# Chapter 3

# Proposed Techniques

In this chapter, we descript four spectrum sensing techniques that we are proposed. Two methods are double-constrain adaptive energy detection (DCAED) and fast spectrum sensing with coordinate system (FSC) for additive white Gaussian noise (AWGN) channel. Two techniques are two-stage spectrum sensing and modified FSC for noise uncertainty and path loss environment.

## 3.1 Double constraints adaptive energy detection

From chapter 2, we known that ED [60-63] is the most widely utilized because it consumes the shortest sensing time with the least complexity. However, the accuracy of detection of ED is unreliable under bad condition of communication channel or at low signal to noise ratio (SNR) condition. In [64, 65], the performance of ED is improved by using an adaptive threshold. In general, the threshold of ED is set by fixing target performance metrics. There are 2 ways to set a threshold for ED. The first way is done by fixing target probability of false alarm which is called “constant false alarm rate (CFAR)”. The other way is done by fixing target probability of detection which is called “constant detection rate (CDR)”. An adaptive threshold energy detection (ATED) changes its decision threshold depending on the condition of communication channel. The system threshold switches between the threshold of CFAR and CDR. Although the detection performance of ED is improved, the false alarm detection rate does not achieve the target performance stated by IEEE 802.22 standard which the spectrum sensing technique has to perform spectrum sensing with probability of false detection less than 0.1.

### 3.1.1 Conventional energy detection technique

As shown in Figure3-1, the PU signal is received by SU. The output from bandpass filter is digitized by analog to digital converter (ADC). The existence of PU is determined by measuring the energy of the received signal and compares to predetermined threshold.

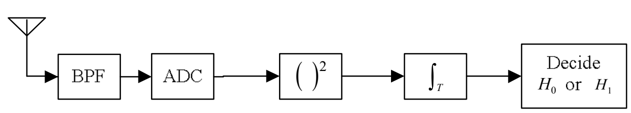


**Figure 3-1** Model of conventional energy detection technique [64].

The decision statistic of ED is given as

$$Y_{\mathrm{ED}} = \frac{1}{N} \sum_{n=1}^{N} |\mathbf{x}(n)|^2. \tag{3-1}$$

When the PU absents, the decision statistic can be represented as

$$Y_{\mathrm{ED}} = \frac{1}{\mathrm{N}} \sum_{\mathrm{n}=1}^{\mathrm{N}} |\boldsymbol{\eta}\ (\mathrm{n})|^2. \tag{3-2}$$

If both of primary signal and noise is an independent and identically distributed (i.i.d.) random process. The mean ($\mu_0$) and variance ${\sigma_0}^2$ under hypothesis $H_0$ can be derived as

$$\mu_0 = E|Y_{\mathrm{ED}}| = \frac{1}{\mathrm{N}} \sum_{\mathrm{n}=1}^{\mathrm{N}} |\boldsymbol{\eta}\ (\mathrm{n})|^2$$

$$= \frac{1}{\mathrm{N}} \sum_{\mathrm{n}=1}^{\mathrm{N}} {\sigma_{\boldsymbol{\eta}}}^2 = {\sigma_{\boldsymbol{\eta}}}^2 \tag{3-3}$$

$${\sigma_0}^2 = E|Y_{\mathrm{ED}} - \mu_0|^2 = \frac{1}{\mathrm{N}} \left|E|\boldsymbol{\eta}\ (\mathrm{n})|^4 - {\sigma_{\boldsymbol{\eta}}}^4\right|. \tag{3-4}$$

If Gaussian noise is real-valued, $E|\boldsymbol{\eta}\ (\mathrm{n})|^4 = 3{\sigma_{\boldsymbol{\eta}}}^2$. The variance ${\sigma_0}^2$ can be expressed as

$${\sigma_0}^2 = \sqrt{\frac{2}{N}} {\sigma_{\boldsymbol{\eta}}}^2. \tag{3-5}$$

Thus, the probability of false alarm ($\mathrm{P_{fa}}$) can be expressed as

$$\mathrm{P_{fa}} = Q\left(\left(\frac{\lambda}{{\sigma_{\boldsymbol{\eta}}}^2} - 1\right)\sqrt{\frac{N}{2}}\right) \tag{3-6}$$

where $\lambda$ is decision threshold, ${\sigma_{\boldsymbol{\eta}}}^2$ is the variance of noise and ${\sigma_{\mathbf{s}}}^2$ is the variance of primary user signal and $Q(\cdot)$ is standard Gauss complementary cumulative distribution function.

When the PU presents ($H_1$), the decision statistic is given as

$$Y_{\mathrm{ED}} = \frac{1}{\mathrm{N}} \sum_{\mathrm{n}=1}^{\mathrm{N}} |\mathbf{s}(\mathrm{n}) + \boldsymbol{\eta}\ (\mathrm{n})|^2 \tag{3-7}$$

Under hypothesis $H_1$, the mean ($\mu_1$) can be derived as

$$\mu_1 = E|Y_{\mathrm{ED}}| = \frac{1}{\mathrm{N}} \sum_{\mathrm{n}=1}^{\mathrm{N}} |\mathbf{s}(\mathrm{n}) + \boldsymbol{\eta}\ (\mathrm{n})|^2$$

$$= {\sigma_{\mathbf{s}}}^2 + {\sigma_{\boldsymbol{\eta}}}^2 = (\gamma + 1){\sigma_{\boldsymbol{\eta}}}^2 \tag{3-8}$$

$$\gamma = \frac{{\sigma_{\mathbf{s}}}^2}{{\sigma_{\boldsymbol{\eta}}}^2} \tag{3-9}$$

where $\gamma$ represents signal-to-noise ratio (SNR). The variance ${\sigma_1}^2$ is given as

$${\sigma_1}^2 = E|Y_{\mathrm{ED}} - \mu_1|^2 \tag{3-10}$$

$$= \frac{1}{\mathrm{N}} \left|E|\mathbf{s}\ (\mathrm{n})|^4 + E|\boldsymbol{\eta}\ (\mathrm{n})|^4 - \left({\sigma_{\mathbf{s}}}^2 - {\sigma_{\boldsymbol{\eta}}}^2\right) + 2{\sigma_{\mathbf{s}}}^2{\sigma_{\boldsymbol{\eta}}}^2\right|$$

If Gaussian noise is real-valued, $E|\mathbf{s}(\mathrm{n})|^4 = 3{\sigma_\mathbf{s}}^2$ and $E|\boldsymbol{\eta}\,(\mathrm{n})|^4 = 3{\sigma_{\boldsymbol{\eta}}}^2$. The variance ${\sigma_1}^2$ can be expressed as

$$ {\sigma_1}^2 = \sqrt{\frac{2}{N}}(\gamma + 1){\sigma_{\boldsymbol{\eta}}}^2. \tag{3-11} $$

Thus, the probability of detection $(\mathrm{P_d})$ can be represented as

$$ \mathrm{P_d} = Q\left[\frac{\sqrt{\mathrm{N}/2}}{\gamma+1}\left(\frac{\lambda}{{\sigma_{\boldsymbol{\eta}}}^2} - \gamma - 1\right)\right]. \tag{3-12} $$

There are 2 ways to set the threshold for ED technique. The first technique is called CFAR which the threshold is set by fixing $\mathrm{P_{fa}}$. Thus, the threshold for CFAR can be computed by

$$ \lambda_{CFAR} = \left(Q^{-1}(\mathrm{P_{fa}})\sqrt{\frac{2}{N}} + 1\right){\sigma_{\boldsymbol{\eta}}}^2. \tag{3-13} $$

To set the threshold by fixing $\mathrm{P_d}$ , which is called CDR, can be done by

$$ \lambda_{CDR} = \left(\sqrt{\frac{2}{N}}(\gamma + 1)Q^{-1}(\mathrm{P_d}) + \gamma + 1\right){\sigma_{\boldsymbol{\eta}}}^2. \tag{3-14} $$

However, it should be realized that the predetermined thresholds ($\lambda_{CDR}$ and $\lambda_{CFAR}$) are set by fixing only a single target performance metric. Thus, there is always be a tradeoff in the performance of ED technique by fixing only a single target performance metric. ED with threshold based on CDR gives poor detection performance in perspective of $\mathrm{P_{fa}}$. Conversely, ED with threshold based on CFAR gives poor detection performance in perspective of $\mathrm{P_d}$.

### 3.1.2 Adaptive threshold energy detection

From [63, 64], the adaptive threshold energy detection (ATED) technique was proposed. The adaptive parameter ($\alpha$) was introduced to vary the threshold depending on the condition of communication channel. As shown is Figure 3-2, the SNR estimator plays as an important part of the system. The SNR estimator estimates the variance noise from the received signal and sends it to the threshold setter device. The threshold setter device generates a new threshold which is appropriate to the communication channel at the period of time

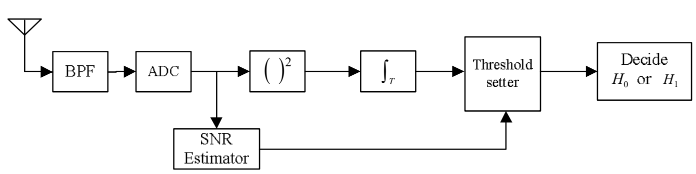


**Figure 3-2** Model of adaptive threshold energy detection technique [64]

The new threshold is given by

$$\lambda = \lambda_{CFAR} + \alpha * (\lambda_{CDR} - \lambda_{CFAR}), 0 \leq \alpha \leq 1. \tag{3-15}$$

The adaptive parameter ($\alpha$) is set depending on the condition of communication channel. Since the SNR of communication channel is estimated, the system calculates the critical sample which is appropriate to the communication channel at the period of time. If the number of sample of the system is lower than the number of critical sample, the adaptive parameter ($\alpha$) is set to be 1. On the other hand, if the number of sample of the system is greater than the number of critical sample, the adaptive parameter ($\alpha$) is set to be 0. In addition, the value of adaptive parameter ($\alpha$) can be change between 0 to 1.

### 3.1.3 Double constraints adaptive energy detection

In this section, double constraint adaptive energy detection (DCAED) is explained. DCAED exploits an interdependent between $\mathrm{P_{fa}}$ and $\mathrm{P_d}$ to generate a new adaptive factor ($\beta$). However, there is no directly way to set the threshold by fixing $\mathrm{P_{fa}}$ and $\mathrm{P_d}$ as the target performance metrics. DCAED sets the adaptive factor ($\beta$) by using the critical sample ($N_c$), since $N_c$ retains the independent between $\mathrm{P_{fa}}$ and $\mathrm{P_d}$. Then adaptive factor is used to set the threshold in order to achieve target performance metrics.

The system model is shown in Figure 3-3. The information from SNR estimator is gathered by adaptive threshold device. The estimated SNR value is compared to critical SNR ($\gamma_c$). If the estimated value is greater than critical value ($\gamma_c$) means that the commination channel is in a good condition which conventional ED offers a reliable detection performance. Thus, the adaptive factor ($\beta$) is set to make the system remains the new threshold as predetermined threshold. On the other hand, if the estimated value is lower than critical value, the new threshold is generated by setting the adaptive factor ($\beta$) depending on the condition of communication channel.

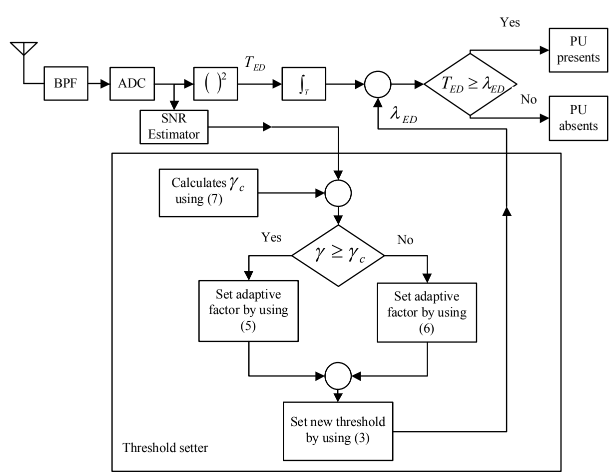


**Figure 3-3** Model of DCAED.

The threshold is given as

$$\lambda_{New} = \beta {\sigma_{\mathbf{est}}}^2 \left(\frac{\lambda_{\mathrm{CFAR}}}{{\sigma_{\boldsymbol{\eta}}}^2} - 1\right) + {\sigma_{\mathbf{est}}}^2 \tag{3-16}$$

where ${\sigma_{\mathbf{est}}}^2$ is an estimated noise variance .

$N_c$ refers to a minimum number of sample that is required by conventional energy detection technique to meet the target performance metrics ($\mathrm{P_{fa}}$ and $\mathrm{P_d}$). By using (3-6) and (3-12), the interdependent between these parameters can be shown as

$$\mathrm{P_{fa}} = Q\left(Q^{-1}(\mathrm{P_d})(\gamma + 1) + \gamma\sqrt{\frac{N}{2}}\right) \tag{3-17}$$

$$\mathrm{P_d} = Q\left(\frac{1}{(\gamma+1)}\left(Q^{-1}(\mathrm{P_{fa}}) - \gamma\sqrt{\frac{N}{2}}\right)\right). \tag{3-18}$$

By solving (3-6) and (3-12), the critical sample ($N_c$) can be expressed as

$$N_c = \frac{2}{\gamma^2}[Q^{-1}(\mathrm{P_{fa}}) - Q^{-1}(\mathrm{P_d})(\gamma + 1)]^2. \tag{3-19}$$

From the definition of critical sample, we can conclude that if we set the new threshold by changing the sample ($N$) to critical sample ($N_c$) in (3-13) or (3-14). The performance of ED will meet the target performance metrics. However, it is not feasible to change the sample to the desired number in practical. Thus, DCAED meets the target accuracy of detection performance metrics as changing critical sample by using the adaptive factor to change the system threshold.

By solving (3-6), (3-17) and (3-19) under condition of the proposed scheme. The adaptive factor ($\beta$) of the system can be expressed as

$$\beta = \begin{cases} \dfrac{\lambda_{\mathrm{CFAR}} - {\sigma_{\mathbf{est}}}^2}{\left(\frac{\lambda_{\mathrm{CFAR}}}{{\sigma_{\boldsymbol{\eta}}}^2} - 1\right){\sigma_{\mathbf{est}}}^2} & , \ \gamma \geq \gamma_c \quad [C_0] \quad (3-20) \\ \dfrac{\gamma\sqrt{N/2}}{(Q^{-1}(\mathrm{P_{fa}}) - Q^{-1}(\mathrm{P_d})(\gamma+1))} & , \ \gamma < \gamma_c \quad [C_1] \quad (3-21) \end{cases}$$

where $C_0$ is the condition that estimated SNR is greater than critical SNR and $C_1$ is the condition that estimated SNR is lower than critical SNR.

In addition, the critical SNR ($\gamma_c$) for the system is given by

$$\gamma_c = \frac{Q^{-1}(\mathrm{P_{fa}}) - Q^{-1}(\mathrm{P_d})}{Q^{-1}(P_{\mathrm{d}}) - \sqrt{\frac{N}{2}}}. \tag{3-22}$$

### 3.2 Fast Spectrum Sensing with Coordinate System

In this section, we describe in detail with mathematical models of the fast spectrum sensing with coordinate system (FSC) algorithm. The FSC algorithm is a spectrum sensing technique that requires prior knowledge of a PU's signals. The framework for the FSC algorithm can be categorized into two phases — coordinate system construction and sensing. The coordinate system must be predetermined from the two most significant features of WM signals and kept in the knowledge base. The sensing phase determines the existence of a PU by comparing the FSC decision statistic ($Y_{\mathrm{FSC}}$) to the FSC threshold ($\gamma_{\mathrm{FSC}}$). The decision statistic is calculated by projecting the PU's signal onto the predetermined coordinate system.

Following the PCA algorithm, the WM signals are first decomposed into a small set of features. The significance of each feature can then be explained by an eigenvector and eigenvalue, where the eigenvector represents the direction of the feature and the eigenvalue explains the variance of the WM signals in that direction. Therefore, the eigenvector corresponding to the highest eigenvalue represents the direction in which most of the data within the WM signals are varying. This eigenvector refers to the most significant feature of WM signals.

#### 3.2.1 Coordinate System Construction

In this section, our coordinate system is introduced. The new coordinate system is of a lower dimension than the original data space. The main objectives of this phase are to select the two most significant features of WM signals and to construct a coordinate system. Our coordinate system construction process (as shown in Figure 3-4) exploits the feature extraction and selection process of a PCA algorithm [66-67] to filter out the two most significant features of WM signals and then uses them as the axes for a new coordinate system. Due to the smaller size of the new coordinate system, the FSC algorithm consumes less memory, has less computational burden, and has a short sensing time.

We assume that the WM signals of a PU are known to an SU. These WM signals are used as the *training* signals. Let the vectors $\mathbf{s}_1$, $\mathbf{s}_2$,..., $\mathbf{s}_M$ represent WM signals. These vectors are referred to as *training* vectors. The training vectors are given by

$$\begin{aligned} \mathbf{s}_1 &= [s_1(1)\ s_1(2) \dots s_1(N)]^{\mathrm{T}}, \\ \mathbf{s}_2 &= [s_2(1)\ s_2(2) \dots s_2(N)]^{\mathrm{T}}, \\ &\vdots \\ \mathbf{s}_M &= [s_M(1)\ s_M(2) \dots s_M(N)]^{\mathrm{T}}. \end{aligned} \tag{3-23}$$

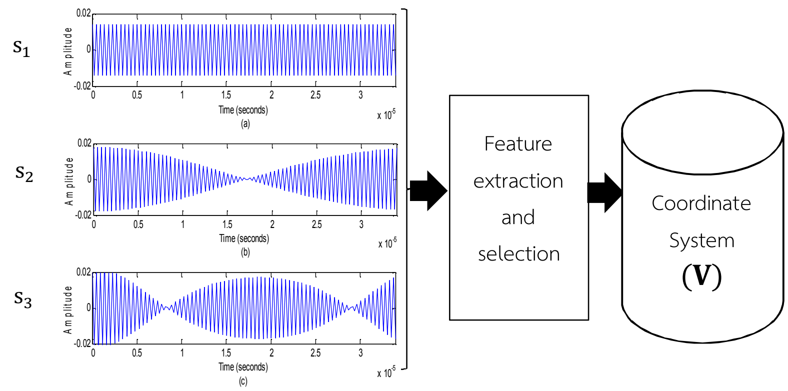


**Figure 3-4** Coordinate system construction phase of FSC algorithm.

The procedure for the coordinate system construction phase is described as follows.

### *A. Feature Extraction*

Firstly, we eliminate the common features of the WM signals by subtracting the average WM signals vector ($\boldsymbol{\varepsilon}$) from each training vector ($\mathbf{s}_i$).

$$\boldsymbol{\beta}_\mathrm{i} = \mathbf{s}_i - \boldsymbol{\varepsilon}, \tag{3-24}$$

where $\boldsymbol{\beta}_i$ is a vector that contains the significant features of the WM signals. The average WM signals vector ($\boldsymbol{\varepsilon}$) can be expressed as

$$\boldsymbol{\varepsilon} = \frac{1}{M}\sum_{i=1}^{M} \mathbf{s}_i. \tag{3-25}$$

Next, we compute the covariance matrix ($\mathbf{C}$) of $\boldsymbol{\beta}_i$, which is given by

$$\mathbf{C} = \frac{1}{M}\sum_{i=1}^{M} \boldsymbol{\beta}_i \boldsymbol{\beta}_i{}^{\mathrm{T}}. \tag{3-26}$$

From the covariance matrix, a matrix of eigenvectors ($\mathbf{V} = [\mathbf{v}_1\ \mathbf{v}_2 \dots \mathbf{v}_d]$) and a vector of corresponding eigenvalues ($\boldsymbol{\lambda} = [\lambda_1\ \lambda_2 \dots \lambda_d]^{\mathrm{T}}$) can be obtained by using the aforementioned eigen-decomposition algorithm.

### *B. Feature Selection*

From the matrix of eigenvectors ($\mathbf{V}$), we keep only the $k$ *best* eigenvectors (that is, those that correspond to the $k$ largest eigenvalues), and the resulting set is then used to form the new coordinate system. The $k$ best eigenvectors are determined by

$$\frac{\sum_{i=1}^{k} \lambda_i}{\sum_{i=1}^{d} \lambda_i} \geq 95\%, \tag{3-27}$$

where $d$ is the number of eigenvalues in set λ.

From our investigation, we found that eigenvectors that had a correspondingly high eigenvalue more effectively represented the features of the WM signals than those eigenvectors that had correspondingly small eigenvalues. It is clear that 95% of the total number of features present in the WM signals is a sufficient amount to be representative of all the existing features. Hence, having decided to only select the $k$ best eigenvectors, the dimension of the WM signals is reduced. Reducing the dimension of the WM signals avoids a huge amount of computational burden. Moreover, the effect of noise from the original signal is avoided due to the reduction in dimension of the WM signals Furthermore, the FSC algorithm is tolerant to noise.

### 3.2.2 Sensing Phase

In the sensing phase (see Figure 3-5), the weight of correspondence between the received WM signal and the new coordinate system is calculated by projecting the received signal onto the coordinate system. This weight describes the distribution of the received signal in the new coordinate system. The weight, given as a vector ($\hat{\mathbf{x}}$), can be expressed as

$$\hat{\mathbf{x}} = \mathbf{V}^{\mathrm{T}}(\mathbf{x} - \boldsymbol{\varepsilon}). \tag{3-28}$$

The magnitude of the weight vector is defined as the FSC decision statistic ($Y_{\mathrm{FSC}}$). The magnitude of the weight vector will rise when a PU is present. Otherwise, the magnitude of the weight vector will fall when a PU is not present. The FSC decision statistic ($Y_{\mathrm{FSC}}$) can be expressed as

$$Y_{\mathrm{FSC}} = \|\hat{\mathbf{x}}\|^2 = \left(\sqrt{\textstyle\sum_{i=1}^{k}(\hat{\mathbf{x}})^2}\right)^2 = \textstyle\sum_{i=1}^{k}(\hat{\mathbf{x}})^2. \tag{3-29}$$

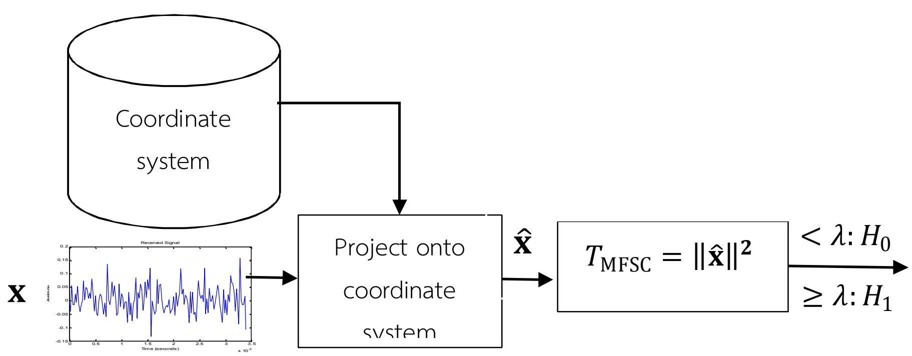


**Figure 3-5** Sensing phase of FSC algorithm.

A mathematical model for the probability of false alarm of the FSC algorithm is given by

$$P_{\mathrm{fa(FSC)}} = P[Y_{\mathrm{FSC}} \geq \gamma_{\mathrm{FSC}} | H_0]. \tag{3-30}$$

Under condition $H_0$,

$$\hat{\mathbf{x}}_{\boldsymbol{\eta}} = \mathbf{V}^T(\mathbf{x}_{\boldsymbol{\eta}} - \boldsymbol{\varepsilon}). \tag{3-31}$$

$$Y_{\mathrm{FSC}} = \left\|\hat{\mathbf{x}}_{\boldsymbol{\eta}}\right\|^2 = \left(\sqrt{\sum_{i=1}^{k}\left(\hat{\mathbf{x}}_{\boldsymbol{\eta}}\right)^2}\right)^2 = \sum_{i=1}^{k}\left(\hat{\mathbf{x}}_{\boldsymbol{\eta}}\right)^2, \tag{3-32}$$

$$\mu_{H_0} = \mathrm{E}[Y_{\mathrm{FSC}}] = \mathrm{E}\left[\sum_{i=1}^{k}\left(\hat{\mathbf{x}}_{\boldsymbol{\eta}}\right)^2\right] = k\mathfrak{m}'_{2,H_0}, \tag{3-33}$$

$$\sigma_{H_0}^2 = \mathrm{Var}\left[\sum_{i=1}^{k}\left(\hat{\mathbf{x}}_{\boldsymbol{\eta}}\right)^2\right] = k\left(\mathfrak{m}'_{4,H_0} - \left(\mathfrak{m}'_{2,H_0}\right)^2\right), \tag{3-34}$$

$$P_{\mathrm{fa(FSC)}} = Q\left[\left(\frac{\gamma_{\mathrm{FSC}} - k\mathfrak{m}'_{2,H_0}}{\sqrt{k\left(\mathfrak{m}'_{4,H_0} - \left(\mathfrak{m}'_{2,H_0}\right)^2\right)}}\right)\right]. \tag{3-35}$$

Note that $\mu_{H_i}$ is the mean value of $H_i$ and that $\mathfrak{m}'_n$ is the $n^{\mathrm{th}}$ order moment of the FSC decision statistic ($Y_{\mathrm{FSC}}$).

Similar to the probability of false alarm, the probability of detection for the FSC algorithm can be expressed as

$$P_{\mathrm{d(FSC)}} = P[Y_{\mathrm{FSC}} \geq \gamma_{\mathrm{FSC}} | H_1]. \tag{3-36}$$

Under condition $H_1$,

$$\hat{\mathbf{x}}_{\mathbf{s}+\boldsymbol{\eta}} = \mathbf{V}^{\mathrm{T}}(\mathbf{x}_{\mathbf{s}+\boldsymbol{\eta}} - \boldsymbol{\varepsilon}), \tag{3-37}$$

$$Y_{\mathrm{FSC}} = \left\|\hat{\mathbf{x}}_{\mathbf{s}+\boldsymbol{\eta}}\right\|^2 = \left(\sqrt{\sum_{i=1}^{k}\left(\hat{\mathbf{x}}_{\mathbf{s}+\boldsymbol{\eta}}\right)^2}\right)^2 = \sum_{i=1}^{k}\left(\hat{\mathbf{x}}_{\mathbf{s}+\boldsymbol{\eta}}\right)^2, \tag{3-38}$$

$$\mu_{H_1} = \mathrm{E}[Y_{\mathrm{FSC}}] = \mathrm{E}\left[\sum_{i=1}^{k}\left(\hat{\mathbf{x}}_{\mathbf{s}+\boldsymbol{\eta}}\right)^2\right] = k\mathfrak{m}'_{2,H_1}, \tag{3-39}$$

$$\sigma_{H_1}^2 = \mathrm{Var}\left[\sum_{i=1}^{k}\left(\hat{\mathbf{x}}_{\mathbf{s}+\boldsymbol{\eta}}\right)^2\right] = k\left(\mathfrak{m}'_{4,H_1} - \left(\mathfrak{m}'_{2,H_1}\right)^2\right), \tag{3-40}$$

$$P_{\mathrm{d(FSC)}} = Q\left[\left(\frac{\gamma_{\mathrm{FSC}} - k\mathfrak{m}'_{2,H_1}}{\sqrt{k\left(\mathfrak{m}'_{4,H_1} - \left(\mathfrak{m}'_{2,H_1}\right)^2\right)}}\right)\right]. \tag{3-41}$$

In addition, the probability of misdetection of the FSC algorithm is given by

$$P_{\mathrm{m(FSC)}} = 1 - P_{\mathrm{d(FSC)}}. \tag{3-42}$$

### 3.3 Two-stage spectrum sensing

In this section, the proposed two-stage spectrum sensing algorithms are explained. Our proposed two-stage spectrum sensing algorithms (as depicted in Figure 3-6) exploit the merits of ED CAV and MME technique.

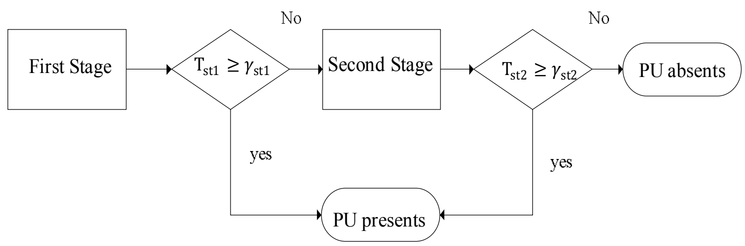


**Figure 3-6** Two-stage spectrum sensing scheme [53].

The scheme of the proposed two-stage spectrum sensing techniques can be separated into 2 stages including coarse sensing stage and fine sensing stage. Mathematical models of overall probability of false alarm and overall probability of detection for two-stage spectrum sensing are given by

$$P_{fa} = P_{fa,1^{st}} + (1 - P_{fa,1^{st}})\, P_{fa,2^{nd}} \tag{3-43}$$

$$P_{d} = P_{d,1^{st}} + (1 - P_{d,1^{st}})\, P_{d,2^{nd}} \tag{3-44}$$

where $P_{fa}$ is $P_{fa}$ of the system, $P_d$ is $P_d$ of the system, $P_{fa,1^{st}}$ is $P_{fa}$ of the first stage, $P_{fa,2^{nd}}$ is $P_{fa}$ of the second stage, $P_{d,1^{st}}$ is $P_d$ of the first stage and $P_{d,2^{nd}}$ is $P_d$ of the second stage.

For a given channel, the existence of primary user is firstly determined by the first stage. Similar to other two-stage spectrum sensing techniques [52, 73], ED is utilized as the first stage. Although ED offers inaccurate detection at low SNR and when uncertainty noise power occurs, it performs spectrum sensing within short time. In addition, at high SNR environment, ED offers an accurate detection. If an average energy of received signal is greater than the threshold ($\gamma_{ED}$) then the spectrum band is declared to be presented. If the average energy of received signal is lower than $\gamma_{ED}$, the second stage is activated. The threshold of the first stage can be expressed as

$$\gamma_{ED} = \left(Q^{-1}\left(\frac{P_{fa,ED}}{\sqrt{N}}\right) + 1\right) \sigma_{\eta}{}^{2} \tag{3-45}$$

In our proposed algorithm, MME and CAV are utilized as a second stage. For ED to CAV two-stage spectrum sensing technique, after the second stage is activated, the statistical covariance of the signal sample is computed by (2-20). If the statistical covariance of the signal sample is lower than the threshold (2-21), the two-stage spectrum sensing technique determines that primary user absents. If the statistical covariance of the signal sample is greater than the threshold, the two-stage spectrum sensing technique determines that primary user presents.

For ED to MME two-stage spectrum sensing technique, after the second stage is activated, the maximum and minimum eigenvalue of covariance matrix of signal sample is computed by (2-14). If the ratio of maximum to minimum eigenvalue is lower than the threshold (2-28), the two-stage spectrum sensing technique determines that primary user absents. Otherwise, the two-stage spectrum sensing technique determines that primary user presents.

### 3.4 Modified- fast spectrum sensing with coordinate system (MFSC)

In this section, we both derive the mathematical model and describe the framework of modified- fast spectrum sensing with coordinate system (MFSC), which is modified from FSC (section 3.2), under path loss effect and noise uncertainty. The framework of MFSC algorithm is separated into two phases including coordinate system construction and sensing like FSC. Firstly, the coordinate system must be predetermined by keeping the two most significant features of WM signals. The sensing phase determines the existence of a PU by comparing the MFSC decision statistic ($T_{\mathrm{MFSC}}$), where $T_{\mathrm{MFSC}}$ is calculated by projecting the received signal onto the coordinate system, to the MFSC threshold ($\lambda_{\mathrm{MFSC}}$).

#### 3.4.1 Coordinate System Construction

To construct a coordinate system, the known WM signals are decomposed into a set of features. Only the two most significant features are obtained and used as the axes of the coordinate system. The significance of each feature is explained by the eigenvector which is corresponding to the maximum eigenvalue.

Lets $\mathbf{s}_{\mathrm{i}}$ is a vector that represents WM signal. This vector is known as *training* vector. The training vectors are given by

$$\begin{aligned} \mathbf{s}_1 &= [s_1(1)\ s_1(2) \dots s_1(N)]^{\mathrm{T}}, \\ \mathbf{s}_2 &= [s_2(1)\ s_2(2) \dots s_2(N)]^{\mathrm{T}}, \\ &\vdots \\ \mathbf{s}_M &= [s_M(1)\ s_M(2) \dots s_M(N)]^{\mathrm{T}}. \end{aligned} \tag{3-46}$$

The procedure of the coordinate system construction can be summarized as the following

Firstly, the common features of the WM signals is eliminated by subtracting the average WM signals vector ($\boldsymbol{\varepsilon}$) from each training vector ($\mathbf{s}_i$).

$$\boldsymbol{\beta}_{\mathrm{i}} = \mathbf{s}_i - \boldsymbol{\varepsilon}, \tag{3-47}$$

where $\boldsymbol{\beta}_i$ is a vector that contains the significant features of the WM signals. The average WM signals vector ($\boldsymbol{\varepsilon}$) can be expressed as

$$\boldsymbol{\varepsilon} = \frac{1}{M}\sum_{i=1}^{M} \mathbf{s}_i. \tag{3-48}$$

Next, the covariance matrix ($\mathbf{C}$) of $\boldsymbol{\beta}_i$ is computed. Therefore, the covariance matrix ($\mathbf{C}$) is given by

$$\mathbf{C} = \frac{1}{M}\sum_{i=1}^{M} \boldsymbol{\beta}_i \boldsymbol{\beta}_i^{\mathrm{T}}. \tag{3-49}$$

Using the eigen-decomposition algorithm, a matrix of eigenvectors ($\mathbf{V} = [\mathbf{v}_1\ \mathbf{v}_2\ \dots\ \mathbf{v}_d]$) and a vector of corresponding eigenvalues ($\boldsymbol{e} = [e_1\ \ e_2\ \dots\ e_d]^{\mathrm{T}}$) are obtained. Finally, only the $k$ *best* eigenvectors corresponding to the $k$ largest eigenvalues are used to form the coordinate system. The number of $k$ can be determined by

$$\frac{\sum_{i=1}^{k} e_i}{\sum_{i=1}^{d} e_i} \geq 95\%, \tag{3-50}$$

where $d$ is the number of eigenvalues in set $\boldsymbol{e}$.

### 3.4.2 Sensing Phase

The weight vector ($\hat{\mathbf{x}}$) is given as

$$\hat{\mathbf{x}} = \mathbf{V}^{\mathrm{T}}(\mathbf{x} - \boldsymbol{\varepsilon}). \tag{3-51}$$

and $\mathbf{x}$ is SU received signal under noise uncertainty.

Finally, the magnitude of the weight vector is calculated and used as the MFSC decision statistic ($T_{\mathrm{MFSC}}$). Therefore, the MFSC decision statistic ($T_{\mathrm{MFSC}}$) can be expressed as

$$T_{\mathrm{MFSC}} = \|\hat{\mathbf{x}}\|^2 = \left(\sqrt{\sum_{i=1}^{k} (\hat{\mathbf{x}})^2}\right)^2 = \sum_{i=1}^{k} (\hat{\mathbf{x}})^2. \tag{3-52}$$

To determine the existence of PU, the MFSC decision statistic is compared to the MFSC threshold ($\lambda_{\mathrm{MFSC}}$).

As mention earlier, the threshold is needed to be vary on the strength of path loss effect. From our investigation, we found that the changing in the signal's amplitude does not affect changing in the signal's feature (eigenvector) but affects changing in the average vector ($\boldsymbol{\varepsilon}$). Thus, the weight vector under path loss effect when the PU does not exist can be expressed as

$$\hat{\mathbf{x}}_{\boldsymbol{\eta}} = \mathbf{V}^{T}(\mathbf{x}_{\boldsymbol{\eta}} - \hat{\boldsymbol{\varepsilon}}), \tag{3-53}$$

where the average vector ($\boldsymbol{\varepsilon}$) under path loss effect is given by

$$\hat{\boldsymbol{\varepsilon}} = \sqrt{PL}\boldsymbol{\varepsilon}. \tag{3-54}$$

The probability of false alarm ($P_{\text{fa}}$) of the MFSC algorithm is given by

$$P_{\text{fa}} = Q\left[\left(\frac{\lambda_{\text{MFSC}} - k\mathfrak{m}'_{2,\hat{\mathbf{x}}_{\boldsymbol{\eta}}}}{\sqrt{k\left(\mathfrak{m}'_{4,\hat{\mathbf{x}}_{\boldsymbol{\eta}}} - \left(\mathfrak{m}'_{2,\hat{\mathbf{x}}_{\boldsymbol{\eta}}}\right)^2\right)}}\right)\right]. \tag{3-55}$$

where $\mu_{\hat{\mathbf{x}}_{\boldsymbol{\eta}}}$ is the mean value of $\hat{\mathbf{x}}_{\boldsymbol{\eta}}$ and that $\mathfrak{m}'_n$ is the $n^{\text{th}}$ order moment of $\hat{\mathbf{x}}_{\boldsymbol{\eta}}$.

In general, the system threshold is set by fixing the target $P_{\text{fa}}$ , then the MFSC threshold ($\lambda_{\text{MFSC}}$) is given by

$$\lambda_{\text{MFSC}} = Q^{-1}(P_{\text{fa}})\sqrt{k\left(\mathfrak{m}'_4 - \left(\mathfrak{m}'_{2,H_0}\right)^2\right)} + k\mathfrak{m}'_2, \tag{3-56}$$

The probability of detection for the MFSC algorithm can be expressed as

$$P_{\text{d}} = Q\left[\left(\frac{\lambda_{\text{MFSC}} - k\mathfrak{m}'_{2,\hat{\mathbf{x}}_{\mathbf{s}+\boldsymbol{\eta}}}}{\sqrt{k\left(\mathfrak{m}'_{4,\hat{\mathbf{x}}_{\mathbf{s}+\boldsymbol{\eta}}} - \left(\mathfrak{m}'_{2,\hat{\mathbf{x}}_{\mathbf{s}+\boldsymbol{\eta}}}\right)^2\right)}}\right)\right]. \tag{3-57}$$