APPENDIX B6

ASSUMPTIONS' CROSS CHECKING FOR THE CASE I.3

Given any value of consumer's location x, the utility from buying the product from new branch of player 2 will be always higher than buying the product from the original branch of player 2 because of the following.

$$u_4 \quad = \quad V - p_2^* - (x - m_2^*)^2 + n_2^* \quad = \quad V - \frac{13}{8} - (x-1)^2 + 1$$

$$> \quad V - \frac{13}{8} - (x-1)^2 + \frac{1}{2}$$

$$> \quad V - p_2^* - (x - x_2^*)^2 + y_2^* \quad = \quad u_3$$

By considering the competition between original branch of player 1 and new branch of player 2, consumer locate at x will buy the product from player 1 if

$$u_1 \quad > \quad u_4$$

$$V - p_1^* - (x - x_1^*)^2 + y_1^* \quad > \quad V - p_2^* - (x - m_2^*)^2 + n_2^*$$

$$V - \frac{9}{8} - (x-0)^2 + \frac{1}{2} \quad > \quad V - \frac{13}{8} - (x-1)^2 + 1$$

$$-x^2 \quad > \quad -x^2 + 2x - 1$$

$$x \quad < \quad \frac{1}{2}$$

Again, by considering the competition between new branch of player 1 and new branch of player 2, consumer locate at x will buy the product from player 1 if

$$u_2 \quad > \quad u_4$$

$$V - p_1^* - (x - m_1^*)^2 + n_1^* \quad > \quad V - p_2^* - (x - m_2^*)^2 + n_2^*$$

$$V - \frac{9}{8} - (x - m_1^*)^2 + 0 \quad > \quad V - \frac{13}{8} - (x-1)^2 + 1$$

$$-\frac{1}{2} - x^2 + 2m_1^* x - m_1^{*2} \quad > \quad -x^2 + 2x - 1$$

$$x < \frac{1/2 - m_1^2}{2(1 - m_1)}$$

Since the value of $m_1 \in [0,1)$, the value of $\left(1/2 - m_1^2\right)\big/\left(2(1-m_1)\right)$ is less than $1/2$. Therefore, in this case, the only market separating line we need to consider is x_{c_2}. By substitute the optimal location, product's quality level, and pricing of both players into $x_{c_2}, \pi_1,$ and π_2, the results are $x_{c_2} = 1/2, \pi_1 = 1/2,$ and $\pi_2 = 1/2$. Since there is only one market separating line, x_{c_2}, and both optimal price and market share of both players are positive, we can conclude that the assumptions are correct.