APPENDIX C

DERIVATION OF AGGREGATE SUPPLY

This appendix shows the derivation of the aggregate supply of the model. Recall that the production function of firm j is

$$Y_t^H(j) = A_t Q\left(V_t^j\left(L_t^j, K_t^j\right), E_t^j\right) \tag{C.1}$$

The demand for output of intermediate firm j is defined as, detailed in Appendix A,

$$Y_t^H(j) = Y_t^H\left(\frac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta}$$

According to the assumption that some firms can choose their price at time t, when firms change their prices, the demand for their outputs changes as a result. Consequently, the input requirement will be affected. Therefore the input requirement function will depend on the produced output at the level of the chosen prices. The labor, capital, and energy input requirement function, therefore, are defined as the following:

$$L_t^j\left(Y_t^H\left(\frac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta}\right) = \frac{1}{A_t}V^{-1}\left[Q^{-1}\left(Y_t^H\left(\frac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta}, E_t^j\left(Y_{t+\tau}^H\left(\frac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta}\right)\right), K_t^j\left(Y_{t+\tau}^H\left(\frac{\tilde{P}_t^H(j)}{P_{t+\tau}^H}\right)^{-\vartheta}\right)\right]$$

$$= \frac{1}{A_t}\mathbb{F}_1\left(Y_t^H\left(\frac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta}\right)$$

$$K_t^j\left(Y_t^H\left(\frac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta}\right) = \frac{1}{A_t}V^{-1}\left[Q^{-1}\left(Y_t^H\left(\frac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta}, E_t^j\left(Y_t^H\left(\frac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta}\right)\right), L_t^j\left(Y_t^H\left(\frac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta}\right)\right]$$

$$= \frac{1}{A_t}\mathbb{F}_2\left(Y_t^H\left(\frac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta}\right)$$

$$E_t^j\left(Y_t^H\left(\frac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta}\right)=\frac{1}{A_t}Q^{-1}\left[Y_t^H\left(\frac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta},V\left(L_t^j\left(Y_t^H\left(\frac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta}\right),K_t^j\left(Y_t^H\left(\frac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta}\right)\right)\right]$$
$$=\frac{1}{A_t}\mathbb{F}_3\left(Y_t^H\left(\frac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta}\right)$$

where $\frac{1}{A_t}\mathbb{F}_1\left(Y_t^H\left(\frac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta}\right)$, $\frac{1}{A_t}\mathbb{F}_2\left(Y_t^H\left(\frac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta}\right)$, and $\frac{1}{A_t}\mathbb{F}_3\left(Y_t^H\left(\frac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta}\right)$ are , respectively, labor, capital and energy input requirement functions and are the same for j^{th} producers.

The wage rate and the capital rental are determined according to the competitive market. Firms have to take as given the price of energy according to the small open economy. The total cost of producing the quantity $Y_t^H(j)$ is, then,

$$\frac{W_t}{P_t^H}\frac{1}{A_t}\mathbb{F}_1\left(Y_t^H\left(\frac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta}\right)+\frac{R_t^K}{P_t^H}\frac{1}{A_t}\mathbb{F}_2\left(Y_t^H\left(\frac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta}\right)+\frac{P_t^E}{P_t^H}\mathbb{F}_3\frac{1}{A_t}\left(Y_t^H\left(\frac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta}\right)$$

When firms set their output prices, assuming Calvo (1983) price setting assumption, firms will take as given the input prices and the demand for their outputs. The firms' price setting problem can be stated as

$$\max_{\tilde{P}_t^H(j)}\mathfrak{L}_t(j)=\mathbb{E}_t\sum_{\tau=0}^{\infty}(\alpha\delta)^{\tau}\Lambda_{t+\tau}\left\{\frac{\tilde{P}_t^H(j)}{P_{t+\tau}^H}Y_{t+\tau}^H(j)-\frac{W_{t+\tau}}{P_{t+\tau}^H}L_{t+\tau}^j-\frac{R_{t+\tau}^K}{P_{t+\tau}^H}K_{t+\tau}^j-\frac{P_{t+\tau}^E}{P_{t+\tau}^H}E_{t+\tau}^j\right\} \tag{C.2}$$

where $L_{t+\tau}^j=\frac{1}{A_t}\mathbb{F}_1\left(\left(\frac{\tilde{P}_t^H(j)}{P_{t+\tau}^H}\right)^{-\vartheta}Y_{t+\tau}^H\right)$, $K_{t+\tau}^j=\frac{1}{A_t}\mathbb{F}_2\left(\left(\frac{\tilde{P}_t^H(j)}{P_{t+\tau}^H}\right)^{-\vartheta}Y_{t+\tau}^H\right)$, and

$$E_{t+\tau}^j=\frac{1}{A_t}\mathbb{F}_3\left(\left(\frac{\tilde{P}_t^H(j)}{P_{t+\tau}^H}\right)^{-\vartheta}Y_{t+\tau}^H\right)$$

subject to the demand for good of firm j , $Y_{t+\tau}^{H}(j)=\left(\frac{\tilde{P}_{t}^{H}(j)}{P_{t+\tau}^{H}}\right)^{-\vartheta}Y_{t+\tau}^{H}$.

The first order condition $\frac{\partial \mathfrak{L}_{t}(j)}{\partial \tilde{P}_{t}^{H}}=0$ is stated as the following

$$\mathbb{E}_{t}\sum_{\tau=0}^{\infty}(\alpha\delta)^{\tau}\Lambda_{t+\tau}\left\{\frac{\tilde{P}_{t}^{H}(j)}{P_{t+\tau}^{H}}-\frac{\vartheta}{\vartheta-1}\frac{1}{A_{t+\tau}}\left[\frac{W_{t+\tau}}{P_{t+\tau}^{H}}\mathbb{F}_{1}^{/}\left(Y_{t+\tau}^{H}\left(\frac{\tilde{P}_{t}^{H}(j)}{P_{t+\tau}^{H}}\right)^{-\vartheta}\right)+\frac{R_{t+\tau}^{K}}{P_{t+\tau}^{H}}\mathbb{F}_{2}^{/}\left(Y_{t+\tau}^{H}\left(\frac{\tilde{P}_{t}^{H}(j)}{P_{t+\tau}^{H}}\right)^{-\vartheta}\right)\right.\right.$$
$$\left.\left.+\frac{P_{t+\tau}^{E}}{P_{t+\tau}^{H}}\mathbb{F}_{3}^{/}\left(Y_{t+\tau}^{H}\left(\frac{\tilde{P}_{t}^{H}(j)}{P_{t+\tau}^{H}}\right)^{-\vartheta}\right)\right]\right\}\left(\frac{\tilde{P}_{t}^{H}(j)}{P_{t+\tau}^{H}}\right)^{-\vartheta}Y_{t+\tau}^{H}=0$$

(C.3)

Firms are identical, we can drop j

$$\mathbb{E}_{t}\sum_{\tau=0}^{\infty}(\alpha\delta)^{\tau}\Lambda_{t+\tau}\left\{\frac{\tilde{P}_{t}^{H}}{P_{t+\tau}^{H}}-\frac{\vartheta}{\vartheta-1}\frac{1}{A_{t+\tau}}\frac{1}{P_{t+\tau}^{H}}\left[W_{t+\tau}\mathbb{F}_{1}^{/}\left(Y_{t+\tau}^{H}\left(\frac{\tilde{P}_{t}^{H}}{P_{t+\tau}^{H}}\right)^{-\vartheta}\right)+R_{t+\tau}^{K}\mathbb{F}_{2}^{/}\left(Y_{t+\tau}^{H}\left(\frac{\tilde{P}_{t}^{H}}{P_{t+\tau}^{H}}\right)^{-\vartheta}\right)\right.\right.$$
$$\left.\left.+P_{t+\tau}^{E}\mathbb{F}_{3}^{/}\left(Y_{t+\tau}^{H}\left(\frac{\tilde{P}_{t}^{H}}{P_{t+\tau}^{H}}\right)^{-\vartheta}\right)\right]\right\}\left(\frac{\tilde{P}_{t}^{H}}{P_{t+\tau}^{H}}\right)^{-\vartheta}Y_{t+\tau}^{H}=0$$

(C.4)

Let define $X_{t}^{H}=\frac{\tilde{P}_{t}^{H}}{P_{t}^{H}}$, and $\Pi_{t}^{H}=\frac{P_{t}^{H}}{P_{t-1}^{H}}$. Rearranging the above equation, yield

$$\mathbb{E}_{t}\sum_{\tau=0}^{\infty}(\alpha\delta)^{\tau}\Lambda_{t+\tau}\left\{\frac{X_{t}}{\prod_{s=1}^{\tau}\Pi_{t+s}}-\frac{\vartheta}{\vartheta-1}\frac{1}{A_{t}}\left[\frac{W_{t+\tau}}{P_{t+\tau}^{H}}\mathbb{F}_{1}^{/}\left(Y_{t+\tau}^{H}\left(\frac{X_{t}}{\prod_{s=1}^{\tau}\Pi_{t+s}}\right)^{-\vartheta}\right)\right.\right.$$
$$\left.\left.+\frac{R_{t+\tau}^{K}}{P_{t+\tau}^{H}}\mathbb{F}_{2}^{/}\left(Y_{t+\tau}^{H}\left(\frac{X_{t}}{\prod_{s=1}^{\tau}\Pi_{t+s}}\right)^{-\vartheta}\right)+\frac{P_{t+\tau}^{E}}{P_{t+\tau}^{H}}\mathbb{F}_{3}^{/}\left(Y_{t+\tau}^{H}\left(\frac{X_{t}}{\prod_{s=1}^{\tau}\Pi_{t+s}}\right)^{-\vartheta}\right)\right]\right\}\left(\frac{\tilde{P}_{t}^{H}(j)}{P_{t+\tau}^{H}}\right)^{-\vartheta}Y_{t+\tau}^{H}=0$$

(C.5)

From the FOC of household, the real wage rate can be determined as following

$$\frac{W_t}{P_t^H} = \frac{P_t^c}{P_t^H}\frac{U_L\left(\cdot,\cdot,L_t\right)}{U_C\left(C_t,\cdot,\cdot\right)} = \frac{P_t^c}{P_t^H}\frac{U_L\left(\cdot,\cdot,\mathbb{F}_1\left(Y_t^H\right)/A_t\right)}{U_C\left(s_c Y_t^H,\cdot,\cdot\right)} \tag{C.6}$$

where s_c is the fraction of consumption in the aggregate output. $U_L\left(\cdot,\cdot,L_t\right)$ is the marginal disutility of labour, and $U_C\left(C_t,\cdot,\cdot\right)$ is the marginal utility of consumption.

Allowing bounded fluctuation in $\left(Y_t^H,\Pi_t,X_t^H,A_t,\Lambda,\frac{P_t^c}{P_t^H},\frac{R_t^K}{P_t^H},\frac{P_t^E}{P_t^H}\right)$ around the steady state $\left(C^H,1,1,1,1,1,\frac{R^K}{P^H},\frac{P^E}{P^H}\right)$.

Under the Calvo (1983) price setting assumption, the domestic price index can be defined as the following:

$$P_t^H = \left[\alpha\left(P_{t-1}^H\right)^{1-\vartheta} + (1-\alpha)\left(\tilde{P}_t^H\right)^{1-\vartheta}\right]^{\frac{1}{1-\vartheta}} \tag{C.7}$$

Multiplying both sides of the previous equation by $\frac{1}{P_{t-1}^H}$ we shall obtain

$$\Pi_t^H = \left[\alpha + (1-\alpha)\left(\Pi_t^H\right)^{1-\vartheta}\left(X_t^H\right)^{1-\vartheta}\right]^{\frac{1}{1-\vartheta}}$$

$$\alpha = \left[1-(1-\alpha)\left(X_t^H\right)^{1-\vartheta}\right]\left(\Pi_t^H\right)^{1-\vartheta}$$

$$\Pi_t^H = \alpha^{\frac{1}{1-\vartheta}}\left[1-(1-\alpha)\left(X_t^H\right)^{1-\vartheta}\right]^{\frac{1}{1-\vartheta}}$$

Taking log-lineraization to the above equation, we get

$$\pi_t^H = \frac{1}{1-\vartheta} - \frac{(1-\alpha)}{1-(1-\alpha)}(1-\vartheta)x_t^H = \frac{1-\alpha}{\alpha}x_t^H \tag{C.8}$$

Log-linearized the FOC of firm,(C.5), yield

$$\mathbb{E}_t\sum_{\tau=0}^{\infty}(\alpha\delta)^{\tau}\left[x_t^H - \sum_{s=1}^{\tau}\pi_{t+s}^H + a_{t+\tau} + \frac{1}{\Lambda_\pi}\left\{-w_{t+\tau} + p_{t+\tau}^H - \varpi_1\left(y_{t+\tau}^H - \vartheta\left(x_t^H - \sum_{s=1}^{\tau}\pi_{t+s}^H\right)\right)\right.\right.$$
$$\left.\left.-\tilde{r}_{t+\tau}^k - \varpi_2\left(y_{t+\tau}^H - \vartheta\left(x_t^H - \sum_{s=1}^{\tau}\pi_{t+s}^H\right)\right) - \tilde{p}_{t+\tau}^E - \varpi_3\left(y_{t+\tau}^H - \vartheta\left(x_t^H - \sum_{s=1}^{\tau}\pi_{t+s}^H\right)\right)\right\}\right] = 0 \tag{C.9}$$

where $f_1^{/}=\varpi_1 y^H(j)$, $f_2^{/}=\varpi_2 y^H(j)$, and $f_3^{/}=\varpi_3 y^H(j)$ are the log linearization of the $\mathbb{F}_1^{/}$, $\mathbb{F}_2^{/}$, and $\mathbb{F}_3^{/}$, respectively. And $\tilde{r}_{t+\tau}^{k}$, and $\tilde{p}_{t+\tau}^{E}$ are the log deviation from the steady of the real capital rental cost and real energy price.

With $\varpi_1 = \dfrac{d\mathbb{F}_1^{/}\left(Y_t^H\left(\dfrac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta}\right)}{dY_t^H\left(\dfrac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta}}\dfrac{Y_t^H\left(\dfrac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta}}{\mathbb{F}_1^{/}\left(Y_t^H\left(\dfrac{\tilde{P}_t^H(j)}{P_t^H}\right)^{-\vartheta}\right)} \geq 0$ is the elasticity of $\mathbb{F}_1^{/}$ with respect to $Y_t^H(j)$. And $\varpi_2 \geq 0$ is the elasticity of $\mathbb{F}_2^{/}$ with respect to $Y_t^H(j)$, and $\varpi_3 \geq 0$ is the elasticity of $\mathbb{F}_3^{/}$ with respect to $Y_t^H(j)$, which are defined analogously with ϖ_1.

It is assumed that ϖ_1 covers the total effects (including cross effects of the remaining two inputs) of inputs change with respect to the change in output. And $\Lambda_\pi = \dfrac{W}{P^H}\mathbb{F}_1^{/}\left(Y^H\right)+\dfrac{R^K}{P^H}\mathbb{F}_2^{/}\left(Y^H\right)+\dfrac{P^E}{P^H}\mathbb{F}_3^{/}\left(Y^H\right)$

Log linearization the wage rate (C.6), yield

$$\ln W_t = \ln P_t^c + \ln U_L\left(\cdot,\cdot,\frac{\mathbb{F}_1\left(Y_t^H\right)}{A_t}\right) - \ln U_C\left(s_c Y_t^H,\cdot,\cdot\right)$$

$$w_t = p_t^c + \frac{1}{U_L}\left(\frac{\partial U_L}{\partial L_t}\frac{\partial L_t}{\partial Y_t^H}dY_t^H - \frac{\partial U_L}{\partial L_t}\frac{\partial L_t}{\partial A_t}dA_t\right) - \frac{1}{U_C}dU_C$$

$$w_t = p_t^c + \frac{L_t}{U_L}\frac{\partial U_L}{\partial L_t}\frac{Y_t^H}{L_t}\frac{\partial L_t}{\partial Y_t^H}\frac{dY_t^H}{Y_t^H} - \frac{L_t}{U_L}\frac{\partial U_L}{\partial L_t}\frac{A_t}{L_t}\frac{\partial L_t}{\partial A_t}\frac{dA_t}{A_t}$$
$$-\frac{Y_t^H}{U_C}\frac{\partial U_C}{\partial C_t}\frac{\partial C_t}{\partial Y_t^H}\left(\frac{dY_t^H}{Y_t^H}\right)$$

where U_C (U_L) is marginal utility of consumption (labor) , $\dfrac{\partial U}{\partial C}$ ($\dfrac{\partial U}{\partial L}$), and

$$U_C = C^{-\rho} \quad , \; dU_C = -\rho C^{-\rho-1}$$

$$U_L = \chi_L L^{\rho_L} \quad , \frac{\partial U_L}{\partial L} = \chi_L \rho_L L^{\rho_L - 1}$$

According to log- approximation of the $\mathbb{F}_1^{/}$ is $f_1^{/} = \varpi_1 y^H(j)$, converting the log form yield $\mathbb{F}_1^{/} = Y^H(j)^{\varpi_1}$. Then, it can be expressed $\mathbb{F}_1 = \frac{Y^H(j)^{1+\varpi_1}}{1+\varpi_1}$, since $L_t^j = \frac{\mathbb{F}_1\left(Y_t^H(j)\right)}{A_t}$, therefore it can be stated $L_t^j = \frac{1}{A_t}\frac{Y^H(j)^{1+\varpi_1}}{1+\varpi_1}$. Finally, we get

$$\frac{L_t}{U_L(\bullet)}\frac{\partial U_L(\bullet)}{\partial L_t}\frac{Y_t^H}{L_t}\frac{\partial L_t}{\partial Y_t^H} = \rho_L(1+\varpi_1)$$

$$\frac{L_t}{U_L(\bullet)}\frac{\partial U_L(\bullet)}{\partial L_t}\frac{A_t}{L_t}\frac{\partial L_t}{\partial A_t} = -\rho_L$$

$$\frac{Y_t^H}{U_C(\bullet)}\frac{\partial U_C(\bullet)}{\partial C_t}\frac{\partial C_t}{\partial Y_t^H} = -\rho$$

The log-linearization of the real wage rate can be stated as

$$\begin{aligned} w_t &= p_t^c + \varpi_{LY} y_t^H - \varpi_{LA} a_t - \varpi_c y_t^H \\ w_t &= p_t^c + (\varpi_{LY} - \varpi_c) y_t^H - \varpi_{LA} a_t \end{aligned} \tag{C.10}$$

where $\varpi_{LY} = \rho_L(1+\varpi_1) > 0$ is the elasticity of marginal disutility of labour ,$U_L(\bullet)$, with respect to domestically produced output, $\varpi_c = -\rho < 0$ is the elasticity of marginal utility of consumption,$U_C(\bullet)$, with respect to domestically produced output, and $(\varpi_{LY} - \varpi_c) > 0$. $\varpi_{LA} = -\rho_L < 0$ is the elasticity of marginal disutility of labour, $U_L(\bullet)$, with respect to productivity shocks.

Let Define $\varpi_{Y^H} = \frac{1}{\Lambda_\pi}(\varpi_1 + \varpi_2 + \varpi_3) \geq 0$ and substituting (C.10), (C.9) can be expressed as

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} (\alpha\delta)^\tau \left[\begin{array}{l} \left(1+\vartheta\varpi_{Y^H}\right)\left(x_t^H - \sum_{s=1}^{\tau}\pi_{t+s}^H\right) - \varpi_{Y^H} y_{t+\tau}^H + \left(1+\frac{\varpi_{LA}}{\Lambda_\pi}\right) a_{t+\tau} \\ + \frac{1}{\Lambda_\pi}\left\{-p_{t+\tau}^c + p_{t+\tau}^H - (\varpi_{LY} - \varpi_c) y_{t+\tau}^H\right\} + \frac{1}{\Lambda_\pi}\left\{-\tilde{r}_{t+\tau}^k - \tilde{p}_{t+\tau}^E\right\} \end{array} \right] = 0$$

Rearranging

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} (\alpha\delta)^{\tau} \left[\begin{aligned} &\left(1+\vartheta\varpi_{Y^H}\right)\left(x_t^H - \sum_{s=1}^{\tau}\pi_{t+s}^H\right) + \left(1+\frac{\varpi_{LA}}{\Lambda_\pi}\right)a_{t+\tau} - \frac{1}{\Lambda_\pi}\left(\varpi_{LY} - \varpi_c + \Lambda_\pi\varpi_{Y^H}\right)y_{t+\tau}^H \\ &+\frac{1}{\Lambda_\pi}\left\{-p_{t+\tau}^c + p_{t+\tau}^H\right\} + \frac{1}{\Lambda_\pi}\left\{-\tilde{r}_{t+\tau}^k - \tilde{p}_{t+\tau}^E\right\} \end{aligned} \right] = 0$$

(C.11)

In the flexible price adjustment equilibrium the potential output is fluctuated only by the productivity shocks. Therefore, it is assumed that the log of the productivity parameter, a_t, fulfill

$$\left(1+\frac{\varpi_{LA}}{\Lambda_\pi}\right)a_t = \tilde{\varpi} y_t^N \qquad \text{(C.12)}$$

where $\tilde{\varpi} = \frac{1}{\Lambda_\pi}\left(\varpi_{LY} - \varpi_c + \Lambda_\pi \varpi_{Y^H}\right) > 0$

According to the relationship of the domestic price and the consumer price index, $p_t^H - p_t^c = -\omega q_t$, and the definition of output gap: $y_t = y_t^H - y_t^N$, where y_t is the deviation form the steady state of output gap and y_t^N is the potential or natural output. The potential output is the output defined in the equilibrium when firms are freely to adjust their prices. Then (C.11) can be expressed as

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} (\alpha\delta)^{\tau} \left[\left(1+\vartheta\varpi_{Y^H}\right)\left(x_t^H - \sum_{s=1}^{\tau}\pi_{t+s}^H\right) - \tilde{\varpi} y_{t+\tau} - \frac{\omega}{\Lambda_\pi} q_t + \frac{1}{\Lambda_\pi}\left\{-\tilde{r}_{t+\tau}^k - \tilde{p}_{t+\tau}^E\right\} \right] = 0$$

(C.13)

Let define

$$\tilde{z}_{t+\tau} = \tilde{\varpi} y_{t+\tau} + \frac{\omega}{\Lambda_\pi} q_t - \frac{1}{\Lambda_\pi}\left\{-\tilde{r}_{t+\tau}^k - \tilde{p}_{t+\tau}^E\right\}$$

Then

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \alpha^{\tau}\delta^{\tau} \left\{ \left(1+\vartheta\varpi_{Y^H}\right)\left(x_t^H - \sum_{s=1}^{\tau}\pi_{t+s}^H\right) - \tilde{z}_{t+\tau} \right\} \right] = 0$$

Change the summation order in the above equation as follows

$$\sum_{\tau=0}^{\infty} \alpha^{\tau}\delta^{\tau} \sum_{s=1}^{\tau} \pi_{t+s}^H = \sum_{s=1}^{\infty} \pi_{t+s}^H \sum_{\tau=s}^{\infty} \alpha^{\tau}\delta^{\tau} = \sum_{s=1}^{\infty} \pi_{t+s}^H \frac{\alpha^s\delta^s}{1-\alpha\delta} = \frac{1}{1-\alpha\delta} \sum_{\tau=1}^{\infty} \alpha^{\tau}\delta^{\tau}\pi_{t+\tau}^H$$

Then the aggregate supply as expressed above can be expressed as

$$\mathbb{E}_t\left\{\left(\frac{1+\vartheta\varpi_{Y^H}}{1-\alpha\delta}\right)x_t^H-\left(\frac{1+\vartheta\varpi_{Y^H}}{1-\alpha\delta}\right)\sum_{\tau=1}^{\infty}\alpha^{\tau}\delta^{\tau}\pi_{t+\tau}^H-\sum_{\tau=0}^{\infty}\alpha^{\tau}\delta^{\tau}\tilde{z}_{t+\tau}\right\}=0$$

$$x_t^H=\mathbb{E}_t\left\{\sum_{\tau=1}^{\infty}\alpha^{\tau}\delta^{\tau}\pi_{t+\tau}^H+\frac{1-\alpha\delta}{1+\vartheta\varpi_{Y^H}}\sum_{\tau=0}^{\infty}\alpha^{\tau}\delta^{\tau}\tilde{z}_{t+\tau}\right\}$$

$$x_t^H=\mathbb{E}_t\left\{\delta\alpha\pi_{t+1}^H+\frac{1-\alpha\delta}{1+\vartheta\varpi_{Y^H}}\tilde{z}_t\right\}+\alpha\delta\mathbb{E}_t x_{t+1}^H$$

Combining the above equation with the domestic price adjustment process, (C.8), the aggregate Supply can be expressed as

$$\pi_t^H=\delta\pi_{t+1|t}^H+\zeta\left(\tilde{z}_t\right) \tag{C.14}$$

where $\zeta=\dfrac{(1-\alpha)(1-\alpha\delta)}{\alpha\left(1+\vartheta\varpi_{Y^H}\right)}>0$

It is assumed that the inflation rate is adjusted in a simple partial adjustment as:

$$\pi_t^H=\alpha_\pi\pi_{t-1}^H+(1-\alpha_\pi)\left(\delta\pi_{t+1|t}^H+\zeta\tilde{z}_t\right)$$

Let π_t^H be predetermine one periods**,** and approximately $\delta\to 1$, so as to ensure the natural-rate hypothesis, then

$$\pi_{t+1|t}^H=\alpha_\pi\pi_t^H+(1-\alpha_\pi)\left(\delta\pi_{t+2|t}^H+\zeta\tilde{z}_{t+1|t}\right)$$

Substituting

$$\tilde{z}_{t+1|t}=\tilde{\varpi}y_{t+1|t}+\frac{\omega}{\Lambda_\pi}q_{t+1|t}-\frac{1}{\Lambda_\pi}\left\{-\tilde{r}_{t+1|t}^k-\tilde{p}_{t+1|t}^E\right\}$$

Then

$$\pi_{t+1|t}^H=\alpha_\pi\pi_t^H+(1-\alpha_\pi)\left[\pi_{t+2|t}^H+\zeta\left\{\tilde{\varpi}y_{t+1|t}+\frac{\omega}{\Lambda_\pi}q_{t+1|t}-\frac{1}{\Lambda_\pi}\left(-\tilde{r}_{t+1|t}^k-\tilde{p}_{t+1|t}^E\right)\right\}\right]$$

Rearranging

$$\pi_{t+1}^H=\alpha_\pi\pi_t^H+(1-\alpha_\pi)\pi_{t+2|t}^H+\alpha_y y_{t+1|t}+\alpha_q q_{t+1|t}+\alpha_p\left(\tilde{r}_{t+1|t}^k+\tilde{p}_{t+1|t}^E\right)+v_{t+1} \tag{C.15}$$

where $\alpha_y \equiv (1-\alpha_\pi)\zeta\tilde{\varpi}$ $\qquad$ $\alpha_q = (1-\alpha_\pi)\dfrac{\zeta\omega}{\Lambda_\pi}$

$\alpha_p = (1-\alpha_\pi)\dfrac{\zeta}{\Lambda_\pi}$

Next, taking the expectation at time t of (C.15)

$$(1-\alpha_\pi)\pi^H_{t+2|t} = \pi^H_{t+1|t} - \alpha_\pi\pi^H_t - \alpha_y y_{t+1|t} - \alpha_q q_{t+1|t} - \alpha_p\left(\tilde{r}^k_{t+1|t} + \tilde{p}^E_{t+1|t}\right) \tag{C.16}$$

The aggregate demand is defined, detailed in Appendix B, as

$$\begin{aligned} y_{t+1|t} \equiv{}& \beta_y y_t - \beta_\varphi\varphi_{t+1|t} + \beta^*_y y^*_{t+1|t} + \beta_q q_{t+1|t} + \beta_{in} in_{t+1|t} - \beta_e\left(p^W_{t+1|t} + q_{t+1|t} + e_{t+1|t}\right) \\ & -\beta_f\left(f_{t+2|t} - \left(i^*_{t+1|t} + f_{t+1}\right)\right) - \left(\gamma^n_y - \beta_y\right)y^n_t \end{aligned}$$

Then (C.16) can be stated as

$$\begin{aligned} (1-\alpha_\pi)\pi^H_{t+2|t} ={}& \pi^H_{t+1|t} - \alpha_\pi\pi^H_t - \alpha_y\Big\{\beta_y y_t - \beta_\varphi\varphi_{t+1|t} + \beta^*_y y^*_{t+1|t} + \beta_q q_{t+1|t} + \beta_{in} in_{t+1|t} - \beta_e\left(p^W_{t+1|t} + q_{t+1|t} + e_{t+1|t}\right) \\ & -\beta_f\left(f_{t+2|t} - \left(i^*_{t+1|t} + f_{t+1}\right)\right) - \left(\gamma^n_y - \beta_y\right)y^n_t\Big\} - \alpha_q q_{t+1|t} - \alpha_p\left(\tilde{r}^k_{t+1|t} + \tilde{p}^E_{t+1|t}\right) \end{aligned}$$

Reformulate the above the equation

$$\begin{aligned} (1-\alpha_\pi)\pi^H_{t+2|t} ={}& \pi^H_{t+1|t} - \alpha_\pi\pi^H_t - \alpha_y\left[\beta_y y_t - \beta_\varphi\varphi_{t+1|t} + \beta^*_y y^*_{t+1|t} + \beta_q q_{t+1|t} + \beta_{in} in_{t+1|t}\right] \\ & -\alpha_y\left[-\beta_e\left(p^W_{t+1|t} + q_{t+1|t} + e_{t+1|t}\right) - \beta_f\left(f_{t+2|t} - \left(i^*_{t+1|t} + f_{t+1}\right)\right)\right] - \alpha_y\left[-\left(\gamma^n_y - \beta_y\right)\right]y^n_t \\ & -\left[\alpha_y\beta_{in}\right]in_{t+1|t} - \alpha_q q_{t+1|t} - \alpha_p\left(\tilde{r}^k_{t+1|t} + \tilde{p}^E_{t+1|t}\right) \end{aligned}$$

Substituting $q_{t+1|t} = q_t + \hat{i}_t - \pi^H_{t+1|t} - \hat{i}^*_t + \pi^*_{t+1|t} - \Omega_t$ and $\varphi_{t+1|t} = \varphi_t - \hat{i}_t + \pi^H_{t+1|t}$ into the above equation. Then, we get

$$\left(1-\alpha_{\pi}\right)\pi^{H}_{t+2|t}$$
$$=\pi^{H}_{t+1|t}-\alpha_{\pi}\pi^{H}_{t}-\alpha_{y}\left[\beta_{y}y_{t}-\beta_{\varphi}\left(\varphi_{t}-\hat{i}_{t}+\pi^{H}_{t+1|t}\right)+\beta^{*}_{y}y^{*}_{t+1|t}+\left(\beta_{q}-\beta_{e}\right)\left(q_{t}+\hat{i}_{t}-\pi^{H}_{t+1|t}-\hat{i}^{*}_{t}+\pi^{*}_{t+1|t}-\Omega_{t}\right)\right]$$
$$+\alpha_{y}\left[\gamma^{n}_{y}-\beta_{y}\right]y^{n}_{t}-\alpha_{y}\beta_{in}in_{t+1|t}-\alpha_{q}\left(q_{t}+\hat{i}_{t}-\pi^{H}_{t+1|t}-\hat{i}^{*}_{t}+\pi^{*}_{t+1|t}-\Omega_{t}\right)-\alpha_{p}\left(\tilde{r}^{k}_{t+1|t}+\tilde{p}^{E}_{t+1|t}\right)$$
$$-\alpha_{y}\left[-\beta_{e}\left(p^{W}_{t+1|t}+e_{t+1|t}\right)-\beta_{f}\left(f_{t+2|t}-\left(i^{*}_{t+1|t}+f_{t+1}\right)\right)\right]$$

Rearranging

$$\begin{aligned}\left(1-\alpha_{\pi}\right)\pi^{H}_{t+2|t}=\;&\pi^{H}_{t+1|t}-\alpha_{\pi}\pi^{H}_{t}-\alpha_{y}\left[\beta_{y}y_{t}-\beta_{\varphi}\varphi_{t}+\beta^{*}_{y}y^{*}_{t+1|t}+\left(\beta_{q}-\beta_{e}\right)\left(q_{t}+\pi^{*}_{t+1|t}-\hat{i}^{*}_{t}-\Omega_{t}\right)\right]\\&-\alpha_{y}\left(\beta_{\varphi}+\beta_{q}-\beta_{e}\right)\left(\hat{i}_{t}-\pi^{H}_{t+1|t}\right)+\alpha_{y}\left[\gamma^{n}_{y}-\beta_{y}\right]y^{n}_{t}-\alpha_{y}\beta_{in}in_{t+1|t}\\&-\alpha_{q}\left(q_{t}+\hat{i}_{t}-\pi^{H}_{t+1|t}+\pi^{*}_{t+1|t}-\hat{i}^{*}_{t}-\Omega_{t}\right)-\alpha_{p}\left(\tilde{r}^{k}_{t+1|t}+\tilde{p}^{E}_{t+1|t}\right)\\&-\alpha_{y}\left[-\beta_{e}\left(p^{W}_{t+1|t}+e_{t+1|t}\right)-\beta_{f}\left(f_{t+2|t}-\left(i^{*}_{t+1|t}+f_{t+1}\right)\right)\right]\end{aligned}$$

Finally, the aggregate supply can be expressed as

$$\begin{aligned}\left(1-\alpha_{\pi}\right)\pi^{H}_{t+2|t}=\;&-\alpha_{\pi}\pi^{H}_{t}-\alpha_{y}\beta_{y}y_{t}+\alpha_{y}\beta_{\varphi}\varphi_{t}-\alpha_{y}\beta^{*}_{y}y^{*}_{t+1|t}-\left[\alpha_{y}\left(\beta_{q}-\beta_{e}\right)+\alpha_{q}\right]\left(\pi^{*}_{t+1|t}-\hat{i}^{*}_{t}-\Omega_{t}\right)\\&+\left[1+\alpha_{y}\left(\beta_{\varphi}+\beta_{q}-\beta_{e}\right)+\alpha_{q}\right]\pi^{H}_{t+1|t}+\alpha_{y}\left[\gamma^{n}_{y}-\beta_{y}\right]y^{n}_{t}-\alpha_{y}\beta_{in}in_{t+1|t}\\&-\left[\alpha_{y}\left(\beta_{\varphi}+\beta_{q}-\beta_{e}\right)+\alpha_{q}\right]\hat{i}_{t}-\left[\alpha_{q}+\alpha_{y}\left(\beta_{q}-\beta_{e}\right)\right]q_{t}-\alpha_{p}\left(\tilde{r}^{k}_{t+1|t}+\tilde{p}^{E}_{t+1|t}\right)\\&-\alpha_{y}\left[-\beta_{e}\left(p^{W}_{t+1|t}+e_{t+1|t}\right)-\beta_{f}\left(f_{t+2|t}-\left(i^{*}_{t+1|t}+f_{t+1}\right)\right)\right]\end{aligned}$$

(C.17)