Songklanakarin J. Sci. Technol.
38 (5), 569-574, Sep. - Oct. 2016

http://www.sjst.psu.ac.th

Original Article

Bivariate copulas on the exponentially weighted moving average control chart

Sasigarn Kuvattana[1], Piyapatr Busababodhin[2], Yupaporn Areepong[1], and Saowanit Sukparungsee[1]*

[1] *Department of Applied Statistics, Faculty of Applied Science, King Mongkut's University of Technology North Bangkok, Bang Sue, Bangkok, 10800 Thailand.*

[2] *Department of Mathematics, Faculty of Science, Mahasarakham University, Kantharawichai, Maha Sarakham, 41150 Thailand.*

Received: 9 October 2015; Accepted: 4 February 2016

Abstract

This paper proposes four types of copulas on the Exponentially Weighted Moving Average (EWMA) control chart when observations are from an exponential distribution using a Monte Carlo simulation approach. The performance of the control chart is based on the Average Run Length (*ARL*) which is compared for each copula. Copula functions for specifying dependence between random variables are used and measured by Kendall's tau. The results show that the Normal copula can be used for almost all shifts.

Keywords: ARL, copula, EWMA control chart, Monte Carlo simulation

1. Introduction

Control charts are statistical and visual tools that are used in the monitoring of the quality of production from industry manufacturing processes. They are designed and evaluated under the assumption of observations by the process. Multivariate Statistical Process Control (MSPC) is a valuable method when several process variables are being monitored (Prabhu and Runger, 1997). The MSPC are used as the relationships between random variables that are sensitive to assignable causes and they are poorly detected by univariate control charts on individual variables. Multivariate control charts are generalizations of their univariate counterparts (Mahmoud and Maravelakis, 2013), e.g. the Hotelling T^2 control chart first introduced by Hotelling (1947); the Multivariate Exponentially Weighted Moving Average (MEWMA) control chart proposed by Lowry *et al.* (1992) and the Multivariate Cumulative Sum (MCUSUM) control chart suggested by Crosier (1988). The MEWMA and MCUSUM control charts are commonly used to detect small or moderate shifts in the mean vectors (see Midi and Shabbak, 2011; Runger *et al.*, 1999). Most multivariate detection procedures are based on a multi-normality assumption and independence but many processes are often non-normal and correlated. Many multivariate control charts have a lack of related joint distribution but copulas can specify this property.

The copula approach is a method for modeling non-linearity, asymmetricality and tail dependence in several fields; it can be used in the study of dependence or association between random variables. The copula approach is based on a representation from Sklar's theorem (Sklar, 1973) and copula theory is the formalization of the separated correlation of a multivariate distribution from the marginal distributions that make up the multivariate distribution. If two or more variables are correlated, a joint distribution can be constructed from the marginal distributions of variables. A bivariate copulas approach is the simplest case for the description of dependent random variables and it can apply to control charts

* Corresponding author.
Email address: saowanit.s@sci.kmutnb.ac.th

(Kuvattana *et al.*, 2015). Recent papers have used copulas on control charts including: copula based bivariate ZIP control chart (Fatahi *et al.*, 2011; Fatahi *et al.*, 2012); copula Markov CUSUM chart (Dokouhaki and Noorossana, 2013); Shewhart control charts for autocorrelated and normal data (Hryniewicz, 2012), and copulas of non-normal multivariate cases for the Hotelling T^2 control chart (Verdier, 2013).

According to above papers, differences in distribution can be found in control charts. In the manufacturing process, the time is used to represent some attributes or variable measures which are observed as consecutive events of concern. When the probability of the event in the next small time interval does not vary through time, the distribution of the time for an event is known as an exponential distribution. It is a continuous distribution and widely known in the monitoring of time for successive occurrences of events. This paper therefore presents work on the Multivariate Exponentially Weighted Moving Average control chart when observation are generated by an exponential distribution and uses bivariate copulas for specifying dependence between random variables.

2. The Multivariate Exponentially Weighted Moving Average Control Chart

The Multivariate Exponentially Weighted Moving Average (MEWMA) chart is a standard tool in statistical quality control introduced by Lowry *et al.* (1992). Given observations W_1, W_2, ... from a d-variate Gaussian distribution $\mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, define recursively, for $i = 1, 2, \ldots$,

$$\mathbf{Z}_i = \Lambda \mathbf{W}_i + (1-\Lambda)\mathbf{Z}_{i-1} \tag{1}$$

where $\mathbf{Z}_0$ is the vector of variable values from the historical data, and Λ is a diagonal matrix with entries $\lambda_1, \ldots, \lambda_d$. The quantity to be plotted is

$$T_i^2 = \mathbf{Z}_i' \textstyle\sum_i^{-1} \mathbf{Z}_i \tag{2}$$

where

$$\sum_i = \frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]\sum \tag{3}$$

when $\lambda_1 = \ldots = \lambda_d = \lambda \in (0,1)$, as assumed in the present paper. The control chart signals a shift in the mean vector when $T_i^2 > h$, where h is the control limit chosen to achieve a desired in-control

The performance of the MEWMA in detecting changes in the mean is generally measured by the Average Run Length (*ARL*) as a function of the difference $\boldsymbol{\mu} - \boldsymbol{\mu}_0$ between the target mean $\boldsymbol{\mu}_0$ and its real value $\boldsymbol{\mu}$. The *ARL* further depends on the degree of dependence between the variables, measured by the covariance matrix $\boldsymbol{\Sigma}$, and the scalar charting weight λ associated to the past observations.

Note that

1. if $\lambda = 1$ in (1), the MEWMA control chart statistic reduces to $T_i^2 = \mathbf{Z}_i' \sum_i^{-1} \mathbf{Z}_i$, the statistic used in the Hotelling T^2 control chart (Runger *et al.*, 1999; Montgomery, c2009).
2. This paper considers the bivariate Exponentially Weighted Moving Average control chart and will extend to the multivariate case for future work.

3. Copula Function

Considering a bivariate case, If $H(x,y)$ is the joint distribution of the random vector (X,Y), with continuous marginal distributions $F(x) = H(x,\infty)$, $G(y) = H(\infty, y)$ of *X, Y*, respectively, then there exists a unique copula $C(u,v)$ such that $H(x,y) = C\left(F(x), G(y); \theta\right)$ where $C(.,.)$ is determined by

$$C(u,v) = H(F^{-1}(u), G^{-1}(v))$$

where $F^{-1}(.)$, $G^{-1}(.)$ are quantiles functions of $F(.)$, $G(.)$, respectively, which are defined as

$$F^{-1} : [0,1] \to \mathbb{R}$$

$$F^{-1}(u) = \inf\left\{x \in \mathbb{R} \ : \ F(x) \ge u\right\}$$

For the purposes of the statistical method, it is desirable to parameterize the copula function. Let θ denote the association parameter of the bivariate distribution, and there exists a copula *C* (Trivedi and Zimmer, 2007). This paper focuses on the Normal copula and three types of Archimedean copulas, namely the Clayton, Frank and Gumbel copulas, respectively because these copulas are well-known. (Genest and MacKay, 1986).

3.1 Normal copula

The Normal copula is an elliptical copula. From the bivariate Normal distribution with zero means, unit variances and 2×2 correlation matrix $\boldsymbol{\Sigma}$, the Normal copula determined by:

$$C(u,v;\ \boldsymbol{\Sigma}) = \Phi(\Phi^{-1}(u), \Phi^{-1}(v);\ \boldsymbol{\Sigma});\ \ -1 \le \theta \le 1, \tag{4}$$

where $\Phi(.,\ \boldsymbol{\Sigma})$ is the bivariate normal cumulative distribution function, Φ is the univariate normal cumulative distribution function, Φ^{-1} is the univariate normal inverse cumulative distribution function or quantiles function $\theta = (\mu_X, \mu_Y, \sigma_X, \sigma_Y, \rho)$ and (Joe, 2015).

3.2 Archimedean copulas

Consider a class Φ of functions $\phi : [0,1] \to [0,\infty]$ with a continuous, strict decrease, such that $\phi(1) = 0$, $\phi'(t) < 0$ and $\phi''(t) > 0$ for all $0 < t < 1$ (Genest and MacKay, 1986; Genest and Rivest, 1993; Nelsen, 2006). There are three types of Archimedean copulas and these types are generated as follows:

3.2.1 Clayton copula

$$C(u,v) = \left[max(u^{-\theta} + v^{-\theta} - 1,\ 0) \right]^{-1/\theta},$$

$$\phi(t) = (t^{-\theta} - 1) / \theta \ ; \ \theta \in [-1, \infty) \setminus 0. \quad (5)$$

3.2.2 Frank copula

$$C(u,v) = -\frac{1}{\theta} ln(1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{e^{-\theta} - 1}),$$

$$\phi(t) = -ln(\frac{e^{-\theta t} - 1}{e^{-\theta} - 1}) \ ; \ \theta \in (-\infty, \infty) \setminus 0. \quad (6)$$

3.2.3 Gumbel copula

$$C(u,v) = \exp(-[(-ln\ \mathrm{u})^{\theta} + (-ln\ \mathrm{v})^{\theta}]^{1/\theta}),$$

$$\phi(t) = [-ln(t)]^{\theta};\ \theta \in [1, \infty). \quad (7)$$

4. Dependence Measures for Data

Theoretically, a parametric measure of the linear dependence between random variables is measured by a correlation coefficient whereas nonparametric measures of dependence are measured by Spearman's rho and Kendall's tau. According to the earlier literature, the copulas can be used in the study of dependence or association between random variables and the values of Kendall's tau are easy to calculate so this measure is used for observation dependencies.

Let X and Y be continuous random variables whose copula is C then Kendall's tau for X and Y is given by $\tau_c = 4\iint_{\mathbf{I}^2} C(u,v)\ dC(u,v)$ - 1 where τ_c is Kendall's tau of copula C and the unit square $\mathbf{I}^2$ is the product $\mathbf{I} \times \mathbf{I}$ where $\mathbf{I} = [0,1]$ and the expected value of the function $C(u,v)$ of uniform (0,1) random variables U and V whose joint distribution function is C i.e., $\tau_c = 4E[C(U,V)] - 1$ (Nelsen, 2006).

Archimedean copula C is generated by ϕ, then $\tau_{Arch} = 4\int_0^1 \frac{\phi(t)}{\phi'(t)}\ dt + 1$ where τ_{Arch} is Kendall's tau of Archimedean copula C (Genest and MacKay, 1986).

From all above literature about dependence measures for data, we can apply the copula function as shown in Table 1.

5. Average Run Length and Monte Carlo Simulation

The performance of a control chart is measured by the Average Run Length (*ARL*). *ARL* is the average number of points that must be plotted before a point indicates an out-of-control condition. *ARL* is classified into ARL_0 and ARL_1, where ARL_0 is the Average Run Length when the process is in-control and ARL_1 is the Average Run Length when the process is out-of-control (Busaba *et al.*, 2012).

In this paper we used a Monte Carlo simulation by using R statistical software (see Yan, 2007; Hofert and Mächler, 2011; Mächler and Zurich, 2011; Hofert *et al.*, 2012) with 50,000 simulations and a sample size of 1,000. Observations were from an exponential distribution with parameter $\alpha = 1$. A shift size is reported in terms of the quantity $\delta = \mu - \mu_0$ and large values of δ correspond to bigger shifts in the mean. The value $\delta = 0$ and the process mean $\mu = 1$ are in-control. The process means are defined as 1.5, 2, 2.5, 3, 4 and 5 for out-of-control processes. The simulation experiments were carried out to assess the performance of the MEWMA control chart with $\lambda = 0.05$ and 0.1. Copula estimations are restricted to the cases of dependence (positive and negative dependence). For all copula models, the setting θ corresponds with Kendall's tau. The level of dependence is measured by Kendall's tau values $(-1 \le \tau \le 1)$, which are defined to 0.5 and -0.5 for moderate dependence.

6. Research Results

The simulation results are presented in Tables 2 to 5 and the results are only empirical. The different values of exponential parameters denote μ_1 for the variables X and μ_2 for the variables Y. For in-control process, the MEWMA control chart was chosen by setting the desired $ARL_0 = 370$ for each copula. Tables 2 and 3 show moderate positive dependence $(\tau = 0.5)$, and Tables 4 and 5 show moderate negative dependence $(\tau = -0.5)$. Tables 2 to 5 show that the ARL_1 values of $\lambda = 0.05$ give less than $\lambda = 0.1$ for all cases. The result in Table 2 shows the mean shifts of $\tau = 0.5$ when

Table 1. Kendall's tau of copula function.

Copula	Kendall's tau	Parameter space of θ
Normal	$arcsin(\theta) / (\pi / 2)$	$[-1, 1]$
Clayton	$\theta / (\theta + 2)$	$[-1, \infty) \setminus \{0\}$
Frank	$1 + 4\left(\frac{1}{\theta} \int_0^{\theta} \frac{\mathrm{t}}{\mathrm{e}^{\mathrm{t}} - 1}\ \mathrm{dt}\ \text{-}1\right) / \theta$	$(-\infty, \infty) \setminus \{0\}$
Gumbel	$(\theta - 1) / \theta$	$[1, \infty)$

Table 2. *ARL* of the MEWMA control chart with Kendall's tau value equal to 0.5 in the case of one exponential parameter.

Parameters		ARL_0 and ARL_1 ($\tau = 0.5$)							
μ_1	μ_2	$\lambda = 0.05$				$\lambda = 0.1$			
		Normal	Clayton	Frank	Gumbel	Normal	Clayton	Frank	Gumbel
1	**1**	**370.156**	**370.060**	**369.857**	**370.002**	**370.087**	**370.172**	**369.823**	**369.830**
1	1.5	**94.668**	100.708	98.999	96.584	110.275	109.723	111.496	114.150
1	2	**29.412**	32.832	31.764	29.536	37.065	40.099	39.587	36.810
1	2.5	12.261	14.116	13.544	**12.166**	15.919	17.862	17.414	15.617
1	3	6.218	7.264	6.921	**6.016**	8.353	9.437	9.218	8.134
1	4	2.135	2.556	2.381	**2.091**	3.055	3.653	3.494	2.935
1	5	1.114	1.282	1.232	**1.100**	1.544	1.769	1.727	1.505
1	1	370.156	370.060	369.857	370.002	370.087	370.172	369.823	369.830
1.5	1	**96.096**	101.038	99.059	97.368	110.439	109.459	110.930	114.103
2	1	29.537	32.924	31.386	**29.447**	37.049	40.526	39.643	37.190
2.5	1	12.305	14.202	13.428	**12.114**	15.894	17.958	17.352	15.761
3	1	6.216	7.399	6.936	**6.039**	8.353	9.626	9.209	8.196
4	1	2.159	2.558	2.376	**2.122**	3.079	3.688	3.451	2.957
5	1	1.123	1.298	1.209	**1.097**	1.559	1.779	1.719	1.501

Table 3. *ARL* of the MEWMA control chart with Kendall's tau value equal to 0.5 in the case of two exponential parameters

Parameters		ARL_0 and ARL_1 ($\tau = 0.5$)							
μ_1	μ_2	$\lambda = 0.05$				$\lambda = 0.1$			
		Normal	Clayton	Frank	Gumbel	Normal	Clayton	Frank	Gumbel
1	**1**	**370.156**	**370.060**	**369.857**	**370.002**	**370.087**	**370.172**	**369.823**	**369.830**
1.5	1.5	**51.746**	51.781	52.322	55.436	64.579	59.223	61.268	71.133
2	2	**13.908**	14.588	14.535	14.792	18.728	18.668	18.977	20.224
2.5	2.5	**5.336**	5.923	5.752	5.669	7.753	8.049	8.110	8.156
3	3	**2.479**	2.912	2.808	2.581	3.895	4.211	4.224	4.064
4	4	**0.765**	0.973	0.889	0.804	1.268	1.503	1.467	1.276
5	5	**0.355**	0.449	0.428	0.380	0.564	0.689	0.662	0.578

μ_1 is fixed at 1, the values of the Normal copula are less than the other copulas for small shifts ($\mu_1 = 1,\ 1.5 \le \mu_2 \le 2$) and the ARL_1 values of the Gumbel copula are less than the other copulas for moderate and large shifts ($\mu_1 = 1,\ 2.5 \le \mu_2 \le 5$). When μ_2 is fixed at 1, the ARL_1 values of the Normal copula are less than the other copulas for small shifts ($\mu_1 = 1.5$, $\mu_2 = 1$) and the ARL_1 values of the Gumbel copula are less than the other copulas for moderate and large shifts ($2 \le \mu_1 \le 5, \mu_2 = 1$). In Table 3, in the case of the same shifts in both exponential parameters of $\tau = 0.5$, the ARL_1 values of the Normal copula are less than the other copulas for all shifts. Table 4 shows that for the mean shifts of $\tau = -0.5$, the ARL_1 values of the Normal copula are less than the other copulas for all shifts. In Table 5, in the case of the same shifts in both exponential parameters of $\tau = -0.5$, the ARL_1 values of the Normal copula are less than the other copulas for small and moderate shifts ($1.5 \le \mu_1 \le 3,\ 1.5 \le \mu_2 \le 3$) and the ARL_1 values of the Frank copula are less than the other copulas for large shifts ($4 \le \mu_1 \le 5,\ 4 \le \mu_2 \le 5$).

7. Conclusions

Dependence measures of two or more variables can be investigated in terms of various copulas. We consider 4 types of copulas because these copulas are well-known for operators and compare with Elliptical and Archimedean copulas. No research can be found regarding the use of the copula function in the literature addressing MEWMA control charts. This paper shows multivariate exponentially weighted moving average (MEWMA) control charts for four types of copulas and the level of dependence is measured by Kendall's tau values. The results revealed that it is necessary to detect

Table 4. *ARL* of the MEWMA control chart with Kendall's tau value equal to -0.5 in the case of one exponential parameter

Parameters		ARL_0 and ARL_1 (τ = -0.5)					
		λ=0.05			λ=0.1		
μ_1	μ_2	Normal	Clayton	Frank	Normal	Clayton	Frank
1	**1**	**370.101**	**370.085**	**370.066**	**369.997**	**370.097**	**370.006**
1	1.5	**95.500**	102.359	97.488	108.255	117.774	110.485
1	2	**30.538**	33.325	31.208	37.381	42.635	38.630
1	2.5	**13.028**	14.552	13.336	16.431	18.485	17.070
1	3	**6.665**	7.433	6.858	8.744	9.749	9.046
1	4	**2.331**	2.537	2.414	3.318	3.772	3.447
1	5	**1.215**	1.303	1.233	1.677	1.784	1.704
1	1	370.101	370.085	370.066	369.997	370.097	370.006
1.5	1	**95.569**	102.952	97.330	108.393	117.551	112.144
2	1	**30.527**	33.831	31.126	37.554	42.599	39.181
2.5	1	**12.993**	14.670	13.479	16.397	18.726	17.214
3	1	**6.668**	7.486	6.839	8.720	9.901	9.132
4	1	**2.355**	2.553	2.395	3.246	3.805	3.438
5	1	**1.219**	1.300	1.243	1.619	1.779	1.686

Table 5. *ARL* of the MEWMA control chart with Kendall's tau value equal to -0.5 in the case of two exponential parameters

Parameters		ARL_0 and ARL_1 (τ = -0.5)					
		λ=0.05			λ=0.1		
μ_1	μ_2	Normal	Clayton	Frank	Normal	Clayton	Frank
1	**1**	**370.101**	**370.085**	**370.066**	**369.997**	**370.097**	**370.006**
1.5	1.5	**48.372**	52.222	49.772	58.604	65.926	60.917
2	2	**12.751**	13.804	13.113	16.948	19.121	17.754
2.5	2.5	**4.785**	5.180	4.890	6.896	7.644	7.199
3	3	**2.230**	2.380	2.279	3.385	3.747	3.521
4	4	0.679	0.523	**0.678**	1.121	1.110	1.147
5	5	0.304	0.209	**0.299**	0.475	0.355	0.481

the dependence of the observation to indicate the copula which fits the observation. For moderate dependence, the Normal copula can be used for almost all shifts.

Acknowledgements

The authors would like to express their gratitude to the Ministry of Science and Technology, Thailand, and the Graduate College, King Mongkut's University of Technology North Bangkok, for their financial support.

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