CHAPTER 3

MODEL

3.1 Definitions of "religious commodity", "secular commodity", and "wisdom capital"

Both religious and secular commodities are neither physical goods nor services. Rather, they are classified as a third category that is called "household commodities" in the concept of household production.

"Religious" or "religious practice" activity involves the development of wisdom. Religious commodity is defined as the non-market *product* of "religious practice" activity which can be viewed as a productive process. In general, religious commodity is abstract and is exemplified by periods of being wholesome, right mindfulness, mental development, freedom from defilements, and so on. Although religious commodities are complex and largely unobservable, the inputs to religious production are measurable and indeed are already routinely measured by researchers. According to Iannacone (1984), purchased inputs, household time, and capital affect a family's capacity to "produce" religious commodity. Time devoted to doctrines study, dhamma CD, and textbooks exemplify religious inputs. Nevertheless, in our model, only time input is included as the factor of production. The reason why wisdom capital is excluded will be stated in section 3.2.

Apart from religious activities, all other commodities may be viewed as an aggregate "secular" activity for the purpose of formal analysis. Secular activities are, for instance, alcohol drinking, dating, musical performance, or shopping.

Secular commodity is defined as the non-market *product* of secular activity that can be viewed as a productive process. Secular products are, for example, attachment, periods of being unwholesome, or pleasant feeling. Although abstract and non-measurable, their inputs are market goods that can be quantified such as liquor, movie tickets, pianos, or name brand clothes.

Besides market inputs, “wisdom” which is defined as a productive skill of familiarity with religious doctrines or ability to understand the real truth of nature enters analyses of secular production because people’s skills critically affect the quality and quantity of what they produce. For instance, with more wisdom, a shopaholic woman could buy less items but hold the same degree of secular product (such as pleasant feeling). Similarly, a drinker uses wisdom to manage how much he drinks at each time.

“Wisdom” is also classified as a type of “capital” because it functions in a productive process by turning/managing market inputs into non-market secular commodity, and because it is accumulated via a process of investment in religious commodity.

3.2 Assumptions of the Model

The general idea of the model is that a rationally forward-looking household of a single person maximizes the 2-period time-separable utility by allocating its time resources to participate both market and non-market activities. Total available time of each period goes for 1) working and 2) religious practice. The market value of time or wage rate is exogenously given and total earnings are for purchasing bonds and market secular inputs. In the first period, a representative agent endows with a certain value of innate initial “wisdom capital”. The wisdom capital is used to turn/manage secular inputs into the production of secular commodity.

Apart from the above commodity, the utility depends on the level of religious practice that may be treated as another commodity. It would be assumed that the production of religious commodity requires no skills or any other capitals. Excluding the “wisdom capital” from the religious commodity production allows us to concentrate only on the investment via cross commodity, not “on the job”.

At the first period, a forward-looking household weighs costs and benefits to choose how much he/she practices in religion, works, purchases secular inputs, and purchases bonds. The current choice in religion practice affects not only the current utility but also the future benefits. The choice in religion practice may be considered partly an investment process to accumulate future wisdom capital. The investments returns appear in terms of higher future utility of the secular production.

At the terminal period, the returns from savings and the accumulated wisdom capital affect the consumptions of the market secular input and religion practice.

3.3 Derivations of the Model

Let R and S denote the quantities of religious and secular commodities that a household produces and consume, respectively. R is produced with inputs of personal time.

For S, its production needs inputs of purchased goods and/or service ("purchased goods" refers to anything bought including physical items and human services) and wisdom capital. Hence,

$$R = R(t_R) \tag{3.1}$$

$$S = S(x;P) \tag{3.2}$$

where t_R is the amount of time devoted to religion, P is a stock of "wisdom capital", and x denotes the amount of purchased goods and/or service. Besides t_R, a household allocates the time resource to working time t_w. The total amount of available time is fixed and is normalized to unity. Each person's time constraint may be written as:

$$t_R + t_w = 1. \tag{3.3}$$

According to (3.3), the time devoted to religion plus the time devoted to work equals to the total available time.

The amount of purchased goods and/or service x devoted to secular activity cannot exceed total income, I :

$$x \leq I. \tag{3.4}$$

Total income equals earning. Hence, letting w denote the person's wage rate, total income may be written as

$$I = w \cdot t_w. \tag{3.5}$$

"Full income," I^* can be defined as the amount of income that would be generated if the household devoted all of its time to the market labor:

$$I^* = w. \tag{3.6}$$

By definition, a consumer seeks to allocate his/her resources so as to maximize his/her utility derived from assumed R and S. The utility would be

$$U = U(R,S). \tag{3.7}$$

To proceed, the log Cobb-Douglas preferences specification is applied:

$$U(S,R) = (1-b)\log S + b\log R, \tag{3.8}$$

where b is the preferences parameter.

For S production, the Cobb-Douglas production function is employed:

$$S = P^{1-a}x^a, \tag{3.9}$$

where a is the output elasticity. For R production, linear production is assumed:

$$R = t_R. \tag{3.10}$$

In order to analyze the consequences for the consumption behavior over time, all variables defined above have to be dated. In this model, an agent is assumed to live for two periods. At the first period, initial wisdom capital, P_1, is exogenously given. At the terminal period, wisdom capital, P_2, is produced through accumulating effects of earlier religious activity. The transition function is assumed with increasing returns as

$$P_2 \quad = \quad \exp\left(P_1 \cdot R_1\right). \tag{3.11}$$

A 2-period lived representative agent with the initial wisdom capital, P_1, wishes to maximize his life-time utility, with β as the discount factor, of the form:

$$V\left(P_1\right) \quad = \quad \max U\left(S\left(x_1;P_1\right),t_{R_1}\right)+\beta U\left(S\left(x_2;P_2\right),t_{R_2}\right) \tag{3.12}$$

subject to the life-time budget constraint.[1] (3.13) which shows an equality between the present values of the amount of purchased inputs and that of wage income.

$$x_1+\frac{x_2}{1+r} \quad = \quad w \cdot t_{w_1}+\frac{w}{1+r} \cdot t_{w_2} \tag{3.13}$$

and the time allocation constraint,

$$t_{w,s}+t_{R,s} \quad = \quad 1, \;\; s \quad = \quad 1,2 \tag{3.14}$$

and the law of motion of,

$$P_2 \quad = \quad \exp\left(P_1 \cdot t_{R_1}\right). \tag{3.15}$$

The nature of this problem is a recursive one. It allows us to solve this problem backward, starting from the terminal period back to the initial period.

[1] r is the market interest rate.

In the period 2 problem, the objective function[2] becomes

$$V_0(P_2,b_2) \quad = \quad \max_{x_2,t_{R_2}} \; a(1-b)\log x_2 + (1-a)(1-b)\log P_2 + b\log t_{R_2} \qquad (3.16)$$

subject to the second-period budget constraint

$$x_2 \quad = \quad w\left(1-t_{R_2}\right)+b_2\left(1+r\right). \qquad (3.17)$$

Equations (3.17) shows the equality between the amount of second-period secular input and the sum of wage income in period 2 and the return of savings.

The FOCs are

$$\frac{a(1-b)}{x_2} \quad = \quad \lambda\,, \qquad (3.18)$$

$$\frac{b}{t_{R_2}} \quad = \quad \lambda w\,. \qquad (3.19)$$

Using (3.18) in (3.19) gives

$$t_{R,2} \quad = \quad \frac{b}{wa(1-b)}\cdot x_2\,. \qquad (3.20)$$

Substituting the above equation into the 2nd-period budget constraint (3.17) gives

[2] The subscript 0 of the notation $V_0(P_2,b_2)$ represents the number of periods left, which is zero in this case. This means the terminal period.

$$x_2 \quad = \quad \frac{w}{\chi}+\frac{(1+r)}{\chi}b_2, \qquad (3.21)$$

$$t_{R,2} \quad = \quad \frac{\chi-1}{\chi}\left(1+\frac{(1+r)}{w}b_2\right), \qquad (3.22)$$

where $\chi \quad = \quad 1+\frac{b}{a(1-b)}$. Equations (3.21) and (3.22) show that demands for secular input and religion time in the second period depend positively on the saving returns.

Therefore, we can substitute (3.21) and (3,22) into (3.16) to obtain the indirect utility function or the value function at the beginning of the second period.

$$V_0(P_2,b_2) \quad = \quad \alpha+\left(a(1-b)+b\right)\log\left(w+(1+r)b_2\right)+(1-a)(1-b)\log P_2, \quad (3.23)$$

where $\alpha \quad = \quad a(1-b)\log\frac{1}{p\chi}+b\log\frac{\chi-1}{\chi w}$.

Now, we go back to the initial date and solve for the period 1 problem,

$$V_1(P_1) \quad = \quad \max_{x_1,t_{R_1},P_2,b_2} a(1-b)\log x_1+(1-a)(1-b)\log P_1+b\log t_{R_1}+\beta V_0(P_2,b_2), \ (3.24)$$

subject to the first-period budget constraint

$$x_1 \quad = \quad w\left(1-t_{R_1}\right)-b_2. \qquad (3.25)$$

Equation (3.25) shows that the wage income in the first period is spent for input and bonds purchase.

The FOCs are

$$\frac{b}{t_{R,1}}+\beta(1-a)(1-b)P_1 \quad = \quad \lambda w, \tag{3.26}$$

$$\frac{a(1-b)}{x_1} \quad = \quad \lambda, \tag{3.27}$$

$$\frac{a(1-b)+b}{w+(1+r)b_2} \quad = \quad \frac{\lambda}{\beta(1+r)}. \tag{3.28}$$

From (3.25), (3.26), (3.27) and (3.28), the demand for first-period secular input x_1 can be solved to express in terms of the initial endowment of wisdom P_1 and all other parameters as

$$x_1 \in \left\{ x > 0 \middle| Ax^2 + Bx + C \quad = \quad 0 \right\},$$

where

$$A \quad = \quad (1/a)\Big(\beta(1-a)\big(a(1-b)+\beta\big(a(1-b)+b\big)\big)\Big)P_1,$$

$$B \quad = \quad -\Big(w\big(a(1-b)+b\big)(1+\beta)+w\beta(1-a)(1-b)\big((2+r)/(1+r)\big)P_1\Big),$$

$$C \quad = \quad w^2 a(1-b)\big(1+1/(1+r)\big). \tag{3.29}$$

Moreover, from (3.25), (3.27) and (3.28), the first-period religion time t_{R_1} can be solved explicitly in terms of x_1.

$$t_{R_1} \quad = \quad 1+\frac{1}{1+r}-\frac{1}{w}\left(1+\frac{\beta\big(a(1-b)+b\big)}{a(1-b)}\right)\cdot x_1 \tag{3.30}$$

As shown in equation (3.30) and it can be seen that t_{R_1} and x_1 can be considered as a pair of substitute inputs.

The amount of inputs ($x_2, t_{R,2}$, and b_2) relates linearly to the demand of the first-period purchased market goods x_1 as shown in (3.31), (3.32), and (3.33) respectively:

$$x_2 = \frac{(1+r)}{\chi} \cdot \frac{\beta\left(a(1-b)+b\right)}{a(1-b)} \cdot x_1 , \tag{3.31}$$

$$t_{R,2} = \frac{\chi-1}{\chi} \cdot \frac{\beta\left(a(1-b)+b\right)}{a(1-b)} \cdot x_1 , \tag{3.32}$$

$$b_2 = \frac{\beta\left(a(1-b)+b\right)}{a(1-b)} \cdot x_1 - \frac{w}{1+r} . \tag{3.33}$$

To compute the solution for x_1, it would be further assumed that the maximum value of the parameter β does not exceed $1/(1+a)$, which is written as $\beta(1+a) \leq 1$ and $\beta \leq a$. This implies that an agent will weigh more on the current utility if the output elasticity of x in a secular productive activity becomes larger. On the other hand, for the case that the elasticity of x in a secular productive activity is small, an agent tends to weigh a lot on the future utility; that is, the maximum value of parameter β turns to be large.

The second assumption is that the discount factor of future utility β is at a market interest rate r, which is $\beta = 1/(1+r)$. The third one concerns nature of an agent that the preferences between the secular and religious commodity are equally weighed; that is, $b = 1-b$.

According to the above assumptions, the following proposition states that the amount of x can be written as a function of initial wisdom P_1. This is derived from all first order necessary conditions represented by equation 3.29.

Proposition 3.1 (x_1 ***and*** P_1 ***nexus)*** *If* $\beta(1+a)\le 1$, $\beta = 1/(1+r)$, *and* $b = 1-b$, *the solution of the amount of first-period purchased market input* x_1 *is unique, for a certain value of all parameters.*

1) For $P_1 \in (0,\omega_0]$, *the amount of first-period purchased market input* x_1 *is dependent of initial wisdom* P_1 *and can be written as*

$$x_1(P_1) = \frac{2wa}{\gamma_3}\left(\frac{\gamma_1}{P_1}+\gamma_2-\sqrt{\left(\frac{\gamma_1}{P_1}+\gamma_2\right)^2-\frac{\gamma_3}{P_1}}\right), \qquad (3.34)$$

where $x_1(P_1)\in\left[\frac{\beta aw}{(a+\beta(a+1))},\frac{wa}{(a+1)}\right)$.

2) For $P_1 > \omega_0$, *the amount of first-period purchased market input* x_1 *is independent of initial wisdom* P_1, *which is*

$$x_1 = \frac{\beta aw}{a+\beta(a+1)} \qquad (3.35)$$

where $0<w<1$, $0<a<1$, $0<\beta<1$,

$\gamma_1 = a+1$,

$\gamma_2 = \beta(1-a)$,

$\gamma_3 = \left(4\beta(1-a)(a+\beta(a+1))\right)/(1+\beta)$,

$\omega_0 = \left(2\gamma_1\gamma_2\left((1-\beta)/(1+\beta)\right)+\gamma_3\right)/\gamma_2^2\left(1-\left((1-\beta)/(1+\beta)\right)^2\right)$.

Proof. See Appendix A □

According to proposition 3.1, how an agent chooses to purchase the first-period market input x_1 is influenced by their initial wisdom capital P_1 for the values of P_1 that falls between 0 and ω_0. The width of the interval is determined by the model parameters β and a. Beyond this interval of P_1, an agent chooses to hold their consumption level of x_1 at the same level as that of $P_1 = \omega_0$. Therefore, $P_1 > \omega_0$ can be considered the interval of independence between P_1 and x_1.

Moreover, the amount of other choice variables of the model (t_{R_1}, x_2, $t_{R,2}$, and b_2) can be derived based on the proposition 3.1 and the relationships between other choices variables and x_1 according to equations (3.30) – (3.33). For $P_1 \in (0, \omega_0]$, with substitution of (3.34) and the three assumptions ($\beta(1+a) \le 1$, $\beta = 1/(1+r)$ and $b = 1-b$) into (3.30) – (3.33), all other choice variables (t_{R_1}, x_2, $t_{R,2}$, and b_2) of the model can be derived as shown in (3.36) - (3.39) respectively:

$$t_{R_1} = 1+\beta-\frac{(1+\beta)}{2\beta(1-a)}\cdot\left(\frac{\gamma_1}{P_1}+\gamma_2-\sqrt{\left(\frac{\gamma_1}{P_1}+\gamma_2\right)^2-\frac{\gamma_3}{P_1}}\right), \tag{3.36}$$

$$x_2 = x_1 = \frac{2wa}{\gamma_3}\left(\frac{\gamma_1}{P_1}+\gamma_2-\sqrt{\left(\frac{\gamma_1}{P_1}+\gamma_2\right)^2-\frac{\gamma_3}{P_1}}\right), \tag{3.37}$$

$$t_{R,2} = \frac{2w\beta}{\gamma_3}\left(\frac{\gamma_1}{P_1}+\gamma_2-\sqrt{\left(\frac{\gamma_1}{P_1}+\gamma_2\right)^2-\frac{\gamma_3}{P_1}}\right), \tag{3.38}$$

$$b_2 = \frac{2w(a+1)\beta}{\gamma_3}\left(\frac{\gamma_1}{P_1}+\gamma_2-\sqrt{\left(\frac{\gamma_1}{P_1}+\gamma_2\right)^2-\frac{\gamma_3}{P_1}}\right)-w\beta. \tag{3.39}$$

However, the dependence between x_1 and P_1 holds for only when $P_1 \in (0, \omega_0]$ due to the following reason.

The objective problem, which is based on equation (3.23)-(3.25) and leads to the result of equation 3.34, does not include the constraint of the 1st-period religion time being greater than zero but less than one. The case of $P_1 = \omega_0$ can be considered where P_1 reaches its critical values since the case reflects that an agent spends the 1st-period religion time at its peak (which is unity). According to equation3.36, if P_1 exceeds its critical value ω_0, an agent would spend the 1st-period religion time more than one. This contradicts to the time constraint. Therefore, it reflects that the equation 3.34 succeeds in explaining the consumption behavior only when P_1 does not exceed its critical value of ω_0 In other words, it implies that the relationships between x_1 and P_1 should be determined by another objective problem when $P_1 > \omega_0$. The new objective problem is the situation when an agent 1) spends all of the available time of the first period to religion and 2) uses partly of the future wage income to purchase current market goods. Therefore, an agent trades off among x_1, x_2, and t_{R_2} to maximize their life-time utility. Clearly, for $P_1 > \omega_0$, the objective problem slightly changes from the case of $P_1 \le \omega_0$ in the sense that an agent always choose t_{R_1} at its peak, which is one, and choose x_1 and b_2 to maximize

$$V_1(P_1) = \max_{x_1,b_2} a(1-b)\log x_1 + (1-a)(1-b)\log P_1 + \beta V_0(P_2,b_2),$$

where $V_0(P_2,b_2) = \alpha + \left(a(1-b)+b\right)\log\left(w+(1+r)b_2\right) + (1-a)(1-b)P_1$. (3.40)[3]

subject to 1) the new 1st-period budget constraint (from 3.25)

$$x_1 = -b_2, \qquad (3.41)$$

which an agent does not work at all in period 1 and uses partly of their future wage income to purchase current market goods.

The other constraint is 2) the constraint of non-negative value of $t_{R,2}$ and x_2 according to the equations 3.21 and 3.22 which implies as $b_2 \ge -\beta w$ or

$$x_1 \le \beta w. \qquad (3.42)$$

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[3] Based on equations (3.21)-(3.25).

The FOC with respect to x_1 is

$$\frac{a(1-b)}{x_1} = \frac{\beta(1+r)\big(a(1-b)+b\big)}{\big(w-(1+r)x_1\big)} \tag{3.43}$$

It can be obtained that $x_1(P_1) = \beta aw \big/ \big(a+\beta(a+1)\big)$ which is obviously lower than βw. Therefore,

$$x_1(P_1) = \frac{\beta aw}{\big(a+\beta(a+1)\big)} \tag{3.44}$$

is the amount of x_1 when $P_1 > \omega_0$, as shown in equation (3.35). With substitution of equation (3.35) into(3.41), the amount of b_2 can be obtained as

$$b_2(P_1) = -\frac{\beta aw}{\big(a+\beta(a+1)\big)}. \tag{3.45}$$

Plugging (3.44) into (3.21) and (3.22) and obtaining x_2 and $t_{R,2}$ respectively as

$$x_2 = \frac{\beta aw}{\big(a+\beta(a+1)\big)}, \tag{3.46}$$

$$t_{R,2} = \frac{\beta}{\big(a+\beta(a+1)\big)}. \tag{3.47}$$

Accordingly, for $P_1 > \omega_0$, t_{R_1} , x_2 , $t_{R,2}$, and b_2 become indifferent to P_1.