

## CHAPTER 6 CONCLUSIONS

In this dissertation, we establish the following results. First of all, we introduce an explicit method for finding the least norm of fixed points for strict pseudo mappings by using the projection technique. We provide algorithm which strong convergence theorems are obtained in Hilbert spaces.

Secondly, we introduce a new iterative method for finding a common element of the set of common solution of fixed point of a pseudo-contractive mapping and the set of common solutions of variational inequalities in Hilbert spaces and we prove a strong convergence theorem for finding a common element of the set of fixed points for a continuous pseudo-contractive mapping and the solution set of a variational inequality problem governed by a continuous monotone mapping.

Thirdly, we introduce a new iterative algorithm for solving a common solution of the set of solutions of fixed point for an infinite family of nonexpansive mappings, the set of solution of a system of mixed equilibrium problems and the set of solutions of the variational inclusion for an  $\beta$ -inverse-strongly monotone mapping in a real Hilbert space and we show a strong convergence theorem which solves the problem of finding a common element of the common fixed points, the common solution of a system of mixed equilibrium problem and variational inclusion of inverse-strongly monotone mappings in a Hilbert space.

Fourthly, we introduce a new iterative scheme that converges strongly to a common fixed point of a countable family of strictly pseudo-contractive mappings in a real Hilbert space which is also a solution to variational inequality problem related to quadratic minimization problems.

Finally, we introduce a new hybrid extragradient iterative algorithm we prove strong convergence of an iterative algorithm for finding a common element of the set of fixed points satisfying the equilibrium problem, variational inequalities and fixed point problems of a strict pseudocontraction mapping by the hybrid extragradient projection method in Hilbert spaces.

The following results are all main theorems of this dissertation:

(1). Let  $C$  be a closed convex of a real *Hilbert space*  $H$ . Let  $S : C \rightarrow C$  be a nonexpansive mapping and  $\Omega := F(S) \cap VI(C, A) \neq \emptyset$ . Assume that a sequence  $\{\alpha_n\} \subset (0, 1)$  satisfies the conditions:

$$(i) \lim_{n \rightarrow \infty} \alpha_n = 0;$$

$$(ii) \sum_{n=0}^{\infty} \alpha_n = +\infty.$$

Then the sequence  $\{x_n\}$  generated by the following algorithm

$$x_{n+1} = (1 - \alpha_n)[\lambda Sx_n + (1 - \lambda)x_n] \quad (6.0.1)$$

converges strongly to a fixed point of  $S$  which is of minimal norm which the unique solution of the variational inequality:

$$x^* \in \Omega, \langle x^*, x - x^* \rangle \geq 0, \forall x \in \Omega.$$

(2). Assume that the solution set of (3.1.1) is nonempty. Let the objective function  $f$  be convex, fréchet differentiable and its gradient  $\nabla f$  is Lipschitz continuous with Lipschitz constant  $L$ . In addition, if  $0 \in C$  or  $C$  is closed convex cone. Let  $\mu \in (0, \frac{2}{L})$  and define a sequence  $\{x_n\}$  by following

$$x_{n+1} = (1 - \alpha_n)((I - \mu \nabla f)(x_n) + (1 - \lambda)x_n), \quad n \geq 0$$

where  $\lambda \in (0, 1)$  and the sequence  $\{\alpha_n\} \subset (0, 1)$  satisfies conditions in Theorem 3.1.1. Then the sequence  $\{x_n\}$  converges strongly to the minimum-norm solution of the minimization (3.1.1)

(3). Let  $C$  be a nonempty closed and convex subset of a real Hilbert space  $H$ . Let  $T : C \rightarrow C$  be a continuous pseudo-contractive mapping and  $A : C \rightarrow H$  be a continuous monotone mapping such that  $\mathfrak{F} := F(T) \cap VI(C, A) \neq \emptyset$ . For  $x \in H$ , define  $T_{r_n}$  and  $F_{r_n}$  as follows:

$$T_r x := \{z \in C : \langle y - z, Tz \rangle + \frac{1}{r} \langle y - z, (1 + r)z - x \rangle \leq 0, \quad \forall y \in C\}$$

and

$$F_r x := \{z \in C : \langle y - z, Az \rangle + \frac{1}{r} \langle y - z, z - x \rangle \geq 0, \quad \forall y \in C\}.$$

Let  $B : H \rightarrow H$  be a strongly positive linear bounded self-adjoint operator with coefficients  $\bar{\beta} > 0$  and let  $\{x_n\}$  be a sequence generated by  $x_1 \in C$  and

$$\begin{cases} y_n = F_{r_n} x_n \\ x_{n+1} = \alpha_n u + \delta_n x_n + [(1 - \delta_n)I - \alpha_n B] T_{r_n} y_n, \end{cases} \quad (6.0.2)$$

where  $\{\alpha_n\}, \{\delta_n\} \subset [0, 1]$  and  $\{r_n\} \subset (0, \infty)$  such that

$$(C1) \quad \lim_{n \rightarrow \infty} \alpha_n = 0, \quad \sum_{n=1}^{\infty} \alpha_n = \infty;$$

$$(C2) \quad \lim_{n \rightarrow \infty} \delta_n = 0, \quad \sum_{n=1}^{\infty} |\delta_{n+1} - \delta_n| < \infty;$$

$$(C3) \quad \liminf_{n \rightarrow \infty} r_n > 0, \quad \sum_{n=1}^{\infty} |r_{n+1} - r_n| < \infty.$$

Then, the sequence  $\{x_n\}_{n \geq 1}$  converges strongly to  $z \in \mathfrak{F}$ , which is the unique solution of the variational inequality:

$$\langle (B - f)z, x - z \rangle \geq 0, \quad \forall x \in \mathfrak{F}. \quad (6.0.3)$$

Equivalently,  $z = P_{\mathfrak{F}}(I - B + f)z$ .

(4). Let  $C$  be a nonempty closed and convex subset of a real Hilbert space  $H$ . Let  $A : C \rightarrow H$  be a continuous monotone mapping such that  $\mathfrak{F} := VI(C, A) \neq \emptyset$ . For  $x \in H$ , define  $F_{r_n}$  as follows:

$$F_r x := \{z \in C : \langle y - z, Az \rangle + \frac{1}{r} \langle y - z, z - x \rangle \geq 0, \quad \forall y \in C\}.$$

Let  $f$  be a contraction of  $H$  into itself with a contraction constant  $\beta$  and let  $B : H \rightarrow H$  be a strongly positive linear bounded self-adjoint operator with coefficients  $\bar{\beta} > 0$  and let  $\{x_n\}$  be a sequence generated by  $x_1 \in C$  and

$$x_{n+1} = \alpha_n \gamma f(x_n) + \delta_n x_n + [(1 - \delta_n)I - \alpha_n B] F_{r_n} x_n, \quad (6.0.4)$$

where  $\{\alpha_n\}, \{\delta_n\} \subset [0, 1]$  and  $\{r_n\} \subset (0, \infty)$  such that

$$(C1) \quad \lim_{n \rightarrow \infty} \alpha_n = 0, \quad \sum_{n=1}^{\infty} \alpha_n = \infty;$$

$$(C2) \quad \lim_{n \rightarrow \infty} \delta_n = 0, \quad \sum_{n=1}^{\infty} |\delta_{n+1} - \delta_n| < \infty;$$

$$(C3) \quad \liminf_{n \rightarrow \infty} r_n > 0, \quad \sum_{n=1}^{\infty} |r_{n+1} - r_n| < \infty.$$

Then, the sequence  $\{x_n\}_{n \geq 1}$  converges strongly to  $z \in \mathfrak{F}$ , which is the unique solution of the variational inequality:

$$\langle (B - \gamma f)z, x - z \rangle \geq 0, \quad \forall x \in \mathfrak{F} \quad (6.0.5)$$

Equivalently,  $z = P_{\mathfrak{F}}(I - B + \gamma f)z$ .

**(5).** Let  $C$  be a nonempty closed and convex subset of a real Hilbert space  $H$ . Let  $T : C \rightarrow C$  be a continuous pseudo-contractive mapping and  $\mathfrak{F} := F(T) \neq \emptyset$ . For  $x \in H$ , define  $T_{r_n}$  as follows:

$$T_r x := \{z \in C : \langle y - z, Tz \rangle + \frac{1}{r} \langle y - z, (1+r)z - x \rangle \leq 0, \quad \forall y \in C\}.$$

Let  $f$  be a contraction of  $H$  into itself with a contraction constant  $\beta$  and let  $B : H \rightarrow H$  be a strongly positive linear bounded self-adjoint operator with coefficients  $\bar{\beta} > 0$  and let  $\{x_n\}$  be a sequence generated by  $x_1 \in C$  and

$$x_{n+1} = \alpha_n \gamma f(x_n) + \delta_n x_n + [(1 - \delta_n)I - \alpha_n B]T_{r_n} x_n, \quad (6.0.6)$$

where  $\{\alpha_n\}, \{\delta_n\} \subset [0, 1]$  and  $\{r_n\} \subset (0, \infty)$  such that

$$(C1) \quad \lim_{n \rightarrow \infty} \alpha_n = 0, \quad \sum_{n=1}^{\infty} \alpha_n = \infty;$$

$$(C2) \quad \lim_{n \rightarrow \infty} \delta_n = 0, \quad \sum_{n=1}^{\infty} |\delta_{n+1} - \delta_n| < \infty;$$

$$(C3) \quad \liminf_{n \rightarrow \infty} r_n > 0, \quad \sum_{n=1}^{\infty} |r_{n+1} - r_n| < \infty.$$

Then, the sequence  $\{x_n\}_{n \geq 1}$  converges strongly to  $z \in F$ , which is the unique solution of the variational inequality:

$$\langle (B - \gamma f)z, x - z \rangle \geq 0, \quad \forall x \in \mathfrak{F}. \quad (6.0.7)$$

Equivalently,  $z = P_{\mathfrak{F}}(I - B + \gamma f)z$ .

**(6).** Let  $H$  be a real Hilbert space. Let  $T_n : H \rightarrow H$  be a continuous pseudo-contractive mapping and  $A : H \rightarrow H$  be a continuous monotone mapping such that  $\mathfrak{F} := F(T) \cap A^{-1}(0) \neq \emptyset$ . For  $x \in H$ , define  $T_{r_n}$  and  $F_{r_n}$  as follows:

$$T_r x := \{z \in C : \langle y - z, Tz \rangle + \frac{1}{r} \langle y - z, (1+r)z - x \rangle \leq 0, \quad \forall y \in C\}$$

and

$$F_r x := \{z \in C : \langle y - z, Az \rangle + \frac{1}{r} \langle y - z, z - x \rangle \geq 0, \quad \forall y \in C\}.$$

Let  $f$  be a contraction of  $C$  into itself with a contraction constant  $\beta$  and let  $B : H \rightarrow H$  be a strongly positive linear bounded self-adjoint operator with coefficients  $\bar{\beta} > 0$  and let  $\{x_n\}$  be a sequence generated by  $x_1 \in C$  and

$$\begin{cases} y_n = F_{r_n} x_n \\ x_{n+1} = \alpha_n \gamma f(x_n) + \delta_n x_n + [(1 - \delta_n)I - \alpha_n B] T_{r_n} y_n \end{cases} \quad (6.0.8)$$

where  $\{\alpha_n\}, \{\delta_n\} \subset [0, 1]$  and  $\{r_n\} \subset (0, \infty)$  such that

$$\begin{aligned} \text{(C1)} \quad & \lim_{n \rightarrow \infty} \alpha_n = 0, \quad \sum_{n=1}^{\infty} \alpha_n = \infty; \\ \text{(C2)} \quad & \lim_{n \rightarrow \infty} \delta_n = 0, \quad \sum_{n=1}^{\infty} |\delta_{n+1} - \delta_n| < \infty; \\ \text{(C3)} \quad & \liminf_{n \rightarrow \infty} r_n > 0, \quad \sum_{n=1}^{\infty} |r_{n+1} - r_n| < \infty. \end{aligned}$$

Then, the sequence  $\{x_n\}_{n \geq 1}$  converges strongly to  $z \in \mathfrak{F}$ , which is the unique solution of the variational inequality:

$$\langle (B - \gamma f)z, x - z \rangle \geq 0, \quad \forall x \in \mathfrak{F} \quad (6.0.9)$$

Equivalently,  $z = P_{\mathfrak{F}}(I - B + \gamma f)z$ .

(7). Let  $H$  be a real Hilbert space,  $C$  a close convex subset of  $H$  and  $B$  be an  $\beta$ -inverse-strongly monotone operator. Let  $\varphi : C \rightarrow \mathbb{R}$  be a convex and lower semicontinuous function,  $f : C \rightarrow C$  be a contraction mapping with coefficient  $\alpha$  ( $0 < \alpha < 1$ ),  $M : H \rightarrow 2^H$  be a maximal monotone operator. Let  $A$  be a strongly positive linear bounded operator of  $H$  into itself with coefficient  $\bar{\gamma} > 0$ . Assume that  $0 < \gamma < \frac{\bar{\gamma}}{\alpha}$  and  $\lambda \in (0, 2\beta)$ . Let  $\{T_n\}$  be a family of nonexpansive mappings of  $H$  into itself such that

$$\theta := \bigcap_{n=1}^{\infty} F(T_n) \cap (\bigcap_{k=1}^N \text{SMEP}(F_k)) \cap I(B, M) \neq \emptyset.$$

Suppose  $\{x_n\}$  is a sequence generated by the following algorithm for  $x_0, u_n \in C$  arbitrarily:

$$\begin{cases} u_n = K_{r_n, n}^{F_N} \cdot K_{r_{n-1}, n}^{F_{N-1}} \cdot K_{r_{n-2}, n}^{F_{N-2}} \cdot \dots \cdot K_{r_2, n}^{F_2} \cdot K_{r_1, n}^{F_1} \cdot x_n, \quad \forall n \in N \\ x_{n+1} = P_C[\epsilon_n f(x_n) + (I - \epsilon_n)W_n J_{M, \lambda}(u_n - \lambda B u_n)] \end{cases} \quad (6.0.10)$$

for all  $n = 0, 1, 2, \dots$ , and the conditions (C1)-(C2) in Theorem 4.1.1 are satisfied.

Then, the sequence  $\{x_n\}$  converges strongly to  $q \in \theta$ , where  $q = P_\theta(f + I)(q)$  which solves the following variational inequality:

$$\langle (f - I)q, p - q \rangle \leq 0, \quad \forall p \in \theta.$$

(8). Let  $H$  be a real Hilbert space,  $C$  a close convex subset of  $H$  and  $B$  be an  $\beta$ -inverse-strongly monotone operator. Let  $\varphi : C \rightarrow \mathbb{R}$  be a convex and lower semicontinuous function,  $f : C \rightarrow C$  be a contraction mapping with coefficient  $\alpha$  ( $0 < \alpha < 1$ ),  $M : H \rightarrow 2^H$  be a maximal monotone operator. Let  $A$  be a strongly positive linear bounded operator of  $H$  into itself with coefficient  $\bar{\gamma} > 0$ . Assume that  $0 < \gamma < \frac{\bar{\gamma}}{\alpha}$  and  $\lambda \in (0, 2\beta)$ . Let  $\{T_n\}$  be a family of nonexpansive mappings of  $H$  into itself such that

$$\theta := \bigcap_{n=1}^{\infty} F(T_n) \cap (\bigcap_{k=1}^N \text{SMEP}(F_k)) \cap I(B, M) \neq \emptyset.$$

Suppose  $\{x_n\}$  is a sequence generated by the following algorithm for  $x_0, u \in C$  and  $u_n \in C$ :

$$\begin{cases} u_n = K_{r_n, n}^{F_N} \cdot K_{r_{n-1}, n}^{F_{N-1}} \cdot K_{r_{n-2}, n}^{F_{N-2}} \cdot \dots \cdot K_{r_2, n}^{F_2} \cdot K_{r_1, n}^{F_1} \cdot x_n, \quad \forall n \in N \\ x_{n+1} = P_C[\epsilon_n u + (I - \epsilon_n)W_n J_{M, \lambda}(u_n - \lambda B u_n)] \end{cases} \quad (6.0.11)$$

for all  $n = 0, 1, 2, \dots$ , and the conditions (C1)-(C2) in Theorem 4.1.1 are satisfied.

Then, the sequence  $\{x_n\}$  converges strongly to  $q \in \theta$ , where  $q = P_\theta(q)$  which solves the following variational inequality:

$$\langle u - q, p - q \rangle \leq 0, \quad \forall p \in \theta.$$

(9). Let  $H$  be a real Hilbert space,  $C$  a close convex subset of  $H$  and  $B$  be an  $\beta$ -inverse-strongly monotone operator. Let  $f : C \rightarrow C$  be a contraction mapping with coefficient  $\alpha$  ( $0 < \alpha < 1$ ),  $M : H \rightarrow 2^H$  be a maximal monotone operator. Let  $A$  be a strongly positive linear bounded operator of  $H$  into itself with coefficient  $\bar{\gamma} > 0$ . Assume that  $0 < \gamma < \frac{\bar{\gamma}}{\alpha}$ . Let  $\{T_n\}$  be a family of nonexpansive mappings of  $H$  into itself such that

$$\theta := \bigcap_{n=1}^{\infty} F(T_n) \cap VI(C, B) \neq \emptyset.$$

Suppose  $\{x_n\}$  is a sequence generated by the following algorithm for  $x_0 \in C$  arbitrarily:

$$x_{n+1} = P_C \left[ \epsilon_n \gamma f(x_n) + (I - \epsilon_n A) W_n P_C(x_n - \lambda B x_n) \right] \quad (6.0.12)$$

for all  $n = 0, 1, 2, \dots$ , and the conditions (C1)-(C2) in Theorem 4.1.1 are satisfied.

Then, the sequence  $\{x_n\}$  converges strongly to  $q \in \theta$ , where  $q = P_\theta(\gamma f + I - A)(q)$  which solves the following variational inequality:

$$\langle (\gamma f - A)q, p - q \rangle \leq 0, \quad \forall p \in \theta.$$

**(10).** Let  $C$  be a nonempty closed and convex subset of a real Hilbert space  $H$ . Let  $f : C \rightarrow H$  be a  $\rho$ -contraction mapping,  $S : C \rightarrow H$  be a nonexpansive mapping and  $T : C \rightarrow C$  be a  $k$ -strict pseudo-contraction mapping such that  $F(T) \neq \emptyset$ . Let  $x_1 \in C$  and define the sequence  $\{x_n\}$  by

$$\begin{cases} y_n = P_C[\beta_n S x_n + (1 - \beta_n)x_n], \\ x_{n+1} = P_C[\alpha_n f(x_n) + (1 - \alpha_n)V y_n], \quad \forall n \geq 1, \end{cases} \quad (6.0.13)$$

where  $V = kI + (1 - k)T$ ,  $\{\alpha_n\} \subset (0, 1)$  and  $\{\beta_n\} \subset (0, 1)$  are sequences satisfying the conditions (H2) with  $\tau = 0$ , (H3), either (H4) and (H5), or (H6) and (H7). Then the sequence  $\{x_n\}$  converges strongly to a point  $z \in F(T)$ , which is the unique solution of the variational inequality:

$$\langle (I - f)z, x - z \rangle \geq 0, \quad \forall x \in F(T).$$

**(11).** Let  $C$  be a nonempty closed and convex subset of a real Hilbert space  $H$ . Let  $f : C \rightarrow H$  be a  $\rho$ -contraction mapping,  $S : C \rightarrow H$  be a nonexpansive mapping and  $\{T_i\}_{i=1}^\infty : C \rightarrow C$  be a countable family of nonexpansive mappings and  $\mathcal{F} = \bigcap_{i=1}^\infty F(T_i) \neq \emptyset$ . Let  $\alpha_0 = 1$ ,  $x_1 \in C$  and define the sequence  $\{x_n\}$  by

$$\begin{cases} y_n = P_C[\beta_n S x_n + (1 - \beta_n)x_n], \\ x_{n+1} = P_C[\alpha_n f(x_n) + \sum_{i=1}^n (\alpha_{i-1} - \alpha_i) T_i y_n], \quad \forall n \geq 1, \end{cases} \quad (6.0.14)$$

where  $\{\alpha_n\} \subset (0, 1)$  and  $\{\alpha_n\}$  is a strictly decreasing sequence,  $\{\beta_n\} \subset (0, 1)$  and  $\{\alpha_n\}$  and  $\{\beta_n\}$  are sequences satisfying the conditions (H2) with  $\tau = 0$ , (H3), either (H4) and (H5), or (H6) and (H7). Then the sequence  $\{x_n\}$  converges strongly to a point  $z \in \mathcal{F}$ , which is the unique solution of the variational inequality:

$$\langle (I - f)z, x - z \rangle \geq 0, \quad \forall x \in \mathcal{F}.$$

(12). Let  $C$  be a nonempty closed and convex subset of a real Hilbert space  $H$ . Let  $f : C \rightarrow H$  be a  $\rho$ -contraction mapping,  $S : C \rightarrow H$  be a nonexpansive mapping and  $T : C \rightarrow C$  be a nonexpansive mapping such that  $F(T) \neq \emptyset$ . Let  $x_1 \in C$  and define the sequence  $\{x_n\}$  by

$$\begin{cases} y_n = P_C[\beta_n Sx_n + (1 - \beta_n)x_n], \\ x_{n+1} = P_C[\alpha_n f(x_n) + (1 - \alpha_n)Ty_n], \quad \forall n \geq 1, \end{cases} \quad (6.0.15)$$

where  $\{\alpha_n\} \subset (0, 1)$ ,  $\{\beta_n\} \subset (0, 1)$  and  $\{\alpha_n\}$  and  $\{\beta_n\}$  are sequences satisfying the conditions (H2) with  $\tau = 0$ , (H3), either (H4) and (H5), or (H6) and (H7). Then the sequence  $\{x_n\}$  converges strongly to a point  $z \in F(T)$ , which is the unique solution of the variational inequality:

$$\langle (I - f)z, x - z \rangle \geq 0, \quad \forall x \in F(T).$$

(13). Let  $C$  be a nonempty closed and convex subset of a real Hilbert space  $H$ . Let  $f : C \rightarrow H$  be a  $\rho$ -contraction mapping,  $S : C \rightarrow C$  be a nonexpansive mapping and  $T : C \rightarrow C$  be a  $k$ -strict pseudo-contraction mapping and  $\mathcal{F} = F(T) \neq \emptyset$ . Let  $x_1 \in C$  and define the sequence  $\{x_n\}$  by

$$\begin{cases} y_n = \beta_n Sx_n + (1 - \beta_n)x_n, \\ x_{n+1} = P_C[\alpha_n f(x_n) + (1 - \alpha_n)Vy_n], \quad \forall n \geq 1, \end{cases} \quad (6.0.16)$$

where  $V = kI + (1 - k)T$ ,  $\{\alpha_n\} \subset (0, 1)$ ,  $\{\beta_n\} \subset (0, 1)$  and  $\{\alpha_n\}$  and  $\{\beta_n\}$  are sequences satisfying the conditions (H2) with  $\tau \in (0, \infty)$ , (H3), (H8) and (H9). Then the sequence  $\{x_n\}$  converges strongly to a point  $x^* \in \mathcal{F}$ , which is the unique solution of the variational inequality:

$$\langle \frac{1}{\tau}(I - f)x^* + (I - S)x^*, x - x^* \rangle \geq 0, \quad \forall x \in \mathcal{F}. \quad (6.0.17)$$

(14). Let  $C$  be a nonempty closed and convex subset of a real Hilbert space  $H$ . Let  $f : C \rightarrow H$  be a  $\rho$ -contraction mapping,  $S : C \rightarrow C$  be a nonexpansive mapping and  $\{T_i\}_{i=1}^{\infty} : C \rightarrow C$  be a countable family of nonexpansive mappings and  $\mathcal{F} = \bigcap_{i=1}^{\infty} F(T_i) \neq \emptyset$ . Let  $\alpha_0 = 1$ ,  $x_1 \in C$  and define the sequence  $\{x_n\}$  by

$$\begin{cases} y_n = \beta_n Sx_n + (1 - \beta_n)x_n, \\ x_{n+1} = P_C[\alpha_n f(x_n) + \sum_{i=1}^n (\alpha_{i-1} - \alpha_i)T_i y_n], \quad \forall n \geq 1, \end{cases} \quad (6.0.18)$$

where  $\{\alpha_n\} \subset (0, 1)$  and  $\{\alpha_n\}$  is a strictly decreasing sequence,  $\{\beta_n\} \subset (0, 1)$  and  $\{\alpha_n\}$  and  $\{\beta_n\}$  are sequences satisfying the conditions (H2) with  $\tau \in (0, \infty)$ , (H3), (H8) and (H9). Then the sequence  $\{x_n\}$  converges strongly to a point  $x^* \in \mathcal{F}$ , which is the unique solution of the variational inequality:

$$\left\langle \frac{1}{\tau}(I - f)x^* + (I - S)x^*, x - x^* \right\rangle \geq 0, \quad \forall x \in \mathcal{F}. \quad (6.0.19)$$

**(15).** Let  $C$  be a nonempty closed and convex subset of a real Hilbert space  $H$ . Let  $f : C \rightarrow H$  be a  $\rho$ -contraction mapping,  $S, T : C \rightarrow C$  be nonexpansive mappings and  $\mathcal{F} = F(T) \neq \emptyset$ . Let  $x_1 \in C$  and define the sequence  $\{x_n\}$  by

$$\begin{cases} y_n = \beta_n Sx_n + (1 - \beta_n)x_n, \\ x_{n+1} = P_C[\alpha_n f(x_n) + (1 - \alpha_n)Ty_n], \quad \forall n \geq 1, \end{cases} \quad (6.0.20)$$

where  $\{\alpha_n\} \subset (0, 1)$ ,  $\{\beta_n\} \subset (0, 1)$  and  $\{\alpha_n\}$  and  $\{\beta_n\}$  are sequences satisfying the conditions (H2) with  $\tau \in (0, \infty)$ , (H3), (H8) and (H9). Then the sequence  $\{x_n\}$  converges strongly to a point  $x^* \in \mathcal{F}$ , which is the unique solution of the variational inequality:

$$\left\langle \frac{1}{\tau}(I - f)x^* + (I - S)x^*, x - x^* \right\rangle \geq 0, \quad \forall x \in \mathcal{F}. \quad (6.0.21)$$

**(16).** Let  $C$  be a nonempty, closed and convex subset of a real Hilbert space  $H$ , let  $F$  be a bifunction  $F : C \times C \rightarrow \mathbb{R}$  satisfying conditions (A1) – (A5), let  $A$  be an  $\alpha$ -inverse-strongly monotone mapping of  $C$  into  $H$ . Let  $S$  be a  $\xi$ -strict pseudocontraction mapping from  $C$  to  $C$  and such that  $\Theta := EP(F) \cap \text{Fix}(S) \cap VI(C, A) \neq \emptyset$ . Suppose that the sequences  $\{\alpha_n\}, \{\beta_n\}, \{\lambda_n\}$  and  $\mu$  satisfying the condition (i) – (iv). Then the sequences  $\{x_n\}$  generated by Algorithm 1 converges strongly to the projection of  $x_0$  on to the set  $\Theta$ .