Chapter 2
Basic Concept

This chapter gives a brief introduction to wireless microphone signals based on the IEEE 802.22 standard and conventional spectrum sensing techniques. After that, two factors that degrade the performance of spectrum sensing techniques, i.e., noise uncertainty and path loss, are briefly reviewed.

2.1 Wireless microphone signals

In this research, a wireless microphone (WM) signal is considered as a PU signal. To evaluate the performance of spectrum sensing techniques for WM signal, the WM signal is modeled by IEEE 802.22 [59]. Therefore, the WM signal is categorized into 3 models — silent, soft speaker and loud speaker. Silent means that the PU transmits only the frequency modulation (FM) carrier and tone key. Soft speaker means that the PU transmits the FM carrier with some moderate amount of deviation. Loud speaker means that the PU transmits the FM carrier with near the maximum amount of deviation.

The WM signal can be expressed as

$$s(t) = A_{\mathrm{c}}\cos(2\pi f_{\mathrm{c}}t + 2\pi k_{\mathrm{f}}\int_0^t m(\tau)\mathrm{d}\tau), \tag{2-1}$$

$$m(\tau) = \sin(f_{\mathrm{m}}t), \tag{2-2}$$

where A_{c} is amplitude of carrier signal, $m(\tau)$ is the modulating signal, f_{m} is message frequency, f_{c} is carrier frequency and k_{f} is frequency modulation (FM) deviation factor.

Based on IEEE 802.22, the parameter of silent, soft speaker and loud speaker of the WM signal are set as shown in Table 2-1.

Table 2-1. Model of wireless microphone signal [59].

	Silent	Soft speaker	Loud speaker
$m(\tau)$ frequency (kHz)	32	3.9	13.4
FM deviation factor (kHz) (k_{f})	±5	±15	±32.6

Figure 2-1 modulation of wireless microphone signals at silent situation, soft speaker situation and loud speaker situation, respectively.

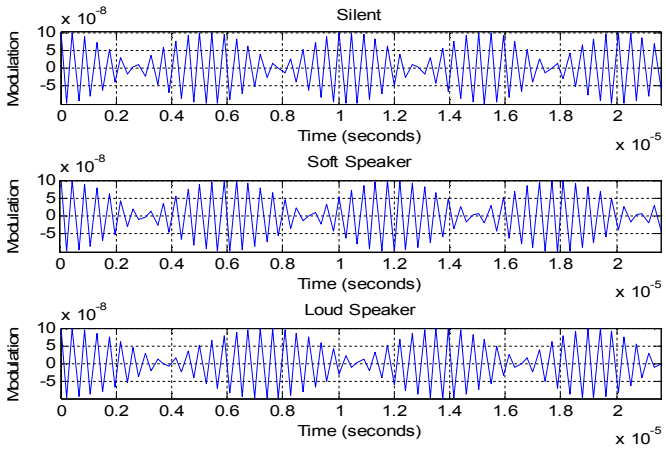

Figure 2-1 Three models of wireless microphone signal

2.2 Spectrum Sensing Techniques

Spectrum sensing is a critical function of CR that periodically detects the existence of a PU during its sensing period. Generally, spectrum sensing techniques can be broadly classified into two types: 1) detection techniques based on prior knowledge about signal, 2) blind detection techniques which do not require any prior knowledge. The detection technique based on prior knowledge requires for the information of primary signal's pattern. This sensing technique normally offers better sensing performance than blind detection technique. Nevertheless, when the secondary user does not have the information about the pattern of PU, the sensing performance of this technique is also drop. The solution of this problem is that the secondary user has to keep various signals' pattern of PU in the database which makes the system requires large size of memory. In addition, the increasing of the information also makes the increasing in the computational burden which effect to the increasing in the complexity and also sensing time. On the contrary, blind detection technique does not require any prior knowledge about primary signal which make it is more flexible. The advantages of blind detection technique are less computational complexity, less time to perform sensing and can be applied to any pattern of primary signal. The disadvantage is the performance of detection, which degrade greatly at low Signal-to-Nosie Ratio (SNR).

Accuracy of detection can be evaluated through statistical models, including probability of detection ($\mathbf{P_d}$), probability of false alarm ($\mathbf{P_{fa}}$) and probability of misdetection ($\mathbf{P_m}$). The probability of detection refers to correct declaration of a secondary user when a primary user actually presents or absents. The probability of false alarm refers to the declaration that a primary user presents when it actually absents. Conversely, the probability

of misdetection refers to the declaration that a primary user absents when it actually presents. Target performance in perspective of an accuracy of detection is to maximize the probability of detection while the probability of false alarm and probability of misdetection should be minimized. The other performance metric is sensing time which is the duration that a secondary user performs spectrum sensing. IEEE 802.22 standard states that the duration to perform spectrum sensing is 2 seconds [43]. However, there is a tradeoff between duration to perform spectrum sensing and an accuracy of detection. In general, the secondary user should delicately perform spectrum sensing to achieve high accuracy of detection. This will make the system consumes more sensing time, more complexity and the system throughput also decreases.

To detect the existence of a PU, there are two hypothesis models of a received signal that are expressed as follows:

$$\mathbf{x} = \begin{cases} \boldsymbol{\eta} & \text{when a PU is absent } [H_0], \\ \mathbf{s} + \boldsymbol{\eta} & \text{when a PU is present } [H_1], \end{cases} \tag{2-3}$$

where $\mathbf{x}$ is the signal an SU receives, $\boldsymbol{\eta}$ is additive white Gaussian noise, and $\mathbf{s}$ is the signal transmitted by a PU.

In this section, we conclude the well-known spectrum sensing techniques including its own operational requirement and merits/demerits. Individual requirements and merits/demerits are briefly reviewed as follows:

A. Energy Detection

Energy detection (ED) is one of the most widely used techniques because it is easy to implement and does not require any prior knowledge about signal's pattern. However, the performance of detection degrade greatly at low SNRs. The average energy of received signal is define as a decision statistic which can be expressed as

$$\mathrm{Y_{ED}} = \frac{1}{N} \sum_{t=1}^{N} |\mathrm{x(t)}|^2 \tag{2-4}$$

where $\mathrm{Y_{ED}}$ and N denote test statistic and the sample interval, respectively. The threshold is determined by using probability of false alarm ($\mathrm{P_{fa}}$). In addition, probability of detection ($\mathrm{P_d}$) can also be used. Mathematical models of probability of false alarm and probability of detection are given by

$$\begin{aligned} \mathrm{P_{fa}} &= P[\mathrm{Y_{ED}} \geq \gamma_{\mathrm{ED}} | H_0] \\ &= \mathrm{Q}\left[\left(\frac{\gamma_{\mathrm{ED}}}{{\sigma_{\mathrm{n}}}^2} - 1\right)\sqrt{N}\right] \\ \mathrm{P_d} &= P[\mathrm{Y_{ED}} \geq \gamma_{\mathrm{ED}} | H_1] \end{aligned} \tag{2-5}$$

$$= \mathrm{Q}\left[\frac{\sqrt{N}}{\alpha+1}\left(\frac{\gamma_{\mathrm{ED}}}{{\sigma_{\mathrm{n}}}^2}-\alpha-1\right)\right] \quad (2\text{-}6)$$

$$\alpha = \frac{{\sigma_{\mathrm{s}}}^2}{{\sigma_{\mathrm{n}}}^2}$$

where γ_{ED} denotes decision threshold, $\mathrm{Q}(.)$ is standard Gauss complementary cumulative distribution function, ${\sigma_{\mathrm{n}}}^2$ is the variance of noise, ${\sigma_{\mathrm{s}}}^2$ is the variance of desired signal. To determine the existence of primary user, the test statistic is compared to the threshold. The spectrum band is vacant if test statistic is less than the threshold

B. Matched Filter Detection

Matched filter detection (MFD) uses the correlation between the received and known signals. The output from MFD is compared to a threshold to determine the existence of a PU. The test statistic of MFD, Y_{MFD}, is given by

$$Y_{\mathrm{MFD}} = \sum_{n=0}^{N-1} x(n)\, s^*(n), \quad (2\text{-}7)$$

where $s^*(n)$ is the conjugate of the known signal. The decision threshold, γ_{MFD}, is determined from the probability of false alarm, $P_{\mathrm{fa(MFD)}}$. Alternatively, the probability of detection, $P_{\mathrm{d(MFD)}}$, can also be used as the decision threshold. Mathematical models for $P_{\mathrm{fa(MFD)}}$ and $P_{\mathrm{d(MFD)}}$ are given as

$$\begin{aligned} P_{\mathrm{fa(MFD)}} &= P[Y_{\mathrm{MFD}} \geq \gamma_{\mathrm{MFD}} | H_0] \\ &= Q\left[\left(\frac{\gamma_{\mathrm{MFD}}}{\sigma_{\eta}\sqrt{E}}\right)\right], \end{aligned} \quad (2\text{-}8)$$

$$\begin{aligned} P_{\mathrm{d(MFD)}} &= P[Y_{\mathrm{MFD}} \geq \gamma_{\mathrm{MFD}} | H_1] \\ &= Q\left[\left(\frac{\gamma_{\mathrm{MFD}}-E}{\sigma_{\eta}\sqrt{E}}\right)\right], \end{aligned} \quad (2\text{-}9)$$

where E is the energy of desired signal.

C. Maximum Eigenvalue Detection

Maximum eigenvalue detection (MED) is the sensing technique based on statistical covariance of the signal. Since the covariance matrix contains the correlation between signal samples, thus this detector calculate the maximum eigenvalue of covariance matrix and used as test statistic in order to determine the existence of primary user. A received signal comprising L consecutive samples is given by

$$\mathbf{x} = [x(n)\ x(n-1) \ldots x(n-L-1)]^{\mathrm{T}}, \quad (2\text{-}10)$$

$$\mathbf{s} = [s(n)\ \ s(n-1) \ldots s(n-L-1)]^{\mathrm{T}}, \quad (2\text{-}11)$$

$$\boldsymbol{\eta} = [\eta(n)\ \eta(n-1) \ldots \eta(n-L-1)]^{\mathrm{T}}, \tag{2-12}$$

where L is a smoothing factor. Since the statistical covariance matrix cannot be directly calculated, the sample covariance matrix of the received signal is computed by the following procedure:

1. The sample auto-correlations of the received signal are firstly expressed as

$$\varphi(l) = \frac{1}{N}\sum_{m=0}^{N-1} x(m)x(m-l)\,, \quad l = 0, 1, 2, \ldots, L-1. \tag{2-13}$$

2. Secondly, the sample covariance matrix of the received signal is calculated as

$$\mathbf{R}_{\mathbf{x}}(N) = \begin{bmatrix} \varphi(0) & \varphi(1) & \cdots & \varphi(l-1) \\ \varphi(1) & \varphi(0) & & \varphi(l-2) \\ \vdots & & \ddots & \vdots \\ \varphi(l-1) & & \cdots & \varphi(0)) \end{bmatrix}. \tag{2-14}$$

Note that the sample covariance matrix is a Toeplitz and symmetric matrix.

3. Thirdly, the eigenvalues of (2-14) are calculated using an eigen-decomposition algorithm. Note that only the maximum eigenvalue of the received signal, $\lambda_{\max}$, is used in step 4 to determine the existence of a PU.

4. Finally, the existence of a PU can now be determined from the value of $\lambda_{\max}$.

$$\lambda_{\max}(N) > \gamma_{\mathrm{MED}}{\sigma_{\boldsymbol{\eta}}}^2 \quad \text{when a PU is present,} \tag{2-15}$$

$$\lambda_{\max}(N) \leq \gamma_{\mathrm{MED}}{\sigma_{\boldsymbol{\eta}}}^2 \quad \text{when a PU is absent,} \tag{2-16}$$

where γ_{MED} denotes a predetermined decision threshold.

Since the sample covariance matrix of the noise is nearly a Wishart random matrix, MED is analyzed using the probability distribution of the normalized largest eigenvalue — referred to as "Tracy–Widom distribution". Thereby, $P_{\mathrm{fa(MED)}}$ can be expressed as

$$\begin{aligned} P_{\mathrm{fa(MED)}} &= P\left[\lambda_{\max}(\mathbf{R}_{\boldsymbol{\eta}}(N)) > \gamma_{\mathrm{MED}}{\sigma_{\boldsymbol{\eta}}}^2\right] \\ &\approx 1 - F\left[\left(\frac{\gamma_{\mathrm{MED}}N-\rho}{v}\right)\right], \end{aligned} \tag{2-17}$$

$$\rho = \left(\sqrt{N-1} + \sqrt{L}\right)^2, \tag{2-18}$$

$$v = \left(\sqrt{N-1} + \sqrt{L}\right)\left(\frac{1}{\sqrt{N-1}} + \frac{1}{\sqrt{L}}\right)^{1/3}. \tag{2-19}$$

D. Covariance Absolute Value Detection

With covariance absolute value detection (CAV), an SU determines the existence of a PU from the received signal. This is done by comparing the auto-correlation of the received signal to the CAV threshold. However, CAV will perform poorly when the auto-correlation of the received signal is low. The test statistic of CAV, Y_{CAV}, is given by

$$Y_{\mathrm{CAV}} = \left(\varphi(0) + \frac{2}{L}\sum_{l=1}^{L-1}(L-l)|\varphi(l)|\right)\left(\varphi(0)\right)^{-1}. \tag{2-20}$$

The threshold for CAV detection, γ_{CAV}, can be expressed as

$$\gamma_{\mathrm{CAV}} = \left(1 + (L-1)\sqrt{\frac{2}{N\pi}}\right)\left(1 - Q^{-1}(P_{\mathrm{fa}})\sqrt{\frac{2}{N}}\right)^{-1}. \tag{2-21}$$

A PU is present if $Y_{\mathrm{CAV}} \geq \gamma_{\mathrm{CAV}}$. Mathematical models for $P_{\mathrm{fa(CAV)}}$ and $P_{\mathrm{d(CAV)}}$ are given as

$$P_{\mathrm{fa(CAV)}} = 1 - Q\left[\frac{\frac{1}{\gamma_{\mathrm{CAV}}}\left(1+(L-1)\sqrt{\frac{2}{N\pi}}\right)-1}{\sqrt{\frac{2}{N}}}\right], \tag{2-22}$$

$$P_{\mathrm{d(CAV)}} = 1 - Q\left[\frac{\frac{1}{\gamma_{\mathrm{CAV}}}+\left(\frac{\gamma_L\,\mathrm{SNR}}{\gamma_{\mathrm{CAV}}(\mathrm{SNR}+1)}\right)-1}{\sqrt{\frac{2}{N}}}\right], \tag{2-23}$$

where γ_L is given by

$$\gamma_L \triangleq \frac{2}{L}\sum_{l=1}^{L-1}(L-1)|\alpha_l| \tag{2-24}$$

and α_l is given by

$$\alpha_l = E[\mathrm{s}(n)\mathrm{s}(n-l)]\Big/_{\sigma_{\mathbf{s}}{}^2}. \tag{2-25}$$

E. Maximum to Minimum Eigenvalue Detection

The procedure of maximum to minimum eigenvalue detection (MME) is similar to MED. However, the MME method determines the existence of a PU by comparing the ratio of the maximum and minimum eigenvalues with the threshold γ_{MME}. MME detection can be calculated using (2-14). The test statistic for the MME detection method is given by

$$Y_{\mathrm{MME}} = {\lambda_{\max}}/{\lambda_{\min}}. \tag{2-26}$$

The probability of false alarm for MME detection is given by

$$P_{\mathrm{fa(MME)}} \approx 1 - F\left[\frac{\gamma_{\mathrm{MME}}\left(\sqrt{N}+\sqrt{L}\right)^2 - \rho}{v}\right]. \tag{2-27}$$

The threshold of the first stage can be expressed as

$$\gamma_{\mathrm{MME}} = \frac{F^{-1}(1-\mathrm{P_{fa}})v+\mu}{\left(\sqrt{\mathrm{N}}+\sqrt{L}\right)^2} \tag{2-28}$$

F. Leading Eigenvector Detection

Leading eigenvector detection (LED) calculates the correlation between the leading eigenvector of the received signal and the leading eigenvector of the known signal. Similar to MFD, the output is compared to a threshold to determine the existence of a PU. Since LED keeps only the most significant feature of the received signal, the technique requires less memory than MFD. However, since the LED technique needs to calculate the leading eigenvector of the received signal, the sensing time and complexity of computation is increased.

Let us define the following PU signals, $\mathbf{x}_i, i = 1, 2, \dots, M$, each of which has d dimensions, as

$$\begin{aligned}
\mathbf{x}_1 &= [x(n)\ x(n+1) \dots\ x(n+d-1)]^{\mathrm{T}}, \\
\mathbf{x}_2 &= [x(n+1)\ x(n+2) \dots\ x(n+d)]^{\mathrm{T}}, \\
&\vdots \\
\mathbf{x}_M &= [x(N+n-d)\ \dots\ x(N+n-1)]^{\mathrm{T}}.
\end{aligned} \tag{2-29}$$

The LED procedure can then be summarized as follows:

1. The sample covariance matrix of a received signal $\mathbf{x}_i$ is given by

$$\mathbf{R}_{\mathbf{x}} = \frac{1}{M}\sum_{i=1}^{M} \mathbf{x}_i \mathbf{x}_i^{\mathrm{T}}. \tag{2-30}$$

Note that we assume the sample mean to be zero.

2. The eigenvalues and eigenvectors of the received signal can be calculated using (2-30). Only an eigenvector corresponding to the largest eigenvalue, $\mathbf{v}_1$, is considered. The test statistic for LED is given by

$$Y_{\mathrm{LED}} = \max_{l=0,\,1,\,2,\,\ldots,\,d} \left| \sum_{j=1}^{d} \mathbf{v}_1[j]\hat{\mathbf{v}}_1[j+l] \right|. \quad (2\text{-}31)$$

3. The existence of a PU can now be determined from the value of Y_{LED}.

$Y_{\mathrm{LED}} > \gamma_{\mathrm{LED}}$ when a PU is present, (2-32)

$Y_{\mathrm{LED}} \leq \gamma_{\mathrm{LED}}$ when a PU is absent, (2-33)

where $\hat{\mathbf{v}}_1$ is the leading eigenvector of the received signal, $\mathbf{v}_1$ is the leading eigenvector of the known signal, and γ_{LED} is a predetermined threshold.

2.3 Noise uncertainty

In practical communication system, noise may occurs from more than one sources. Then the variance of noise is difficult to be exactly estimated. Once noise occurs from various sources, the disturbance of noise is undesirable that is referred to an "uncertain behavior" or "noise uncertainty" [75]. The noise uncertainty may occur from the time-varying of thermal noise in a receiver and the non-linearity of the receiver. In addition, the transmission of other users also causes the noise uncertainty. When the uncertainty of noise occurs, the variance of noise distributes within range of $\left[\alpha{\sigma_{\boldsymbol{\eta}}}^2, \frac{1}{\alpha}{\sigma_{\boldsymbol{\eta}}}^2\right]$. Then, an estimated noise power can be expressed as

$${\hat{\sigma}_{\boldsymbol{\eta}}}^2 = \alpha{\sigma_{\boldsymbol{\eta}}}^2 \quad (2\text{-}34)$$

where α is a noise uncertainty interval and ${\sigma_{\boldsymbol{\eta}}}^2$ is a noise variance. Then, noise uncertainty factor (in dB) distributes within range $[-B, B]$ when noise uncertainty factor (in dB) is given as

$$B = max\,\{10\log_{10}\alpha\}. \quad (2\text{-}35)$$

2.4 Path loss

In practical communication networks, the received signal power of the transmitted signal may be lower than its transmitted power due to an attenuation of signal strength (power) due to the propagation distance between PU and SU. This is referred to path loss [76-77]. The mathematical model of path loss is derived as

$$PL \equiv Cd^{-\aleph} \quad (2\text{-}36)$$

where PL is path loss, d is distance between PU and SU, C is loss constant and $\aleph$ is path loss exponent.

Then, the received SNR due to path loss effect can be expressed as

$$\tilde{\gamma}_{PL} = \frac{PL \cdot \sigma_{\mathbf{s}}{}^2}{\sigma_{\boldsymbol{\eta}}{}^2} \quad (2\text{-}37)$$

where $\sigma_{\mathbf{s}}{}^2$ is a signal variance.

2.5 Principal Component Analysis

Principal component analysis is a main trend in classical feature extraction and data compression method which data is represented in lower dimensionality (subspace) through linear transformation technique. PCA algorithm commonly used in the field of pattern recognition, such as face recognition and vehicle license plate recognition. The main objective of PCA algorithm is to reduce original data dimensionality by performing a covariance analysis between factors and eliminating the extrinsic features (or later principal components). In other words, PCA algorithm attempts to find significant features (or principal components) of the distribution of data. Through the computation of linear transformation, a new coordinate system is chosen for the data set comes to lie on the axis. Mathematical theory that used in PCA algorithm including standard deviation, covariance, eigenvectors, eigenvalues and also linear transformation.

PCA algorithm reduced the dimension of data while the variance in the original-dimensional space is preserved as much as possible. In addition, PCA algorithm perform this reduction with minimum mean square error compared to the desired data. In term of computation, the principal component can be found by performing computed the eigenvector and eigenvalue of covariance matrix of the data. Eigenvector corresponding to the largest eigenvalue represented the most significant feature of the data (principal component). In other words, the principal component is the direction (or axis on a new coordinate) of greatest variation which data can relied on. The second component is the orthogonal direction with the next highest variation (or eigenvalue) and so on.

Referring to face recognition, eigenface is a vital element that effectively represent face image using PCA algorithm. The main concept of eigenface is to reconstruct any collected face images using the weight combination of significant features of images which obtained from the collection. Thus, eigenfaces can be defined as the principal directions of all possible face images in a new coordinate systems. Referring to face recognition, eigenface is a vital element that effectively represent face image using PCA algorithm. The main concept of eigenface is to reconstruct any collected face images using the weight combination of significant features of images which obtained from the collection. Thus, eigenfaces can be defined as the principal directions of all possible face images in a new coordinate systems. Training operations of face recognition can be summarized as the following:

The PCA algorithm can be summarized as follow.

1. Obtain images $U_1, U_2, \ldots, U_N$

2. Represent every image U_i as vector I_i
3. Compute the average of image vector (μ):

$$\mu = \frac{1}{N}\sum_{i=1}^{N} I_i \tag{2-38}$$

4. Subtract the mean image (γ_i):

$$\gamma_i = I_i - \mu \tag{2-39}$$

5. Compute the covariance matrix (C) :

$$C = \frac{1}{N}\sum_{i=1}^{N} \gamma_i {\gamma_i}^T \tag{2-40}$$

6. Compute the eigenvectors ($V = [v_1, v_2, \dots, v_K]$) and eigenvalues ($u$) of C. Where eigenvectors (V) known as eigenfaces or eigenspace.
7. Keep only K best eigenvectors corresponding to the K largest eigenvalues.
8. Each image (subtract the mean image: γ_i) in the training set can be represented as a linear combination of the K best eigenvectors:

$$\gamma_i - \mu = \sum_{j=1}^{K} \tilde{x}_j v_i \tag{2-41}$$

or

$$\tilde{x}_j = {v_i}^T \gamma_i \tag{2-42}$$

9. Represent γ_i as $\tilde{x} = \begin{bmatrix} {\tilde{x}_1}^i \\ {\tilde{x}_2}^i \\ \vdots \\ {\tilde{x}_K}^i \end{bmatrix}$ (2-43)

It is clear that 95% of the total number of features present in the images is a sufficient amount to be representative of all the existing features. Hence, having decided to only select the k best eigenvectors, the dimension of the images is reduced. Reducing the dimension of the WM signals avoids a huge amount of computational burden. Moreover, the effect of noise from the original signal is avoided due to the reduction in dimension of the images.

For given an unknown image (I_{test}) follows these procedure.

1. Normalize I: $\gamma_{test} = I_{test} - \mu$ (2-44)
2. Project on the eigenspace: $\gamma_{test} - \mu = \sum_{j=1}^{K} \tilde{x}_j v_i$ (2-45)

3. Represent γ_{test} as $\tilde{x}_{test} = \begin{bmatrix} \tilde{x}_{1,test} \\ \tilde{x}_{2,test} \\ \vdots \\ \tilde{x}_{K,tset} \end{bmatrix}$.