Executive Summary

1. Introduction to the research problem and its significance

In mathematics, a nonlinear system is a system which is not linear; that is, a system which does not satisfy the superposition principle, or whose output is not directly proportional to its input. Less technically, a nonlinear system is any problem where the variables to be solved for cannot be written as a linear combination of independent components.

Nonlinear problems are of interest to engineers, physicists and mathematicians because most physical systems are inherently nonlinear in nature. Nonlinear equations are difficult to solve and give rise to interesting phenomena. While solving problems related to nonlinear ordinary differential equations, it is often expedient to simplify equations by a suitable change of variables. One of the fundamental methods to solve this relies upon the transformation of a given equation to another equation of standard form. The transformation may be to an equation of equal order or of greater or lesser order. In particular, the possibility that a given equation could be linearized, i.e., transformed to a linear equation, was a most attractive proposition due to the special properties of linear differential equations. The reduction of an ordinary differential equation to a linear ordinary differential equation besides simplification allows us to construct an exact solution of the original equation.

One type of the classification problem is the equivalence problem. Two systems of differential equations are said to be equivalent if there exists an invertible transformation which transforms any solution of one system to a solution of the other system and vice versa. The linearization problem is a particular case of the equivalence problem, where one of the systems is a linear system. It is one of the essential parts in the study of nonlinear equations.

One of the main motivations for studying the linearization problem is the possibility of finding the general solution. Notice that even after finding the linearizing transformation one has to solve a linear system of second-order ordinary differential equations.

Any matrix over C is similar to an upper triangular matrix, but not necessarily similar to a diagonal matrix. Despite this we can still demand that it be similar to a matrix which is as 'nice as possible', which is the Jordan Normal Form. This has applications to systems of difference or differential equations, which can be represented by matrices - putting the matrix in Jordan Normal Form makes it easier to find solutions

to the system of difference or differential equations. Because of these advantages, the constant matrix in Jordan Normal Form will be used to be the linear coefficient of our linear system.

The main difficulty in solving the linearization problem comes from the large number of complicated calculations. Because of this difficulty, there are no one attempts to solve this problem for nonlinear system of equations higher than two. However if we can solve the linearization problem of system of three second-order ordinary differential equations, then we should set a new process to solve the problems in Physics or Engineering.

2. Objectives

2.1 To find the necessary and sufficient conditions for a general system of three second-order ordinary differential equations

$$y_1'' = F_1\left(x, y_1, y_2, y_3, y_1', y_2', y_3'\right),$$
$$y_2'' = F_2\left(x, y_1, y_2, y_3, y_1', y_2', y_3'\right),$$
$$y_3'' = F_3\left(x, y_1, y_2, y_3, y_1', y_2', y_3'\right)$$

to be equivalent to a linear system

$$\ddot{u} + Ku = 0,$$

with respect to a point transformation

$$t = \varphi(x, y_1, y_2, y_3), u_1 = \psi_1(x, y_1, y_2, y_3), u_2 = \psi_2(x, y_1, y_2, y_3), u_3 = \psi_3(x, y_1, y_2, y_3),$$

where K are in Jordan Normal Forms matrices

$$K = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}, \quad K = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_2 \end{bmatrix}, \quad K = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 1 \\ 0 & 0 & \lambda_2 \end{bmatrix},$$

$$K = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}, \quad K = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}, \quad K = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix},$$

and $\lambda, \lambda_1, \lambda_2$ and λ_3 are constants.

2.2 To find the linearizing point transformation.

2.3 To find some examples and applications.

2.4 To construct the testing program to check the linearizable criteria.

3. Methodology

3.1 Study the structure and properties of system of three second-order ordinary differential equations, transformations, Jordan Normal Form and corresponding research.

3.2 Find the necessary conditions for linearization by using changed of derivatives.

3.3 Find the sufficient conditions for linearization by using compatibility theory.

3.4 Find the point transformation.

3.5 Construct the testing program for checking the linearizable criteria.

3.6 Find some examples and applications.

3.7 Summary, preparing documents for publications and writing a research report.

4. Schedule for the entire project and expected outputs

1st year (2012)

Activities and procedures	2012 (The first 6 months)					
	1	2	3	4	5	6
1. Study the structure and properties of system of three second-order ordinary differential equations, transformations, Jordan Normal Form and corresponding research.						
2. Find the necessary conditions for linearization for all cases.						
3. Report the progress of the project in the first 6 months.						
Activities and procedures	**2012 (The last 6 months)**					
	7	8	9	10	11	12
1. Classify all possible cases for linearization.						
2. Invent and research on the sufficient conditions for linearization in some cases.						
3. Find the linearizing point transformation in some cases.						
4. Find some examples.						
5. Prepare documents for publications and submit to publication in international journals.						

6. Report the progress of the project in the last 6 months.						

2nd year (2013)

Activities and procedures	2013 (The first 6 months)					
	1	2	3	4	5	6
1. Continue solve the sufficient conditions for linearization in the remaining cases.						
2. Find the linearizing point transformation in the remaining cases.						
3. Report the progress of the project in the first 6 months						
Activities and procedures	**2013 (The last 6 months)**					
	7	8	9	10	11	12
1. Solve all cases of sufficient conditions for linearization.						
2. Construct the testing program for checking the linearizable criteria.						
3. Find some examples and applications.						
4. Prepare documents for publications and submit to publication in international journals.						
5. Report the completed results to TRF.						

5. Expected result to be published in international journals

Title : Linearization of system of three second-order ordinary differential equations via point transformations.

International journals: Communications in Nonlinear Science and Numerical Simulations

Title : Completed criteria for system of three second-order ordinary differential equations to be linearizable by point transformations.

International journals : Journal of Applied Mathematics

6. Budget details

List	1st year	2nd year	Total
1. Honorarium			
- compensation for the principal investigator	120,000	120,000	240,000
2. Materials			
- office material	5,000	5,000	10,000
- Mathematical books and journals	8,000	8,000	16,000
- computer material***	10,000	10,000	20,000
(Ram, Harddisk, Sound Cards, VGA Cards)			
3. Expenses			
- Travel expenses for research and presentations in the country			
transportation	15,000	15,000	30,000
accommodation	11,000	11,000	22,000
food expenses	3,500	3,500	7,000
- the postage	1,000	1,000	2,000
- the copier	20,000	20,000	40,000
- the report	1,500	1,500	3,000
- page charge for publication	20,000	20,000	40,000
4. Wage			
- daily wages pay for student to do research (2 people x 200 bath x 125 days)	25,000	25,000	50,000
Total project budget	240,000	240,000	480,000

เนื้อหางานวิจัย

เนื้อหางานวิจัยจะแบ่งเป็น 3 ส่วนใหญ่ ได้แก่

1. ความรู้พื้นฐาน
2. ผลลัพธ์ในส่วนของ Linearization of system of three second-order ordinary differential equations via point transformations
3. ผลลัพธ์ในส่วนของ Group classification of systems of three second-order ordinary differential equations

(ดูรายระเอียดทั้งหมดดังเอกสารแนบหน้าถัดไป)