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ภาคผนวก

ในส่วนของภาพผนวกจะแสดงรายละเอียดของโปรแกรม SCILAB ที่ใช้ในการคำนวณเพื่อแก้ปัญหา ศักย์ของสมการปัวซง โดยแบ่งเนื้อหาออกเป็น 2 ส่วนคือ ส่วนโปรแกรมหลัก และส่วนของโปรแกรมย่อย ดังต่อไปนี้

ส่วนที่ 1 โปรแกรมหลัก

```

N=10;
[ep,w] = Gaussip(N);
n=16 ;// number of boundary node
Xb=coorx(n);
Yb=coory(n);
for i=1:n
X(i,1)=Xb(i,1); // Only on boundary
Y(i,1)=Yb(i,1); // Only on boundary
end
l=7;
Xin1=[1.5 1.2 .6 0 .9 .3 0];
Yin1=[0 -.35 -.45 -.45 0 0 0];
Xin=Xin1';
Yin=Yin1';
Xbi=[-1.5;-1.2;-0.6;-0.6;1.2;1.5;1.2;0.6;0;-0.6;-1.2;-0.9;-0.3;0;0.3;0.9];
Ybi=[0;-0.35;-0.45;-0.45;-0.45;-0.35;0;0.35;0.45;0.45;0.45;0.35;0;0;0;0];
for i=1:17
Xb(n+i,1)=Xbi(i);
Yb(n+i,1)=Ybi(i);
end
// Select the kind of right hand side of POISSON's equation
disp(' Enter type of function(b) in RHS of POISSON eqs by ');
disp(' Enter 1 if b=-2, 2 if b=-x and 3 if b=-x^2: ');
typ=input('->');
if typ==1
disp(' Welcome to subprogram LqBEM 1.1 ');
disp(' The right hand side of the equation is b=-2 ');
elseif typ==2
disp(' Welcome to subprogram LqBEM 1.2 ');
disp(' The right hand side of the equation is b=-x ');
else

```

```

disp(' Welcome to subprogram LqBEM 1.3 ');
disp(' The right hand side of the equation is b=-x^2');
end
// compute alpha's step
b=valueb(Xb,Yb,typ);
phi1=matrixphi(Xb,Yb,typ);
alpha=inv(phi1)*b;
//*****
// Construct distance d(i,j) from a node i to an element j
// and l(j) is a length of element j
//*****
d=dist(n,X,Y);
L=lenet(n,X,Y);
//Construct the matrix H,G,Uhat and Qhat for solving equation
Hhatq=matrixhq(n,X,Y,ep,w)/2;
Gq=matrixgq(n,X,Y,ep,w)/2;
Qhatin=matrixqhatin(Xb,Yb,X,Y,L);
Uhat=matrixuhat(Xb,Yb,X,Y);
Qhat=matrixqhat(Xb,Yb,X,Y,L);
// Computing the value of RHS of equation
RHS=(Hhatq*Uhat-Gq*Qhatin)*alpha;
// Because of the given boundary condition , HU(on LHS of )=0
// So we next compute Q by follows:
U=zeros(n,1); // by boundary condition
Qq=-(inv(Gq)*RHS);
nod=1:1:n;
node=(nod');
disp('_____')
disp(' node X Y potential pot. deriv.')
```

```

for i=1:n
solu(i,1)=node(i,1);
solu(i,2)=Xb(i,1);
solu(i,3)=Yb(i,1);
solu(i,4)=U(i,1);
solu(i,5)=Q(i,1);
end
solu
```

```

// ----- step 2 : Construct matrix H and G -----
din=distin(n,l,Xin,Yin,X,Y);// is dist (xin,yin) to each element
H2=matrixhq2(n,l,X,Y,Xin,Yin,ep,w)/2;
G2=matrixgq2(n,l,X,Y,Xin,Yin,ep,w)/2;
Ual=marixUnew(n,l,Xin,Yin,Xb,Yb);
// ----- step 3 : compute the solutions at each int node -----
Uval=G2*Qq-H2*U+(H2*Uhat-G2*Qhatin)*alpha+(Ual*alpha);
// ----- Consider the Exact solution -----
if typ==1
for i=1:l
Uvalex(i,1)=-0.8*(Xin(i)^2/4+Yin(i)^2-1);
end
elseif typ==2
for i=1:l
Uvalex(i,1)=-(2*Xin(i)/7)*(Xin(i)^2/4+Yin(i)^2-1);
end
else
for i=1:l
Uvalex(i,1)=(-1/246)*(50*Xin(i)^2-8*Yin(i)^2+33.6)*(Xin(i)^2/4+Yin(i)^2-1);
end
end
disp('----- The solutions are shown as follows -----')
nod=1:l;
node=(nod');
disp(' node X Y U-Solutions. U-Exact.')
for i=1:l
solu1(i,1)=node(i,1);
solu1(i,2)=Xin(i,1);
solu1(i,3)=Yin(i,1);
solu1(i,4)=Uval(i,1);
solu1(i,5)=Uvalex(i,1);
end
solu1
disp('*****')
////////// Graph of Normal derivative //////////
D=[1:l];
grph1=Q;

```

```

if typ==1
for i=1:n
grph2(i)=-0.2*(X(i)^2+8*Y(i)^2);
end
elseif typ==2
for i=1:n
grph2(i)=(X(i)*((-6*X(i)^2/28)-(2*Y(i)^2/7)+(2/7))-(16*X(i)*(Y(i)^2/7))/sqrt(X(i)^2+16*Y(i)^2);
end
else
for i=1:n
grph2(i)=(1/246)*(-50*X(i)^3-96*X(i)*Y(i)^2+83.2*X(i))*(X(i)/2)+(1/246)*(-96*X(i)^2*Y(i)+32*Y(i)^3-
83.2*Y(i))*Y(i);
end
end
plot(D,grph1,'r-.s',D,grph2,'b:');
xlabel('Node');ylabel('Normal derivative');
if typ==1
title('Normal Derivative from QBEM31');
legend('QBEM 3.1','Exact');
elseif typ==2
title('Normal Derivative from QBEM32');
legend('QBEM 3.2','Exact');
else
title('Normal Derivative from QBEM33');
legend('QBEM 3.3','Exact');
end

```

ส่วนที่ 2 โปรแกรมย่อย

```

function Xb=coorx(n)
k=[0:360/n:360];
for i=1:(n/4)
Xb(i)=-2*sin((90-k(i))*%pi/180);
end
q2=1;
for i=(n/4)+2:2*(n/4)
Xb(i)=-Xb((n/4)+1-q2);

```

```

q2=q2+1;
end
q3=1;
for i=2*(n/4)+2:3*(n/4)
Xb(i)=Xb(2*(n/4)+1-q3);
q3=q3+1;
end
q4=1;
for i=3*(n/4)+2:n
Xb(i)=-Xb(3*(n/4)+1-q4);
q4=q4+1;
end
Xb(n/4+1)=0;
Xb(2*(n/4)+1)=-Xb(1);
Xb(3*(n/4)+1)=0;
endfunction

```

```

function Yb=coory(n)
for i=1:2*(n/4)+1
Yb(i)=-sqrt(1-((Xb(i)^2)/4));
end
for i=2*(n/4)+1:n
Yb(i)=sqrt(1-((Xb(i)^2)/4));
end
endfunction

```

```

function [p] = Lgndrp(N)
if N <= 0 then p = 1;
elseif N == 1 then p = [1,0];
else
p = ((2*N-1)*[Lgndrp(N-1) 0]-(N-1)*[0 0 Lgndrp(N-2)])/N;
end;
endfunction

```

```

function [ep,w] = Gaussip(N)
if N < 0 then

```

```

else
ep = roots(Lgndrp(N))';
A(1,:) = ones(1,N); b(1) = 2;
for n = 2:N
A(n,:) = A(n-1,:).*ep;
if modulo(n,2)==0 then b(1,n) = 0;
else b(1,n) = 2/n;
end;
end;
w = b/A';
end;
endfunction

function [Hhatq]=matrixhq(n,X,Y,ep,w)
Hhatq=zeros(n);
// Next we shall construct H-hat by considering in 3 parts
// first for R(i,j+1), second for R(i,j) and three for R(i,j-1)
// We use 10-point Gaussian quadrature method to compute
for g=1:10
F1(g)=0.5*((ep(g))^2-ep(g));
F2(g)=1-(ep(g))^2;
F3(g)=0.5*((ep(g))^2+ep(g));
end
//***** j=1 *****
for i=1:n
for j=1
// **** first consider for R(i,j+1)
for g=1:10
A=X(j)-2*X(j+1)+X(j+2);
B=0.5*(X(j+2)-X(j));
C=Y(j)-2*Y(j+1)+Y(j+2);
D=0.5*(Y(j+2)-Y(j));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j)*F1(g)+X(j+1)*F2(g)+X(j+2)*F3(g);
y1(g)=Y(j)*F1(g)+Y(j+1)*F2(g)+Y(j+2)*F3(g);
R2ij1(g)=(X(i)-x1(g))^2+(Y(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;

```

```

dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);
dudx(g)=x1(g)-X(i); dudy(g)=y1(g)-Y(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S1(g)=(func(g)*Jco(g)*w(g)*F1(g))/R2ij1(g);
end
ss1=sum(S1);
// **** next consider for R(i,j)
for g=1:10
A=X(n)-2*X(j)+X(j+1);
B=0.5*(X(j+1)-X(n));
C=Y(n)-2*Y(j)+Y(j+1);
D=0.5*(Y(j+1)-Y(n));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(n)*F1(g)+X(j)*F2(g)+X(j+1)*F3(g);
y1(g)=Y(n)*F1(g)+Y(j)*F2(g)+Y(j+1)*F3(g);
R2ij1(g)=(X(i)-x1(g))^2+(Y(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);
dudx(g)=x1(g)-X(i); dudy(g)=y1(g)-Y(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S2(g)=(func(g)*Jco(g)*w(g)*F2(g))/R2ij1(g);
end
ss2=sum(S2);
// **** next consider for R(i,j-1)
for g=1:10
A=X(n-1)-2*X(n)+X(j);
B=0.5*(X(j)-X(n-1));
C=Y(n-1)-2*Y(n)+Y(j);
D=0.5*(Y(j)-Y(n-1));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(n-1)*F1(g)+X(n)*F2(g)+X(j)*F3(g);
y1(g)=Y(n-1)*F1(g)+Y(n)*F2(g)+Y(j)*F3(g);
R2ij1(g)=(X(i)-x1(g))^2+(Y(i)-y1(g))^2;

```

```

dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);
dudx(g)=x1(g)-X(i); dudy(g)=y1(g)-Y(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S3(g)=(func(g)*Jco(g)*w(g)*F3(g))/R2ij1(g);
end
ss3=sum(S3);
Hhatq(i,j)=-0.5*(ss1+ss2+ss3)/%pi;
end
end
//***** j=2 ***
for i=1:n
for j=2
// **** first consider for R(i,j+1)
for g=1:10
A=X(j)-2*X(j+1)+X(j+2);
B=0.5*(X(j+2)-X(j));
C=Y(j)-2*Y(j+1)+Y(j+2);
D=0.5*(Y(j+2)-Y(j));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j)*F1(g)+X(j+1)*F2(g)+X(j+2)*F3(g);
y1(g)=Y(j)*F1(g)+Y(j+1)*F2(g)+Y(j+2)*F3(g);
R2ij1(g)=(X(i)-x1(g))^2+(Y(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);
dudx(g)=x1(g)-X(i); dudy(g)=y1(g)-Y(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S1(g)=(func(g)*Jco(g)*w(g)*F1(g))/R2ij1(g);
end
ss1=sum(S1);
// **** next consider for R(i,j)
for g=1:10
A=X(j-1)-2*X(j)+X(j+1);

```

```

B=0.5*(X(j+1)-X(j-1));
C=Y(j-1)-2*Y(j)+Y(j+1);
D=0.5*(Y(j+1)-Y(j-1));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-1)*F1(g)+X(j)*F2(g)+X(j+1)*F3(g);
y1(g)=Y(j-1)*F1(g)+Y(j)*F2(g)+Y(j+1)*F3(g);
R2ij1(g)=(X(i)-x1(g))^2+(Y(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);
dudx(g)=x1(g)-X(i); dudy(g)=y1(g)-Y(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S2(g)=(func(g)*Jco(g)*w(g)*F2(g))/R2ij1(g);
end
ss2=sum(S2);
/**** next consider for R(i,j-1)
for g=1:10
A=X(n)-2*X(j-1)+X(j);
B=0.5*(X(j)-X(n));
C=Y(n)-2*Y(j-1)+Y(j);
D=0.5*(Y(j)-Y(n));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(n)*F1(g)+X(j-1)*F2(g)+X(j)*F3(g);
y1(g)=Y(n)*F1(g)+Y(j-1)*F2(g)+Y(j)*F3(g);
R2ij1(g)=(X(i)-x1(g))^2+(Y(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);
dudx(g)=x1(g)-X(i); dudy(g)=y1(g)-Y(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S3(g)=(func(g)*Jco(g)*w(g)*F3(g))/R2ij1(g);
end
ss3=sum(S3);
/*****
Hhatq(i,j)=-0.5*(ss1+ss2+ss3)/%pi;

```

```

end
end
//***** j=3-n-2 *****
for i=1:n
for j=3:n-2
// **** first consider for R(i,j+1)
for g=1:10
A=X(j)-2*X(j+1)+X(j+2);
B=0.5*(X(j+2)-X(j));
C=Y(j)-2*Y(j+1)+Y(j+2);
D=0.5*(Y(j+2)-Y(j));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j)*F1(g)+X(j+1)*F2(g)+X(j+2)*F3(g);
y1(g)=Y(j)*F1(g)+Y(j+1)*F2(g)+Y(j+2)*F3(g);
R2ij1(g)=(X(i)-x1(g))^2+(Y(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);
dudx(g)=x1(g)-X(i); dudy(g)=y1(g)-Y(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S1(g)=(func(g)*Jco(g)*w(g)*F1(g))/R2ij1(g);
end
ss1=sum(S1);
// **** next consider for R(i,j)
for g=1:10
A=X(j-1)-2*X(j)+X(j+1);
B=0.5*(X(j+1)-X(j-1));
C=Y(j-1)-2*Y(j)+Y(j+1);
D=0.5*(Y(j+1)-Y(j-1));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-1)*F1(g)+X(j)*F2(g)+X(j+1)*F3(g);
y1(g)=Y(j-1)*F1(g)+Y(j)*F2(g)+Y(j+1)*F3(g);
R2ij1(g)=(X(i)-x1(g))^2+(Y(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);

```

```

ny(g)=- (dxds(g))/Jco(g);
dudx(g)=x1(g)-X(i); dudy(g)=y1(g)-Y(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S2(g)=(func(g)*Jco(g)*w(g)*F2(g))/R2ij1(g);
end
ss2=sum(S2);
//**** next consider for R(i,j-1)
for g=1:10
A=X(j-2)-2*X(j-1)+X(j);
B=0.5*(X(j)-X(j-2));
C=Y(j-2)-2*Y(j-1)+Y(j);
D=0.5*(Y(j)-Y(j-2));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-2)*F1(g)+X(j-1)*F2(g)+X(j)*F3(g);
y1(g)=Y(j-2)*F1(g)+Y(j-1)*F2(g)+Y(j)*F3(g);
R2ij1(g)=(X(i)-x1(g))^2+(Y(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=- (dxds(g))/Jco(g);
dudx(g)=x1(g)-X(i); dudy(g)=y1(g)-Y(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g); S3(g)=(func(g)*Jco(g)*w(g)*F3(g))/R2ij1(g);
end
ss3=sum(S3);
//*****
Hhatq(i,j)=-0.5*(ss1+ss2+ss3)/%pi;
end
end
//***** j= n-1 *****
for i=1:n
// when j=2...n-2
for j=n-1
// **** first consider for R(i,j+1)
for g=1:10
A=X(j)-2*X(j+1)+X(1);
B=0.5*(X(1)-X(j));
C=Y(j)-2*Y(j+1)+Y(1);

```



```

D=0.5*(Y(1)-Y(j));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j)*F1(g)+X(j+1)*F2(g)+X(1)*F3(g);
y1(g)=Y(j)*F1(g)+Y(j+1)*F2(g)+Y(1)*F3(g);
R2ij1(g)=(X(i)-x1(g))^2+(Y(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);
dudx(g)=x1(g)-X(i); dudy(g)=y1(g)-Y(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S1(g)=(func(g)*Jco(g)*w(g)*F1(g))/R2ij1(g);
end
ss1=sum(S1);
//**** next consider for R(i,j)
for g=1:10
A=X(j-1)-2*X(j)+X(j+1);
B=0.5*(X(j+1)-X(j-1));
C=Y(j-1)-2*Y(j)+Y(j+1);
D=0.5*(Y(j+1)-Y(j-1));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-1)*F1(g)+X(j)*F2(g)+X(j+1)*F3(g);
y1(g)=Y(j-1)*F1(g)+Y(j)*F2(g)+Y(j+1)*F3(g);
R2ij1(g)=(X(i)-x1(g))^2+(Y(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);
dudx(g)=x1(g)-X(i); dudy(g)=y1(g)-Y(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S2(g)=(func(g)*Jco(g)*w(g)*F2(g))/R2ij1(g);
end
ss2=sum(S2);
//**** next consider for R(i,j-1)
for g=1:10
A=X(j-2)-2*X(j-1)+X(j);
B=0.5*(X(j)-X(j-2));

```

```

C=Y(j-2)-2*Y(j-1)+Y(j);
D=0.5*(Y(j)-Y(j-2));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-2)*F1(g)+X(j-1)*F2(g)+X(j)*F3(g);
y1(g)=Y(j-2)*F1(g)+Y(j-1)*F2(g)+Y(j)*F3(g);
R2ij1(g)=(X(i)-x1(g))^2+(Y(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);
dudx(g)=x1(g)-X(i); dudy(g)=y1(g)-Y(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S3(g)=(func(g)*Jco(g)*w(g)*F3(g))/R2ij1(g);
end
ss3=sum(S3);
//*****
Hhatq(i,j)=-0.5*(ss1+ss2+ss3)/%pi;
end
end
//***** j= n *****
for i=1:n
// when j=2...n-2
for j=n
// **** first consider for R(i,j+1)
for g=1:10
A=X(j)-2*X(1)+X(2);
B=0.5*(X(2)-X(j));
C=Y(j)-2*Y(1)+Y(2);
D=0.5*(Y(2)-Y(j));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j)*F1(g)+X(1)*F2(g)+X(2)*F3(g);
y1(g)=Y(j)*F1(g)+Y(1)*F2(g)+Y(2)*F3(g);
R2ij1(g)=(X(i)-x1(g))^2+(Y(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);

```

```

dudx(g)=x1(g)-X(i); dudy(g)=y1(g)-Y(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S1(g)=(func(g)*Jco(g)*w(g)*F1(g))/R2ij1(g);
end
ss1=sum(S1);
// **** next consider for R(i,j)
for g=1:10
A=X(j-1)-2*X(j)+X(1);
B=0.5*(X(1)-X(j-1));
C=Y(j-1)-2*Y(j)+Y(1);
D=0.5*(Y(1)-Y(j-1));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-1)*F1(g)+X(j)*F2(g)+X(1)*F3(g);
y1(g)=Y(j-1)*F1(g)+Y(j)*F2(g)+Y(1)*F3(g);
R2ij1(g)=(X(i)-x1(g))^2+(Y(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);
dudx(g)=x1(g)-X(i); dudy(g)=y1(g)-Y(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S2(g)=(func(g)*Jco(g)*w(g)*F2(g))/R2ij1(g);
end
ss2=sum(S2);
// **** next consider for R(i,j-1)
for g=1:10
A=X(j-2)-2*X(j-1)+X(j);
B=0.5*(X(j)-X(j-2));
C=Y(j-2)-2*Y(j-1)+Y(j);
D=0.5*(Y(j)-Y(j-2));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-2)*F1(g)+X(j-1)*F2(g)+X(j)*F3(g);
y1(g)=Y(j-2)*F1(g)+Y(j-1)*F2(g)+Y(j)*F3(g);
R2ij1(g)=(X(i)-x1(g))^2+(Y(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);

```

```

ny(g)=- (dxds(g))/Jco(g);
dudx(g)=x1(g)-X(i); dudy(g)=y1(g)-Y(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S3(g)=(func(g)*Jco(g)*w(g)*F3(g))/R2ij1(g);
end
ss3=sum(S3);
/*****

Hhatq(i,j)=-0.5*(ss1+ss2+ss3)/%pi;
end
end
for i=1:n
Hhatq(i,i)=0;
end
// Next we consider Hhat's values when i=j
Hhat1=sum(Hhatq','m');
for i=1:n
Hhatq(i,i)=-Hhat1(1,i);
end
endfunction

/*****

function [Gq]=matrixgq(n,X,Y,ep,w,L)
/*****
Gq=zeros(n,n);
// Next we shall construct G by considering in 3 parts
// first for R(i,j+1), second for R(i,j) and three for R(i,j-1)
// We use 10-point Gaussian quadrature method to compute
/*****
for g=1:10
F1(g)=0.5*((ep(g))^2-ep(g));
F2(g)=1-(ep(g))^2;
F3(g)=0.5*((ep(g))^2+ep(g));
end
/***** j=1 *****/
for i=1:n
for j=1

```

```

// **** first consider for R(i,j+1)
for g=1:10
A=X(j)-2*X(j+1)+X(j+2);
B=0.5*(X(j+2)-X(j));
C=Y(j)-2*Y(j+1)+Y(j+2);
D=0.5*(Y(j+2)-Y(j));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j)*F1(g)+X(j+1)*F2(g)+X(j+2)*F3(g);
y1(g)=Y(j)*F1(g)+Y(j+1)*F2(g)+Y(j+2)*F3(g);
R2ij1(g)=sqrt((X(i)-x1(g))^2+(Y(i)-y1(g))^2);
if i ==j
S1(g)=(-0.5*Jco(g)*w(g)*F1(g)*log(R2ij1(g)))/%pi;
SS1=0;
else
dxds=-0.5*(3*X(j)-4*X(j+1)+X(j+2));
dyds=-0.5*(3*Y(j)-4*Y(j+1)+Y(j+2));
jacob1=sqrt((dxds)^2+(dyds)^2);
SS1=(jacob1)/%pi;
red(g)=0.5*(1+ep(g));
SS2(g)=-0.5*w(g)*log(red(g))*(jacob1)/%pi;
P1(g)=(-0.5*Jco(g)*w(g)*F1(g)*log(R2ij1(g)))/%pi;
S1(g)=P1(g)-SS2(g) ;
end
end
ss1=sum(S1)+SS1;
// **** next consider for R(i,j)
for g=1:10
A=X(n)-2*X(j)+X(j+1);
B=0.5*(X(j+1)-X(n));
C=Y(n)-2*Y(j)+Y(j+1);
D=0.5*(Y(j+1)-Y(n));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(n)*F1(g)+X(j)*F2(g)+X(j+1)*F3(g);
y1(g)=Y(n)*F1(g)+Y(j)*F2(g)+Y(j+1)*F3(g);
R2ij1(g)=sqrt((X(i)-x1(g))^2+(Y(i)-y1(g))^2);
if i ==j
S2(g)=(-0.5*Jco(g)*w(g)*F2(g)*log(R2ij1(g)))/%pi;

```

```

SS1=0;
else
dxds=0.5*(X(j+1)-X(n));
dyds=0.5*(Y(j+1)-Y(n));
jacob1=sqrt((dxds)^2+(dyds)^2);
SS1=(jacob1)/%pi;
red(g)=abs(ep(g));
SS2(g)=-0.5*w(g)*log(red(g))*(jacob1)/%pi;
P2(g)=-0.5*Jco(g)*w(g)*F2(g)*log(R2ij1(g))/%pi;
S2(g)=P2(g)- SS2(g);
end
end
ss2=sum(S2)+SS1;
**** next consider for R(i,j-1)
for g=1:10
A=X(n-1)-2*X(n)+X(j);
B=0.5*(X(j)-X(n-1));
C=Y(n-1)-2*Y(n)+Y(j);
D=0.5*(Y(j)-Y(n-1));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(n-1)*F1(g)+X(n)*F2(g)+X(j)*F3(g);
y1(g)=Y(n-1)*F1(g)+Y(n)*F2(g)+Y(j)*F3(g);
R2ij1(g)=sqrt((X(i)-x1(g))^2+(Y(i)-y1(g))^2);
if i =j
S3(g)=-0.5*Jco(g)*w(g)*F3(g)*log(R2ij1(g))/%pi;
SS1=0;
else
dxds=0.5*(X(n-1)-4*X(n)+3*X(j));
dyds=0.5*(Y(n-1)-4*Y(n)+3*Y(j));
jacob1=sqrt((dxds)^2+(dyds)^2);
SS1=(jacob1)/%pi;
red(g)=0.5*(1-ep(g));
SS3(g)=-0.5*w(g)*log(red(g))*(jacob1)/%pi;
P3(g)=-0.5*Jco(g)*w(g)*F3(g)*log(R2ij1(g))/%pi;
S3(g)=P3(g)- SS3(g);
end
end

```

```

ss3=sum(S3)+SS1;
//*****
Gq(i,j)=(ss1+ss2+ss3);
end
end
//***** j=2 *****
for i=1:n
for j=2
// **** first consider for R(i,j+1)
for g=1:10
A=X(j)-2*X(j+1)+X(j+2);
B=0.5*(X(j+2)-X(j));
C=Y(j)-2*Y(j+1)+Y(j+2);
D=0.5*(Y(j+2)-Y(j));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j)*F1(g)+X(j+1)*F2(g)+X(j+2)*F3(g);
y1(g)=Y(j)*F1(g)+Y(j+1)*F2(g)+Y(j+2)*F3(g);
R2ij1(g)=sqrt((X(i)-x1(g))^2+(Y(i)-y1(g))^2);
if i ==j
S1(g)=(-0.5*Jco(g)*w(g)*F1(g)*log(R2ij1(g)))/%pi;
SS1=0;
else
dxds=-0.5*(3*X(j)-4*X(j+1)+X(j+2));
dyds=-0.5*(3*Y(j)-4*Y(j+1)+Y(j+2));
jacob1=sqrt((dxds)^2+(dyds)^2);
SS1=(jacob1)/%pi;
red(g)=0.5*(1+ep(g));
SS2(g)=-0.5*w(g)*log(red(g))*jacob1/%pi;
P1(g)=(-0.5*Jco(g)*w(g)*F1(g)*log(R2ij1(g)))/%pi;
S1(g)=P1(g)-SS2(g) ;
end
end
ss1=sum(S1)+SS1;
// **** next consider for R(i,j)
for g=1:10
A=X(j-1)-2*X(j)+X(j+1);
B=0.5*(X(j+1)-X(j-1));

```

```

C=Y(j-1)-2*Y(j)+Y(j+1);
D=0.5*(Y(j+1)-Y(j-1));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-1)*F1(g)+X(j)*F2(g)+X(j+1)*F3(g);
y1(g)=Y(j-1)*F1(g)+Y(j)*F2(g)+Y(j+1)*F3(g);
R2ij1(g)=sqrt((X(i)-x1(g))^2+(Y(i)-y1(g))^2);
if i =j
S2(g)=(-0.5*Jco(g)*w(g)*F2(g)*log(R2ij1(g)))/%pi;
SS1=0;
else
dxds=0.5*(X(j+1)-X(j-1));
dyds=0.5*(Y(j+1)-Y(j-1));
jacob1=sqrt((dxds)^2+(dyds)^2);
SS1=(jacob1)/%pi;
red(g)=abs(ep(g));
SS2(g)=-0.5*w(g)*log(red(g))*(jacob1)/%pi;
P2(g)=-0.5*Jco(g)*w(g)*F2(g)*log(R2ij1(g))/%pi;
S2(g)=P2(g)- SS2(g);
end
end
ss2=sum(S2)+SS1;
/**** next consider for R(i,j-1)
for g=1:10
A=X(n)-2*X(j-1)+X(j);
B=0.5*(X(j)-X(n));
C=Y(n)-2*Y(j-1)+Y(j);
D=0.5*(Y(j)-Y(n));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(n)*F1(g)+X(j-1)*F2(g)+X(j)*F3(g);
y1(g)=Y(n)*F1(g)+Y(j-1)*F2(g)+Y(j)*F3(g);
R2ij1(g)=sqrt((X(i)-x1(g))^2+(Y(i)-y1(g))^2);
if i =j
S3(g)=-0.5*Jco(g)*w(g)*F3(g)*log(R2ij1(g))/%pi;
SS1=0;
else
dxds=0.5*(X(n)-4*X(j-1)+3*X(j));
dyds=0.5*(Y(n)-4*Y(j-1)+3*Y(j));

```

```

jacob1=sqrt((dxds)^2+(dyds)^2);
SS1=(jacob1)/%pi;
red(g)=0.5*(1-ep(g));
SS3(g)=-0.5*w(g)*log(red(g))*(jacob1)/%pi;
P3(g)=-0.5*Jco(g)*w(g)*F3(g)*log(R2ij1(g))/%pi;
S3(g)=P3(g)- SS3(g);
end
end
ss3=sum(S3)+SS1;
//*****
Gq(i,j)=(ss1+ss2+ss3);
end
end
//***** j=2-(n-2) *****
for i=1:n
for j=3:n-2
// **** first consider for R(i,j+1)
for g=1:10
A=X(j)-2*X(j+1)+X(j+2);
B=0.5*(X(j+2)-X(j));
C=Y(j)-2*Y(j+1)+Y(j+2);
D=0.5*(Y(j+2)-Y(j));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j)*F1(g)+X(j+1)*F2(g)+X(j+2)*F3(g);
y1(g)=Y(j)*F1(g)+Y(j+1)*F2(g)+Y(j+2)*F3(g);
R2ij1(g)=sqrt((X(i)-x1(g))^2+(Y(i)-y1(g))^2);
if i ==j
S1(g)=(-0.5*Jco(g)*w(g)*F1(g)*log(R2ij1(g)))/%pi;
SS1=0;
else
dxds=-0.5*(3*X(j)-4*X(j+1)+X(j+2));
dyds=-0.5*(3*Y(j)-4*Y(j+1)+Y(j+2));
jacob1=sqrt((dxds)^2+(dyds)^2);
SS1=(jacob1)/%pi;
red(g)=0.5*(1+ep(g));
SS2(g)=-0.5*w(g)*log(red(g))*(jacob1)/%pi;
P1(g)=(-0.5*Jco(g)*w(g)*F1(g)*log(R2ij1(g)))/%pi;

```

```

S1(g)=P1(g)-SS2(g) ;
end
end
ss1=sum(S1)+SS1;
// **** next consider for R(i,j)
for g=1:10
A=X(j-1)-2*X(j)+X(j+1);
B=0.5*(X(j+1)-X(j-1));
C=Y(j-1)-2*Y(j)+Y(j+1);
D=0.5*(Y(j+1)-Y(j-1));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-1)*F1(g)+X(j)*F2(g)+X(j+1)*F3(g);
y1(g)=Y(j-1)*F1(g)+Y(j)*F2(g)+Y(j+1)*F3(g);
R2ij1(g)=sqrt((X(i)-x1(g))^2+(Y(i)-y1(g))^2);
if i ==j
S2(g)=(-0.5*Jco(g)*w(g)*F2(g)*log(R2ij1(g)))/%pi;
SS1=0;
else
dxds=0.5*(X(j+1)-X(j-1));
dyds=0.5*(Y(j+1)-Y(j-1));
jacob1=sqrt((dxds)^2+(dyds)^2);
SS1=(jacob1)/%pi;
red(g)=abs(ep(g)); SS2(g)=-0.5*w(g)*log(red(g))*(jacob1)/%pi;
P2(g)=-0.5*Jco(g)*w(g)*F2(g)*log(R2ij1(g))/%pi;
S2(g)=P2(g)- SS2(g);
end
end
ss2=sum(S2)+SS1;
// **** next consider for R(i,j-1)
for g=1:10
A=X(j-2)-2*X(j-1)+X(j);
B=0.5*(X(j)-X(j-2));
C=Y(j-2)-2*Y(j-1)+Y(j);
D=0.5*(Y(j)-Y(j-2));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-2)*F1(g)+X(j-1)*F2(g)+X(j)*F3(g);
y1(g)=Y(j-2)*F1(g)+Y(j-1)*F2(g)+Y(j)*F3(g);

```

```

R2ij1(g)=sqrt((X(i)-x1(g))^2+(Y(i)-y1(g))^2);
if i =j
S3(g)=-0.5*Jco(g)*w(g)*F3(g)*log(R2ij1(g))/%pi;
SS1=0;
else
dxds=0.5*(X(j-2)-4*X(j-1)+3*X(j));
dyds=0.5*(Y(j-2)-4*Y(j-1)+3*Y(j));
jacob1=sqrt((dxds)^2+(dyds)^2);
SS1=(jacob1)/%pi;
red(g)=0.5*(1-ep(g));
SS3(g)=-0.5*w(g)*log(red(g))*(jacob1)/%pi;
P3(g)=-0.5*Jco(g)*w(g)*F3(g)*log(R2ij1(g))/%pi;
S3(g)=P3(g)- SS3(g);
end
end
ss3=sum(S3)+SS1;
//*****
Gq(i,j)=(ss1+ss2+ss3);
end
end
//***** j=n-1 *****
for i=1:n
for j=n-1
// **** first consider for R(i,j+1)
for g=1:10
A=X(j)-2*X(j+1)+X(1);
B=0.5*(X(1)-X(j));
C=Y(j)-2*Y(j+1)+Y(1);
D=0.5*(Y(1)-Y(j));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j)*F1(g)+X(j+1)*F2(g)+X(1)*F3(g);
y1(g)=Y(j)*F1(g)+Y(j+1)*F2(g)+Y(1)*F3(g);
R2ij1(g)=sqrt((X(i)-x1(g))^2+(Y(i)-y1(g))^2);
if i =j S1(g)=(-0.5*Jco(g)*w(g)*F1(g)*log(R2ij1(g)))/%pi;
SS1=0;
else
dxds=-0.5*(3*X(j)-4*X(j+1)+X(1));

```

```

dyds=-0.5*(3*Y(j)-4*Y(j+1)+Y(1));
jacob1=sqrt((dxds)^2+(dyds)^2);
SS1=(jacob1)/%pi;
red(g)=0.5*(1+ep(g));
SS2(g)=-0.5*w(g)*log(red(g))*(jacob1)/%pi;
P1(g)=(-0.5*Jco(g)*w(g)*F1(g)*log(R2ij1(g)))/%pi;
S1(g)=P1(g)-SS2(g) ;
end
end
ss1=sum(S1)+SS1;
// **** next consider for R(i,j)
for g=1:10
A=X(j-1)-2*X(j)+X(j+1);
B=0.5*(X(j+1)-X(j-1));
C=Y(j-1)-2*Y(j)+Y(j+1);
D=0.5*(Y(j+1)-Y(j-1));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-1)*F1(g)+X(j)*F2(g)+X(j+1)*F3(g);
y1(g)=Y(j-1)*F1(g)+Y(j)*F2(g)+Y(j+1)*F3(g);
R2ij1(g)=sqrt((X(i)-x1(g))^2+(Y(i)-y1(g))^2);
if i ==j
S2(g)=(-0.5*Jco(g)*w(g)*F2(g)*log(R2ij1(g)))/%pi;
SS1=0;
else
dxds=0.5*(X(j+1)-X(j-1));
dyds=0.5*(Y(j+1)-Y(j-1));
jacob1=sqrt((dxds)^2+(dyds)^2);
SS1=(jacob1)/%pi;
red(g)=abs(ep(g));
SS2(g)=-0.5*w(g)*log(red(g))*(jacob1)/%pi;
P2(g)=-0.5*Jco(g)*w(g)*F2(g)*log(R2ij1(g)))/%pi;
S2(g)=P2(g)- SS2(g);
end
end
ss2=sum(S2)+SS1;
//**** next consider for R(i,j-1)
for g=1:10

```

```

A=X(j-2)-2*X(j-1)+X(j);
B=0.5*(X(j)-X(j-2));
C=Y(j-2)-2*Y(j-1)+Y(j);
D=0.5*(Y(j)-Y(j-2));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-2)*F1(g)+X(j-1)*F2(g)+X(j)*F3(g);
y1(g)=Y(j-2)*F1(g)+Y(j-1)*F2(g)+Y(j)*F3(g);
R2ij1(g)=sqrt((X(i)-x1(g))^2+(Y(i)-y1(g))^2);
if i =j
S3(g)=-0.5*Jco(g)*w(g)*F3(g)*log(R2ij1(g))/%pi;
SS1=0;
else
dxds=0.5*(X(j-2)-4*X(j-1)+3*X(j));
dyds=0.5*(Y(j-2)-4*Y(j-1)+3*Y(j));
jacob1=sqrt((dxds)^2+(dyds)^2);
SS1=(jacob1)/%pi;
red(g)=0.5*(1-ep(g));
SS3(g)=-0.5*w(g)*log(red(g))*(jacob1)/%pi;
P3(g)=-0.5*Jco(g)*w(g)*F3(g)*log(R2ij1(g))/%pi;
S3(g)=P3(g)- SS3(g);
end
end
ss3=sum(S3)+SS1;
/*****
Gq(i,j)=(ss1+ss2+ss3);
end
end
/***** j=n *****/
for i=1:n
for j=n
// **** first consider for R(i,j+1)
for g=1:10
A=X(j)-2*X(1)+X(2);
B=0.5*(X(2)-X(j));
C=Y(j)-2*Y(1)+Y(2);
D=0.5*(Y(2)-Y(j));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);

```

```

x1(g)=X(j)*F1(g)+X(1)*F2(g)+X(2)*F3(g);
y1(g)=Y(j)*F1(g)+Y(1)*F2(g)+Y(2)*F3(g);
R2ij1(g)=sqrt((X(i)-x1(g))^2+(Y(i)-y1(g))^2);
if i =j
S1(g)=(-0.5*Jco(g)*w(g)*F1(g)*log(R2ij1(g)))/%pi;
SS1=0;
else
dxds=-0.5*(3*X(j)-4*X(1)+X(2));
dyds=-0.5*(3*Y(j)-4*Y(1)+Y(2));
jacob1=sqrt((dxds)^2+(dyds)^2);
SS1=(jacob1)/%pi;
red(g)=0.5*(1+ep(g));
SS2(g)=-0.5*w(g)*log(red(g))*(jacob1)/%pi;
P1(g)=(-0.5*Jco(g)*w(g)*F1(g)*log(R2ij1(g)))/%pi;
S1(g)=P1(g)-SS2(g) ;
end
end
ss1=sum(S1)+SS1;
// **** next consider for R(i,j)
for g=1:10
A=X(j-1)-2*X(j)+X(1);
B=0.5*(X(1)-X(j-1));
C=Y(j-1)-2*Y(j)+Y(1);
D=0.5*(Y(1)-Y(j-1));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-1)*F1(g)+X(j)*F2(g)+X(1)*F3(g);
y1(g)=Y(j-1)*F1(g)+Y(j)*F2(g)+Y(1)*F3(g);
R2ij1(g)=sqrt((X(i)-x1(g))^2+(Y(i)-y1(g))^2);
if i =j
S2(g)=(-0.5*Jco(g)*w(g)*F2(g)*log(R2ij1(g)))/%pi;
SS1=0;
else
dxds=0.5*(X(1)-X(j-1));
dyds=0.5*(Y(1)-Y(j-1));
jacob1=sqrt((dxds)^2+(dyds)^2);
SS1=(jacob1)/%pi;
red(g)=abs(ep(g));

```

```

SS2(g)=-0.5*w(g)*log(red(g))*(jacob1)/%pi;
P2(g)=-0.5*Jco(g)*w(g)*F2(g)*log(R2ij1(g))/%pi;
S2(g)=P2(g)- SS2(g);
end
end
ss2=sum(S2)+SS1;
/**** next consider for R(i,j-1)
for g=1:10
A=X(j-2)-2*X(j-1)+X(j);
B=0.5*(X(j)-X(j-2));
C=Y(j-2)-2*Y(j-1)+Y(j);
D=0.5*(Y(j)-Y(j-2));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-2)*F1(g)+X(j-1)*F2(g)+X(j)*F3(g);
y1(g)=Y(j-2)*F1(g)+Y(j-1)*F2(g)+Y(j)*F3(g);
R2ij1(g)=sqrt((X(i)-x1(g))^2+(Y(i)-y1(g))^2);
if i =j
S3(g)=-0.5*Jco(g)*w(g)*F3(g)*log(R2ij1(g))/%pi;
SS1=0;
else
dxds=0.5*(X(j-2)-4*X(j-1)+3*X(j));
dyds=0.5*(Y(j-2)-4*Y(j-1)+3*Y(j));
jacob1=sqrt((dxds)^2+(dyds)^2);
SS1=(jacob1)/%pi;
red(g)=0.5*(1-ep(g));
SS3(g)=-0.5*w(g)*log(red(g))*(jacob1)/%pi;
P3(g)=-0.5*Jco(g)*w(g)*F3(g)*log(R2ij1(g))/%pi;
S3(g)=P3(g)- SS3(g);
end
end
ss3=sum(S3)+SS1;
/*****
Gq(i,j)=(ss1+ss2+ss3);
end
end
endfunction
/*****

```

```

function [Hhat]=matrixh(n,X,Y,d,ep,w,L)
Hhat=zeros(n);
x=zeros(n,1);
y=zeros(n,1);
// Compute x,y midpoint of each element (see p.39 Davies )
for i=1:n-1
x(i)=x(i)+(X(i)+X(i+1))/2;
y(i)=y(i)+(Y(i)+Y(i+1))/2;
end
x(n)=x(n)+(X(n)+X(1))/2;
y(n)=y(n)+(Y(n)+Y(1))/2;
// Next we shall construct H-hat by considering in 2 parts
// first for R(i,j) and second, for R(i,j-1)
// ***** For the element 1 to n-1 we obtain *****
for i=1:n
// when j=2...n-1
for j=2:n-1
// first consider for R(i,j)
for g=1:10
x1(g)=x(j)+(1/2)*((X(j+1)-X(j))*ep(g));
y1(g)=y(j)+(1/2)*((Y(j+1)-Y(j))*ep(g));
R2ij1(g)=(X(i)-x1(g))^2+(Y(i)-y1(g))^2;
S1(g)=w(g)*((1-ep(g))/R2ij1(g));
end
s1=sum(S1);
ss1=(-L(j)*d(i,j)/(4))*(s1);
// ***** next consider for R(i,j-1)
for g=1:10
x3(g)=x(j-1)+(1/2)*((X(j)-X(j-1))*ep(g));
y3(g)=y(j-1)+(1/2)*((Y(j)-Y(j-1))*ep(g));
R2ij3(g)=(X(i)-x3(g))^2+(Y(i)-y3(g))^2;
S3(g)=w(g)*((1+ep(g))/R2ij3(g));
end
s3=sum(S3);
//——
ss2=(-L(j)*d(i,j-1)/(4))*(s3);

```

```

//*****
Hhat(i,j)=(1/(2*%pi))*(ss1+ss2);
end
end
// ***** For an element j=1 *****
// first consider for R(i,j)
for i=1:n
for g=1:10
x1(g)=x(1)+(1/2)*((X(2)-X(1))*ep(g));
y1(g)=y(1)+(1/2)*((Y(2)-Y(1))*ep(g));
R2ij1(g)=(X(i)-x1(g))^2+(Y(i)-y1(g))^2;
S1(g)=w(g)*((1-ep(g))/R2ij1(g));
end
s1=sum(S1);
ss1=(-L(1)*d(i,1)/(4))*(s1);
// **** next consider for R(i,j-1)
for g=1:10
x3(g)=x(n)+(1/2)*((X(1)-X(n))*ep(g));
y3(g)=y(n)+(1/2)*((Y(1)-Y(n))*ep(g));
R2ij3(g)=(X(i)-x3(g))^2+(Y(i)-y3(g))^2;
S3(g)=w(g)*((1+ep(g))/R2ij3(g));
end
s3=sum(S3);
//—
ss2=(-L(1)*d(i,n)/(4))*(s3);
//*****
Hhat(i,1)=(1/(2*%pi))*(ss1+ss2);
end
//***** For an element j=n *****
for i=1:n
// first for R(i,j)
for g=1:10
x1(g)=x(n)+(1/2)*((X(1)-X(n))*ep(g));
y1(g)=y(n)+(1/2)*((Y(1)-Y(n))*ep(g));
R2ij1(g)=(X(i)-x1(g))^2+(Y(i)-y1(g))^2;
S1(g)=w(g)*((1-ep(g))/R2ij1(g));
end

```



```

s1=sum(S1);
//—
ss1=(-L(n)*d(i,n)/(4))*(s1);
// Next for R(i,j)
for g=1:10
x3(g)=x(n-1)+(1/2)*((X(n)-X(n-1))*ep(g));
y3(g)=y(n-1)+(1/2)*((Y(n)-Y(n-1))*ep(g));
R2ij3(g)=(X(i)-x3(g))^2+(Y(i)-y3(g))^2;
S3(g)=w(g)*((1+ep(g))/R2ij3(g));
end
s3=sum(S3);
//—
ss2=(-L(n)*d(i,n-1)/(4))*(s3);
//——
Hhat(i,n)=(1/(2*pi))*(ss1+ss2);
end
// Next we consider Hhat's values when i=j
Hhat1=sum(Hhat','m');
for i=1:n
Hhat(i,i)=-Hhat1(1,i);
end
endfunction

//*****

function [G]=matrixg(n,X,Y,d,ep,w,L)
G=zeros(n,2*n);
x=zeros(n,1);
y=zeros(n,1);
for i=1:n-1
x(i)=x(i)+(X(i)+X(i+1))/2;
y(i)=y(i)+(Y(i)+Y(i+1))/2;
end
x(n)=x(n)+(X(n)+X(1))/2;
y(n)=y(n)+(Y(n)+Y(1))/2;
for i=1:n
for j=1:n-1

```

```

for g=1:10
x1(g)=x(j)+(1/2)*((X(j+1)-X(j))*ep(g));
y1(g)=y(j)+(1/2)*((Y(j+1)-Y(j))*ep(g));
Rij1(g)=sqrt((X(i)-x1(g))^2+(Y(i)-y1(g))^2);
S1(g)=w(g)*(1-ep(g))*log(Rij1(g));
S2(g)=w(g)*(1+ep(g))*log(Rij1(g));
end
s1=sum(S1);
s2=sum(S2);
G(i,2*j-1)=(1/(2*pi))^(-L(j)/4)*s1;
G(i,2*j)=(1/(2*pi))^(-L(j)/4)*s2;
end
end
//consider for j=n
for i=1:n
for g=1:10
x1(g)=x(n)+(1/2)*((X(1)-X(n))*ep(g));
y1(g)=y(n)+(1/2)*((Y(1)-Y(n))*ep(g));
Rij1(g)=sqrt((X(i)-x1(g))^2+(Y(i)-y1(g))^2);
S1(g)=w(g)*(1-ep(g))*log(Rij1(g));
S2(g)=w(g)*(1+ep(g))*log(Rij1(g));
end
s1=sum(S1);
s2=sum(S2);
G(i,2*n-1)=(1/(2*pi))^(-L(n)/4)*s1;
G(i,2*n)=(1/(2*pi))^(-L(n)/4)*s2;
end
//In the case of i=j
for i=1:n
G(i,2*i-1)=(1/(2*pi))^(-L(i)/2)*(log(L(i))-(3/2));
G(i,2*i)=(1/(2*pi))^(-L(i)/2)*(log(L(i))-(1/2));
end
//From node i to an element i-1
for i=2:n
G(i,2*i-2)=(1/(2*pi))^(-L(i-1)/2)*(log(L(i-1))-(3/2));
G(i,2*i-3)=(1/(2*pi))^(-L(i-1)/2)*(log(L(i-1))-(1/2));
end
end

```

```

G(1,2*n)=(1/(2*%pi))*(-L(n)/2)*(log(L(n))-(3/2));
G(1,2*n-1)=(1/(2*%pi))*(-L(n)/2)*(log(L(n))-(1/2));
endfunction

//*****

function [Grd]=reduceG(n,X,Y,d,G,L,ep,w)
Grd=zeros(n);
x=zeros(n,1);
y=zeros(n,1);
for i=1:n-1
x(i)=x(i)+(X(i)+X(i+1))/2;
y(i)=y(i)+(Y(i)+Y(i+1))/2;
end
x(n)=x(n)+(X(n)+X(1))/2;
y(n)=y(n)+(Y(n)+Y(1))/2;
// Next we shall construct H-hat by considering in 2 parts
// first for R(i,j) and second, for R(i,j-1)
// ***** For the element 1 to n-1 we obtain *****
for i=1:n
// when j=2...n-1
for j=2:n-1
// first consider for R(i,j)
for g=1:10
x1(g)=x(j)+(1/2)*((X(j+1)-X(j))*ep(g));
y1(g)=y(j)+(1/2)*((Y(j+1)-Y(j))*ep(g));
Rij1(g)=sqrt((X(i)-x1(g))^2+(Y(i)-y1(g))^2);
S1(g)=w(g)*(1-ep(g))*log(Rij1(g));
end
s1=sum(S1);
ss1=(-L(j)/4)*(s1);
// **** next consider for R(i,j-1)
for g=1:10
x3(g)=x(j-1)+(1/2)*((X(j)-X(j-1))*ep(g));
y3(g)=y(j-1)+(1/2)*((Y(j)-Y(j-1))*ep(g));
Rij3(g)=sqrt((X(i)-x3(g))^2+(Y(i)-y3(g))^2);
S3(g)=w(g)*(1+ep(g))*log(Rij3(g));

```

```

end
s3=sum(S3);
//—
ss2=(-L(j)/4)*(s3);
//*****
Grd(i,j)=(1/(2*%pi))*(ss1+ss2);
end
end
// ***** For an element j=1 *****
// first consider for R(i,j)
for i=1:n
for g=1:10
x1(g)=x(1)+(1/2)*((X(2)-X(1))*ep(g));
y1(g)=y(1)+(1/2)*((Y(2)-Y(1))*ep(g));
Rij1(g)=sqrt((X(i)-x1(g))^2+(Y(i)-y1(g))^2); S1(g)=w(g)*(1-ep(g))*log(Rij1(g));
end
s1=sum(S1);
ss1=(-L(1)/4)*(s1);
// **** next consider for R(i,j-1)
for g=1:10
x3(g)=x(n)+(1/2)*((X(1)-X(n))*ep(g));
y3(g)=y(n)+(1/2)*((Y(1)-Y(n))*ep(g));
Rij3(g)=sqrt((X(i)-x3(g))^2+(Y(i)-y3(g))^2); S3(g)=w(g)*(1+ep(g))*log(Rij3(g));
end
s3=sum(S3);
ss2=(-L(1)/4)*(s3);
//*****
Grd(i,1)=(1/(2*%pi))*(ss1+ss2);
end
//***** For an element j=n *****
for i=1:n
// first for R(i,j)
for g=1:10
x1(g)=x(n)+(1/2)*((X(1)-X(n))*ep(g));
y1(g)=y(n)+(1/2)*((Y(1)-Y(n))*ep(g));
Rij1(g)=sqrt((X(i)-x1(g))^2+(Y(i)-y1(g))^2);
S1(g)=w(g)*(1-ep(g))*log(Rij1(g));

```

```

end
s1=sum(S1);
//—
ss1=(-L(n)/4)*(s1);
// Next for R(i,j)
for g=1:10
x3(g)=x(n-1)+(1/2)*((X(n)-X(n-1))*ep(g));
y3(g)=y(n-1)+(1/2)*((Y(n)-Y(n-1))*ep(g));
Rij3(g)=sqrt((X(i)-x3(g))^2+(Y(i)-y3(g))^2);
S3(g)=w(g)*(1+ep(g))*log(Rij3(g));
end
s3=sum(S3);
//—
ss2=(-L(n)/4)*(s3);
//——
Grd(i,n)=(1/(2*pi))*(ss1+ss2);
end
// Next we consider Hhat's values when i=j
for i=1:n
Grd(i,i)=(1/(2*pi))*(-(L(i)/2)*(log(L(i))-(3/2))-(L(i)/2)*(log(L(i))-(3/2)));
end
endfunction

//*****

function [H2]=matrixhq2(n,l,X,Y,Xin,Yin,ep,w)
H2=zeros(1,n);
// Next we shall construct H2 by considering in 3 parts
// first for R(i,j+1), second for R(i,j) and three for R(i,j-1)
// We use 10-point Gaussian quadrature method to compute
//*****

for g=1:10
F1(g)=0.5*((ep(g))^2-ep(g));
F2(g)=1-(ep(g))^2;
F3(g)=0.5*((ep(g))^2+ep(g));
end
//***** j=1 *****

```

```

for i=1:l
// when j=2...n-2
for j=1
// **** first consider for R(i,j+1)
for g=1:10
A=X(j)-2*X(j+1)+X(j+2);
B=0.5*(X(j+2)-X(j));
C=Y(j)-2*Y(j+1)+Y(j+2);
D=0.5*(Y(j+2)-Y(j));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j)*F1(g)+X(j+1)*F2(g)+X(j+2)*F3(g);
y1(g)=Y(j)*F1(g)+Y(j+1)*F2(g)+Y(j+2)*F3(g);
R2ij1(g)=(Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);
dudx(g)=x1(g)-Xin(i); dudy(g)=y1(g)-Yin(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S1(g)=(func(g)*Jco(g)*w(g)*F1(g))/R2ij1(g);
end
ss1=sum(S1);
// **** next consider for R(i,j)
for g=1:10
A=X(n)-2*X(j)+X(j+1);
B=0.5*(X(j+1)-X(n));
C=Y(n)-2*Y(j)+Y(j+1);
D=0.5*(Y(j+1)-Y(n));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(n)*F1(g)+X(j)*F2(g)+X(j+1)*F3(g);
y1(g)=Y(n)*F1(g)+Y(j)*F2(g)+Y(j+1)*F3(g);
R2ij1(g)=(Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);
dudx(g)=x1(g)-Xin(i); dudy(g)=y1(g)-Yin(i);

```



```

func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S2(g)=(func(g)*Jco(g)*w(g)*F2(g))/R2ij1(g);
end
ss2=sum(S2);
//**** next consider for R(i,j-1)
for g=1:10
A=X(n-1)-2*X(n)+X(j);
B=0.5*(X(j)-X(n-1));
C=Y(n-1)-2*Y(n)+Y(j);
D=0.5*(Y(j)-Y(n-1));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(n-1)*F1(g)+X(n)*F2(g)+X(j)*F3(g);
y1(g)=Y(n-1)*F1(g)+Y(n)*F2(g)+Y(j)*F3(g);
R2ij1(g)=(Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);
dudx(g)=x1(g)-Xin(i); dudy(g)=y1(g)-Yin(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S3(g)=(func(g)*Jco(g)*w(g)*F3(g))/R2ij1(g);
end
ss3=sum(S3);
//*****
H2(i,j)=-0.5*(ss1+ss2+ss3)/%pi;
end
end
//***** j=2 *****
for i=1:l
// when j=2...n-2
for j=2
// **** first consider for R(i,j+1)
for g=1:10
A=X(j)-2*X(j+1)+X(j+2);
B=0.5*(X(j+2)-X(j));
C=Y(j)-2*Y(j+1)+Y(j+2);
D=0.5*(Y(j+2)-Y(j));

```

```

Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j)*F1(g)+X(j+1)*F2(g)+X(j+2)*F3(g);
y1(g)=Y(j)*F1(g)+Y(j+1)*F2(g)+Y(j+2)*F3(g);
R2ij1(g)=(Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);
dudx(g)=x1(g)-Xin(i); dudy(g)=y1(g)-Yin(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S1(g)=(func(g)*Jco(g)*w(g)*F1(g))/R2ij1(g);
end
ss1=sum(S1);
// **** next consider for R(i,j)
for g=1:10
A=X(j-1)-2*X(j)+X(j+1);
B=0.5*(X(j+1)-X(j-1));
C=Y(j-1)-2*Y(j)+Y(j+1);
D=0.5*(Y(j+1)-Y(j-1));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-1)*F1(g)+X(j)*F2(g)+X(j+1)*F3(g);
y1(g)=Y(j-1)*F1(g)+Y(j)*F2(g)+Y(j+1)*F3(g);
R2ij1(g)=(Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);
dudx(g)=x1(g)-Xin(i); dudy(g)=y1(g)-Yin(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S2(g)=(func(g)*Jco(g)*w(g)*F2(g))/R2ij1(g);
end
ss2=sum(S2);
// **** next consider for R(i,j-1)
for g=1:10
A=X(n)-2*X(j-1)+X(j);
B=0.5*(X(j)-X(n));
C=Y(n)-2*Y(j-1)+Y(j);

```

```

D=0.5*(Y(j)-Y(n));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(n)*F1(g)+X(j-1)*F2(g)+X(j)*F3(g);
y1(g)=Y(n)*F1(g)+Y(j-1)*F2(g)+Y(j)*F3(g);
R2ij1(g)=(Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);
dudx(g)=x1(g)-Xin(i); dudy(g)=y1(g)-Yin(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S3(g)=(func(g)*Jco(g)*w(g)*F3(g))/R2ij1(g);
end
ss3=sum(S3);
//*****
H2(i,j)=-0.5*(ss1+ss2+ss3)/%pi;
end
end
//***** j=3-n-2 *****
for i=1:l
    /
    // when j=2...n-2
    for j=3:n-2
        // **** first consider for R(i,j+1)
        for g=1:10
            A=X(j)-2*X(j+1)+X(j+2);
            B=0.5*(X(j+2)-X(j));
            C=Y(j)-2*Y(j+1)+Y(j+2);
            D=0.5*(Y(j+2)-Y(j));
            Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
            x1(g)=X(j)*F1(g)+X(j+1)*F2(g)+X(j+2)*F3(g);
            y1(g)=Y(j)*F1(g)+Y(j+1)*F2(g)+Y(j+2)*F3(g);
            R2ij1(g)=(Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2;
            dxds(g)=A*ep(g)+B;
            dyds(g)=C*ep(g)+D;
            nx(g)=(dyds(g))/Jco(g);
            ny(g)=-(dxds(g))/Jco(g);
            dudx(g)=x1(g)-Xin(i); dudy(g)=y1(g)-Yin(i);

```

```

func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S1(g)=(func(g)*Jco(g)*w(g)*F1(g))/R2ij1(g);
end
ss1=sum(S1);
//**** next consider for R(i,j)
for g=1:10
A=X(j-1)-2*X(j)+X(j+1);
B=0.5*(X(j+1)-X(j-1));
C=Y(j-1)-2*Y(j)+Y(j+1);
D=0.5*(Y(j+1)-Y(j-1));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-1)*F1(g)+X(j)*F2(g)+X(j+1)*F3(g);
y1(g)=Y(j-1)*F1(g)+Y(j)*F2(g)+Y(j+1)*F3(g);
R2ij1(g)=(Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);
dudx(g)=x1(g)-Xin(i); dudy(g)=y1(g)-Yin(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S2(g)=(func(g)*Jco(g)*w(g)*F2(g))/R2ij1(g);
end
ss2=sum(S2);
//**** next consider for R(i,j-1)
for g=1:10
A=X(j-2)-2*X(j-1)+X(j);
B=0.5*(X(j)-X(j-2));
C=Y(j-2)-2*Y(j-1)+Y(j);
D=0.5*(Y(j)-Y(j-2));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-2)*F1(g)+X(j-1)*F2(g)+X(j)*F3(g);
y1(g)=Y(j-2)*F1(g)+Y(j-1)*F2(g)+Y(j)*F3(g);
R2ij1(g)=(Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);

```

```

dudx(g)=x1(g)-Xin(i); dudy(g)=y1(g)-Yin(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S3(g)=(func(g)*Jco(g)*w(g)*F3(g))/R2ij1(g);
end
ss3=sum(S3);
//*****
H2(i,j)=-0.5*(ss1+ss2+ss3)/%pi;
end
end
//***** j= n-1 *****
for i=1:l
// when j=2...n-2
for j=n-1
// **** first consider for R(i,j+1)
for g=1:10
A=X(j)-2*X(j+1)+X(1);
B=0.5*(X(1)-X(j));
C=Y(j)-2*Y(j+1)+Y(1);
D=0.5*(Y(1)-Y(j));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j)*F1(g)+X(j+1)*F2(g)+X(1)*F3(g);
y1(g)=Y(j)*F1(g)+Y(j+1)*F2(g)+Y(1)*F3(g);
R2ij1(g)=(Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);
dudx(g)=x1(g)-Xin(i); dudy(g)=y1(g)-Yin(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S1(g)=(func(g)*Jco(g)*w(g)*F1(g))/R2ij1(g);
end
ss1=sum(S1);
// **** next consider for R(i,j)
for g=1:10
A=X(j-1)-2*X(j)+X(j+1);
B=0.5*(X(j+1)-X(j-1));
C=Y(j-1)-2*Y(j)+Y(j+1);

```

```

D=0.5*(Y(j+1)-Y(j-1));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-1)*F1(g)+X(j)*F2(g)+X(j+1)*F3(g);
y1(g)=Y(j-1)*F1(g)+Y(j)*F2(g)+Y(j+1)*F3(g);
R2ij1(g)=(Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);
dudx(g)=x1(g)-Xin(i); dudy(g)=y1(g)-Yin(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S2(g)=(func(g)*Jco(g)*w(g)*F2(g))/R2ij1(g);
end
ss2=sum(S2);
/**** next consider for R(i,j-1)
for g=1:10
A=X(j-2)-2*X(j-1)+X(j);
B=0.5*(X(j)-X(j-2));
C=Y(j-2)-2*Y(j-1)+Y(j);
D=0.5*(Y(j)-Y(j-2));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-2)*F1(g)+X(j-1)*F2(g)+X(j)*F3(g);
y1(g)=Y(j-2)*F1(g)+Y(j-1)*F2(g)+Y(j)*F3(g);
R2ij1(g)=(Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);
dudx(g)=x1(g)-Xin(i); dudy(g)=y1(g)-Yin(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S3(g)=(func(g)*Jco(g)*w(g)*F3(g))/R2ij1(g);
end
ss3=sum(S3);
/*****
H2(i,j)=-0.5*(ss1+ss2+ss3)/%pi;
end
end

```

```

//***** j= n *****
for i=1:l
// when j=2...n-2
for j=n
// **** first consider for R(i,j+1)
for g=1:10
A=X(j)-2*X(1)+X(2);
B=0.5*(X(2)-X(j));
C=Y(j)-2*Y(1)+Y(2);
D=0.5*(Y(2)-Y(j));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j)*F1(g)+X(1)*F2(g)+X(2)*F3(g);
y1(g)=Y(j)*F1(g)+Y(1)*F2(g)+Y(2)*F3(g);
R2ij1(g)=(Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);
dudx(g)=x1(g)-Xin(i); dudy(g)=y1(g)-Yin(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S1(g)=(func(g)*Jco(g)*w(g)*F1(g))/R2ij1(g);
end
ss1=sum(S1);
// **** next consider for R(i,j)
for g=1:10
A=X(j-1)-2*X(j)+X(1);
B=0.5*(X(1)-X(j-1));
C=Y(j-1)-2*Y(j)+Y(1);
D=0.5*(Y(1)-Y(j-1));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-1)*F1(g)+X(j)*F2(g)+X(1)*F3(g);
y1(g)=Y(j-1)*F1(g)+Y(j)*F2(g)+Y(1)*F3(g);
R2ij1(g)=(Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);

```

```

dudx(g)=x1(g)-Xin(i); dudy(g)=y1(g)-Yin(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S2(g)=(func(g)*Jco(g)*w(g)*F2(g))/R2ij1(g);
end
ss2=sum(S2);
//**** next consider for R(i,j-1)
for g=1:10
A=X(j-2)-2*X(j-1)+X(j);
B=0.5*(X(j)-X(j-2));
C=Y(j-2)-2*Y(j-1)+Y(j);
D=0.5*(Y(j)-Y(j-2));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-2)*F1(g)+X(j-1)*F2(g)+X(j)*F3(g);
y1(g)=Y(j-2)*F1(g)+Y(j-1)*F2(g)+Y(j)*F3(g);
R2ij1(g)=(Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2;
dxds(g)=A*ep(g)+B;
dyds(g)=C*ep(g)+D;
nx(g)=(dyds(g))/Jco(g);
ny(g)=-(dxds(g))/Jco(g);
dudx(g)=x1(g)-Xin(i); dudy(g)=y1(g)-Yin(i);
func(g)=nx(g)*dudx(g)+ny(g)*dudy(g);
S3(g)=(func(g)*Jco(g)*w(g)*F3(g))/R2ij1(g);
end
ss3=sum(S3);
//*****

H2(i,j)=-0.5*(ss1+ss2+ss3)/%pi;
end
end
endfunction

//*****

function [G2]=matrixgq2(n,l,X,Y,Xin,Yin,ep,w)
//*****
G2=zeros(l,n);
// Next we shall construct G2 by condidering in 3 parts
// first for R(i,j+1), second for R(i,j) and three for R(i,j-1)

```

```
// We use 10-point Gaussian quadrature method to compute
```

```
//*****
```

```
for g=1:10
```

```
F1(g)=0.5*((ep(g))^2-ep(g));
```

```
F2(g)=1-(ep(g))^2;
```

```
F3(g)=0.5*((ep(g))^2+ep(g));
```

```
end
```

```
//***** j=1 *****
```

```
for i=1:l
```

```
// when j=2...n-2
```

```
for j=1
```

```
// **** first consider for R(i,j+1)
```

```
for g=1:10
```

```
A=X(j)-2*X(j+1)+X(j+2);
```

```
B=0.5*(X(j+2)-X(j));
```

```
C=Y(j)-2*Y(j+1)+Y(j+2);
```

```
D=0.5*(Y(j+2)-Y(j));
```

```
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
```

```
x1(g)=X(j)*F1(g)+X(j+1)*F2(g)+X(j+2)*F3(g);
```

```
y1(g)=Y(j)*F1(g)+Y(j+1)*F2(g)+Y(j+2)*F3(g);
```

```
R2ij1(g)=sqrt((Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2);
```

```
S1(g)=Jco(g)*w(g)*F1(g)*log(R2ij1(g));
```

```
end
```

```
ss1=sum(S1);
```

```
// **** next consider for R(i,j)
```

```
for g=1:10
```

```
A=X(n)-2*X(j)+X(j+1);
```

```
B=0.5*(X(j+1)-X(n));
```

```
C=Y(n)-2*Y(j)+Y(j+1);
```

```
D=0.5*(Y(j+1)-Y(n));
```

```
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
```

```
x1(g)=X(n)*F1(g)+X(j)*F2(g)+X(j+1)*F3(g);
```

```
y1(g)=Y(n)*F1(g)+Y(j)*F2(g)+Y(j+1)*F3(g);
```

```
R2ij1(g)=sqrt((Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2);
```

```
S2(g)=Jco(g)*w(g)*F2(g)*log(R2ij1(g));
```

```
end
```

```
ss2=sum(S2);
```

```

//**** next consider for R(i,j-1)
for g=1:10
A=X(n-1)-2*X(n)+X(j);
B=0.5*(X(j)-X(n-1));
C=Y(n-1)-2*Y(n)+Y(j);
D=0.5*(Y(j)-Y(n-1));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(n-1)*F1(g)+X(n)*F2(g)+X(j)*F3(g);
y1(g)=Y(n-1)*F1(g)+Y(n)*F2(g)+Y(j)*F3(g);
R2ij1(g)=sqrt((Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2);
S3(g)=Jco(g)*w(g)*F3(g)*log(R2ij1(g));
end
ss3=sum(S3);
//*****
G2(i,j)=-0.5*(ss1+ss2+ss3)/%pi;
end
end
//***** j=2 *****
for i=1:l
// when j=2...n-2
for j=2
// **** first consider for R(i,j+1)
for g=1:10
A=X(j)-2*X(j+1)+X(j+2);
B=0.5*(X(j+2)-X(j));
C=Y(j)-2*Y(j+1)+Y(j+2);
D=0.5*(Y(j+2)-Y(j));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j)*F1(g)+X(j+1)*F2(g)+X(j+2)*F3(g);
y1(g)=Y(j)*F1(g)+Y(j+1)*F2(g)+Y(j+2)*F3(g);
R2ij1(g)=sqrt((Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2);
S1(g)=Jco(g)*w(g)*F1(g)*log(R2ij1(g));
end
ss1=sum(S1);
// **** next consider for R(i,j)
for g=1:10
A=X(j-1)-2*X(j)+X(j+1);

```

```

B=0.5*(X(j+1)-X(j-1));
C=Y(j-1)-2*Y(j)+Y(j+1);
D=0.5*(Y(j+1)-Y(j-1));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-1)*F1(g)+X(j)*F2(g)+X(j+1)*F3(g);
y1(g)=Y(j-1)*F1(g)+Y(j)*F2(g)+Y(j+1)*F3(g);
R2ij1(g)=sqrt((Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2);
S2(g)=Jco(g)*w(g)*F2(g)*log(R2ij1(g));
end
ss2=sum(S2);
//**** next consider for R(i,j-1)
for g=1:10
A=X(n)-2*X(j-1)+X(j);
B=0.5*(X(j)-X(n));
C=Y(n)-2*Y(j-1)+Y(j);
D=0.5*(Y(j)-Y(n));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(n)*F1(g)+X(j-1)*F2(g)+X(j)*F3(g);
y1(g)=Y(n)*F1(g)+Y(j-1)*F2(g)+Y(j)*F3(g);
R2ij1(g)=sqrt((Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2);
S3(g)=Jco(g)*w(g)*F3(g)*log(R2ij1(g));
end
ss3=sum(S3);
//*****
G2(i,j)=-0.5*(ss1+ss2+ss3)/%pi;
end
end
//***** j=3-n-2 *****
for i=1:l
// when j=2...n-2
for j=3:n-2
// **** first consider for R(i,j+1)
for g=1:10
A=X(j)-2*X(j+1)+X(j+2);
B=0.5*(X(j+2)-X(j));
C=Y(j)-2*Y(j+1)+Y(j+2);
D=0.5*(Y(j+2)-Y(j));

```

```

Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j)*F1(g)+X(j+1)*F2(g)+X(j+2)*F3(g);
y1(g)=Y(j)*F1(g)+Y(j+1)*F2(g)+Y(j+2)*F3(g);
R2ij1(g)=sqrt((Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2);
S1(g)=Jco(g)*w(g)*F1(g)*log(R2ij1(g));
end
ss1=sum(S1);
//**** next consider for R(i,j)
for g=1:10
A=X(j-1)-2*X(j)+X(j+1);
B=0.5*(X(j+1)-X(j-1));
C=Y(j-1)-2*Y(j)+Y(j+1);
D=0.5*(Y(j+1)-Y(j-1));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-1)*F1(g)+X(j)*F2(g)+X(j+1)*F3(g);
y1(g)=Y(j-1)*F1(g)+Y(j)*F2(g)+Y(j+1)*F3(g);
R2ij1(g)=sqrt((Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2);
S2(g)=Jco(g)*w(g)*F2(g)*log(R2ij1(g));
end
ss2=sum(S2);
//**** next consider for R(i,j-1)
for g=1:10
A=X(j-2)-2*X(j-1)+X(j);
B=0.5*(X(j)-X(j-2));
C=Y(j-2)-2*Y(j-1)+Y(j);
D=0.5*(Y(j)-Y(j-2));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-2)*F1(g)+X(j-1)*F2(g)+X(j)*F3(g);
y1(g)=Y(j-2)*F1(g)+Y(j-1)*F2(g)+Y(j)*F3(g);
R2ij1(g)=sqrt((Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2);
S3(g)=Jco(g)*w(g)*F3(g)*log(R2ij1(g));
end
ss3=sum(S3);
//*****
G2(i,j)=-0.5*(ss1+ss2+ss3)/%pi;
end
end

```

```

//***** j= n-1 *****
for i=1:l
// when j=2...n-2
for j=n-1
// **** first consider for R(i,j+1)
for g=1:10
A=X(j)-2*X(j+1)+X(1);
B=0.5*(X(1)-X(j));
C=Y(j)-2*Y(j+1)+Y(1);
D=0.5*(Y(1)-Y(j));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j)*F1(g)+X(j+1)*F2(g)+X(1)*F3(g);
y1(g)=Y(j)*F1(g)+Y(j+1)*F2(g)+Y(1)*F3(g);
R2ij1(g)=sqrt((Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2);
S1(g)=Jco(g)*w(g)*F1(g)*log(R2ij1(g));
end
ss1=sum(S1);
// **** next consider for R(i,j)
for g=1:10
A=X(j-1)-2*X(j)+X(j+1);
B=0.5*(X(j+1)-X(j-1));
C=Y(j-1)-2*Y(j)+Y(j+1);
D=0.5*(Y(j+1)-Y(j-1));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-1)*F1(g)+X(j)*F2(g)+X(j+1)*F3(g);
y1(g)=Y(j-1)*F1(g)+Y(j)*F2(g)+Y(j+1)*F3(g);
R2ij1(g)=sqrt((Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2);
S2(g)=Jco(g)*w(g)*F2(g)*log(R2ij1(g));
end
ss2=sum(S2);
//**** next consider for R(i,j-1)
for g=1:10
A=X(j-2)-2*X(j-1)+X(j);
B=0.5*(X(j)-X(j-2));
C=Y(j-2)-2*Y(j-1)+Y(j);
D=0.5*(Y(j)-Y(j-2));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);

```



```

x1(g)=X(j-2)*F1(g)+X(j-1)*F2(g)+X(j)*F3(g);
y1(g)=Y(j-2)*F1(g)+Y(j-1)*F2(g)+Y(j)*F3(g);
R2ij1(g)=sqrt((Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2);
S3(g)=Jco(g)*w(g)*F3(g)*log(R2ij1(g));
end
ss3=sum(S3);
//*****
G2(i,j)=-0.5*(ss1+ss2+ss3)/%pi;
end
end
//***** j= n *****
for i=1:l
// when j=2...n-2
for j=n
// **** first consider for R(i,j+1)
for g=1:10
A=X(j)-2*X(1)+X(2);
B=0.5*(X(2)-X(j));
C=Y(j)-2*Y(1)+Y(2);
D=0.5*(Y(2)-Y(j));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j)*F1(g)+X(1)*F2(g)+X(2)*F3(g);
y1(g)=Y(j)*F1(g)+Y(1)*F2(g)+Y(2)*F3(g);
R2ij1(g)=sqrt((Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2);
S1(g)=Jco(g)*w(g)*F1(g)*log(R2ij1(g));
end
ss1=sum(S1);
// **** next consider for R(i,j)
for g=1:10
A=X(j-1)-2*X(j)+X(1);
B=0.5*(X(1)-X(j-1));
C=Y(j-1)-2*Y(j)+Y(1);
D=0.5*(Y(1)-Y(j-1));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-1)*F1(g)+X(j)*F2(g)+X(1)*F3(g);
y1(g)=Y(j-1)*F1(g)+Y(j)*F2(g)+Y(1)*F3(g);
R2ij1(g)=sqrt((Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2);

```

```

S2(g)=Jco(g)*w(g)*F2(g)*log(R2ij1(g));
end
ss2=sum(S2);
**** next consider for R(i,j-1)
for g=1:10
A=X(j-2)-2*X(j-1)+X(j);
B=0.5*(X(j)-X(j-2));
C=Y(j-2)-2*Y(j-1)+Y(j);
D=0.5*(Y(j)-Y(j-2));
Jco(g)=sqrt((A*ep(g)+B)^2+(C*ep(g)+D)^2);
x1(g)=X(j-2)*F1(g)+X(j-1)*F2(g)+X(j)*F3(g);
y1(g)=Y(j-2)*F1(g)+Y(j-1)*F2(g)+Y(j)*F3(g);
R2ij1(g)=sqrt((Xin(i)-x1(g))^2+(Yin(i)-y1(g))^2);
S3(g)=Jco(g)*w(g)*F3(g)*log(R2ij1(g));
end
ss3=sum(S3);
*****
G2(i,j)=-0.5*(ss1+ss2+ss3)/%pi;
end
end
endfunction

*****

// Construct distance matrix d(i,j)(=distance from node i to an element j)
function [d]=dist(n,X,Y)
d=zeros(n);
for i=1:n
for j=1:n-1
ax=X(j+1)-X(j);
ay=Y(j+1)-Y(j);
if abs(ax)>=0.0001
ta=(ay/ax);
d(i,j)=abs(ta*X(i)-Y(i)+Y(j)-ta*X(j))/(sqrt(ta^2+1));
else
d(i,j)=abs(X(i)-X(j));
end
end

```

```

end
end
for i=1:n
ax=X(1)-X(n);
ay=Y(1)-Y(n);
if abs(ax)>=0.0001
ta=(ay/ax);
d(i,n)=abs(ta*X(i)-Y(i)+Y(n)-ta*X(n))/(sqrt(ta^2+1));
else
d(i,n)=abs(X(i)-X(n));
end
end
endfunction

```

```

//*****

```

```

function L=lenet(n,X,Y)
L=zeros(n,1);
for j=2:n-1
L(j)=L(j)+sqrt((X(j+1)-X(j))^2+(Y(j+1)-Y(j))^2);
end
L(n)=sqrt((X(1)-X(n))^2+(Y(1)-Y(n))^2);
L(1)=sqrt((X(2)-X(1))^2+(Y(2)-Y(1))^2);
endfunction

```

```

//*****

```

```

// Construct distance matrix d(i,j)(=distance from node i to an element j)
function [din]=distin(n,l,Xin,Yin,X,Y)
din=zeros(1,n);
for i=1:l
for j=1:n-1
ax=X(j+1)-X(j);
ay=Y(j+1)-Y(j);
if abs(ax)>=0.0001
ta=(ay/ax);
din(i,j)=abs(ta*Xin(i)-Yin(i)+Y(j)-ta*X(j))/(sqrt(ta^2+1));

```

```

else
din(i,j)=abs(Xin(i)-X(j));
end
end
end
for i=1:l
ax=X(1)-X(n);
ay=Y(1)-Y(n);
if abs(ax)>=0.0001
ta=(ay/ax);
din(i,n)=abs(ta*Xin(i)-Yin(i)+Y(n)-ta*X(n))/(sqrt(ta^2+1));
else
din(i,n)=abs(Xin(i)-X(n));
end
end
endfunction

```

```

//*****

```

```

function [b]=valueb(Xb,Yb,typ)

```

```

b=zeros(n+17,1);
if typ==1
for i=1:n+17
b(i)=b(i)-2;
end
elseif typ==2
for i=1:n+17
b(i)=b(i)-Xb(i);
end
else
for i=1:n+17
b(i)=b(i)-(Xb(i)^2);
end
end
endfunction

```

```

//*****

```

```

function [phi1]=matrixphi(Xb,Yb,typ);
phi1=zeros(n+17,n+17);
for i=1:n+17
for j=1:n+17
phi1(i,j)=phi1(i,j)+1+sqrt((Xb(i)-Xb(j))^2+(Yb(i)-Yb(j))^2);
end
end
endfunction

```

```

//*****

```

```

function [Uhat]=matrixuhat(Xb,Yb,X,Y)
Uhat=zeros(n,n+17);
for i=1:n
for j=1:n+17
r(i,j)=sqrt((X(i)-Xb(j))^2+(Y(i)-Yb(j))^2);
Uhat(i,j)=Uhat(i,j)+(r(i,j)^2)/4+(r(i,j)^3)/9;
end
end
endfunction

```

```

//*****

```

```

function [Qhat]=matrixqhat(Xb,Yb,X,Y,L)
Qhat=zeros(2*n,n+17);
for k=2:n
Xt(2*k-1)=X(k);
Xt(2*k-2)=X(k);
Yt(2*k-1)=Y(k);
Yt(2*k-2)=Y(k);
end
Xt(1)=X(1);
Yt(1)=Y(1);
Xt(2*n)=X(1);
Yt(2*n)=Y(1);
for k=1:n

```

```

for j=1:n+17
X1=Xt(2*k-1);
X2=Xt(2*k);
Y1=Yt(2*k-1);
Y2=Yt(2*k);
Xp=Xb(j);
Yp=Yb(j);
r1=sqrt((X1-Xp)^2+(Y1-Yp)^2);
r2=sqrt((X2-Xp)^2+(Y2-Yp)^2);
Qhat(2*k-1,j)=(0.5+r1/3)*((X1-Xp)*(Y2-Y1)-(Y1-Yp)*(X2-X1))/L(k);
Qhat(2*k,j)=(0.5+r2/3)*((X2-Xp)*(Y2-Y1)-(Y2-Yp)*(X2-X1))/L(k);
end
end
endfunction

```

```

function [Qhatin]=matrixqhatin(Xb,Yb,X,Y,L)
Qhatin=zeros(n,n+17);
for k=1:n
Xt(k)=X(k);
Yt(k)=Y(k);
end
Xt(n+1)=X(1);
Yt(n+1)=Y(1);
for k=1:n
for j=1:n+17
X1=Xt(k);
X2=Xt(k+1);
Y1=Yt(k);
Y2=Yt(k+1);
Xp=Xb(j);
Yp=Yb(j);
r1=sqrt((X1-Xp)^2+(Y1-Yp)^2);
Qhatin(k,j)=(0.5+r1/3)*((X1-Xp)*(Y2-Y1)-(Y1-Yp)*(X2-X1))/L(k);
end
end
endfunction

```

```
/**/
```

```
function [Ual]=marixUnew(n,l,Xin,Yin,Xb,Yb)
```

```
Ual=zeros(l,n+17);
```

```
for i=1:l
```

```
for j=1:n+17
```

```
r(i,j)=sqrt((Xin(i)-Xb(j))^2+(Yin(i)-Yb(j))^2);
```

```
Ual(i,j)=Ual(i,j)+(r(i,j)^2)/4+(r(i,j)^3)/9;
```

```
end
```

```
end
```

```
endfunction
```



การเผยแพร่ผลงานวิทยานิพนธ์

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