

APPENDIX B14

ASSUMPTIONS' CROSS CHECKING FOR THE CASE V

By considering the competition between original branch of player 1 and original branch of player 2, consumer locate at x will buy the product from player 1 if

$$\begin{array}{rcl}
 u_1 & > & u_3 \\
 V - p_1^* - (x - x_1^*)^2 + y_1^* & > & V - p_2^* - (x - x_2^*)^2 + y_2^* \\
 V - \frac{9}{8} - (x - 0)^2 + \frac{1}{2} & > & V - \frac{9}{8} - (x - 1)^2 + \frac{1}{2} \\
 -x^2 & > & -x^2 + 2x - 1 \\
 x & < & \frac{1}{2}
 \end{array}$$

By considering the competition between new branch of player 1 and original branch of player 2, consumer locate at x will buy the product from player 1 if

$$\begin{array}{rcl}
 u_2 & > & u_3 \\
 V - p_1^* - (x - m_1^*)^2 + n_1^* & > & V - p_2^* - (x - x_2^*)^2 + y_2^* \\
 V - \frac{9}{8} - (x - m_1^*)^2 + 0 & > & V - \frac{9}{8} - (x - 1)^2 + \frac{1}{2} \\
 -x^2 + 2m_1^*x - m_1^{*2} & > & -x^2 + 2x - \frac{1}{2} \\
 x & < & \frac{1 - 2m_1^2}{4(1 - m_1)}
 \end{array}$$

Since the value of $m_1 \in [0,1]$, the value of $(1 - 2m_1^2)/(4(1 - m_1))$ is less than $1/2$. Thus, the consumer who locate at x where $x < 1/2$ will buy the product from player 1 anyway. By considering the competition between original branch of player 1

and new branch of player 2, consumer locate at x will buy the product from player 2 if

$$\begin{array}{rcl}
 u_1 & < & u_4 \\
 V - p_1^* - (x - x_1^*)^2 + y_1^* & < & V - p_2^* - (x - m_2^*)^2 + n_2^* \\
 V - \frac{9}{8} - (x - 0)^2 + \frac{1}{2} & < & V - \frac{9}{8} - (x - m_2^*)^2 + 0 \\
 -x^2 + \frac{1}{2} & < & -x^2 + 2m_2^*x - m_2^{*2} \\
 x & > & \frac{1 + 2m_2^2}{4m_2}
 \end{array}$$

Since the value of $m_2 \in [0,1]$, the value of $(1 + 2m_2^2)/(4m_2)$ is greater than $1/2$. Thus, the consumer who locates at x where $1/2 < x$ will buy the product from player 2 anyway. Therefore, in this case, the only market separating line we need to consider is x_c . By substitute the optimal location, product's quality level, and pricing of both players into x_c, π_1 , and π_2 , the results are $x_c = 1/2, \pi_1 = 1/2$, and $\pi_2 = 1/2$. Since there is only one market separating line, x_c , and both optimal price and market share of both players are positive, we can conclude that the assumptions are correct.