P_1}+\gamma_2-\sqrt{\left(\dfrac{\gamma_1}{P_1}+\gamma_2\right)^2-\dfrac{\gamma_3}{P_1}}\right)$ units.

[17] b_2 increases by $\dfrac{\partial b_2(P_1)}{\partial w} = \dfrac{2(a+1)\beta}{\gamma_3}\left(\dfrac{\gamma_1}{P_1}+\gamma_2-\sqrt{\left(\dfrac{\gamma_1}{P_1}+\gamma_2\right)^2-\dfrac{\gamma_3}{P_1}}\right)-\beta$ units.

input of the 2nd period[18] and the religion time of the 2nd period[19], x_2 and t_{R_2} respectively. One difference from the first period is that t_{R_2} is now sensitive to the wage change. This is due to the fact that the wealth effect of the terminal period comes from not only the wage change of that period but also that of the previous period via the return of the increasing purchased bonds. This amplifies the wealth effect to overcome the effect of increasing price of t_{R_2}. Therefore, the absolute demand for t_{R_2} would rise with wage. Next, for the case of $P_1 \geq \omega_0$, the first-period budget constraint is as

$$x_1 + b_2 = 0 \qquad (4.6)$$

This is the case where an agent spends all of their 1st-period time for religion and does not work at all. Therefore, in order to analyze the effect of wage change, equation (4.6) will be plugged into the second-period constraint to obtain

$$x_1(1+r) + x_2 + wt_{R_2} = w \qquad (4.7)$$

According to equation (4.7), an increase in wage results in price and wealth effect that raises the amount of x_1[20] and x_2[21]. However, t_{R_1} is insensitive to any change in wage as shown in equation 3.47. This is due to the fact that the rising wage also implies higher indirect price of t_R that consequently inhibits the wealth effect.

---

[18] x_2 increases by $\frac{\partial x_2(P_1)}{\partial w} = \frac{2a}{\gamma_3}\left(\frac{\gamma_1}{P_1} + \gamma_2 - \sqrt{\left(\frac{\gamma_1}{P_1} + \gamma_2\right)^2 - \frac{\gamma_3}{P_1}}\right)$ units.

[19] $t_{R,2}$ x_2 increases by $\frac{\partial t_{R,2}(P_1)}{\partial P_1} = \frac{2\beta}{\gamma_3}\left(\frac{\gamma_1}{P_1} + \gamma_2 - \sqrt{\left(\frac{\gamma_1}{P_1} + \gamma_2\right)^2 - \frac{\gamma_3}{P_1}}\right)$ units.

[20] x_1 increases by $\frac{\partial x_1(P_1)}{\partial w} = \frac{\beta a}{(a + \beta(a+1))}$ units

[21] x_2 increases by $\frac{\partial x_2(P_1)}{\partial w} = \frac{\beta a}{(a + \beta(a+1))}$ units.