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LIST OF SYMBOLS

SYMBOL

\mathbb{R}^d	d-dimensional real space
α	penalty parameter
δ	the Kronecker delta
$\phi(\mathbf{x}), \mathbf{\Phi}(\mathbf{x})$	shape function and its vector form
Ω	problem domain
Ω_s	local sub-domain
$\partial\Omega_s$	boundary of local sub-domain
Γ	boundary of domain
Γ_u	essential boundary
Γ_q	natural boundary
Γ_l	non-local boundary
λ	the thermal diffusivity
L_s	the part of boundary L_s which is inside the global domain
Γ_{su}	the part of overlap between the boundary of the local sub-domain and the essential boundary
Γ_{sq}	the part of overlap between the boundary of the local sub-domain and the natural boundary
\mathbf{n}	is the outward unit normal direction to the boundary $\partial\Omega$
n_x	the outward unit normal vector in x -space
n_y	the outward unit normal vector in y -space
t	time variable
Δt	time-step
T	final time
Δx	mesh size in x -direction
Δy	mesh size in y -direction
J	the residual function

u	temperature
u^h	trial function
\hat{u}	approximation value
$p(\mathbf{x}), \mathbf{p}(\mathbf{x})$	basis function and its vector form
m	the number of basis
N, n	the number of nodes
k	the number of supporting points influencing \mathbf{x}
s	the normalized distance between the interpolation point and the considered supporting point
ε	the regularization parameter
$a(\mathbf{x}), \mathbf{a}(\mathbf{x})$	coefficients of basis function and its vector form
d_i	distance from node \mathbf{x}_i
r_i	the size of the support for the test functions
w_i	weight function
K	the diffusivity coefficient
Q	source/sink term
v_i	test function
T	transpose