APPENDIX B

DERIVATION OF AGGREGATE DEMAND

This appendix shows the derivation of the aggregate demand function. Recalling that the consumption Euler equation is:

$$C_{t+1|t} = \left(\delta(1+i_t) \frac{P_t^c}{P_{t+1|t}^c} \right)^{\frac{1}{\rho}} C_t \qquad \text{(B.1)}$$

Loglinear approximation (B.1), taking log both sides yield:

$$\begin{aligned} \ln\left(C_{t+1|t}\right) &= \left(\frac{1}{\rho}\right) \ln\left(\delta(1+i_t) \frac{P_t^c}{P_{t+1|t}^c} \right) + \ln(C_t) \\ &= \left(\frac{1}{\rho}\right) \ln \delta + \left(\frac{1}{\rho}\right) \ln(1+i_t) + \left(\frac{1}{\rho}\right) \ln\left(\frac{P_t^c}{P_{t+1|t}^c} \right) + \ln C_t \end{aligned}$$

Taking the total differentiation of the above equation, we shall obtain:

$$c_t = c_{t+1|t} - \left(\frac{1}{\rho}\right)\left(\hat{i}_t - \pi_{t+1|t}^c\right) \qquad \text{(B.2)}$$

Define $d \ln \Pi_{t+1}^c = d \ln P_{t+1}^c - d \ln P_t^c = \pi_{t+1}^c = p_{t+1}^c - p_t^c$ is the deviation of CPI inflation. Note here, again, that the lowercase letters denote for the log deviation level from the non-stochastic steady state of the corresponding uppercase letters, excluding i_t, for example, $c_t \equiv d \ln C_t$. And $\hat{i}_t = d \ln(i_t)$ denotes the deviation of the nominal interest rate from its steady state, and $\left(\hat{i}_t - \pi_{t+1|t}^c\right)$ is the deviation of the real CPI interest rate.

From Appendix A, we know that the optimal allocations among a variety of goods are the following[1]:

$$C_t^H(j) = \left(\frac{P_t^H(j)}{P_t^H}\right)^{-\vartheta}\left(\frac{P_t^H}{P_t^c}\right)^{-\xi} C_t \quad \text{and} \quad C_t^F(j) = \left(\frac{P_t^F(j)}{P_t^F}\right)^{-\vartheta}\left(\frac{P_t^F}{P_t^c}\right)^{-\xi} C_t$$

In order to get the total consumption of home and foreign produced goods, we need to aggregate among differentiated goods. Therefore the index of home and imported goods can be expressed as:

$$C_t^H = (1-\omega)\left(\frac{P_t^H}{P_t^c}\right)^{-\xi} C_t \qquad \text{and} \qquad C_t^F = \omega\left(\frac{P_t^F}{P_t^c}\right)^{-\xi} C_t$$

where $P_t^F(j) = S_t P_t^{*F}(j)$ and $P_t^F = S_t P_t^{*F}$ according to the assumption of law of one price and PPP.

Taking log-linearization the two equations above yield:

$$c_t^H = c_t - \xi\left(p_t^H - p_t^c\right), \tag{B.3}$$

and

$$c_t^F = c_t - \xi\left(p_t^F - p_t^c\right).$$

We assume that ω is the share of imported goods in the CPI inflation, $\pi_t^c = d\ln\Pi_t^c$. This can be expressed as:

$$\pi_t^c = (1-\omega)\pi_t^H + \omega\pi_t^F \tag{B.4}$$

---

[1]For foreign country, given the fact that the consumption index C is common across countries, the demand for home produced good j is defined as $C_t^{H*}(j) = \left(\frac{P_t^{H*}(j)}{P_t^{H*}}\right)^{-\vartheta}\left(\frac{P_t^{H*}}{P_t^{c*}}\right)^{-\xi} C_t^*$, and the demand for foreign good j is defined as $C_t^{F*}(j) = \left(\frac{P_t^{F*}(j)}{P_t^{F*}}\right)^{-\vartheta}\left(\frac{P_t^{F*}}{P_t^{c*}}\right)^{-\xi} C_t^*$, where $P_t^{H*}(j), \left(P_t^{F*}(j)\right)$ is foreign currency of domestic (foreign) goods.

where $d\ln\left(\Pi_t^H\right) = \pi_t^H$ denotes the domestically produced good inflation defined as

$$\pi_{t+1}^H = \pi_{t+1|t}^H + v_{t+1} \tag{B.5}$$

π^F denotes the domestic-currency inflation of the imported goods defined as

$$\begin{aligned} \pi_t^F &= p_t^F - p_{t-1}^F \\ &= s_t + p_t^{F*} - s_{t-1} - p_{t-1}^{F*} \\ &= \pi_t^{F*} + s_t - s_{t-1} \end{aligned} \tag{B.6}$$

where $p_t^F = s_t + p_t^{F*}$ and $\pi_t^F = p_t^F - p_{t-1}^F$

Defining the deviation of the real exchange rate, $d\ln\left(Q_t\right) = q_t$, as

$$q_t \equiv s_t + p_t^{F*} - p_t^H \tag{B.7}$$

where $p_t^H \equiv d\ln P_t^H$ is the (log) domestic price index, and $p_t^{F*} \equiv d\ln P_t^{F*}$ is the (log) foreign-currency of imported goods price index. Making use of (B.7), (B.6) can be written as:

$$\begin{aligned} \pi_t^F &= \pi_t^{F*} + \left(q_t - p_t^{F*} + p_t^H\right) - \left(q_{t-1} - p_{t-1}^{F*} + p_{t-1}^H\right) \\ &= \pi_t^H + q_t - q_{t-1} \end{aligned} \tag{B.8}$$

Substituting (B.8) into (B.4) yields:

$$\pi_t^c = \pi_t^H + \omega\left(q_t - q_{t-1}\right) \tag{B.9}$$

Reformulating the above equation, we get

$$p_t^H - p_t^c = p_{t-1}^H - p_{t-1}^c - \omega\left(q_t - q_{t-1}\right) \tag{B.10}$$

Substituting (B.10) into (B.3) yields:

$$c_t^H = c_t + \xi\omega\left(q_t - q_{t-1}\right) - \xi\left(p_{t-1}^H - p_{t-1}^c\right), \tag{B.11}$$

Back (B.10) one period and substitute in (B.11), we obtain

$$c_t^H = c_t + \xi\omega\left(q_t - q_{t-1}\right) + \xi\omega\left(q_{t-1} - q_{t-2}\right) - \xi\left(p_{t-2}^H - p_{t-2}^c\right),$$

Iterating the same process to the above equation, given that $q = p^H = p^c = 0$, we get

$$c_t^H = c_t + \xi\omega q_t, \tag{B.12}$$

$$c_t = c_t^H - \xi\omega q_t. \tag{B.13}$$

Substituting (B.9) and (B.13)into(B.2), we shall obtain

$$c_t^H = c_{t+1|t}^H - \xi\omega q_{t+1|t} + \xi\omega q_t - \left(\frac{1}{\rho}\right)\left(\hat{i}_t - \pi_{t+1|t}^H\right) + \left(\frac{1}{\rho}\right)\omega\left(q_{t+1|t} - q_t\right) \quad \text{(B.14)}$$

Rearranging the above equation, we shall get

$$c_t^H = c_{t+1|t}^H - \left(\frac{1}{\rho}\right)\left(\hat{i}_t - \pi_{t+1|t}^H\right) + \left(\frac{1}{\rho}\right)\omega\left(q_{t+1|t} - q_t\right) - \xi\omega\left(q_{t+1|t} - q_t\right) \quad \text{(B.15)}$$

We assume that there is a steady state for c_t^H then $\lim_{\tau\to\infty} c_{t+\tau|t}^H = 0$. Iterating (B.15) we obtain the following expression:

$$c_t^H = c_{t+1|t}^H - \left(\frac{1}{\rho}\right)\sum_{\tau=0}^{\infty}\left\{\hat{i}_{t+\tau} - \pi_{t+\tau+1|t}^H\right\} + \omega\left(\frac{1}{\rho} - \xi\right)\sum_{\tau=0}^{\infty}\left\{q_{t+\tau+1|t} - q_{t+\tau|t}\right\} \quad \text{(B.16)}$$

Given the assumption of $\lim_{\tau\to\infty} q_{t+\tau|t} = 0,$, then (B.16) is given by:

$$c_t^H = -\left(\frac{1}{\rho}\right)\varphi_t - \left(\frac{1}{\rho} - \xi\right)\omega q_t \quad \text{(B.17)}$$

where

$$\varphi_t \equiv \sum_{\tau=0}^{\infty}\left(\hat{i}_{t+\tau|t} - \pi_{t+\tau+1|t}^H\right) \quad \text{(B.18)}$$

Assuming that the Fisher parity holds, the domestic nominal interest rate is given by:

$$\hat{i}_t \equiv r_t + \pi_{t+1|t}^H \quad \text{(B.19)}$$

where r_t is the (log) real interest rate and $\hat{i}_t$ is the (log) of nominal interest rate.

Next, following in Svensson (2000), assuming c_t^{*H} is exogenous and given by

$$\begin{aligned} c_t^{*H} &= c_t^* + \xi^*\omega^* q_t \\ &= \overline{\beta}_y^* y_t^* + \xi^*\omega^* q_t \end{aligned} \quad \text{(B.20)}$$

where c_t^* denotes the foreign aggregate consumption. Here $\xi^* > 1$ denotes the foreign elasticity of substitution between foreign and domestic goods, and ω^* denotes the fraction of domestically produced in foreign aggregate consumption. The coefficient $\overline{\beta}_y^*$ is the income elasticity of foreign consumption and y_t^* denotes the foreign output.

The home produced goods market clearing conditions

$$Y_t^H = C_t^H + IN_t + C_t^{H*}$$

The net export is defined as

$$XN_t = P_t^H C_t^{H*} - P_t^E E_t - P_t^F C_t^F - \left(F_{t+1} - \left(1+i_t^*\right) F_t\right) \tag{B.21}$$

where $P_t^E = P_t^W S_t$ is the domestic currency oil price and P_t^W is the world oil price. Note here that, the net export , XN_t , is equal to 0 for all t .

Finally, we can express the deviation from the steady state of the total aggregate demand for home produced good in period t , $d\ln\left(Y_t^H\right) = y_t^H$, as the following:

$$\begin{aligned} y_t^H &= \frac{C}{Y^H}\left(\left(1-\omega\right)c_t^H + \omega c_t^F\right) + \frac{IN}{Y^H} in_t + \frac{C^{H*}}{Y^H} c_t^{H*} - \frac{C^F}{Y^H}\left(c_t^F\right) - \frac{E}{Y^H} p_t^E - \frac{E}{Y^H} e_t \\ &\quad - \frac{F}{Y^H}\left(f_{t+1} - \left(i_t^* + f_t\right)\right) \\ y_t^H &= s_c c_t^H + s_{in} in_t + s_x c_t^{H*} - s_e p_t^E - s_e e_t - s_f\left(f_{t+1} - \left(1+i^*\right) f_t\right) \end{aligned} \tag{B.22}$$

where $\kappa = \left(1-\omega\right)$ be the share of the domestic aggregate consumption in the total aggregate consumption of home produced goods, in_t denotes the (log) domestic investment defined as $\delta_K in_t = k_{t+1} - \left(1-\delta_K\right) k_t$. With $s_c = \dfrac{C^H}{Y^H}$, $s_{in} = \dfrac{IN}{Y^H}$, $s_e = \dfrac{E}{Y^H}$, $s_x = \dfrac{C^{H*}}{Y^H}$, $s_f = \dfrac{F}{Y^H}$ denote, respectively, steady state share of goods consumption, and domestic investment, energy consumption, export, and foreign bonds holding. With the variables without time scripts denote the steady sate value of the corresponding variables.

Substituting (B.17) and (B.20) into (B.22) yields:

$$\begin{aligned} y_t^H &= s_c\left(1-\omega\right)\left(-\left(\frac{1}{\rho}\right)\varphi_t - \left(\frac{1}{\rho} - \xi\right)\omega q_t\right) + s_x\left(\overline{\beta}_y^* y_t^* + \xi^* \omega^* q_t\right) + s_{in} in_t - s_e p_t^E - s_e e_t \\ &\quad - s_f\left(f_{t+1} - \left(i_t^* + f_t\right)\right) \end{aligned} \tag{B.23}$$

Rearranging the above equation, we obtain:

$$y_t^H = -\tilde{\beta}_\varphi \varphi_t + \tilde{\beta}_y^* y_t^* + \tilde{\beta}_q q_t + s_{in} in_t - s_e p_t^E - s_e e_t - s_f \left(f_{t+1} - \left(i_t^* + f_t \right) \right) \qquad \text{(B.24)}$$

where $\tilde{\beta}_\varphi \equiv s_c \dfrac{\kappa}{\rho}$, $\tilde{\beta}_y^* \equiv s_c (1-\kappa)\overline{\beta}_y^*$, $\tilde{\beta}_q \equiv s_c \left[(1-\kappa)\xi^* \omega^* - \kappa \left(\dfrac{1}{\rho} - \xi \right) \omega \right]$

It is assumed that the real consumption and the aggregate demand are predetermined one period, following Svensson (1998, 2000). Hence, the equation (B.24) can be written as:

$$y_{t+1|t}^H = -\tilde{\beta}_\varphi \varphi_{t+1|t} + \tilde{\beta}_y^* y_{t+1|t}^* + \tilde{\beta}_q q_{t+1|t} + \beta_{in} in_{t+1|t} - s_e \left(p_{t+1|t}^W + q_{t+1|t} + e_{t+1|t} \right) - s_f \left(f_{t+2|t} - \left(i_{t+1|t}^* + f_{t+1} \right) \right) \qquad \text{(B.25)}$$

We further assume that the aggregate demand and output are adjusted by weighting between y_t^H and $y_{t+1|t}^H$ then we get

$$y_{t+1}^H = \beta_y y_t^H + \left(1-\beta_y\right) \begin{bmatrix} -\tilde{\beta}_\varphi \varphi_{t+1|t} + \tilde{\beta}_y^* y_{t+1|t}^* + \tilde{\beta}_q q_{t+1|t} + s_{in} in_{t+1|t} + s_e \left(p_{t+1|t}^W + q_{t+1|t} + e_{t+1|t} \right) \\ -s_f \left(f_{t+2|t} - \left(i_{t+1}^* + f_{t+1} \right) \right) + \eta_{t+1}^H \end{bmatrix} \qquad \text{(B.26)}$$

where $0 < \beta_y < 1$ is the weight between y_t^H and $y_{t+1|t}^H$, and η_t^H is the serially uncorrelated zero-mean domestic demand shocks.

Rearranging the above equation, we get

$$\begin{aligned} y_{t+1}^H \equiv\ & \beta_y y_t^H - \beta_\varphi \varphi_{t+1|t} + \beta_y^* y_{t+1|t}^* + \beta_q q_{t+1|t} + \beta_{in} in_{t+1|t} - \beta_e \left(p_{t+1|t}^W + q_{t+1|t} + e_{t+1|t} \right) \\ & - \beta_f \left(f_{t+2|t} - \left(i_{t+1|t}^* + f_{t+1} \right) \right) + \eta_{t+1}^H - \eta_{t+1}^n \end{aligned} \qquad \text{(B.27)}$$

where

$$\beta_\varphi \equiv \left(1-\beta_y\right)\tilde{\beta}_\varphi \qquad \beta_{in} \equiv \left(1-\beta_y\right) s_{in} \qquad \beta_e \equiv \left(1-\beta_y\right) s_e$$

$$\beta_y^* \equiv \left(1-\beta_y\right)\tilde{\beta}_y^* \qquad \beta_q \equiv \left(1-\beta_y\right)\tilde{\beta}_q \qquad \beta_f \equiv \left(1-\beta_y\right) s_f$$

Let the domestic output gap is defined as $y_{t+1} \equiv y_{t+1}^H - y_{t+1}^n$. Equation (B.27) then can be written in term of the output gap as follows:

$$y_{t+1} \equiv y_{t+1}^H - y_{t+1}^n$$

$$\begin{aligned} y_{t+1|t} \equiv\ & \beta_y y_t - \beta_\varphi \varphi_{t+1|t} + \beta_y^* y_{t+1|t}^* + \beta_q q_{t+1|t} + \beta_{in} in_{t+1|t} - \beta_e \left(p_{t+1|t}^W + q_{t+1|t} + e_{t+1|t} \right) \\ & - \beta_f \left(f_{t+2|t} - \left(i_{t+1|t}^* + f_{t+1} \right) \right) - \left(\gamma_y^n - \beta_y \right) y_t^n \end{aligned} \qquad \text{(B.28)}$$