CHAPTER 1 INTRODUCTION

This dissertation is divided in three topics on geometrical properties of Banach spaces, fixed point theory in modular spaces and best proximity points in metric spaces as follows:

First, we study some geometrical properties of Banach spaces. Geometrical properties is a research topic which interest from a lot of mathematicians. The investigations of metric geometry of normed spaces, date back to 1936, when Clarkson [1] introduced the notion of the uniform convexity property (UC) or the uniform rotun property (UR) of Banach spaces, and it was shown that L_p with $1 < p < \infty$ are examples of such space. Metric geometry of normed spaces has many applications in areas of mathematics, among others in probability, ergodic, control, operator theories, approximation, optimization and fixed point (see e.g. [2, 3, 4, 5]). There are many mathematicians who are interested in studying geometric properties of Banach spaces, because the geometric properties were identified as important characteristics and properties of the Banach spaces. For example, if Banach spaces have some geometric properties such as uniform rotund, P- convexity, Q- convexity, Banach-Saks property then they are reflexive spaces. Also, a lot of mathematicians are also interested in geometric properties related to fixed point theory (see e.g. [6, 7, 8, 9]).

Second, we prove some fixed point theorems for generalized contractive mappings in modular spaces. It is well known that fixed point theory is very useful because it is interesting in mathematics and applied mathematics and can be applied in engineering, physics, economics and others. Fixed point theory is a branch of nonlinear functional analysis. Mathematicians continue to study and research in these areas to find new knowledge and theory. It is accepted that theory and knowledge of the research are particularly useful in the development of academic knowledge in fields. Sometimes it can be applied to other fields and is salient basis for developing basic sciences for the development of the country. Fixed point theory can be applied extensively in some kinds of spaces. In particular, the study of the existence of solution and uniqueness of solution as well as search about how to find answers of equations. Therefore, the study of various theories involved various methods to be used to estimate answers. Fixed point theorems of different mapping is a topic which a lot of mathematicians are interested to apply the solution of nonlinear operator equations such as nonlinear operator equations, variational inequality problem (VIP), equilibrium problems (EP), optimizations problems as well as minimizations problems in Hilbert spaces and Banach spaces. This issue is important and useful in various fields such as physics, applied mathematics, engineering, transport, economics and appropriate management system, etc. (see e.g. [10, 11, 12]). For example, fixed point theorems are incredibly useful when it comes to prove the existence of various types of Nash equilibria (see e.g. [10]) in economics. The study of fixed point theorems in Banach spaces is to find sufficient condition for a mapping T from a subset K of Banach spaces to itself, which has fixed point. A lot of mathematicians have khonw the existence of fixed points for nonlinear mappings in Banach spaces, metric space, modular spaces. The theory of modulars on linear spaces and the corresponding theory of modular linear spaces were founded by Nakano [13, 14] and redefined by Musielak and orlicz [15]. In the present time the theory of modulars and modular spaces have been extensively applied, in particular, in the study of various Orlicz spaces which in their turn have broad applications. In many cases, particularly in applications to integral operators, approximation and fixed point theory, modular type conditions are much more natural as modular type assumptions can be more easily verified than their metric or norm counterparts. Even though a metric is not defined, many problems in metric fixed point theory can be reformulated in modular spaces. For instance, fixed point theorems were proved in [16, 17] for nonexpansive mappings. Later a number of mathematician find sufficient condition to prove fixed point theory for generalized nonlinear mapping. Since 1922, Banach [18] established theorem, which is called Banach's contraction principle. Because of its simplicity and usefulness, it has become a very popular tool in solving existence problems in many branches of mathematical analysis. Banach contraction principle has been extended in many different directions, see [19, 20, 21, 22, 23].

Finally, we show the existence of best proximity points for generalized contractive mappings in metric spaces. Many problems can be formulated as equations of the form $Tx = x$, where T is a self-mapping in some suitable framework. From the fact that fixed point theory find into the existence of a solution to such generic equations and brings out the iterative algorithms to compute a solution to such equations. However, in the case that T is non-self mapping; the aforementioned equation does not necessarily has a solution. Best proximity point theorems provide sufficient conditions that ensure the existence of approximate solutions which are optimal as well. In fact, if there is no solution to the fixed point equation $Tx = x$ for a non-self mapping $T : A \to B$, then it is desirable to determine an approximate solution x such that the error $d(x, Tx)$ is minimum. Best proximity point theorems for several types of contractions have been established in [24, 25, 26, 27].

In this dissertation, motivated and inspired from raised above, we will study some geometric properties of some Banach spaces and procedure for prove the existence of fixed point and best proximity point theorems. Moreover, we extend the classical Banach's contraction principle to new generalized contraction mappings for self mappings and non-self mappings. and give illustrate examples to validate the some results in this dissertation. We introduce the dissertation consists of six chapters as follows:

In Chapter 1, we review the background of this dissertation for geometric properties, fixed point theorems and best proximity point theorems.

In Chapter 2, we give basic definitions, notations and preliminaries which will be used in the later chapters.

In Chapter 3, we study some geometric properties of Lacunany sequence spaces and generalized Cesàro sequence spaces.

In Chapter 4, we study and prove the existence of fixed point and common fixed point theorems for generalized contractions in modular spaces. Furthermore, we prove fixed point theorems for contraction mappings in modular metric spaces and present some examples corresponding to the main theorems.

In Chapter 5, we prove the existence theorems of best proximity point and common best proximity point theorems which is a generalized contraction for non-self mapping and also give some examples to validate some results in this section.

Finally, In Chapter 6, we give the conclusion of this dissertation.