

Chapter 2

Main Results

2.1 On characterizations of regular ordered semigroups in terms of Q -fuzzy subsets

We published the first paper on this topic. Our main results of this topic is to introduced the concept of Q -fuzzy generalized bi-ideals of an ordered semigroup. Regular ordered semigroups are characterized by means of Q -fuzzy left ideals, Q -fuzzy right ideals and Q -fuzzy (generalized) bi-ideals.

Main Theorems

- 2.1.1) Let S be an ordered semigroup and Q be a non-empty set. Then the following statements are equivalent:
- (1) S is regular,
 - (2) $f = f \circ S \circ f$ for any Q -fuzzy bi-ideal f of S .
- 2.1.2) Let S be an ordered semigroup and Q be a non-empty set. Then the following statements are equivalent:
- (1) S is regular,
 - (2) $f = f \circ S \circ f$ for any Q -fuzzy generalized bi-ideal f of S .
- 2.1.3) Let S be an ordered semigroup and Q be a non-empty set. Then the following statements are equivalent:
- (1) S is regular,
 - (2) $f \circ g \circ f = f \cap g$ for any Q -fuzzy bi-ideal f and any Q -fuzzy ideal g of S .
- 2.1.4) Let S be an ordered semigroup and Q be a non-empty set. Then the following statements are equivalent:
- (1) S is regular,
 - (2) $f \circ g \circ f = f \cap g$ for any Q -fuzzy generalized bi-ideal f and any Q -fuzzy ideal g of S .

2.1.5) Let S be an ordered semigroup and Q be a non-empty set. Then the following statements are equivalent:

- (1) S is regular,
- (2) $f \cap g \subseteq f \circ g$ for every Q -fuzzy generalized bi-ideal f and every Q -fuzzy left ideal g of S .
- (3) $f \cap g \subseteq f \circ g$ for every Q -fuzzy bi-ideal f and every Q -fuzzy left ideal g of S .
- (4) $h \cap f \cap g = h \circ f \circ g$ for every Q -fuzzy generalized bi-ideal f , every Q -fuzzy left ideal g and every Q -fuzzy right ideal h of S .
- (5) $h \cap f \cap g = h \circ f \circ g$ for every Q -fuzzy bi-ideal f , every Q -fuzzy left ideal g and every Q -fuzzy right ideal h of S .

2.2 On Q -fuzzy ideals in ordered semigroups

We published the first paper on this topic. In this topic we shows that in ordered groupoids the Q -fuzzy right (resp. Q -fuzzy left) ideals are Q -fuzzy quasi-ideals, in ordered semigroups the Q -fuzzy quasi-ideals are Q -fuzzy bi-ideals, and in regular ordered semigroups the Q -fuzzy quasi-ideals and the Q -fuzzy bi-ideals coincide and show that if S is an ordered semigroup, then a Q -fuzzy subset f is a quasi-ideal of S if and only if there exist a Q -fuzzy right ideal g and a Q -fuzzy left ideal h of S such that $f = g \cap h$.

Main Theorems

2.2.1) If S is an ordered semigroup and Q is a non-empty set, then the Q -fuzzy right (resp. left) ideals of S are Q -fuzzy quasi-ideals of S .

2.2.2) If S is an ordered semigroup and Q is a non-empty set, then the Q -fuzzy quasi-ideals of S are Q -fuzzy bi-ideals of S .

2.2.3) If S is a regular ordered semigroup and Q is a non-empty set, then the Q -fuzzy bi-ideals of S are Q -fuzzy quasi-ideals of S .

2.2.4) Let S and Q be an ordered semigroup and a non-empty set, respectively. Then a Q -fuzzy subset of S is Q -fuzzy quasi-ideals of S if and only if there exist a Q -fuzzy right ideal g and a Q -fuzzy left ideal h of S such that $f = g \cap h$.