CHAPTER 3 MIMO STRATEGIES

This chapter provides a comprehensive overview and investigation of recent results on outage probabilities for MRC in an additive white Gaussian noise. Desired signal is subject to Nakagami fading with arbitrary fading parameters. Using the moment generating function based method, the probability density function (pdf) is derived in terms of output SNR for MRC. The approximate analytical results show the accurate way to access the outage probability for MRC with the Nakagami fading parameters are arbitrary. The performance evaluation of wireless digital communication systems for linear minimum mean square error diversity is also investigated. This approach and its main contribution are to model the non-Gaussian fading channel of wireless systems. A new expression of outage probability in a non-Gaussian fading environment is studied by modeling fading channel as a spherically invariant random process , also the average probability of error then is obtained. As a special case of this work, it is straightforward to consider the situation where the interferers have equal powers and unequal powers. Finally, some numerical results show the impact of the number of antenna elements, degrees of freedom (DOF), the number of interferers and noise.

3.1 Nakagami-*m* Fading

There has been a continuing interest in modeling various propagation channels with the Nakagami-*m* model, which describes multipath scattering with relatively large delay time spreads, Nakagami distributed signal envelope with the pdf is described by [56]

$$f_{\gamma}(\gamma) = \frac{2r^{2m-1}}{\Gamma(m)\Omega^{m}} \exp\left(-\frac{r^2}{\Omega}\right), \qquad \gamma \geq 0 \tag{3.1}$$

where $\Gamma(\cdot)$ is the Gamma function, $\Omega = \overline{r^2}/m$ is average power and m is fading parameter, which must satisfy $m \geq 1/2$ describing the fading severity. The smaller that m is, the more severe fading that occurs, with the special case $m=1$ and $m=1/2$ corresponding to the Raleigh distribution and one-sided Gaussian distribution, respectively. The pdf of the instantaneous SNR, γ_k on *k*th branch of MRC system is given by

$$f_{\gamma_k}(\gamma_k) = \frac{\left(\frac{m_k}{\overline{\gamma_k}}\right)^{m_k}}{\Gamma(m_k)} \gamma_k^{m_k-1} \exp^{\left(-\frac{m_k\gamma_k}{\overline{\gamma_k}}\right)}, \qquad \gamma_k \geq 0 \tag{3.2}$$

The MGF of γ_k is given by

$$M_{\gamma_k}(t) = \frac{\left(\frac{m_k}{\overline{\overline{\gamma_k}}}\right)^{m_k}}{\Gamma(m_k)} \int_0^{\infty} e^{-xt} x^{m_k - 1} e^{\left(-\frac{m_k x}{\overline{\overline{\gamma_k}}}\right)} dx. \tag{3.3}$$

The integral in (3.3) can be evaluated by using the contour integral for exponential function [57]

$$e^{(-x)} = \frac{1}{2\pi j} \int_{-j\infty}^{+j\infty} \Gamma(-s) x^s ds \tag{3.4}$$

where $j = \sqrt{-1}$ and the MGF can be written as

$$M_{\gamma_k}(t) = \frac{1}{\Gamma(m_k)} \left(\frac{m_k}{\overline{\overline{\gamma_k}} t}\right)^{m_k} \frac{1}{2\pi j} \int_{\mathbb{C}} \Gamma(-s)\Gamma(m_k + s) \left(\frac{m_k}{\overline{\overline{\gamma_k}} t}\right)^{s} ds \tag{3.5}$$

The integral in the part of (3.5) can be written in the *L* product resulting as

$$\begin{aligned} M_{\gamma}(t) &= \prod_{k=1}^{L} M_{\gamma_k}(t) \\ &= \left[\prod_{k=1}^{L} \frac{1}{\Gamma(m_k)} \left(\frac{m_k}{\overline{\overline{\gamma_k}} t}\right)^{m_k}\right] \left(\frac{1}{2\pi j}\right)^{L} \int_{\mathbb{C}_1} \int_{\mathbb{C}_2} \cdots \\ &\quad \int_{\mathbb{C}_L} \left\{ \prod_{k=1}^{L} \Gamma(-s_j)\Gamma(m_k + s_k) \left(\frac{m_k}{\overline{\overline{\gamma_k}} t}\right)^{s_k} \right\} ds_1 ds_2 \ldots ds_L \end{aligned} \tag{3.6}$$

which, using the inverse Laplace transform, the corresponding pdf of γ can be written as

$$f_{\gamma}(\gamma) = \frac{1}{2\pi j}\int_{\mathbb{C}} M_{\gamma}(t)e^{\gamma t}dt \tag{3.7}$$

averaging (3.6) with respect to (3.7) we can write

$$\begin{aligned} f_{\gamma}(\gamma) = {} & \left[\prod_{k=1}^{L}\frac{1}{\Gamma(m_k)}\left(\frac{m_k}{\overline{\gamma}_k}\right)^{m_k}\right]\left(\frac{1}{2\pi j}\right)^{L}\int_{\mathbb{C}_1}\int_{\mathbb{C}2}\cdots \\ & \int_{\mathbb{C}_L}\left\{\prod_{k=1}^{L}\Gamma(-s_k)\Gamma(m_k+s_k)\left(\frac{m_k}{\overline{\gamma}_k}\right)^{s_k}\right\} \\ & \times\left(\frac{1}{2\pi j}\int_{\mathbb{C}} t^{-\sum_{k=1}^{L}(m_k+s_k)}e^{\gamma t}dt\right)ds_1 ds_2 \ldots ds_L \end{aligned} \tag{3.8}$$

using result [57, eq. (8-315.1)] to give

$$\begin{aligned} f_{\gamma}(\gamma) = {} & \frac{1}{\Gamma\left(\sum_{k=1}^{L} m_k\right)}\left[\prod_{i=1}^{L}\left(\frac{m_k}{\overline{\gamma}_k}\right)^{m_k}\right] \\ & \times\gamma^{\left(\sum_{k=1}^{L} m_k\right)-1}\left(\frac{1}{2\pi j}\right)^{L}\int_{\mathbb{C}_1}\int_{\mathbb{C}_2}\cdots\int_{\mathbb{C}_L}\frac{\Gamma\left(\sum_{k=1}^{L} m_k\right)}{\Gamma\left(\sum_{k=1}^{L}(m_k+s_k)\right)} \\ & \times\left\{\prod_{k=1}^{L}\Gamma\left(-s_k\right)\frac{\Gamma\left(m_k+s_k\right)}{\Gamma\left(m_k\right)}\left(\frac{m_k\gamma}{\overline{\gamma}_k}\right)^{s_k}\right\}ds_1 ds_2 \ldots ds_L \end{aligned} \tag{3.9}$$

$$f_\gamma(\gamma) = \frac{1}{\Gamma\left(\sum_{k=1}^{L} m_k\right)} \left[\prod_{k=1}^{L}\left(\frac{m_k}{\gamma_k}\right)^{m_k}\right] \gamma^{(\sum_{k=1}^{L} m_k)-1} \sum_{n_1=0}^{\infty} \cdots \sum_{n_L=0}^{\infty} \frac{\left[\prod_{k=1}^{L} (m_k)_{n_k} \left(-\frac{m_k \gamma}{\gamma_k}\right)^{n_k} \frac{1}{n_k!}\right]}{\left(\sum_{k=1}^{L} m_k\right)_{n_T}}$$

(3.10)

where $n_T = \sum_{i=1}^{L} n_i$ and $(\alpha)_k = \Gamma(\alpha+k)/\Gamma(\alpha)$ with $(a)_0 = 1$. The result in (10) can be expressed as hypergeometric function [57]

$$f_\gamma(\gamma) = \frac{1}{\Gamma\left(\sum_{k=1}^{L} m_k\right)} \left[\prod_{k=1}^{L}\left(\frac{m_k}{\gamma_k}\right)^{m_k}\right] \gamma^{\left(\sum_{k=1}^{L} m_k\right)-1} \times \phi_2^{(L)}\left(m_1, m_2, \ldots m_L; \sum_{k=1}^{L} m_k; -\frac{m_1}{\gamma_1}\gamma, -\frac{m_2}{\gamma_2}\gamma, \ldots, -\frac{m_L}{\gamma_L}\gamma\right), \gamma \geq 0$$

(3.11)

where $\phi(\cdot)$ is the confluent hypergeometric function. Computation of $\phi(\cdot)$ can be achieved through various series method [58].

An important performance criterion which is useful for evaluating the quality of radio reception under consideration is the outage probability. The SNR at the output of MRC is the probability of failing to achieve adequate reception of wanted signal, is a useful statistical parameter for assessing the quality of service (QoS) in digital mobile communications. From (3.11) the probability of outage can be described mathematically by

$$P_o\left(\gamma_{th}\right)=\int_0^{\gamma_{th}} f_\gamma\left(x\right)dx$$
$$=\frac{1}{\Gamma\left(1+\sum_{k=1}^{L}\left(m_k\right)\right)}\left[\prod_{k=1}^{L}\left(\frac{m_k\gamma_{th}}{\gamma_k}\right)^{m_k}\right]$$
$$\times\phi_2^{(L)}\left(m_1,m_2,...m_L;1+\sum_{k=1}^{L}m_i;-\frac{m_1}{\gamma_1}\gamma_{th},-\frac{m_2}{\gamma_2}\gamma_{th},...-\frac{m_L}{\gamma_L}\gamma_{th}\right)$$

(3.12)

In general, it is difficult to derive a closed form expression and the integral must be evaluated numerically. For arbitrary numbers of antennas, the result in (3.12) usually performed the series in (3.10). Some numerical results of outage probability for MRC are provided. In Figure 3.1, the probability density function is plotted, it also has the advantage of including the Rayleigh (m = 1) and the one sided Gaussian distribution (m = ½) as special case. Figure 3.2 shows the outage probability versus the SNR for L = 1, 2, and 3, fading parameter $m=0.5$. The outage probability is plotted versus the SNR in Figure 3.3 and Figure 3.4 for dual MRC and for several values of m = 0.5, 9/16, 5/8, 3/4, 1, 3/2, 2, and 3, respectively. It is evident that the outage probability degrades as the fading severity increase as well as the improvement offered by the triple MRC, compared to the dual one.

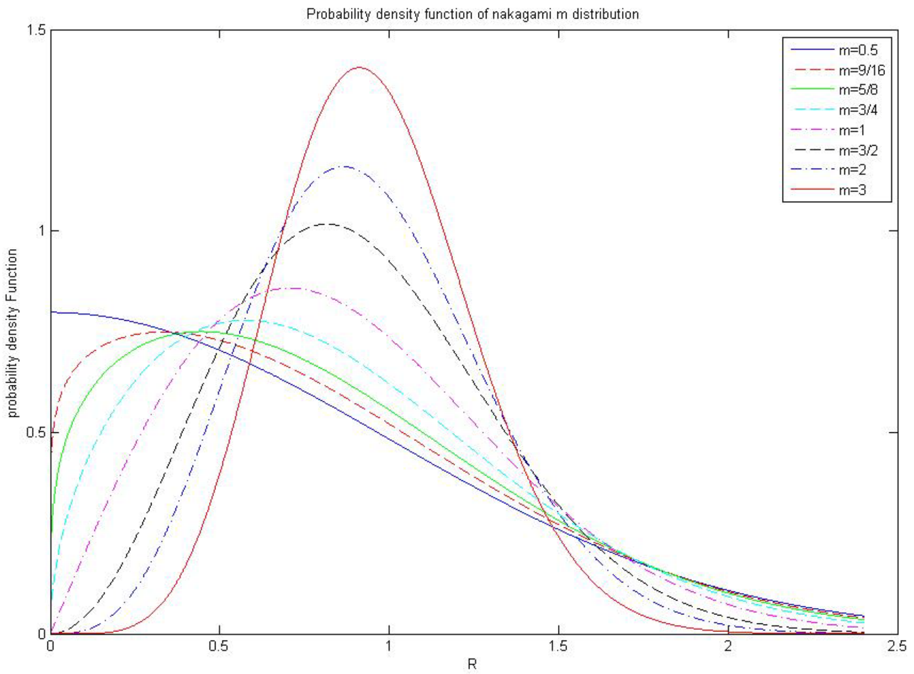

Figure 3.1 The probability density function of Nakagami-*m* distribution

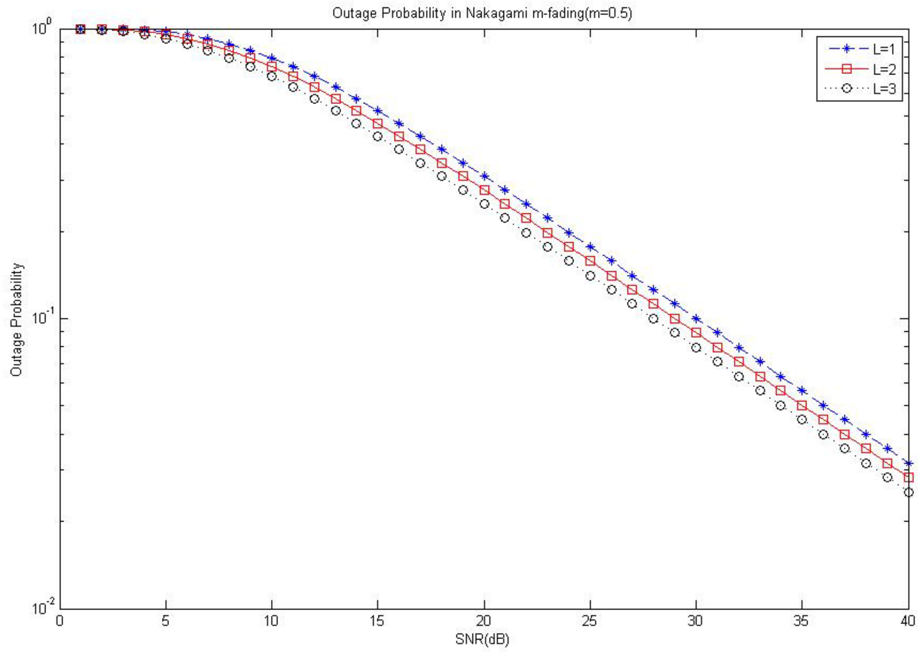

Figure 3.2 Outage probability versus SNR for MRC branchs L = 1, 2, 3, and value of m = 0.5 and SNR = 10 dB

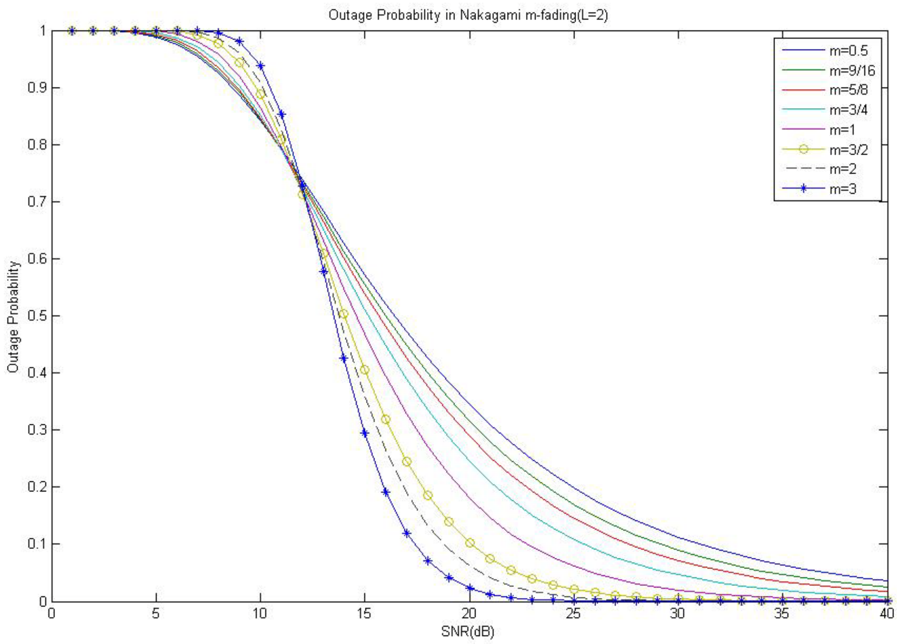

Figure 3.3 Outage probability versus SNR for dual MRC, and several values of m and SNR = 10 dB

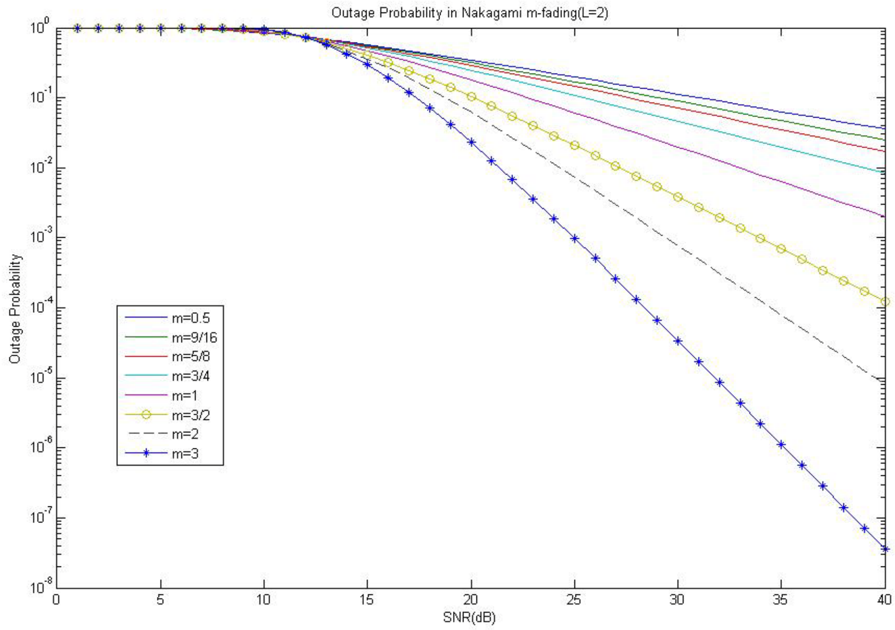

Figure 3.4 Outage probability versus SNR for dual MRC in finescale, and several values of *m* and SNR = 10 dB

3.2 Non-Gaussian Fading

The outage probability is the probability of failing to achieve adequate reception of wanted signal, is a useful statistical parameter for assessing the quality of service (QoS) in digital mobile communication systems. It can be described mathematically by

$$P_{out} = \Pr(0 \le \gamma \le t) = \int_0^t f_\gamma(\gamma) d\gamma \tag{3.13}$$

where $p_\gamma(\gamma)$ is the pdf of the desired and interfering signals power. To carry out (3.13), one needs $f_\gamma(\gamma)$. Its performance now depends on the choice of the pdf of r. The choice of the characteristic pdf correspond to assuming that the random matrix $\mathbf{u}_i$ follows a student-*t* distribution is used in the analysis [59], which is given by

$$f_r(r) = \frac{\Gamma(2\nu) r^{\nu-1}}{\Gamma^2(\nu)(1+r)^{2\nu}} \tag{3.14}$$

where ν is corresponding degrees of freedom (DOF). Note that the student-*t* distribution becomes a standard Gaussian distribution as a special case when $\nu \to \infty$. Therefore, the student-t distribution is expected to be an appropriate non-Gaussian distribution to model an environment with a small number of the scattering signals, with the DOF of the distribution relating to the number of scattering signals in the channel. It can be shown that the conditional outage probability can be written as

$$P_{out} = \int_0^\infty f(\gamma|r) f_r(r) dr \tag{3.15}$$

Substituting (3.13) and (3.14) into (3.15) and carry out the integration [58], P_{out} can be written as

$$P_\gamma(\gamma) = 1 + \exp(-\sigma_n^2 \gamma) \left(\frac{\gamma^\alpha}{2} \right)^{L-M+k} \times (-1)^k \sum \left(\prod_{i=(L,L-M+k)} \left(\frac{P_0}{P_i} \right)^{-\alpha} {}_2F_1 \left(2\alpha, \alpha+1; 2\alpha+1; 1 - \frac{\gamma P_i}{P_0} \right) \right) \tag{3.16}$$

where

$${}_2F_1(a,b;c;x)_n = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{(c)_k} \frac{x^k}{k!} = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{k=0}^{\infty} \frac{\Gamma(a+k)\Gamma(b+k)}{\Gamma(c+k)} \frac{x^k}{k!}$$

By utilizing the series illustration of the hypergeometric function [58] and the α-terms classification, it can be illustrate that, when DOF is very large. These agree with the special case of student-t distribution, for Rayleigh fading channel. Furthermore, the closed form expression are differs from the results of [60].

After obtaining the expressions for SINR as shown above, in this section, we present some numerical results by plotting the outage probability SINR when the channel undergoes spherically invariant fading that is characterized by student-*t* distribution.

Figure 3.5 represents, the outage probability versus average SINR with *L* equal mean power SIRP faded interferers as a parameter, in the presence of 2 antenna elements and DOF $\alpha = 3$. Along with the increasing of number of CCI's, the outage probability relative increase, i.e., the system performance deteriorated. Figure 3.6 represents the outage probability, in presence of 8 equal mean power interferers with number of antenna element as a parameter and the SINR threshold is set as 20 dB. As expect the system performance increases significantly as the number of antennas increase.

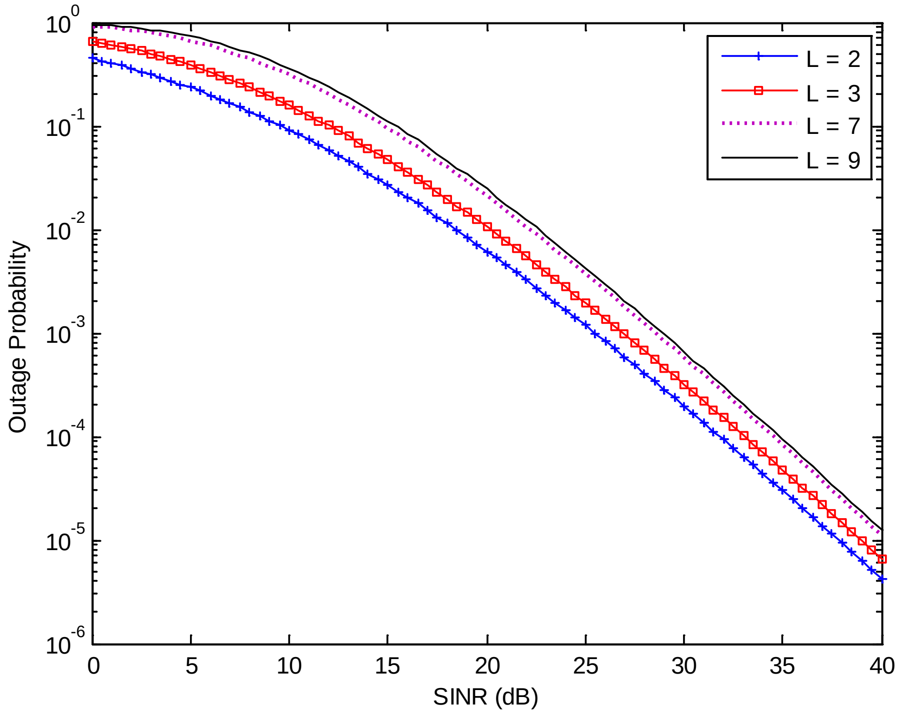

Figure 3.5 Outage probability versus average SINR with *L* equal mean power SIRP faded interferers as a parameter, in the presence of 2 antenna elements and DOF $\alpha = 3$

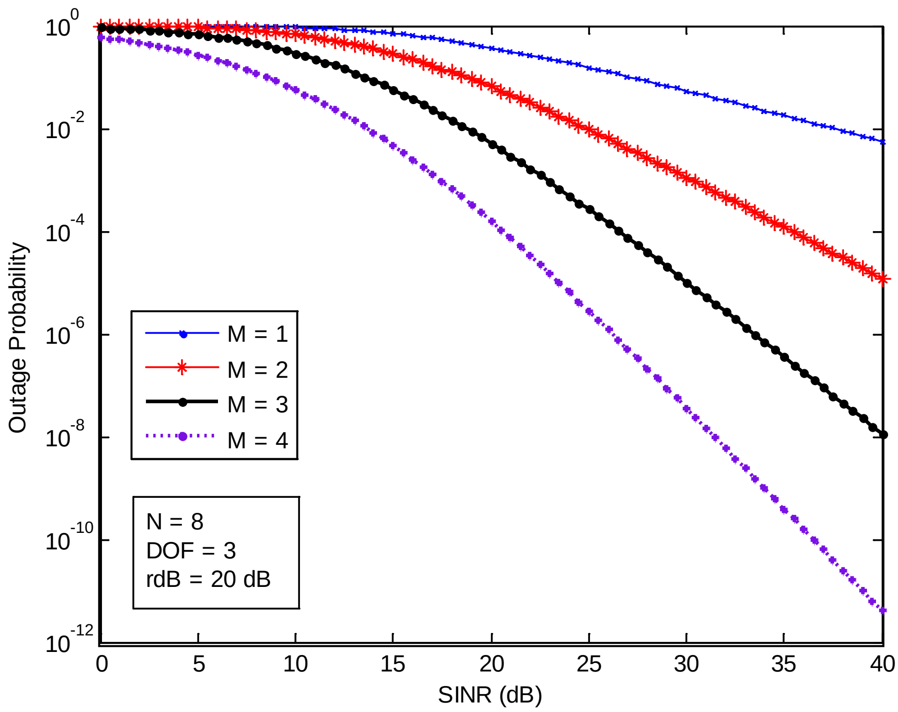

Figure 3.6 Outage probability versus average SINR with antenna arrays as a parameter, in the presence of 8 equal mean powers SIRP faded interferers and DOF $\alpha = 3$

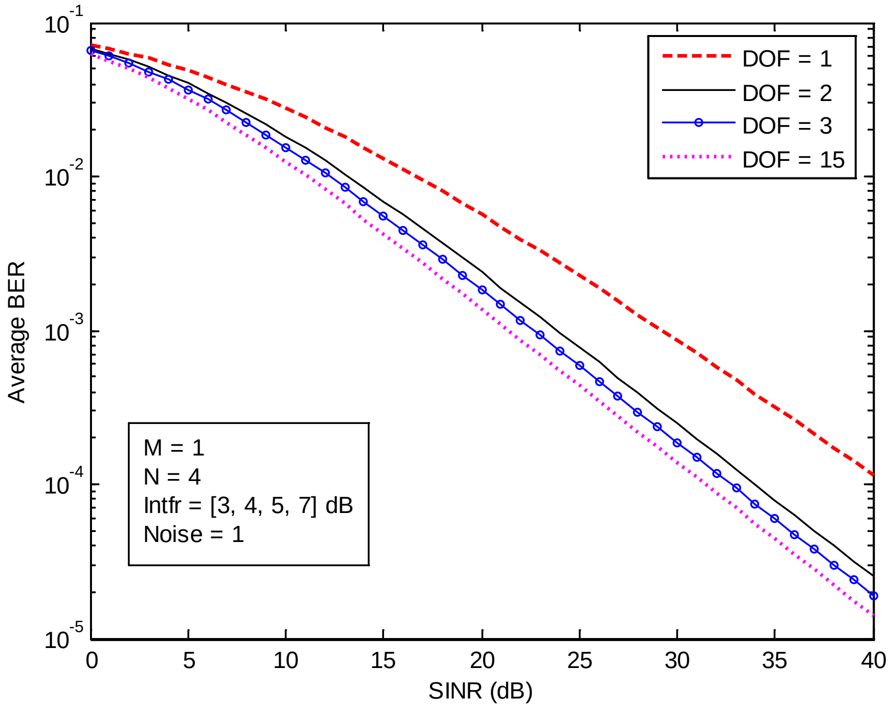

Figure 3.7 Probability of error versus average SINR in presence of four unequal power interferers with DOF as a parameter (DOF = 1, 2, 3, 15) without diversity order

The investigation is done by deriving the pdfs of SINR, in which the multipath fading and co-channel interferers are assumed to be the *t* distribution. Then, the closed form expressions for outage probability in a non-Gaussian fading environment of digital cellular mobile radio communication systems employing MMSE linear diversity is computed with derived in this paper. The numerical results show that this model gives an alternative for fading channel. The system model may be applied with an indoor environment and some other environments that Gaussian model cannot be applied.

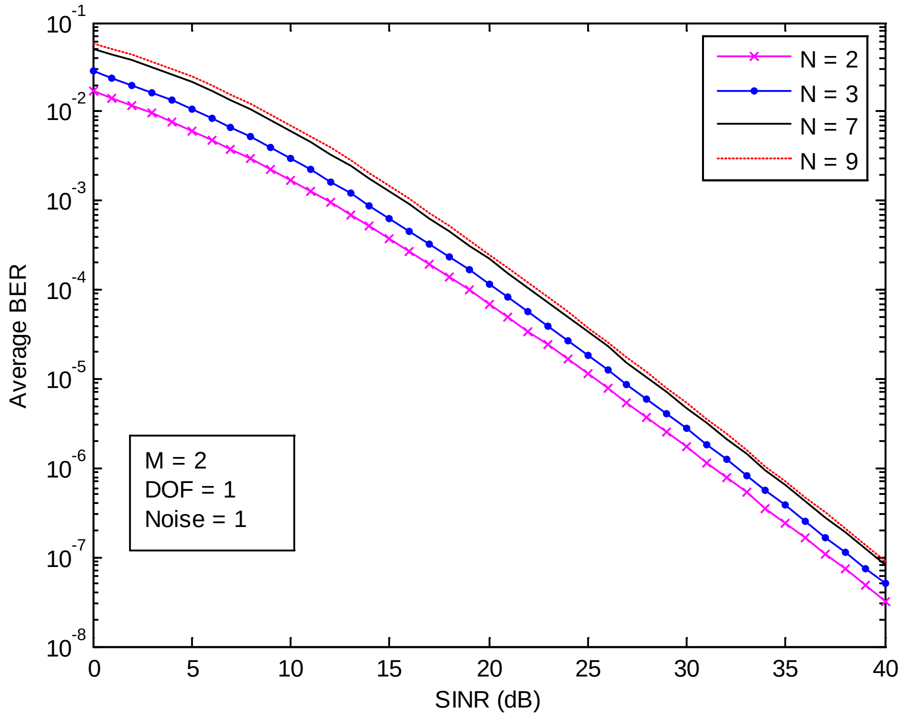

Figure 3.8 Probability of error versus average SINR in presence of two antenna elements with equal power interferers as a parameter

The expressions for the outage probability are given in (3.16). After obtaining the expressions for SINR as shown above, in this section, we present some numerical results by plotting the probability of error SINR when the channel undergoes spherically invariant fading that is characterized by student-*t* distribution. We observe that even when the system becomes interference limited, increasing the order of diversity (dual antenna) still improves system performance.

Figure 3.7 represents, the probability of error versus average SINR with *N* unequal mean power SIRP faded interferers (3, 4, 5, and 7 dB.) as parameters and the DOF as a parameter (DOF =1, 2, 3, 15), in the presence of 1 antenna element (no antenna diversity). Along with the increasing of number of co-channel interferers, the probability of error relative increase, i.e., the system performance deteriorated. Figure 3.8 represents the probability of error, in presence of 2 antenna elements with equal mean power interferers (2, 3, 7, and 9 interferers) as a parameter and the DOF = 1. As expect the system performance increases significantly as the number of antennas increase. As shown in Figures, these favorable results can be explained by the fact that the co-channel interference much deeper fading than desired signal. The diversity improvement is much better as the probability of error decrease.