APPENDIX A

DERIVATION OF HOUSEHOLD'S PROBLEM

This appendix shows the derivation of the demand for the differentiated goods and the representative household's utility maximization problem. In order to get the demands, the representative household finds, first, the optimal allocation for goods across countries which yields C_t^H and C_t^F. The second, the representative household finds the optimal allocation among a variety of goods with in country which yield $C_t^H(j)$ and $C_t^F(j)$.

Household's Consumption Cost minimization Problem

The representative household solves the cost minimization problem:

$$Min\left\{P_t^H C_t^H + P_t^F C_t^F\right\}$$

subject to the following constraints:

$$\left[(1-\omega)^{\frac{1}{\xi}}\left[C_t^H\right]^{\frac{(\xi-1)}{\xi}} + (\omega)^{\frac{1}{\xi}}\left[C_t^F\right]^{\frac{(\xi-1)}{\xi}}\right]^{\frac{\xi}{\xi-1}} = C_t \qquad \text{(A.1)}$$

$$C_t^H = \left[\left(\frac{1}{1-\omega}\right)^{\frac{1}{\vartheta}}\int_{\omega}^{1} C_t^H(j)^{\left(\frac{\vartheta-1}{\vartheta}\right)} dj\right]^{\frac{\vartheta}{(\vartheta-1)}} \qquad P_t^H = \left[\frac{1}{1-\omega}\int_{\omega}^{1} P_t^H(j)^{1-\vartheta} dj\right]^{\frac{1}{(1-\vartheta)}}$$

$$C_t^F = \left[\left(\frac{1}{\omega}\right)^{\frac{1}{\vartheta}}\int_{0}^{\omega} C_t^F(j)^{\left(\frac{\vartheta-1}{\vartheta}\right)} dj\right]^{\frac{\vartheta}{(\vartheta-1)}} \qquad P_t^F = \left[\left(\frac{1}{\omega}\right)\int_{0}^{\omega} S_t P_t^F(j)^{1-\vartheta} dj\right]^{\frac{1}{(1-\vartheta)}}$$

with $(1-\omega)$ is the share of the domestically produced good in the Home consumption expenditure. There is no bias in consumption.

Taking differentiate of the above problem with respect to C_t^H, we get the first order condition as follows:

$$\left(P_t^H\right)=\lambda_t\left[(1-\omega)^{\frac{1}{\xi}}\left(C_t^H\right)^{\frac{(\xi-1)}{\xi}}+(\omega)^{\frac{1}{\xi}}\left(C_t^F\right)^{\frac{(\xi-1)}{\xi}}\right]^{\left(\frac{1}{\xi-1}\right)}(1-\omega)^{\frac{1}{\xi}}\left(C_t^H\right)^{-\left(\frac{1}{\xi}\right)} \tag{A.2}$$

where λ_t is the Lagrange multiplier. The first order condition with respect to C_t^F is given by:

$$P_t^F=\lambda_t\left[(1-\omega)^{\frac{1}{\xi}}\left(C_t^H\right)^{\frac{(\xi-1)}{\xi}}+(\omega)^{\frac{1}{\xi}}\left(C_t^F\right)^{\frac{(\xi-1)}{\xi}}\right]^{\left(\frac{1}{\xi-1}\right)}\omega^{\frac{1}{\xi}}\left(C_t^F\right)^{-\left(\frac{1}{\xi}\right)} \tag{A.3}$$

Dividing (A.2) by (A.3) yields:

$$\frac{P_t^H}{P_t^F}=\left[\frac{1-\omega}{\omega}\right]^{\frac{1}{\xi}}\left[\frac{C_t^F}{C_t^H}\right]^{\frac{1}{\xi}}$$

$$C_t^H=C_t^F\left[\frac{1-\omega}{\omega}\right]\left[\frac{P_t^F}{P_t^H}\right]^{\xi} \tag{A.4}$$

Substituting (A.4) into (A.1), we get

$$C_t=\left[(1-\omega)^{\frac{1}{\xi}}\left[C_t^F\left(\frac{1-\omega}{\omega}\right)\left[\frac{P_t^F}{P_t^H}\right]^{\xi}\right]^{\frac{(\xi-1)}{\xi}}+(\omega)^{\frac{1}{\xi}}\left(C_t^F\right)^{\frac{(\xi-1)}{\xi}}\right]^{\frac{\xi}{\xi-1}}$$

Rearranging the above equation yields

$$C_t=\left[(1-\omega)^{\frac{1}{\xi}}\left[\left(C_t^F\right)^{\frac{(\xi-1)}{\xi}}\left(\frac{1-\omega}{\omega}\right)^{\frac{(\xi-1)}{\xi}}\left[\frac{P_t^F}{P_t^H}\right]^{\xi-1}\right]+\omega^{\frac{1}{\xi}}\left(C_t^F\right)^{\frac{(\xi-1)}{\xi}}\right]^{\frac{\xi}{\xi-1}}$$

$$=\omega^{\frac{1}{\xi-1}}C_t^F\left[\left(\frac{1-\omega}{\omega}\right)\left[\frac{P_t^F}{P_t^H}\right]^{\xi-1}+1\right]^{\frac{\xi}{\xi-1}}$$

$$=\omega^{\frac{1}{\xi-1}}C_t^F\left[\frac{(1-\omega)\left(P_t^H\right)^{1-\xi}+\omega\left(P_t^F\right)^{1-\xi}}{\omega\left(P_t^F\right)^{1-\xi}}\right]^{\frac{\xi}{\xi-1}}$$

Reformulating the latest expression yields

$$C_t^F = \frac{\omega\left(P_t^F\right)^{-\xi}}{\left[(1-\omega)\left(P_t^H\right)^{1-\xi} + \omega\left(P_t^F\right)^{1-\xi}\right]^{-\left(\frac{\xi}{1-\xi}\right)}} C_t \qquad \text{(A.5)}$$

Multiplying both sides of the above equation by P_t^F yields

$$P_t^F C_t^F = \frac{\omega\left(P_t^F\right)^{1-\xi}}{\left[(1-\omega)\left(P_t^H\right)^{1-\xi} + \omega\left(P_t^F\right)^{1-\xi}\right]^{-\left(\frac{\xi}{1-\xi}\right)}} C_t \qquad \text{(A.6)}$$

Solving the problem analogously for C_t^H , we shall obtain

$$C_t^H = \frac{(1-\omega)\left(P_t^H\right)^{-\xi}}{\left[(1-\omega)\left(P_t^H\right)^{1-\xi} + \omega\left(P_t^F\right)^{1-\xi}\right]^{-\left(\frac{\xi}{1-\xi}\right)}} C_t \qquad \text{(A.7)}$$

Or

$$C_t^H = (1-\omega)\left(\frac{P_t^H}{P_t^c}\right)^{-\xi} C_t \qquad \text{(A.8)}$$

Multiplying the above equation by P_t^H , we obtain:

$$P_t^H C_t^H = \frac{(1-\omega)\left(P_t^H\right)^{1-\xi}}{\left[(1-\omega)\left(P_t^H\right)^{1-\xi} + \omega\left(P_t^F\right)^{1-\xi}\right]^{-\left(\frac{\xi}{1-\xi}\right)}} C_t \qquad \text{(A.9)}$$

Combining (A.9) and (A.6) yields:

$$\begin{aligned} P_t^H C_t^H + P_t^F C_t^F &= \frac{(1-\omega)\left(P_t^H\right)^{1-\xi} + \omega\left(P_t^F\right)^{1-\xi}}{\left[(1-\omega)\left(P_t^H\right)^{1-\xi} + \omega\left(P_t^F\right)^{1-\xi}\right]^{-\left(\frac{\xi}{1-\xi}\right)}} C_t \\ &= \left[(1-\omega)\left(P_t^H\right)^{1-\xi} + \omega\left(P_t^F\right)^{1-\xi}\right]^{\frac{1}{1-\xi}} C_t \qquad \text{(A.10)} \\ &= P_t^c C_t \end{aligned}$$

The representative household further solves the cost minimization problem for the differentiated product. This problem can be stated as follow:

$$Min\left\{\int_{\omega}^{1} P_t^H(j) C_t^H(j)\, dj\right\}$$

subject to the following constraint:

$$C_t^H = \left[\left(\frac{1}{1-\omega} \right)^{\frac{1}{\vartheta}} \int_{\omega}^{1} C_t^H(j)^{\left(\frac{\vartheta-1}{\vartheta}\right)} dj \right]^{\frac{\vartheta}{(\vartheta-1)}}, \quad \vartheta > 1. \tag{A.11}$$

The first order condition with respect to $C_t^H(j)$ is:

$$P_t^H(j) = \lambda_t \left[\left(\frac{1}{1-\omega} \right)^{\frac{1}{\vartheta}} \int_{\omega}^{1} C_t^H(j)^{\left(\frac{\vartheta-1}{\vartheta}\right)} dj \right]^{\frac{1}{(\vartheta-1)}} \left(\frac{1}{1-\omega} \right)^{\frac{1}{\vartheta}} C_t^H(j)^{-\frac{1}{\vartheta}}, \tag{A.12}$$

where λ_t is the Lagrange multiplier.

Analogously, for others goods j' the first order condition with respect to $C_t^H(j')$ is

$$P_t^H(j') = \lambda_t \left[\left(\frac{1}{1-\omega} \right)^{\frac{1}{\vartheta}} \int_{\omega}^{1} C_t^H(j')^{\left(\frac{\vartheta-1}{\vartheta}\right)} dj \right]^{\frac{1}{(\vartheta-1)}} \left(\frac{1}{1-\omega} \right)^{\frac{1}{\vartheta}} C_t^H(j')^{-\frac{1}{\vartheta}}, \tag{A.13}$$

By dividing (A.12) by (A.13), for any two goods j and j', then we can show that

$$C_t^H(j) = C_t^H(j') \left[\frac{P_t^H(j')}{P_t^H(j)} \right]^{\vartheta} \tag{A.14}$$

Substituting (A.14) into (A.11) yields:

$$C_t^H = \left[\left(\frac{1}{1-\omega} \right)^{\frac{1}{\vartheta}} \int_{\omega}^{1} \left[C_t^H(j') \left[\frac{P_t^H(j')}{P_t^H(j)} \right]^{\vartheta} \right]^{\left(\frac{\vartheta-1}{\vartheta}\right)} dj \right]^{\frac{\vartheta}{(\vartheta-1)}}$$

$$= C_t^H(j') P_t^H(j')^{\vartheta} \left(\frac{1}{1-\omega} \right)^{\frac{1}{\vartheta-1}} \left[\int_{\omega}^{1} P_t^H(j)^{1-\vartheta} dj \right]^{\frac{\vartheta}{\vartheta-1}}$$

Manipulating the above equation yields:

$$C_t^H(j') = \frac{P_t^H(j')^{-\vartheta}}{\left(\frac{1}{1-\omega} \right)^{\frac{1}{\vartheta-1}} \left[\int_{\omega}^{1} P_t^H(j)^{1-\vartheta} dj \right]^{\frac{\vartheta}{\vartheta-1}}} C_t^H \tag{A.15}$$

Substituting (A.8) in to (A.15) yields:

$$C_t^H(j') = \frac{P_t^H(j')^{-\vartheta}}{\left(\frac{1}{1-\omega}\right)^{\frac{1}{\vartheta-1}}\left[\int_\omega^1 P_t^H(j)^{1-\vartheta}\,dj\right]^{\frac{\vartheta}{\vartheta-1}}}\left[\frac{(1-\omega)\left(P_t^H\right)^{-\xi}}{\left(P_t^c\right)^{-\xi}}C_t\right]$$

$$C_t^H(j') = \frac{P_t^H(j')^{-\vartheta}}{\left(\frac{1}{1-\omega}\right)^{\frac{\vartheta}{\vartheta-1}}\left[\int_\omega^1 P_t^H(j)^{1-\vartheta}\,dj\right]^{\frac{\vartheta}{\vartheta-1}}}\left[\frac{P_t^H}{P_t^c}\right]^{-\xi}C_t$$

$$= \frac{P_t^H(j')^{-\vartheta}}{\left[\left(\frac{1}{1-\omega}\right)\int_\omega^1 P_t^H(j)^{1-\vartheta}\,dj\right]^{\frac{\vartheta}{\vartheta-1}}}\left[\frac{P_t^H}{P_t^c}\right]^{-\xi}C_t$$

Manipulating the above equation yields:

$$C_t^H(j') = \left[\frac{P_t^H(j')}{P_t^H}\right]^{-\vartheta}\left[\frac{P_t^H}{P_t^c}\right]^{-\xi}C_t \qquad \text{(A.16)}$$

Household Utility Maximization Problem

This section shows the first order conditions generated from the representative domestic household's utility maximization problem. The representative household maximizes the discounted life time utility subject to constraints as described in the main text. The problem can be expressed as:

$$\max_{C_t, L_t, B_{t+1}, F_{t+1}, K_{t+1}, M_{t+1}} U_t = \mathbb{E}_t \sum_{\tau=0}^{\infty}\left\{\delta^\tau\left[\frac{C_{t+\tau}^{1-\rho}}{1-\rho} - \frac{\chi_L}{1+\rho_L}L_{t+\tau}^{1+\rho_L} + \frac{\chi}{1-\rho_M}\left(\frac{M_{t+\tau}}{P_{t+\tau}^c}\right)^{1-\rho_M}\right]\right\}$$

subject to

$$P_t^c C_t + M_{t+1} + S_t F_{t+1} + B_{t+1} + P_t^c K_{t+1} = M_t + \left(1+i_{t-1}\right)B_t + \left(1+i_t^*\right)S_t F_t$$
$$+\left(R_t^K + \left(1-\delta_K\right)P_t^c\right)K_t + W_t L_t + \Phi_t$$

The first order conditions are given by:

$$\frac{\partial U}{\partial C_t} \Rightarrow \qquad C_t^{-\rho} - \breve{\lambda}_t P_t^c = 0$$

$$\frac{C_t^{-\rho}}{P_t^c} = \breve{\lambda}_t \tag{A.17}$$

$$\frac{\partial U}{\partial K_{t+1}} \Rightarrow \quad \breve{\lambda}_{t+1|t}\delta\left(R_{t+1|t}^K + (1-\delta_K)P_{t+1|t}^c\right) = P_t^c \breve{\lambda}_t \tag{A.18}$$

$$\frac{\partial U}{\partial B_{t+1}} \Rightarrow \quad -\breve{\lambda}_t + \delta(1+i_t)\breve{\lambda}_{t+1|t} = 0$$

$$\delta = \frac{\breve{\lambda}_t}{(1+i_t)\breve{\lambda}_{t+1|t}} \tag{A.19}$$

$$\frac{\partial U}{\partial L_t} \Rightarrow \quad -\chi_L (L_t)^{\rho_L} = -\breve{\lambda}_t W_t \tag{A.20}$$

$$\frac{\partial U}{\partial M_{t+1}} \Rightarrow \quad \chi\left(\frac{M_{t+1|t}}{P_{t+1|t}^c}\right)^{-\rho_M}\frac{1}{P_t^c} - \breve{\lambda}_t + \delta\breve{\lambda}_{t+1|t} = 0 \tag{A.21}$$

$$\frac{\partial U}{\partial F_{t+1}} \Rightarrow \quad -S_t\breve{\lambda}_t + \delta S_{t+1|t}\breve{\lambda}_{t+1|t}\left(1+i_t^*\right) = 0 \tag{A.22}$$

According to (A.19) and (A.22), the standard uncovered interest rate parity condition can be expressed as the following:

$$1+i_t = \frac{S_{t+1|t}}{S_t}\left(1+i_t^*\right) \tag{A.23}$$

where $\breve{\lambda}_t$ denote the Lagrange multiplier with respect to the household budget constraint.

Making use (A.17) and (A.19), (A.20)-(A.21) can be expressed as

$$\frac{W_t}{P_t^c} = \chi_L \frac{(L_t)^{\rho_L}}{(C_t)^{-\rho}} \tag{A.24}$$

$$\begin{gathered}\chi\left(\frac{M_{t+1|t}}{P_{t+1|t}^c}\right)^{-\rho_M}\frac{1}{P_t^c} = \frac{i_t}{1+i_t}\breve{\lambda}_t \\ \left(\frac{M_{t+1|t}}{P_{t+1|t}^c}\right)^{\rho^M} = \chi\left(\frac{1+i_{t+1|t}}{i_t}\right)C_t^{\rho}\end{gathered} \tag{A.25}$$

$$\frac{C_t}{C_{t+1|t}} = \left(\delta \left(1+i_t\right) \frac{P_t^c}{P_{t+1|t}^c} \right)^{-\frac{1}{\rho}} \tag{A.26}$$

$$\left(\frac{C_{t+1|t}}{C_t} \right)^{-\rho} \delta \left[\left(\frac{R_{t+1|t}^K}{P_{t+1|t}^c} \right) + \left(1-\delta_K\right) \right] = 1 \tag{A.27}$$