APPLICATION OF PASSIVITY CONCEPT FOR SPLIT RANGE CONTROL OF HEAT EXCHANGER NETWORKS

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A SPECIAL RESEARCH PROJECT SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR
THE DEGREE OF MASTER OF ENGINEERING (CHEMICAL ENGINEERING)
FACULTY OF ENGINEERING
KING MONGKUT'S UNIVERSITY OF TECHNOLOGY THONBURI
2011

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Special Research Project Title	Application of Passivity Concept for Split Range Control of Heat Exchanger Networks
Special Research Project Credits	6
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Program	Master of Engineering
Field of Study	Chemical Engineering
Department	Chemical Engineering
Faculty	Engineering
B.E.	2554

Abstract

Split range control design of heat exchanger networks is widely used for heat integrated plants. This design provides a stable and optimal operation for targeted temperatures, while the utility cost is minimized for all possible disturbance variations. Concerning to the performance of stability, an effective controller design named passivity method is presented to regulate the stability for split range control of heat exchanger networks in this work. The dynamic models of the proposed system are developed in state space formulas. The state space models of heat exchanger networks can represent both the process transfer function and the disturbance transfer function. These transfer functions are formulated to determine whether the heat exchanger networks is passive by analyzing the passivity index. As a result, the split range control of heat exchanger networks is a non-passive system. Therefore, the introduction of the weighting function is proposed to adapt the heat exchanger networks into strictly a passive system. The passive controller is then designed in order to keep the heat exchanger networks at the stable operation. The proposed method is tested and compared with PI controller and illustrates that the passivity approach can give better performance over conventional PI controllers. In addition, the dynamic responses of target temperature with passive controllers have a lower oscillation when compared to PI controllers.

Keywords: Split control / Heat exchanger networks / State space / Passivity method

หัวข้อโครงการศึกษาวิจัย	การประยุกต์ใช้แนวคิดพาสสิวิตี้สำหรับระบบควบคุมแบบแบ่งส่วนของเครือข่ายแลกเปลี่ยนความร้อน
หน่วยกิต	6
ผู้เขียน	นาย ณัฐพงษ์ ธาราภูมิ
อาจารย์ที่ปรึกษา	รศ.ดร. ธงไชย ศรีนพคุณ
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บทคัดย่อ

การออกแบบระบบควบคุมแบบแบ่งส่วนของเครือข่ายแลกเปลี่ยนความร้อนถูกใช้กันอย่างกว้างขวางสำหรับโรงงานที่มีการแลกเปลี่ยนความร้อน ในการออกแบบนั้นจะคำนึงถึงความเสถียรภาพ, การควบคุมอุณหภูมิให้เป็นไปตามที่ต้องการและใช้ต้นทุนด้านพลังงานน้อยที่สุดในขณะที่ระบบถูกรบกวน ในการวิจัยนี้การออกแบบระบบควบคุมที่มีประสิทธิภาพที่เรียกว่าวิธีพาสสิวิตี้ได้ถูกนำมาใช้เพื่อออกแบบระบบให้มีความเสถียรภาพ จากนั้นสมการแบบจำลองเชิงพลวัตสำหรับระบบควบคุมแบบแบ่งส่วนของเครือข่ายแลกเปลี่ยนความร้อนจะถูกนำเสนอในรูปแบบของสมการขั้นพื้นฐาน สมการขั้นพื้นฐานที่ได้จะถูกนำไปคำนวนหาฟังก์ชันส่งผ่านของระบบและฟังก์ชันส่งผ่านของตัวแปรรบกวน ซึ่งฟังก์ชันทั้งสองจะถูกนำไปวิเคราะห์หาว่าระบบควบคุมแบบแบ่งส่วนของเครือข่ายแลกเปลี่ยนความร้อนนั้นเป็นระบบพาสซีฟหรือไม่ โดยใช้ดัชนีพาสสิวิตี้เป็นตัวบ่งชี้ จากผลการวิจัยพบว่าระบบควบคุมแบบแบ่งส่วนของเครือข่ายแลกเปลี่ยนความร้อนนั้นไม่เป็นระบบพาสซีฟ จึงได้มีการเสนอฟังก์ชันถ่วงน้ำหนักมาปรับปรุงให้ระบบที่ต้องการเป็นระบบพาสซีฟ จากนั้นตัวควบคุมแบบพาสซีพจะถูกออกแบบเพื่อที่จะควบคุมระบบให้มีความเสถียรภาพ เมื่อนำวิธีการออกแบบดังกล่าวมาทดสอบและเปรียบเทียบกับตัวควบคุมแบบพีไอ พบว่าในการออกแบบระบบควบคุมแบบพาสสิวิตี้มีคุณภาพมากกว่าและการเปลี่ยนแปลงของอุณหภูมิที่ต้องการมีการแกว่งน้อยกว่าการออกแบบระบบควบคุมแบบพีไอ

คำสำคัญ : การควบคุมแบบแบ่งส่วน / เครือข่ายแลกเปลี่ยนความร้อน / สมการขั้นพื้นฐาน / วิธีพาสสิวิตี้

ACKNOWLEDGEMENTS

This project would not have been possible without direct and indirect assistance from many people. Foremost, the author would like to express his gratitude to the advisor of this project, Assoc. Prof. Dr. Thongchai Srinophakun for all technical support, valuable guidance and encouragement. The author also appreciates the supervisor, Prof. Sigurd Skogestad for his valuable suggestions and comments during the development of this overseas study. In addition, appreciation is also extended to Asst. Prof. Dr. Veara Loha and Asst. Prof. Dr. Asawin Meechai, project committee members, for their attention to this research project and giving me useful recommendations.

Also, my sincere thank is also given to my friends, colleagues, and teams in Chemical Engineering Practice School (ChEPS), Miss Nichaporn Kanjanabat, Miss. Kawisra Sompech and Mr. Piroon Samanot, who helped the author with communication and any important information used to finish the master’s degree. Lastly, this project would not have been completed without lots of support from the author’s family and friends.

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NOMENCLATURES

ABBREVIATIONS

Diag	Diagonal
CV	Controlled Variable
HEN	Heat Exchanger Networks
Max	Maximum
Min	Minimum
MIMO	Multi Input Multi Output
MINLP	Mixed Integer Nonlinear Programming
MV	Manipulated Variable
NS	Number of Stage
PI	Proportional Integral
PR	Positive Real
RHP	Right Half Plane
SISO	Single Input Single Output
SPR	Strictly Positive Real

GREEK LETTERS

ρ	Density
ω	Frequency
ν	Passivity index
k_c	Proportional gain of PI controller
τ_I	Time integral of PI controller
γ	Scalar decision variable
Σ	Time invariant dynamical system
$\subset$	Subset
$\lambda_{\min}$	Minimum eigenvalue
$\forall$	For all
$\in$	For some

NOMENCLATURES (CONT.)

SUBSCRIPTS

C	Cold process stream
H	Hot process stream
in	Input
m	*m* matrix dimension
n	number of cell
out	Output
0	Initial state

SUPERSCRIPTS

in	Input
out	Output
T	Transpose
n	*n* matrix dimension
m	*m* matrix dimension
*	Conjugate transpose

SYMBOLS

G(s)	Transfer function of process
K(s)	Transfer function of controller
Re(A)	Real part of complex matrix A
w(s)	Weighting function

UNITS

ºC	Degree celsius
s	Seconds
K	Kelvin
kg	Kilogram
kW	Kilowatt
m^2	Square meter
m^3	Cubic meter

NOMENCLATURES (CONT.)

VARIABLES

a, b, c, k	Decision variables of weighting function
A, B, C, D, E	Matrix coefficient of state space equation
C_p	Specific heat capacity
u	Split fraction of bypasses heat exchanger
F	Volumetric flowrate
FC_p	Heat capacity flowrate
h	Heat transfer coefficient
H	Hot process stream
I	Identity matrix
Im	Imaginary number
j	Square root of minus one (-1)
n	Matrix dimension
R	Real number
s	Frequency domain (or called *s*-domain)
t	Time
T	Temperature
U	Overall heat transfer coefficient
V	Volume of compartment of a heat exchanger
x	State variable
X	Space of state
y	Output variable
Y	Space of output

CHAPTER 1 INTRODUCTION

1.1 Background

Currently, heat exchanger networks (HEN) are widely used in many industries and also published in many researches. However, there are a few literatures, which considers about the minimizing utility consumption and the control system of heat exchanger networks. Nevertheless, a suitable control strategy is needed in the control system of heat exchanger networks. The control system of heat exchanger networks should be optimal operation, which can regulate the temperature at their certain level while utility cost is minimized for all possible disturbances. This is the main motivation of this thesis.

Heat exchanger is the device, which facilitates the heat transfer between hot side stream and cold side stream that have different temperature. The single heat exchanger is commonly used in a wide range of application as heaters or air conditioners in a household. Conversely, the single heat exchanger cannot support for the chemical processing plant or power production plant. Therefore, the heat exchanger networks are required to operate systems. *Glemmestad* [1] proposed three requirements for the optimum operation of heat exchanger networks; the satisfied target temperatures, the minimum utility cost, and the robust dynamic behavior. In order to control the target temperature of heat exchanger networks, the bypassed heat exchanger is required by varies split fractions. In some case studies, these split fractions of bypassed heat exchanger may be saturated and cannot be maintained the target temperatures. These problems can handle by attempting to find a simple operation policy for split range control as presented by *Lersbamrungsuk* [2]. The split range control is the policy for switching between regions. The second requirement is minimized utility cost as studied by *Aguilera* [3] and *Glemmestad et al.* [4]. Not only the satisfied target temperature and the minimized utility cost are interesting but also the stability of heat exchanger networks is very important.

Recently, the one technique to analyze the stability of general processes is the passivity method as presented by *Bao* [5] and *Bao and Lee* [6]. The passive systems are the systems that do not generate energy with respect to the given input and output ports. It is attractive to apply passivity method with split range control of heat exchanger networks to achieve both stable and controllable process. Moreover, the stability analysis of split range control of heat exchanger networks using the passivity method has not been researched yet.

1.2 Objectives

1. To study and develop the state space models for split range control of heat exchanger networks.
2. To apply passivity method for split range control of heat exchanger networks.

1.3 Scopes of work

1. Only water is used as the component and no phase change.
2. The pairing between controlled and manipulated variables using split range control design are solved by GAMs [7].
3. The transfer function equations are solved via MATLAB^{TM}.
4. The passivity method is applied to check the passive of the system and to design passive controller for split range control of heat exchanger networks.
5. The results are verified and compared with PI controller by MATLAB-SIMULINK [1].

CHAPTER 2 THEORY AND LITERATURE REVIWES

This chapter is separated into two sections; the theory and the literature. Each section composes of three main parts which are the heat exchanger networks, the split range control of heat exchanger networks, and the passivity method.

2.1 Theory reviews

The first part reviews the related works to the heat exchanger networks. The split range control of heat exchanger networks is reviewed in the second part and the following part is the review of the passivity method**.**

2.1.1 Heat exchanger networks [1]

This part describes the development of the dynamic model of heat exchanger networks. The general heat exchanger model is applied to simplify the equations. Some fundamentals of heat transfer and steady state properties of ideal countercurrent heat exchanger are described. The formulations of dynamic equations of controlled heat exchanger networks are performed.

The dynamic model of a single heat exchanger is developed from a general heat exchanger model in forms of ordinary differential equations (ODE). The following assumptions are made for the general heat exchanger model:

1. Constant densities of the two fluids (ρ_i).
2. Constant specific heat capacities of the two fluids ($C_{V,i}$).
3. Constant thermal efficiency ($P_{h,\ i}$).
4. Constant flow independent heat transfer coefficient.
5. Lumped models.
6. No phase changes.

A countercurrent heat exchanger networks can be modeled from the assumption as shown in Figure 2.1.

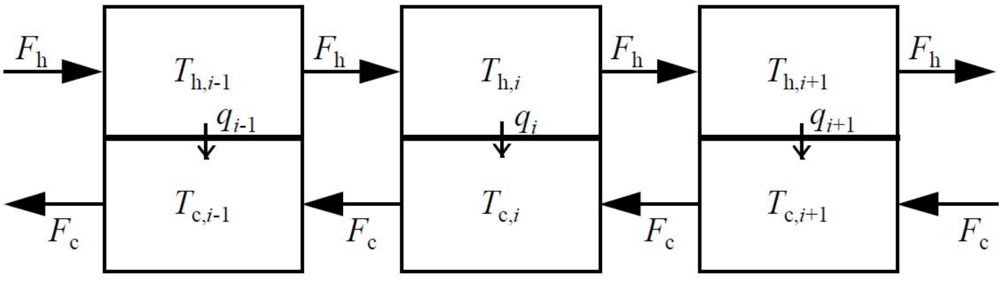

Figure 2.1 The countercurrent heat exchanger networks model [1].

The dynamic equations for each stage becomes

$$\frac{\partial T_{h,i}}{\partial t}=\frac{F_h}{V_{h,i}}\left(T_{h,i-1}-T_{h,i}\right)-\frac{UA_i}{\rho_h V_{h,i} C_{V,h}}\Delta T_{m,i} \qquad (2.1)$$

$$\frac{\partial T_{c,i}}{\partial t}=\frac{F_c}{V_{c,i}}\left(T_{c,i+1}-T_{c,i}\right)+\frac{UA_i}{\rho_c V_{c,i} C_{V,c}}\Delta T_{m,i} \qquad (2.2)$$

Where A_i is the heat exchanger area at stage *i*. $V_{h,\,i}$ and $V_{c,\,i}$ are the volumes of each compartment at the hot side and the cold side streams. The number of each stage is arranged from 1 to *i*, and the heat exchanger model consists of 2*i* ordinary differential equations. At the inlet, the boundary conditions are such that $T_{h,0}$ equal to $T_{h,\,in}$ and $T_{c,\,i+1}$ equal to $T_{c,\,in}$, and at the outlet we have $T_{h,\,i}$ equal to $T_{h,out}$ and $T_{h,1}$ equal to $T_{h,out}$. $\Delta T_{m,\,i}$ is the temperature driving force at stage *i*. Three alternatives for this expression are:

1. Lumped models: $\Delta T_{\mathrm{m},i} = T_{\mathrm{h},i} - T_{\mathrm{c},i}$ (2.3)

2. AMTD: $\Delta T_{\mathrm{m},i} = 0.5\left[\left(T_{\mathrm{h},i-1}-T_{\mathrm{c},i}\right)+\left(T_{\mathrm{h},i}-T_{\mathrm{c},i+1}\right)\right]$ (2.4)

3. LMTD: $\Delta T_{m,i}=\dfrac{(T_{h,i-1}-T_{c,i})-(T_{h,i}-T_{c,i+1})}{\ln\left[(T_{h,i-1}-T_{c,i})/(T_{h,i}-T_{c,i+1})\right]}$ (2.5)

In order to control the target temperature, the bypassed heat exchanger is required by suitable split fractions. This is implemented directly into the heat exchanger model. The variable *u* denotes the split fraction of the inlet stream being bypassed heat exchanger on the hot side or the cold side streams. The outlet temperature after mixing as follows:

$$T^{out} = uT^{in} + (1-u)T^{'out} \qquad (2.6)$$

Where $T^{'out}$ is the outlet temperature of heat exchanger before mixing with the bypassed stream.

2.1.2 Split range control of heat exchanger networks [2]

The operation of heat exchanger networks is optimal if the following three main goals, which are:

1. Primary goal: Target temperature is satisfied
2. Secondary goal: Utility cost is minimized
3. Third goal: Dynamic behavior is satisfied

In order to perform optimization during operation, there has to be degree of freedom (DOF) after regulatory control is implemented. [1]

The number of degrees of freedom ($N_{\text{DOF, U}}$) for heat exchanger networks, which is used for utility cost optimization, is given by:

$$N_{DOF,\,U} = R + N_U - N_t \tag{2.7}$$

Where R is the dimensional space spanned by the manipulated variables in the inner heat exchanger networks. N_{U} is the number of utility exchanger unit and N_{t} is the number of target temperatures.

$N_{DOF,U}<0$: The operation of the heat exchanger networks is not feasible.
$N_{DOF,U}=0$: The operation of the heat exchanger networks is structurally feasible. There is no degree of freedom available for utility cost optimization.
$N_{DOF,U}>0$: The operation of the heat exchanger networks is structurally feasible. There are some degrees of freedom for utility cost optimization.

There are two important manipulated variables of heat exchanger networks:

Single bypasses: Target temperatures are controlled by the split fraction of the bypassed heat exchanger.

Utility duties: Target temperatures are controlled by the duty of the final utility exchanger.

The corresponding steady state optimal operation of heat exchanger networks can be formulated as a linear programming (LP) problem: [3]

Objective function:

$$\text{Min } c^T x$$

Subject to:

$$\begin{aligned} &Ax \le b \\ &A_{eq} x = b_{eq} \\ &x_{\min} \le x \le x_{\max} \end{aligned}$$

Where x consists of the inlet and outlet temperatures on the hot side stream ($T^{in}_{h,i}$ and $T^{out}_{h,i}$) and cold side stream ($T^{in}_{c,i}$ and $T^{out}_{c,i}$) of all exchangers, as well as the duties of all exchangers, which are duties transfer in heat exchanger (Q_i), utility duties of cooler ($Q_{c,i}$), and utility duties of heater ($Q_{h,i}$). The equality constraints include the process models, the internal connections, the supplied temperatures ($T_{s,i}$) and the target temperatures ($T_{t,i}$). The inequality constraints include the lower and upper bounds on the duties of all heat exchangers. In this part, the objective function is minimizing the utility cost. Where C is cost parameter related to the utility duties of cooler and heater.

The LP problem formulation for optimal operation of heat exchanger networks is given in equations 2.8-2.22: [4]

Objective function:

$$\text{Min} \sum_{i \in CU} C_{cu,i} Q_{c,i} + \sum_{j \in HU} C_{hu,j} Q_{h,j} \tag{2.8}$$

Subject to:

a) Conservation of heat transfer

Heat exchanger *i*:

$$Q_i - (mC_P)_{c,i}\left(T_{c,i}^{out} - T_{c,i}^{in}\right) = 0 \qquad i \in PHX \tag{2.9}$$

$$Q_i - (mC_P)_{h,i}\left(T_{h,i}^{in} - T_{h,i}^{out}\right) = 0 \qquad i \in PHX \tag{2.10}$$

Cooler *i*:

$$Q_{c,i} - (mC_P)_{h,i}\left(T_{h,i}^{in} - T_{h,i}^{out}\right) = 0 \qquad i \in CU \tag{2.11}$$

Heater *i*:

$$Q_{h,i} - (mC_P)_{c,i}\left(T_{c,i}^{out} - T_{c,i}^{in}\right) = 0 \qquad i \in HU \tag{2.12}$$

b) Connecting equations

Supply connection:

$$T_{h,i}^{in} = T_{s,i} \qquad i \in HXHS \tag{2.13}$$

$$T_{c,i}^{in} = T_{s,i} \qquad i \in HXCS \tag{2.14}$$

Internal connection:

$$T_{h,i}^{out} - T_{h,j}^{in} = 0 \qquad i \in HXHO,\ j \in HXHI \tag{2.15}$$

$$T_{c,i}^{out} - T_{c,j}^{in} = 0 \qquad i \in HXCO,\ j \in HXCI \tag{2.16}$$

Target connection:

$$T_{h,i}^{out} = T_{t,i} \qquad i \in HXHT \cup CUT \tag{2.17}$$

$$T_{c,i}^{out} = T_{t,i} \qquad i \in HXCT \cup HUT \tag{2.18}$$

c) Lower and upper bounds of heat exchanger

Lower bound:

$$-Q_i \leq 0 \qquad\qquad i \in PHX \cup CU \cup HU \quad (2.19)$$

Upper bound:

$$Q_i \leq P_{h,i}(mC_P)_{h,i}(T_{h,i}^{in} - T_{c,i}^{in}) \qquad\qquad i \in PHX \cup CU \cup HU \quad (2.20)$$

$$P_{h,i} = \frac{NTU_{h,i}(1 - e^{(NTU_{c,i} - NTU_{h,i})})}{NTU_{h,i} - NTU_{c,i}e^{(NTU_{c,i} - NTU_{h,i})}} \quad (2.21)$$

$$NTU_{h,i} = \frac{(UA)_i}{(mC_p)_i^{hot}}, \; NTU_{c,i} = \frac{(UA)_i}{(mC_p)_i^{cold}} \quad (2.22)$$

Where *PHX*: set of all process heat exchangers

CU: set of utility duties of cooler

HU: set of utility duties of heater

HXHT: subset of *PHX* with hot side outlet is a controlled target

HXCT: subset of *PHX* with cold side outlet is a controlled target

CUT: subset of *CU* with outlet is a controlled target

HUT: subset of *HU* with outlet is a controlled target

HXHO:subset of *PHX* with hot side outlet entering a hot side inlet of the adjacent exchanger

HXCO:subset of *PHX* with cold side outlet entering a cold side inlet of the adjacent exchanger

HXHI: subset of *PHX* with hot side inlet coming from a hot side outlet of the adjacent exchanger

HXCI: subset of *PHX* with cold side inlet coming from a cold side outlet of the adjacent exchanger

HXHS: subset of *PHX* with hot side inlet directly coming from a hot supply

HXCS: subset of *PHX* with cold side inlet directly coming from a cold supply

The LP formulation implies that the optimal solutions are always at an intersection of constraints. The inequality constraints in the above LP formulation imply that active constraints occur on manipulated variables (i.e. Lower boundary of utility duties of cooler or heater, lower and upper boundary of split fraction of bypassed heat exchanger). After the target temperatures are controlled by the manipulated variables and the result of optimal operation to keep all remaining manipulated variables at constraints. However, under the variation of operating conditions, the optimal vertex (set of active constraints) may change. For a given operating window, it may have several optimal vertices for active constraint regions. Therefore, to obtain optimality, one needs a good control policy for disturbance tracking to the change of active constraints during the operation. One solution is the switching between active constraints regions called split range control. [8]

The split range control describes the possible methods to implement optimal policy by disturbance tracking to set of active constraints. The assumptions are: [2]

1) The target temperatures are feasible for the given disturbance window.

2) The output constraints do not change and are always active. The optimal point is a vertex and a certain number of inputs are at the constraints.

From the assumptions, the optimal solution has the following properties:

a) The set of active constraints remains constant in a certain region of the disturbance space.

b) The system has two or more critical regions in the disturbance window, it follows that the set of constraints are different.

The optimal solution can be achieve by the following properties:

a) In a given critical region R_o, it is possible to operate the heat exchanger networks optimally using a control structure where the output constraints are controlled some manipulated variables by using SISO control loops with zero steady state error, for example, PI controllers.

b) The system is switched form R_o to a different region R_1 or change active constraint, when the system has disturbance.

The example of split range control has three manipulated variables (MV_1, MV_2, and MV_3) and two controlled variables (CV_1 and CV_2). In general, two manipulated variables are needed to control two controlled variables as SISO control loop. Furthermore, since one optimal solution is always at input constraints, the remaining one manipulated variables may be at active constraints or saturated. For operating window, the active constraint regions can be found by parametric programming and the results are shown in Table 2.1 and Figure 2.2.

Table 2.1 The set of active constraints for example.

Region	**MV_1**	**MV_2**	**MV_3**
1	S	U	U
2	U	U	S
3	U	S	U

U-Unsaturated manipulated variable (inactive constraint)
S-Saturated manipulated variable (active constraint)

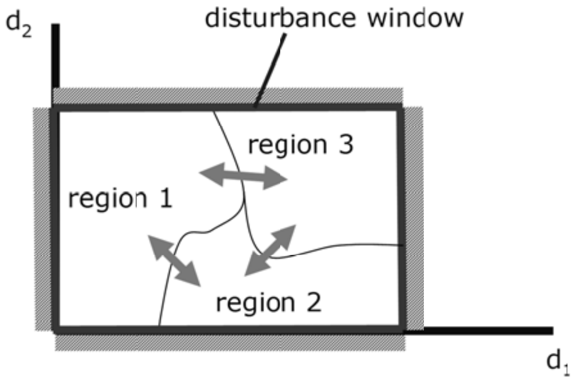

Figure 2.2 The active constraint regions [7].

According to these, the manipulated variable one (MV_1) is saturated as an active constraint in region 1. The manipulated variable two and three (MV_2 and MV_3) are therefore used in order to control the controlled variables one and two (CV_1 and CV_2) as SISO control loop. Moreover, the system will be switched form R_1 to a different region R_2 or change active constraint, when the system has disturbance.

The split range control is the technique of using structural information to find a control structure for optimal operation of heat exchanger networks was proposed by *Mathisen and Glemmestad* [9]. Figure 2.3 shows split range control that are commonly used to control two or more manipulated variables with a single controller, one of them is referred as primary manipulated variable and the other as a secondary manipulated variable. The primary manipulated variable is used to control a target under the nominal condition. However, the final choice of primary and secondary manipulated variables can be based on other considerations also. This flexibility will be exploited in the final control structure design.

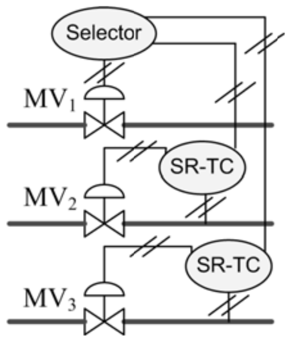

Figure 2.3 The split range control (SR-TC is split range temperature controller) [7].

For determination of optimal split range control structure is used for paring between controlled variables and manipulated variables. The approaches for determining optimal split range control structure identifies the set of active constraints and uses the information of directional effects between controlled variables and manipulated variables. However, the approach must be based on an optimization formulation to determine an optimal split range control structure.

Integer linear programming (ILP) formulation [7], which is the approach based on an optimization formulation, is used to determine split range control structure. The assumptions are:

1) Target temperatures are feasible for the given disturbance window.
2) The output constraints do not change and are always active. The optimal point is a vertex and a certain number of inputs are at the constraints.
3) One split range combination contains only two manipulated variables.
4) Only one manipulated variable is saturation.

The integer linear programming (ILP) formulation for the design of an optimal split range control structure can be formulated as follows:

Definition 1: Set of controlled and manipulated variables

CV:	set of controlled variables
MV:	set of manipulated variables
RS:	set of active constraint regions
MVAAT:	subset of *MV* with manipulated variables which are always active constraints
MVINAT:	subset of *MV* with manipulated variables which are always inactive constraints
MVAT:	subset of *MV* with manipulated variables which change between being active and inactive constraints

Definition 2: Primary and secondary manipulated variables

Primary manipulated variable is a manipulated variable that is used for controlling a target, except when it is saturated. Secondary manipulated variable is used to control the target when primary manipulated variable is saturated.

Definition 3: Relationship between primary and secondary manipulated variables

Let $x_{i,j}$ be a binary variable that relationship between MV_i and MV_j

For $i=j$, $x_{i,i}$ = 1 implies MV_i is a primary manipulated variable

$x_{i,i}$ = 0 implies MV_i is a secondary manipulated variable or unused

For $i\neq j$, $x_{i,j}$ = 1 implies MV_j is a secondary manipulated variable for MV_i

$x_{i,j}$ = 0 implies MV_j is not a secondary manipulated variable for MV_i

Definition 4: Relative order between controlled variables and manipulated variables

Let $r_{k,j}$ be relative order between CV_k and MV_j.

Definition 5: Relationship between controlled variables and manipulated variables

Let $z_{k,j}$ be a binary variable that relationship between CV_k and MV_j

$z_{k,j}$ = 1 implies CV_k is paired with MV_j

$z_{k,j}$ = 0 implies CV_k is not paired with MV_j

Objective function I: Minimizing the number of complexity of control structure

$$\min J_I = \sum_{i \in MV} \sum_{j \in MV, j \neq i} x_{i,j} \tag{2.23}$$

Constraint 1: Assign one primary manipulated variable to each control objective

The number of primary manipulated variables is equal to the number of controlled variables (N_{CV})

$$\sum_{i \in MV} x_{i,i} = N_{CV} \tag{2.24}$$

Constraint 2: MV_i is always an active constraint

Manipulated variable MV_i is active constraint

$$x_{i,i} = 0 \qquad i \in MVAAT \tag{2.25}$$

Manipulated variable MV_i has no need for a secondary manipulated variable

$$\sum_{j \in MV, j \neq i} x_{i,j} = 0 \qquad i \in MVAAT \tag{2.26}$$

Manipulated variable MV_i is not used as a secondary manipulated variable

$$\sum_{j \in MV, j \neq i} x_{j,i} = 0 \qquad i \in MVAAT \tag{2.27}$$

Constraint 3: MV_i is never an active constraint is used as a primary manipulated variable with no need for a secondary manipulated variable

Manipulated variable MV_i is a primary manipulated variable

$$x_{i,i} = 1 \qquad i \in MVINAT \tag{2.28}$$

Manipulated variable MV_i has no need for a secondary manipulated variable

$$\sum_{j \in MV, j \neq i} x_{i,j} = 0 \qquad i \in MVINAT \tag{2.29}$$

Manipulated variable MV_i is not used as a secondary manipulated variable

$$\sum_{j \in MV, j \neq i} x_{j,i} = 0 \qquad i \in MVINAT \tag{2.30}$$

Constraint 4: MV_i changes between active and inactive constraint may be a primary or secondary manipulated variable.

Manipulated variable MV_i is chosen as a primary manipulated variable that can be active constraint, and then a secondary manipulated variable is needed

$$x_{i,i} = 1 \text{ Then } \sum_{j \in MVAT, j \neq i} x_{i,j} = 1 \qquad i \in MVAT \tag{2.31}$$

Manipulated variable MV_i is not chosen as a primary manipulated variable, and then it has no need for a secondary manipulated variable

$$x_{i,i} = 0 \text{ Then } \sum_{j \in MVAT, j \neq i} x_{i,j} = 0 \qquad i \in MVAT \tag{2.32}$$

The above two statements can be written

$$-x_{i,i} + \sum_{j \in MVAT, j \neq i} x_{i,j} = 0 \qquad i \in MVAT \tag{2.33}$$

Manipulated variable MV_j is chosen as a primary manipulated variable, and then it is not used as a secondary manipulated variable for other manipulated variables

$$x_{j,j} = 1 \text{ Then } \sum_{i \in MVAT, i \neq j} x_{i,j} = 0 \qquad j \in MVAT \tag{2.34}$$

Manipulated variable MV_j is chosen as a secondary manipulated variable, and then it is used for at least one primary manipulated variable

$$x_{j,j} = 0 \text{ Then } \sum_{i \in MVAT, i \neq j} x_{i,j} \geq 1 \quad j \in MVAT \tag{2.35}$$

The above two statements can be written

$$x_{j,j} + \sum_{i \in MVAT, i \neq j} x_{i,j} \geq 1 \qquad j \in MVAT \tag{2.36}$$

$$M(x_{j,j} - 1) + \sum_{i \in MVAT, i \neq j} x_{i,j} \leq 0 \quad j \in MVAT \tag{2.37}$$

Where M is a positive integer which is greater than the number of members in $MVAT$

Constraint 5: Possible and impossible split range combination of manipulated variables

a) Impossible split range combination of manipulated variables, two manipulated variables which are active constraints or saturated at the same time cannot be combined as a split range pair.

For an active constraint region R, we have

$$\sum_{i \in MVAT^{A,R}} \sum_{j \in MVAT^{A,R}, j \neq i} x_{i,j} = 0 \qquad R \in RS \tag{2.38}$$

Where $MVAT^{A,R}$ is the subset of $MVAT$ with manipulated variables being active constraints in region R.

b) Possible split range combination of manipulated variables, two manipulated variables which are not active constraint at the same time may be combined as a split range pair.

For an active constraint region R, we have

$$x_{j,j} + \sum_{i \in MVAT^{I,R}} x_{i,j} \geq 1 \qquad j \in MVAT^{A,R},\ R \in RS \tag{2.39}$$

$$x_{i,i} + \sum_{j \in MVAT^{A,R}} x_{j,i} \geq 1 \qquad i \in MVAT^{I,R},\ R \in RS \tag{2.40}$$

Where $MVAT^{I,R}$ is the subset of $MVAT$ with manipulated variables being inactive constraints in region R.

Combining objective function I and constraints 1-5, Problem P1 can be written,

Problem P1

$$\min J_I = \sum_{i \in MV} \sum_{j \in MV, j \neq i} x_{i,j} \tag{2.41}$$

Subject to

Equations 2.23 to 2.40

By solving Problem P1, one obtains split range pairs that can provide optimal switching between active constraint regions. However, the solution of Problem P1 may be non unique. Therefore, relative orders are introduced as an additional criterion for screening the set of poorly controllable structure solutions. The additional objective function and constraints are as follows:

Objective function II: Minimizing the sum of relative orders of the control pairs

$$\min J_{II} = \sum_{k \in CV} \sum_{j \in MV} r_{k,j} z_{k,j} \tag{2.42}$$

Constraint 6: One manipulated variable to each control objective

$$\sum_{j \in MV} z_{k,j} = 1 \qquad k \in CV \tag{2.43}$$

Constraint 7: Only primary manipulated variables are paired with controlled variables.

Manipulated variable MV_j is a primary manipulated variable, it must be paired with a controlled variable

$$x_{j,j} = 1 \text{ Then } \sum_{k \in CV} z_{k,j} = 1 \qquad j \in MV \tag{2.44}$$

Manipulated variable MV_j is not a primary manipulated variable, it must not be paired

$$x_{j,j} = 0 \text{ then } \sum_{k \in CV} z_{k,j} = 0 \qquad j \in MV \tag{2.45}$$

Therefore,

$$-x_{j,j} + \sum_{k \in CV} z_{k,j} = 0 \qquad j \in MV \tag{2.46}$$

The ILP problem now concerns two objective functions that can be solved using lexicographic optimization. In lexicographic optimization, the objectives are arranged in decreasing order of preference; and objectives with a higher preference are considered to be infinitely more important than those with lower orders. Among the solutions that are optimal with respect to the first objective, solutions that are optimal with respect to the second objective are chosen.

Using the idea of lexicographic optimization, we first solve Problem P1:

$$J_I^* = \min_x J_I(x) \qquad x \in S \tag{2.47}$$

Where S is the feasible set and then solves an associated Problem P1′:

$$\min_x J_{II}(x) \qquad x \in S\,,\ J_I = J_I^*(x) \tag{2.48}$$

This ensures that among the minimized JI solutions, the minimized J_{II} solutions are chosen. In principle, we need to solve 2 optimization problems in sequence. However, it is possible to solve P1 and P1′ as a single optimization problem by minimizing a weighted objective function $wJ_I + J_{II}$, where w is a sufficiently large positive number chosen appropriately. Hence, we solve the following Problem P2:

Problem P2

$$J = \min(wJ_I + J_{II}) \tag{2.49}$$

$$J_I = \sum_{i \in MV} \sum_{j \in MV, j \neq i} x_{i,j}\ ,\ J_{II} = \sum_{k \in CV} \sum_{j \in MV} r_{k,j} z_{k,j} \tag{2.50}$$

Subject to

Equations 2.23 to 2.46

It can be seen that constraints 6 and 7 do not alter the feasible set for the ILP Problem P1. The ILP Problem P2 consists of two objective functions with a weighting factor (w)

between the two. The first objective is used to minimize complexity when changing between active constraints whereas the second objective is used to select the most controllable control structure. A large value of w will imply that the second objective will only be considered when there are multiple solutions.

It is possible for the ILP to have no feasible solution, that is, no optimal split range control structure can be found. This may happen when there are conflicts among the equations in constraint 5. In this case, an online optimization may be suggested for implementing optimal operation.

2.1.3 Passivity method [6]

The basic of passivity method are introduced in this section and developed to a major role in stability theory, the passive system is guarantee asymptotically stable. The concept of passivity is fundamentally connected with dissipative and is a special case of dissipative. First of all, the state apace model is explained in passivity method.

1) State space [10]

Dynamic models derived from physical principles typically consist of one or more ordinary differential equations (ODEs). If the dynamic model is nonlinear, then it must be linearized first, as will be demonstrated in Taylor Series method [10]. The linear ODEs models referred to as state space models. State space models have an important advantage, they provide a compact and useful representation of dynamic model that are in the form of a set of linear ODEs

The linear state space model:

$$\dot{X} = Ax + Bu + Ex_0 \tag{2.51}$$

$$y = Cx + Du \tag{2.52}$$

Where	$x \in X \subset R^n$	=	State vector
	$u \in U \subset R^m$	=	Manipulated vector
	$x_0 \in X_0 \subset R^k$	=	Disturbance vector
	$y \in Y \subset R^j$	=	Controlled vector
	$A, B, C, DandE$	=	Constant matrix

The representation $x(t) = \Phi(t, t_0, x_0, u)$ is used to denote the state at time t reached from the initial state x_0 at t_0.

The state space models can be converted to transfer function models. The equations 2.51-2.52 are represented in the form of standard state space model which is normally used in the modern control system.

Where the *process transfer function matrix*, $G_p(s)$ is defined as

$$G_p(s) = C(sI - A)^{-1}B + D \tag{2.53}$$

Where the *disturbance transfer function matrix*, $G_d(s)$ is defined as

$$G_d(s) = C(sI - A)^{-1}E \tag{2.54}$$

According to passivity method [11], the *process transfer function matrix* must be n×n matrix in the form of equation 2.53 is said to be passive system if the *process transfer function matrix* is positive real (PR). There are three conditions:

(1) $G(s)$ is analytical in $\mathrm{Re} > 0$
(2) $G(j\omega) + G^*(j\omega) \geq 0$ for all frequency ω that ω is not a pole of $G(s)$
(3) If there are poles $p_{1,} p_{2,\ldots,} p_m$ of $G(s)$ on the imaginary axis, they are non-repeated and the residue matrices at the poles $\lim_{s \to p_i}(s \to p_i)G(s)$ are Hermitian and positive semi definite.

In addition, the *process transfer function matrix* is said to be strictly passive or strictly positive real (SPR) when the above two conditions are changed to:

(1) $G(s)$ is analytical in $\mathrm{Re} \geq 0$
(2) $G(j\omega) + G^*(j\omega) > 0$ for $\omega \in (-\infty, +\infty)$

To analyze the stability of the linear system, based on the passivity method, the feedback system comprises of a passive system and strictly passive system is asymptotically stable. This method is used for the multi loop control system as show in Figure 2.4.

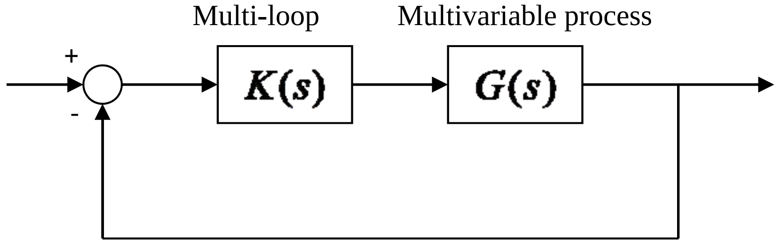

Figure 2.4 The multi loop control systems [5].

The system can be verified by using passivity index (ν_F) from the passivity method in order to analyze whatever the system is passive or not.

2) Passivity Index (ν_F) [5]

Passivity index $\nu_F[G,\omega]$ indicates how far the system $G(s)$ is from being passive the system is passive or stable when the passivity index $\nu_F[G,\omega]$ is negative. The passivity index can be defined as follows:

$$\nu_F(G,\omega) = -\lambda_{\min}\left(\frac{1}{2}\left[G(j\omega)+G^*\left(j\omega\right)\right]\right) \tag{2.55}$$

Where $G^*(j\omega)$ is the complex conjugate transpose of process transfer function $G(j\omega)$.

However, if the system is non-passive with positive of passivity index, the weighting function $w(s)$ or minimum phase transfer function is needed to add into the system in order to change non-passive system to strictly passive system. The equation 2.57 shows the strictly passive system $H(s)$ after the system is added by weighting function.

$$\nu(w(s),\omega) < -\nu(G(s),\omega) \tag{2.56}$$

$$H(s) = G(s) + w_p(s)I \tag{2.57}$$

Where I is the identity matrix of size n which is the $n\times n$ square matrix with ones on the main diagonal and other is zeros.

Then $w_p(s)$ can be chosen to have the following form,

$$w_p(s) = \frac{ks(s+a)}{(s+b)(s+c)} \tag{2.58}$$

Where a, b, c and k are positive real parameters to be determined by solving the following optimization obtained from the passivity concept.

Objective function

$$\min_{a,b,c,k} \sum_{i=1}^{m} (\mathrm{Re}(w_p(j\omega_i)) - \nu_s(G^+(s),\omega_i))^2 \tag{2.59}$$

Subject to

$$\mathrm{Re}(w_p(j\omega_i)) > \nu_s(G^+(s),\omega_i) \qquad \forall i = 1,\ldots,m \tag{2.60}$$

3) Passive controller [6]

For the passive system, the controller achieves the following condition will be the passive controller with decentralized unconditional stability.

1. $\mathrm{Re}\left\{ \dfrac{k_i\,(j\omega)}{1-\nu_s(G(s),\omega)k_i\,(j\omega)} \right\} \geq 0 \qquad \forall\,\omega \in R, i = 1,\ldots,n$ (2.61)
2. $K(s)$ is analytic in $\mathrm{Re}(s) > 0$ (2.62)

The decentralized unconditional stability condition given in equations 2.61-2.62 implies:

1. $k_i\,(s)$ is passive and
2. $\left| k_i\,(j\omega) - \dfrac{1}{2\nu_s(G(s),\omega)} \right| \leq \left| \dfrac{1}{2\nu_s(G(s),\omega)} \right| \qquad \forall\,\omega \in R, i = 1,\ldots,n$ (2.63)

For multi loop control system, the multi loop controllers can be designed based on the proposed stability condition. To achieve decentralized unconditional stability of the close loop system as well as good performance, a controller tuning method is proposed to minimize the sensitivity function of each loop, subject to equations 2.61-2.62. For multi loop PI controller synthesis, this tuning problem is converted into the following optimizations:

Objective function

$$\min_{k_{c,i}^{+},\tau_{I,i}} (-\gamma_i) \tag{2.64}$$

Such that

$$\left| \frac{1}{1+G_{ii}^{+}(j\omega)k_{c,i}^{+}[1+1/\tau_{I,i} \times j\omega)]} \frac{\gamma_i}{j\omega} \right| < 1 \tag{2.65}$$

$$\tau_{I,i}^{2} \geq \frac{k_{c,i}^{+}\nu_s(\omega)}{[1-k_{c,i}^{+}\nu_s(\omega)]\omega^2} \qquad \forall\,\omega \in R, i = 1,\ldots,n \tag{2.66}$$

For the multi loop control system as shown in Figure 2.4 comprising a stable subsystem $G(s)$ and a multi loop controller $K(s) = diag\{k_i(s)\}$, i = 1,..., n, if a stable and minimum phase transfer function $w(s)$ is chosen such that equation 2.58, then the closed loop system will be stable. For any loop i = 1,..., n, the passive controllers are design as follow:

$$k_i^{'}(s) = k_i\,(s)[1-w(s)k_i\,(s)]^{-1} \tag{2.67}$$

2.2 Literature reviews

For the literature reviews section, the first part is reviewed the works related the heat exchanger networks, the split range control of heat exchanger networks, and the passivity method.

2.2.1 Heat exchanger networks

Wolff [12] presented various dynamic models of single heat exchangers and the heat exchanger networks which may also include stream splitting, mixing and bypasses. They studied where bypasses should be placed in the network by dynamics considerations.

Glemmestad [1] discussed a procedure for optimal operation of heat exchanger networks. This procedure is based on structural information of the heat exchanger networks, and is used to find which bypass manipulation that should be adjusted in order to compensate for deviation in the output temperature so that utility consumption is minimized.

2.2.2 Split range control of heat exchanger networks

Lersbamrungsuk [2] studied a simple split range control structure to implement the optimal operation of heat exchanger networks when only single bypasses and utility duties of cooler and heater are used as manipulated variablrs. The optimal operation of heat exchanger networks can be formulated as a linear programming implying the operation always lies at some input constraints.

2.2.3 Passivity method

Guillemin [13] and *Weinberg* [14]. The concept of passive systems originally arose in the context of electrical circuit theory. In such electrical systems, no energy is generated, e.g. a network consisting of only inductors, resistors and capacitors.

Bao [5], *Bao and Lee* [6], *Bao et al.* [11] provided a new approach to stability analysis for multi loop control systems based on passivity theorem. The destabilizing effect of interactions of a multi input multi output system was studied using the passivity index. The decentralized unconditional stability condition which implies closed loop stability of decentralized control systems under control loop failure was derived. An easy to use stability test was performed on open loop stable process transfer matrices, independent of controller design.

CHAPTER 3 METHODOLOGY

The aims of this work are to apply the stability analysis based on passivity method to split range control of heat exchanger networks and to design the passive controller for this process. There are 8 steps in the analysis as shown in Figure 3.1. The details of each step are described below.

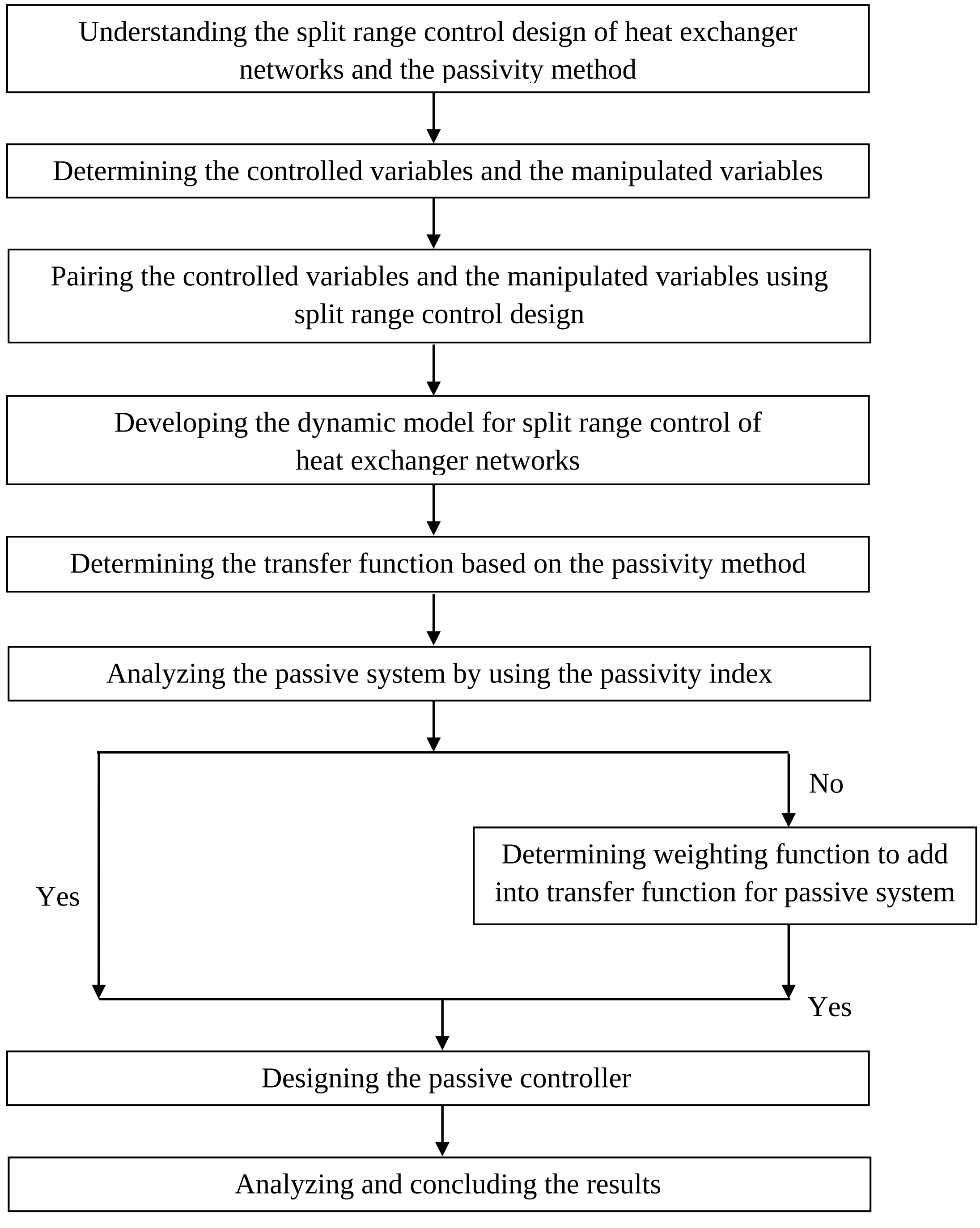

Figure 3.1 The methodology.

3.1 Concept of the split range control design of heat exchanger networks and the passivity method.

First of all, the overview of split range control design of heat exchanger networks was studied from the literature reviews. There were two sections in split range control design of heat exchanger networks which are heat exchanger networks concept and split range control of heat exchanger networks concept. Then, the dynamic behavior of heat exchanger networks was studied from the thesis of *Glemmestad* [1], which is optimal operation of integrated processes, and simulation using MATLAB^{TM}. Moreover, split range control design of heat exchanger networks theories was studies from the thesis of *Lersbamrungsuk* [2]. The split range control was applied in heat exchanger networks, describes possible methods to implement the optimal policy. After that, the overview of passivity method was studied, which is developed by *Bao and Lee* [6]. Passivity method has played a major role in stability theory that passive system is stable.

3.2 Determination the controlled variables and the manipulated variables.

In this step, the controlled variables and manipulated variables for split range control design of heat exchanger networks were determined. In this work, the controlled variables are studied by target temperatures. The manipulated variables are split fraction of bypassed heat exchanger and utility duties of cooler and heater.

3.3 System pairing the controlled variables and the manipulated variables by split range control design.

After the controlled variables and manipulated variables were obtained, the split range control of heat exchanger networks was proposed. The optimal split range control structure is used for suitable pairing the controlled variables and the manipulated variables via GAMs.

3.4 Development the dynamic model for split range control of heat exchanger networks.

After the pairing the controlled variables and the manipulated variables were obtained, the dynamic models of heat exchanger networks were proposed. The dynamic model of heat exchanger networks was presented in the form of energy balance equation. However, there were some assumptions used in this work in order to derive a simplified nonlinear dynamic model of the heat exchanger networks as follows:

Assumption:

1. Constant densities of the two fluids (ρ_i).
2. Constant specific heat capacities of the two fluids ($C_{V,i}$).
3. Constant thermal efficiency ($P_{h,i}$).
4. Constant flow independent heat transfer coefficient.

5. Lumped models.
6. The water is used as the only one component and no phase changes.

The dynamic models of heat exchanger can be illustrated as follows:

Hot side: $$\frac{\partial T_{h,i}}{\partial t} = \frac{F_h}{V_{h,i}}\left(T_{h,i-1} - T_{h,i}\right) - \frac{UA_i}{\rho_h V_{h,i} C_{V,h}}(T_{\mathrm{h},i} - T_{\mathrm{c},i}) \tag{3.1}$$

Cold side: $$\frac{\partial T_{c,i}}{\partial t} = \frac{F_c}{V_{c,i}}\left(T_{c,i+1} - T_{c,i}\right) + \frac{UA_i}{\rho_c V_{c,i} C_{V,c}}(T_{\mathrm{h},i} - T_{\mathrm{c},i}) \tag{3.2}$$

The dynamic models of utility exchanger can be illustrated as follows:

Cooler: $$\frac{\partial T_{h,i}}{\partial t} = \frac{F_h}{V_{h,i}}\left(T_{h,i-1} - T_{h,i}\right) + \frac{Q_c}{\rho_h V_{h,i} C_{V,h}} \tag{3.3}$$

Heater: $$\frac{\partial T_{c,i}}{\partial t} = \frac{F_c}{V_{c,i}}\left(T_{c,i-1} - T_{c,i}\right) - \frac{Q_h}{\rho_c V_{c,i} C_{V,c}} \tag{3.4}$$

Where A_i is the heat exchanger area at stage *i.* $V_{h,\ i}$ and $V_{c,\ i}$ are the volumes of each compartment at hot side stream and cold side stream, respectively. The numbering of each stage from 1 to *i*, the heat exchanger model consists of 2*i* ordinary differential equations.

3.5 Determination the transfer function based on the passivity method.

Since the dynamic models of heat exchanger networks were one or more ordinary differential equations (ODEs). If the dynamic model is nonlinear system, linearization of all equations should be done before finding the transfer function. First of all, the linear dynamic models were written in the form of the state space model using equations 2.51-52. After that, the process and disturbance transfer functions of heat exchanger networks in matrix form were found out using MATLAB.

3.6 Consideration of the passive system by passivity index.

The passivity index was used to indicate that the system was passive or not by using equation 2.57. If the system is non-passive, it was necessary to add a stable and weighting function $w_p(s)$ into process transfer function of the system for passive system. Therefore, this process would be guaranteed stable from the passive method.

3.7 Designing the passive controller.

After the heat exchanger networks were passive system, the multi loop PI controller was designed for each loop control. The parameters of PI controller that make the heat exchanger networks be stable are obtained by minimizing the sensitivity function. Then, the weighting function was absorbed into the controller to make system was stable. Finally, the process was analyzed and verified via MATLAB-SIMULINK**.**

CHAPTER 4 RESULTS AND DISCUSSION

In this chapter, two classical case studies of heat exchanger networks from *Glemmestad* [1] and *Biegler* [15] are further implemented with the proposed method.

4.1 Case study 1

4.1.1 The split range control of heat exchanger networks

The heat exchanger networks from *Glemmestad* [1] as shown in Figure 4.1 are studied. This network has two heat exchangers and two utility exchangers. In addition, four manipulated variables are the split fraction of bypassed heat exchangers 1 and 2 (u_{b1} and u_{b2}) and utility duties of cooler and heater (Q_c and Q_h). These are available for control of target temperatures including outlet temperature of stream H1, outlet temperatures of stream C1 and C2. The disturbance is varied ±10 $^{\circ}$C in the inlet temperature of stream H1.

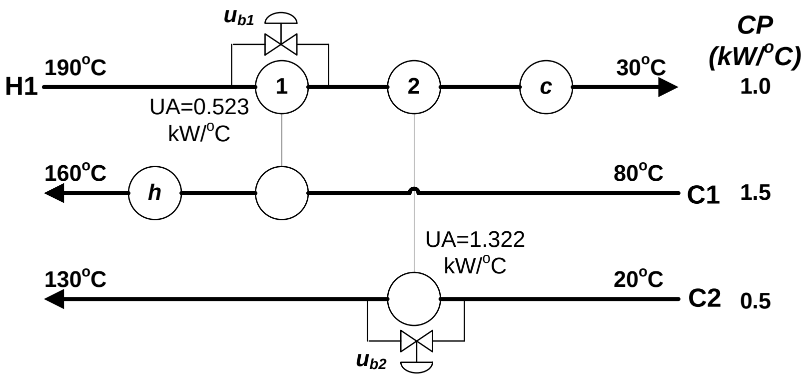

Figure 4.1 The heat exchanger networks in case study 1.

The optimal operation is provided to analyze the split range control of heat exchanger networks. The degrees of freedom must be firstly checked. These are two manipulated variables in the inner and outer heat exchanger networks while there are three for the dimensional spaces spanned by the controlled variable. Therefore, the degree of freedom is one. This can imply that the operation is structurally feasible.

The heat exchanger networks are generated into two active constraint regions after a step change the inlet hot temperature of stream H1 in this case study as shown in Table 4.1.

Table 4.1 The set of active constraints in case study 1.

Region	Manipulated variables			
	Q_c	Q_h	u_{b1}	u_{b2}
1	U	U	S_L	U
2	U	U	U	S_L

U – Unsaturated manipulated variable (inactive constraint)

S_L – Saturated manipulated variable (active constraint) at the lower bound

According to this table, the manipulated variables; u_{b1} and u_{b2}, can become the active constraints and implies that these two manipulated variables should be combined as a split range control. On the other hand, the manipulated variables Q_c and Q_h cannot be the active constraints; therefore, these two manipulated variables should not be considered into split range control.

4.1.2 Pairing system for split range control.

In case study 1, the integer linear programming will be used to propose an optimal split range control structure using GAMs; an optimization software. The solutions from split range control structure identify in terms of the sum of relative orders. The result of optimal split range control structure is shown in Figure 4.2. There are three control loops as follows:

- First loop: Outlet hot temperature of stream H1 is controlled by utility duties of cooler.
- Second loop: Outlet cold temperature of stream C1 is controlled by utility duties of heater.
- Third loop: Outlet cold temperature of stream C2 is controlled by switching between split fraction of bypassed heat exchanger 1 and bypassed heat exchanger 2.

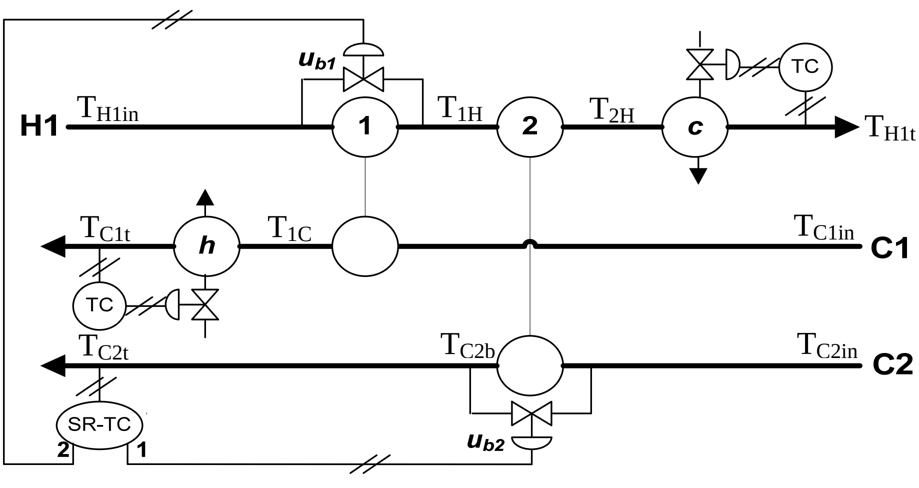

Figure 4.2 The split range control structure of heat exchanger networks in case study 1.

4.1.3 The dynamic model for split range control of heat exchanger networks.

The heat exchanger networks can be classified into 2 networks; the inner and outer heat exchanger networks. Outer heat exchanger networks include cooler and heater.

The dynamic models of heat exchanger are considered to transfer the heat between hot side stream and cold side stream as follows equations 3.1-2. The dynamic model of cooler and heater can be illustrated as follows equations 3.3-4; respectively. The dynamic models are developed based on the controlled, manipulated and disturbance variables of the unit. Since the dynamic models of heat exchanger are the non-linear model, a linearization by using Taylor's series expansion is necessary before generating the state space of heat exchanger. The linear dynamic models are derived in Appendix A.1.

4.1.4 The transfer function based on the passivity method.

The transient response of heat exchanger networks are studied by using the passivity method, the linear dynamic model is first represented in the state space form. State space models are proved by rearranging linear dynamic modes and substituting the numerical values at steady state into matrix A, B, C, D and E of the state space form which can be illustrated in Appendix B.1 in order to find out the transfer function of the system as follows:

- The state space model of inner heat exchanger networks

$$\begin{bmatrix} \dot{T}_{1H} \\ \dot{T}_{2H} \\ \dot{T}_{1C} \\ \dot{T}_{C2b} \end{bmatrix} = \begin{bmatrix} -0.003626 & 0.001245 & 0 & 0 \\ 0.001245 & -0.004817 & 0 & 0 \\ 0.002381 & 0 & -0.005529 & 0.003148 \\ 0 & 0 & 0.003148 & -0.004338 \end{bmatrix} \begin{bmatrix} T_{1H} \\ T_{2H} \\ T_{1C} \\ T_{C2b} \end{bmatrix}$$
$$+ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.1320 \end{bmatrix} \left[f_{C2} \right] + \begin{bmatrix} 0.0024 \\ 0 \\ 0 \\ 0 \end{bmatrix} \left[T_{H1in} \right] \tag{4.1}$$

$$\left[T_{C2t} \right] = \begin{bmatrix} 0 & 0 & 0 & 0.9999 \end{bmatrix} \begin{bmatrix} T_{1H} \\ T_{2H} \\ T_{1C} \\ T_{C2b} \end{bmatrix} + \left[-110.9 \right] \left[f_{C2} \right] \tag{4.2}$$

- The state space model of cooler

$$\left[\dot{T}_{H1t}\right] = \left[\text{-0.002381}\right]\left[T_{H1t}\right] + \left[\text{-0.002381}\right]\left[Q_c\right] + \left[0.0024\right]\left[T_{2H}\right] \tag{4.3}$$

$$\left[T_{H1t}\right] = \left[1\right]\left[T_{H1t}\right] \tag{4.4}$$

- The state space model of heater

$$\left[\dot{T}_{C1t}\right] = \left[\text{-0.003571}\right]\left[T_{C1t}\right] + \left[0.002381\right]\left[Q_h\right] + \left[0.0036\right]\left[T_{1C}\right] \tag{4.5}$$

$$\left[T_{C1t}\right] = \left[1\right]\left[T_{C1t}\right] \tag{4.6}$$

The characteristics of heat exchanger networks are analyzed. The transfer functions including the process transfer function and the disturbance transfer function are solved by using equations referred from

$$G_p(s) = C(sI - A)^{-1}B + D$$
$$G_d(s) = C(sI - A)^{-1}E$$

The matrix transfer function can be determined via MATLAB as follows:

- The transfer function of inner heat exchanger networks

Process: $$\left[T_{C2t}\right] = \left[\frac{-110.9s^2 - 0.9621s - 0.0008311}{s^2 + 0.009867s + 1.408e^{-005}}\right]\left[f_{C2}\right] \tag{4.7}$$

Disturbance

$$\left[T_{C2t}\right] = \left[\frac{1.784\text{e}^{-008}\ \text{s} + 8.594\text{e}^{-011}}{s^4 + 0.01831s^3 + 0.0001133s^2 + 2.759e^{-007}s + 2.24e^{-010}}\right]\left[T_{H2in}\right] \tag{4.8}$$

- The transfer function of cooler

Process: $$\left[T_{H1t}\right] = \left[\frac{\text{-0.002381}}{\text{s + 0.002381}}\right]\left[Q_c\right] \tag{4.9}$$

Disturbance: $$\left[T_{H1t}\right] = \left[\frac{0.002381}{\text{s + 0.002381}}\right]\left[T_{2H}\right] \tag{4.10}$$

- The transfer function of heater

Process: $$\left[T_{C1t}\right] = \left[\frac{0.002381}{\text{s + 0.003571}}\right]\left[Q_h\right] \tag{4.11}$$

Disturbance: $$\left[T_{C1t}\right] = \left[\frac{0.003571}{\text{s + 0.003571}}\right]\left[T_{1C}\right] \tag{4.12}$$

4.1.5 The passivity index.

After the process and disturbance transfer functions are obtained from MATLAB, the passivity index is employed to analyze the passivity behavior of the process.

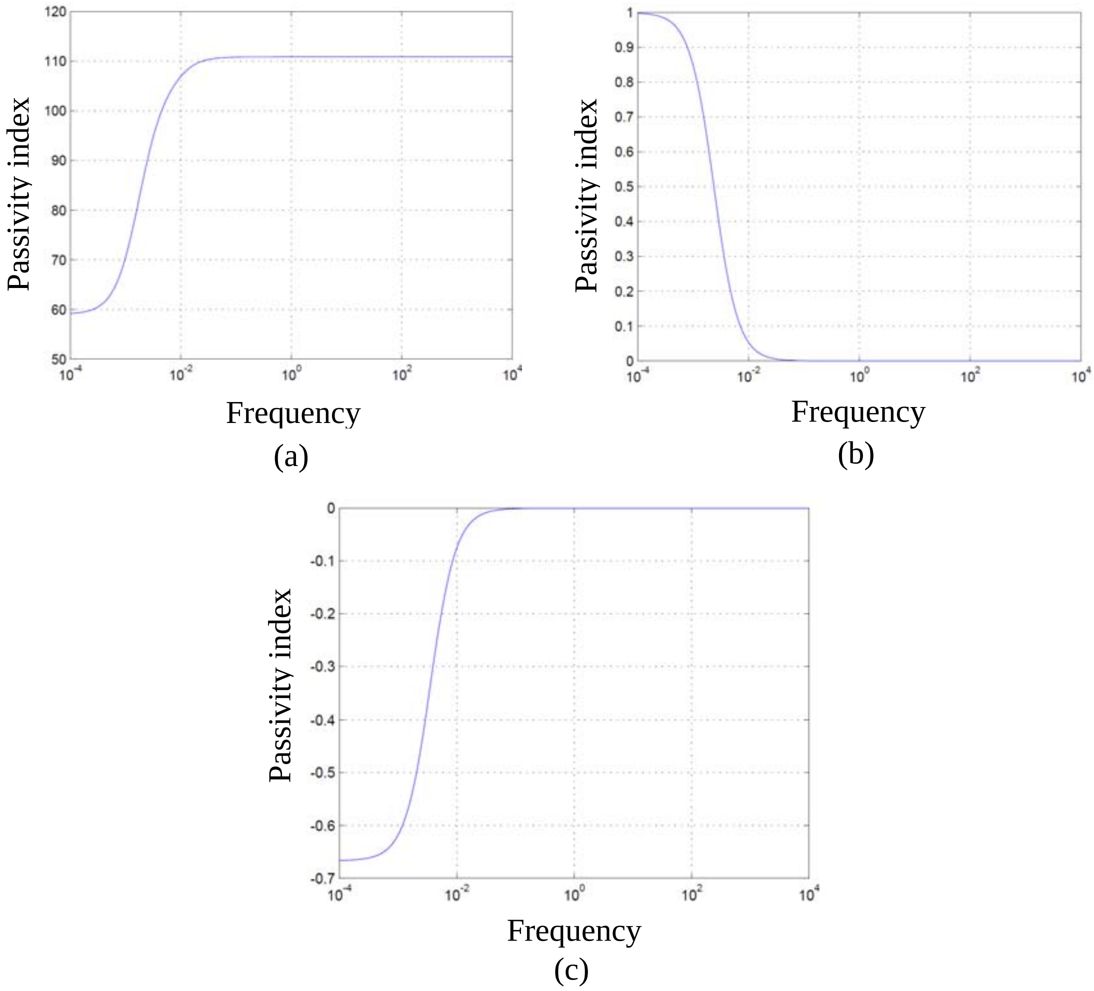

Figure 4.3 The passivity index in case study 1 **(a)** Inner heat exchanger networks, **(b)** cooler and **(c)** heater.

Figure 4.3 shows the individual passivity index. The passivity index of heater is a negative value, the system is passive system. On the other hand, the inner heat exchanger networks and cooler are non-passive due to positive values of passivity index. According to the passivity concept, these non-passive processes can be shifted to the passive regions by adding weighting function $w_p(s)$. The results of the weighting functions make the processes shift into the passive system presented following:

- The weighting function of inner heat exchanger networks

$$w_p(s) = \frac{111.4442 \cdot s \cdot (s + 0.6338)}{(s + 0.000093) \cdot (s + 0.6373)} \tag{4.13}$$

- The weighting function of cooler

$$w_p(s) = \frac{0.1309 \cdot s \cdot (s + 0.2539)}{(s + 1.2 \times 10^{-7}) \cdot (s + 0.0333)} \tag{4.14}$$

The weighing functions from equations 4.13-14 are added to the transfer function of inner heat exchanger networks and cooler; respectively, as presented by $H(s) = G(s) + w_p(s)I$. As a result, the non-passive cooler shifts to the passive system and the passivity index of passive system is illustrated in Figure 4.4.

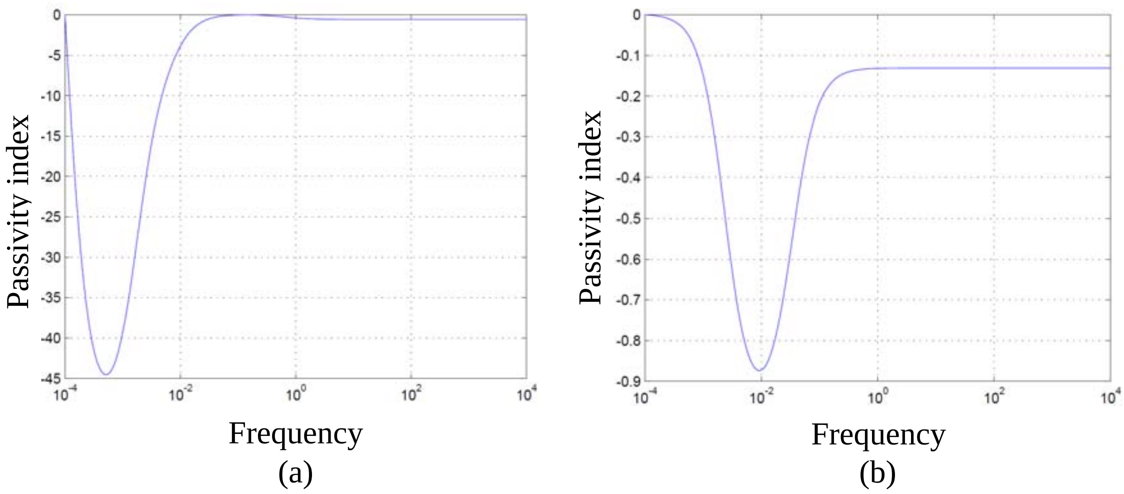

Figure 4.4 The passivity index of passive system in case study 1
(a) Inner heat exchanger networks and **(b)** cooler.

In principle, the weighting function cannot be added into the system directly because the system must be the same. Therefore, the weighting function will be absorbed into the controller to design the passive controller; instead.

4.1.6 The passive controller.

Based on the passivity concept, the feedback system comprises of a passive system and strictly passive system is asymptotically stable. In this work, the heat exchanger networks are already shifted to passive system; therefore, the passive controller for each loop control is designed to make the heat exchanger networks asymptotically stable.

Based on the split range control, the passive controller of third loop, which is control outlet cold temperature of stream C2 by manipulating between split fraction of bypassed heat exchanger 1 and bypassed heat exchanger 2, is used only one controller.

The passive controllers with PI controller are designed from the optimization. The results of the $k_{c,i}$ and $\tau_{I,i}$ for each control loop is shown in Table 4.2.

Table 4.2 The results of the $k_{c,i}$ and $\tau_{I,i}$ for each control loop in case study 1.

Loop		$k_{c,i}$	$\tau_{I,i}$ **(second)**
1.	$\boldsymbol{Q_c}$**-** T_{H1t}	0.6000	10.0000
2.	$\boldsymbol{Q_h}$**-** T_{C1t}	0.7098	0.2200
3.	$\boldsymbol{u_{b2}}$**-** T_{C2t}	0.0080	49.9826
3.	$\boldsymbol{u_{b1}}$**-** T_{C2t}	0.0080	49.9826

Since the controllers used in multi control system are PI controller, the values of $k_{c,i}$ and $\tau_{I,i}$ from Table 4.2 are plugged in the simple form of PI controller as follow:

$$k_i = k_{c,i}\left(1+\frac{1}{\tau_{I,i}s}\right)$$

The result for k_i of each loop control is shown in the following.

$$k_{Q_c-T_{h1t}} = 0.6000\left(1+\frac{1}{10.0000s}\right) \tag{4.15}$$

$$k_{Q_h-T_{h2t}} = 0.7098\left(1+\frac{1}{0.2200s}\right) \tag{4.16}$$

$$k_{u_{b1}-T_{c2t}} = 0.0080\left(1+\frac{1}{49.9826s}\right) \tag{4.17}$$

$$k_{u_{b2}-T_{c2t}} = 0.0080\left(1+\frac{1}{49.9826s}\right) \tag{4.18}$$

According to the passivity method, the closed loop system of inner heat exchanger networks and cooler will be stable when the weighting function is absorbed into the controller.

$$k_i^{'}(s) = k_i\,(s)[1-w_p(s)k_i\,(s)]^{-1}$$

As a result, these three passive controllers will be used in the system.

$$k^{'}_{Q_c-T_{h1t}}(s) = \frac{6.0000s+0.7998}{9.2144s+0.055} \tag{4.19}$$

$$k^{'}_{u_{b1}-T_{c2t}}(s) = \frac{0.3999s+0.2629}{5.4204s+2.7221} \tag{4.20}$$

$$k^{'}_{u_{b2}-T_{c2t}}(s) = \frac{0.3999s+0.2629}{5.4204s+2.7221} \tag{4.21}$$

4.1.7 Verification of heat exchanger networks model.

Heat exchanger networks model verification are separated into 3 parts: 1) opened loop control of heat exchanger networks, 2) closed loop control of heat exchanger networks, and 3) closed loop control of heat exchanger network with noise and time delay.

1) Opened loop control of heat exchanger networks.

The developed heat exchanger networks model is an integration of all possible dynamic equations by including the dynamic models of lump heat exchangers, cooler, and heater. This model can descript the transient responses of target temperature. The dynamic models of heat exchanger networks are implemented by MATLAB-SIMULINK.

Figure 4.5 shows the opened loop control of heat exchanger networks including two heat exchangers and two utility exchangers (one cooler and one heater). This network is three inputs (T_{h1in}, T_{c1in}, and T_{c2in}) and three outputs (T_{h1t}, T_{c1t}, and T_{c2t}) with one disturbance (T_{h1in}). In this part of study, the results of outlet hot temperature of stream H1 profile, outlet cold temperature of stream C1 and C2 profile are shown in Figure 4.6.

After changing of inlet hot temperature of stream H1 form 190 to 200 ºC at the time of 1,500 seconds, the outlet hot temperature of stream H1, the outlet cold temperature of stream C1, and the outlet cold temperature of stream C2 are increased suddenly to their new steady state, as seen in Figure 4.6. Moreover, the outlet hot temperature of stream H1 profile, outlet cold temperature of stream C1 profile, and outlet cold temperature of stream C2 are decreased suddenly to their new steady state after the inlet hot temperature of stream H1 is changed from 200 to 180 ºC at the time of 3,000 seconds. Therefore, there is no need to establish the control system. This result shows that the model under identical conditions can predict the system behavior accurately.

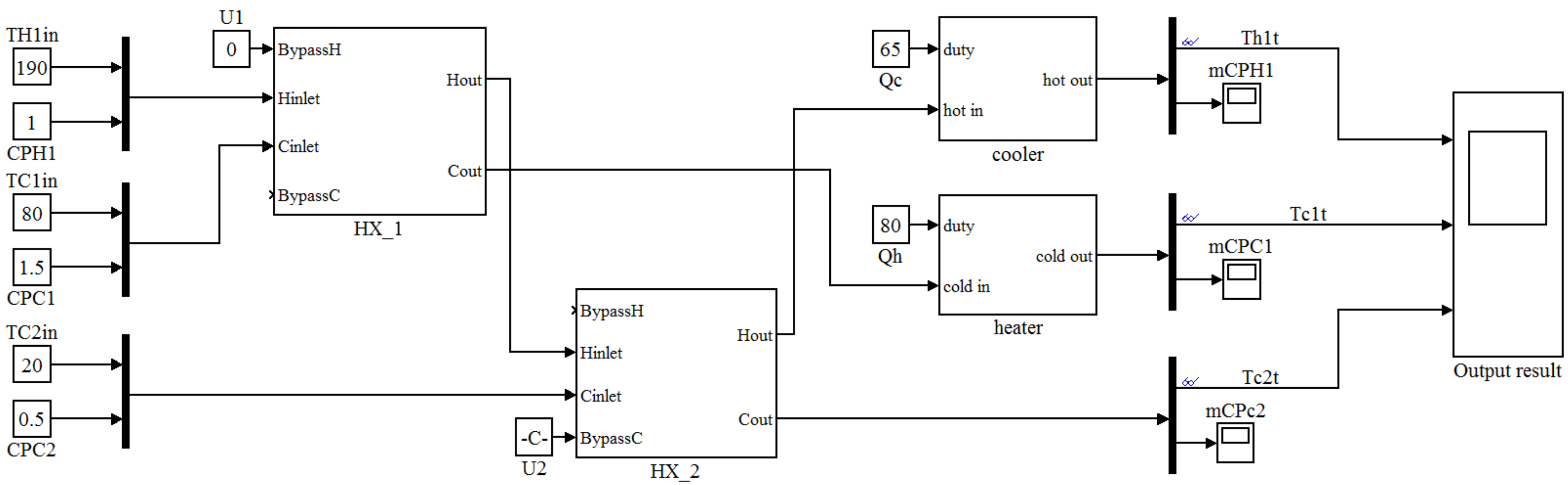

Figure 4.5 The opened loop control of heat exchanger networks in case study 1.

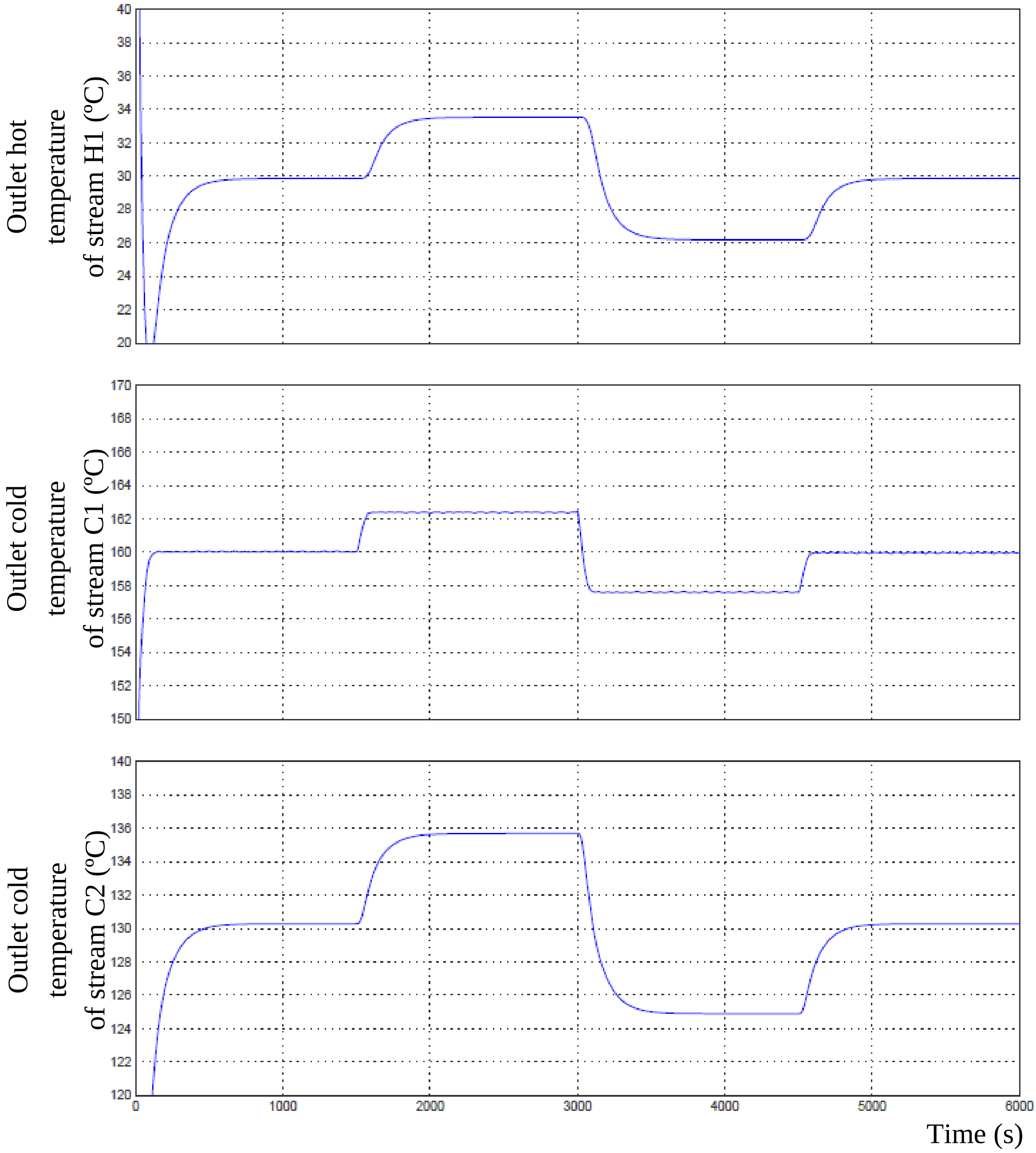

Figure 4.6 The transient responses of opened loop control in case study 1.

2) Closed loop control of heat exchanger networks.

In case study 1, there are 4 available manipulated variables for controlling of all target outlet temperatures, the bypasses of heat exchangers 1 and 2 (u_{b1} and u_{b2}) and the utility duties of cooler and heater (Q_c and Q_h)**.** Next, the passive controllers will be designed by using the passivity concept. In order to stabilize the closed loop system, the process system (*G(s)*) must be strictly passive and the multi loop controller (*K(s)*) must be passive too. For heat exchanger networks, the process renders strictly passive and the controller renders passive by absorbing the minimum phase transfer function $(w_p(s))$, to stabilize this system. The stability of heat exchanger networks is verified with the disturbance. For this study, the disturbances are both increasing and decreasing of temperatures of inlet hot temperature of stream H1 at time 1,500 and 3,000 seconds; from steady state condition. Figure 4.7 shows the closed loop control of heat exchanger networks with passivity concept.

According to Figure 4.7, there are three control loops. The outlet hot temperature of stream H1 is controlled by utility duties of cooler for the first loop. In the second loop, the outlet cold temperature of stream C1 is controlled by utility duties of heater. The out cold temperature of stream C2 is controlled by the split fraction of bypassed heat exchanger 1 and bypassed heat exchanger 2 in last loop.

The result from passive controller process will be compared with PI auto tuning via MATLAB-SIMULINK in every control loops. As follow in Figure 4.8, the disturbance is applied to the process. The transient responses are presented in Figure 4.9. Figure 4.10 shows also the result of manipulated variables compared between passive controller and PI controller.

The results of controlled variable responses are considered after changing the inlet hot temperature of stream H1 at the time of 1,500 and of 3,000 seconds, respectively. Both controllers can adjust the target temperatures. When system is a disturbance at time of 1,500 seconds, the system will be changing from steady state operation to active constraint region 1 as shown in Figure 4.10. Therefore, the split fraction of bypassed heat exchanger 1 will be operated from the saturated or active constraint in order to control the outlet cold temperature of stream C2. Likewise, after system operate at the time of 3,000 seconds, the system will be switched from active constraint region 1 to the active constraint region 2. Figure 4.10 shows the split fraction of bypassed heat exchanger 1 becomes the saturated or active constraint. On the other hand, the split fraction of bypassed 2 is operated in order it control the outlet cold temperature of stream C2. The responses of outlet hot temperature of stream H1, the outlet cold temperature of stream C1, and outlet cold temperature of stream C2 with PI controller are appear to be more oscillated than passive controller. Moreover, the rising time of system with PI controller is more than passive controller. As a result, the split range control of heat exchanger networks designed by the passivity method can be adjusted the target temperature and guarantee stable.

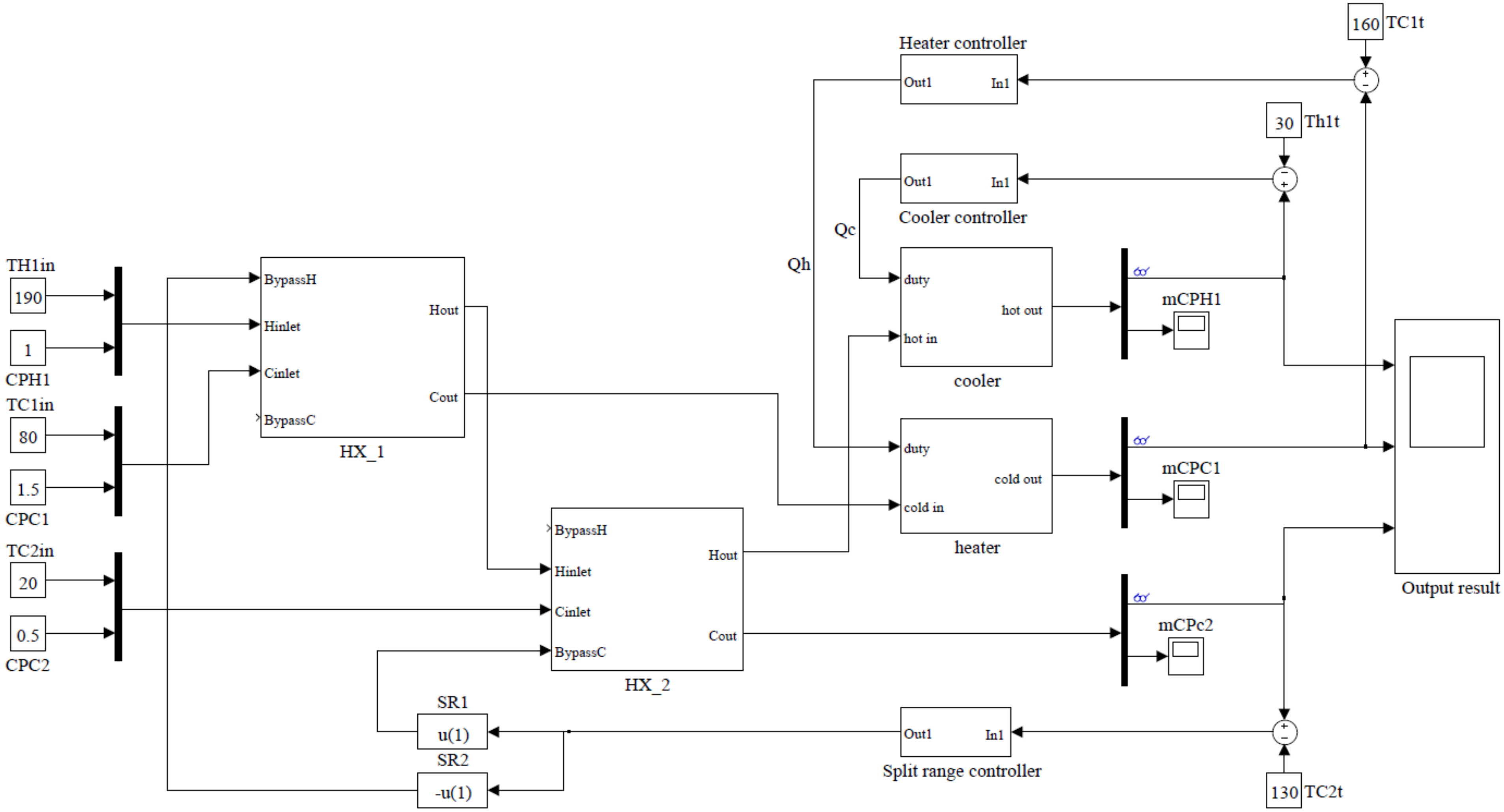

Figure 4.7 The closed loop control of heat exchanger networks with passive controller in case study 1.

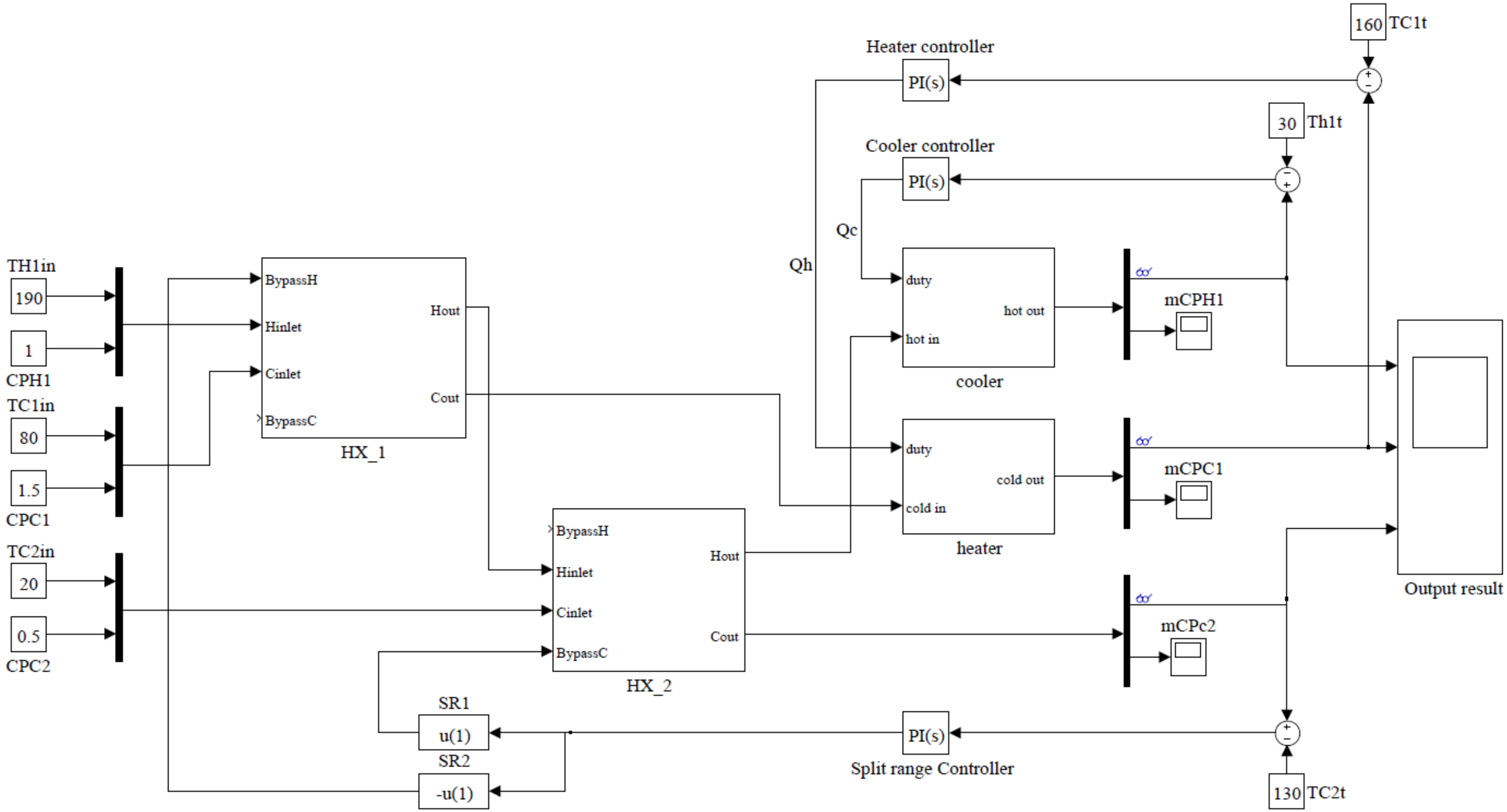

Figure 4.8 The closed loop control of heat exchanger networks with PI controller in case study 1.

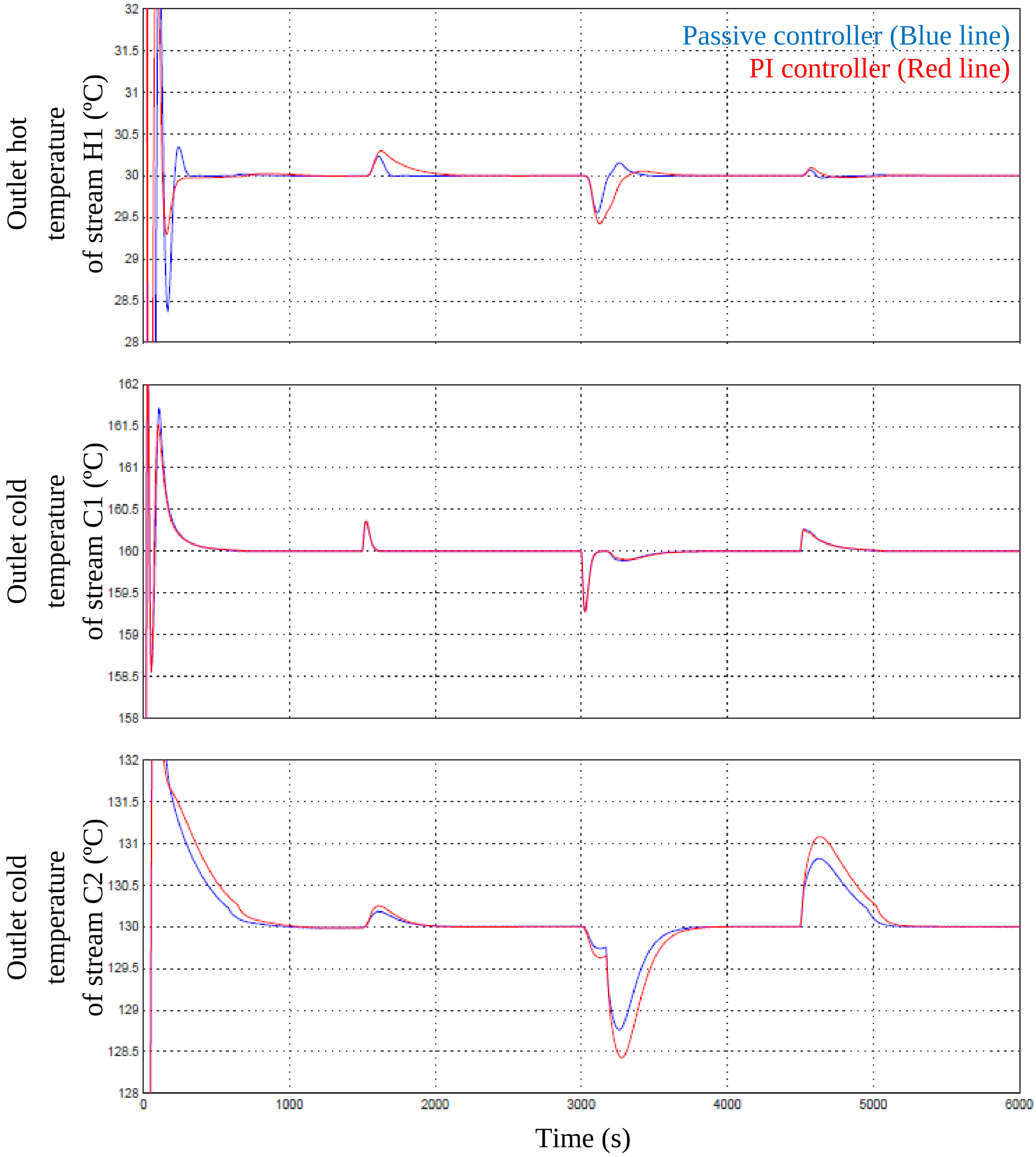

Figure 4.9 The comparison of controlled variable responses between passive controller and PI controller with load disturbance in case study 1.

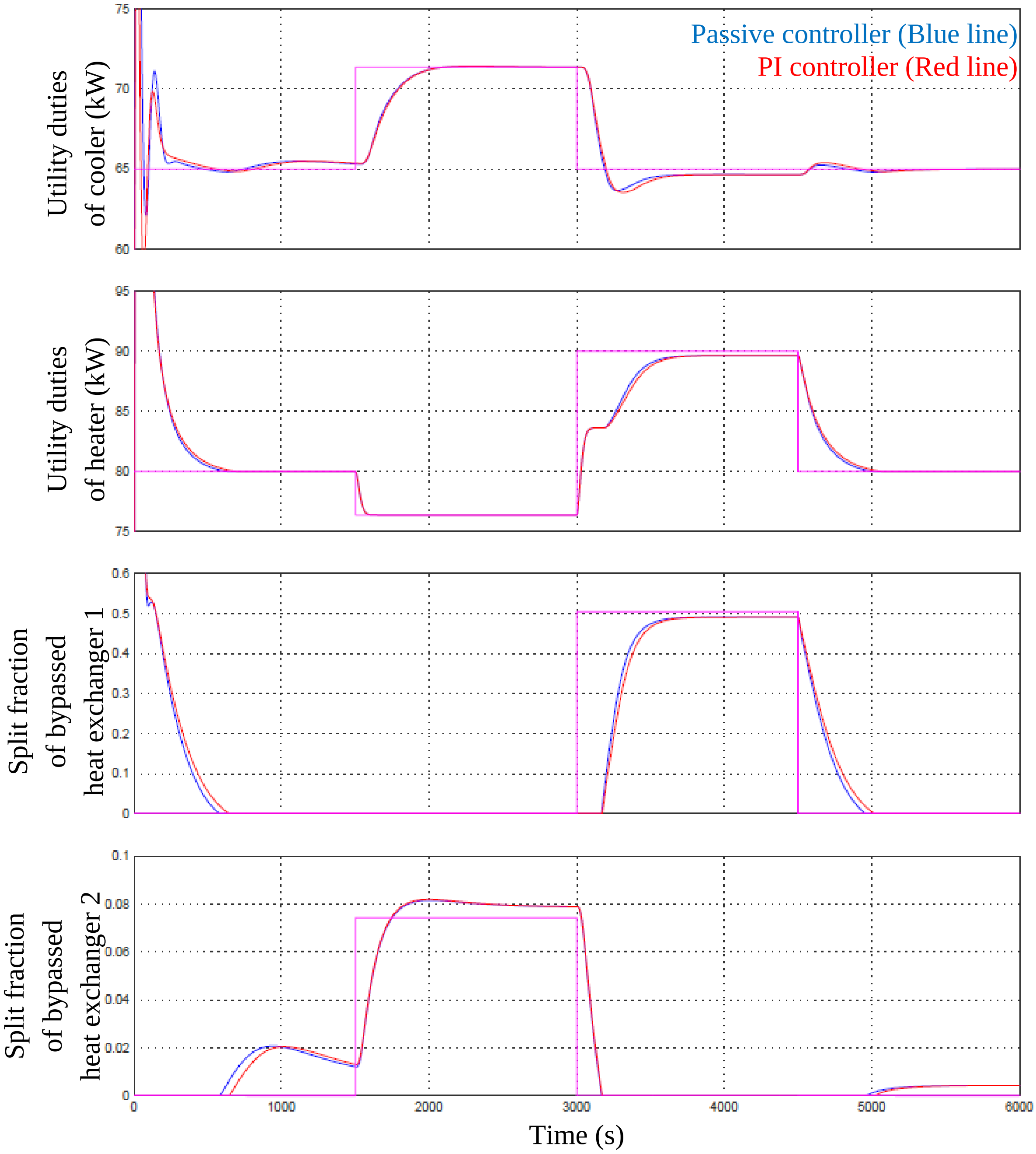

Figure 4.10 The comparison of manipulated variable responses between passive controller and PI controller with load disturbance in case study 1.

After load disturbance are studied for closed loop system of the heat exchanger networks, the passive controller shows better than PI controller. In addition, the set point tracking are considered in order to verify the performance of the designed closed loop system. For this study, the set points are step up and step down 10°C of all target temperatures (H1, C1, and C2) at the time of 1,000, 1,500, 2,500, 3,000, 4000 and 4,500 seconds.

The result from passive controller process will be compared with PI auto tuning via MATLAB-SIMULINK. Figure 4.11 shows the results of controlled variable responses after step change tracking of outlet hot temperature of stream H1, outlet cold temperature of stream C1, and outlet cold temperature of stream C2 at the time of 1,000, 1,500, 2,500, 3,000, 4000 and 4,500 seconds, respectively. Figure 4.12 shows the result of manipulated variables that are compared between passive controller and PI controller.

According to Figure 4.11, the closed loop system of heat exchanger networks from passivity method and PI auto tuning can control the all target temperature and guarantee the stability. In addition, all controlled variable responses with passive controller are similar to PI controller and provide the low overshoot and move to its set point quickly after step up and down 10°C for all set point tracking.

According to Figure 4.12, the manipulated variables do not use the split range control policy when the closed loop system of heat exchanger networks are verified with set point tracking. Likewise, the comparison of manipulated variable responses between passive controller and PI controller are not significantly difference. After set point tracking at the time of 1,000 seconds, the utility duties of cooler and heater are changed in order to maintain the outlet hot temperature of stream H1 and the outlet cold temperature of stream C1 to a new set point; respectively. On the other hand, the split fraction of bypassed heat exchanger 2, the utility duties of cooler and heater are changing in order to control only one outlet cold temperature of stream C2 to a new set point after set point tracking at the time of 4,000 seconds.

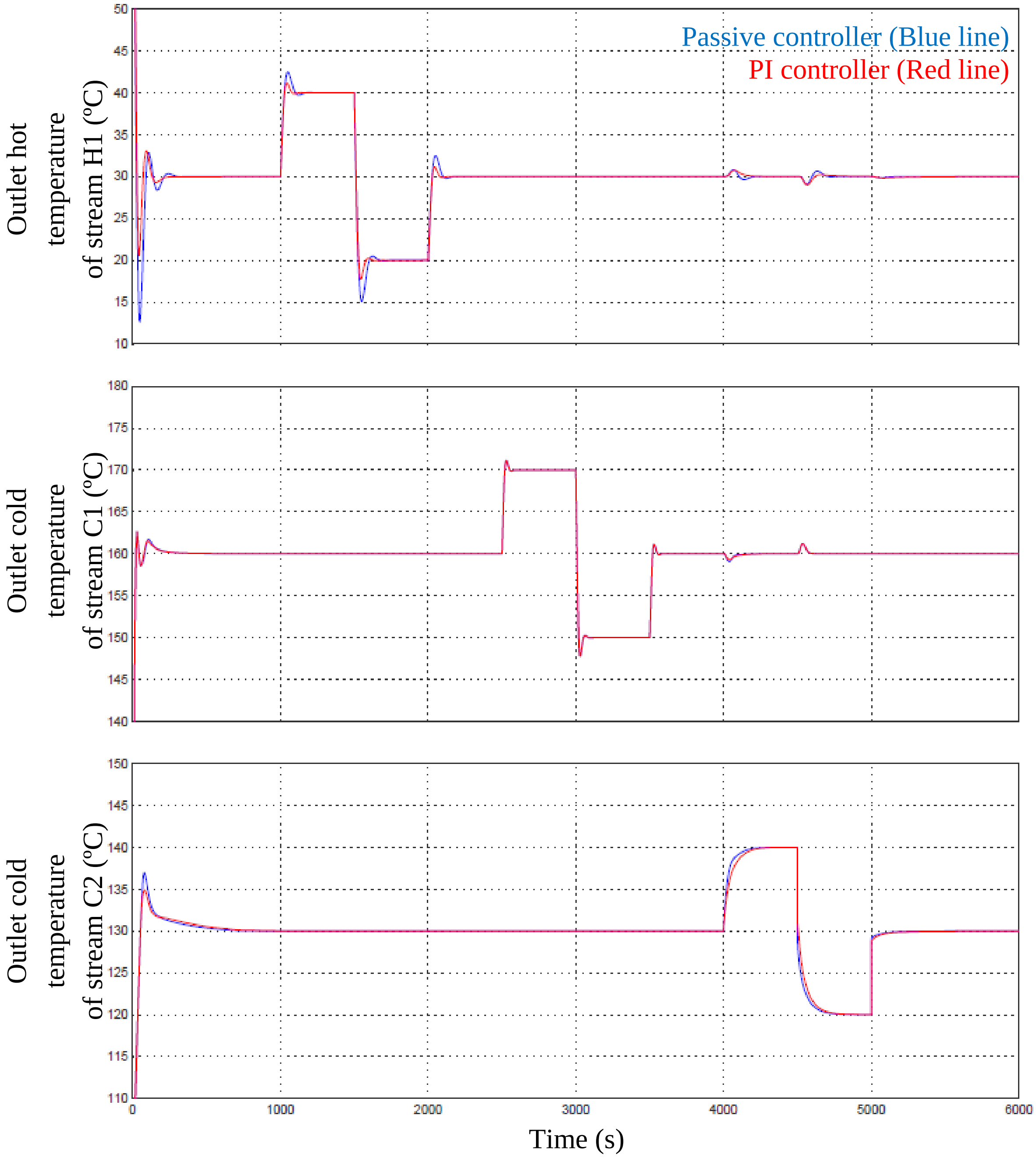

Figure 4.11 The comparison of control variable responses between passive controller and PI controller with set point tracking in case study 1.

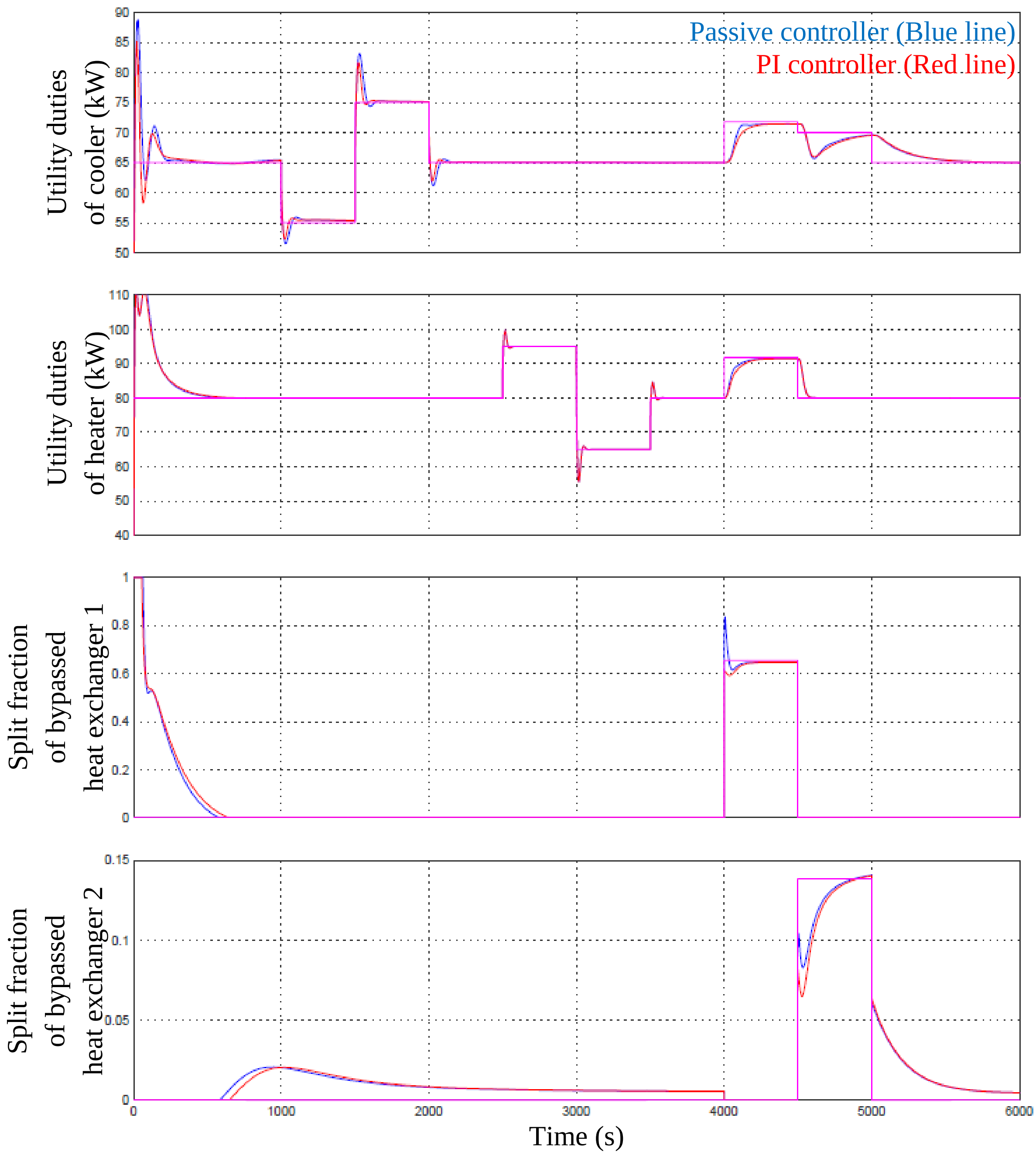

Figure 4.12 The comparison of manipulated variable responses between passive controller and PI controller with set point tracking in case study 1.

3) Closed loop control of heat exchanger networks with noise and time delay.

Normally, the output data from measurement contain with noise. Noise can occur from normal of process operation. Moreover, time delays commonly found in the process industries because of the presence velocity lags and the analysis time associated with measurement. Therefore, the noise and time delays are added to the feedback loop, which adversely affects closed loop stability. That can cause some problems during the analysis.

This part will compare the control variable responses between passive controller and PI controller under noise and time delays. Figures 4.13-14 shows the closed loop control of heat exchanger networks (with noise and time delay) with passive controller and PI controller; respectively.

After the disturbance is applied to process, the transient responses are presented in Figure 4.15. Figure 4.16 shows the result of manipulated variables that are compared between passive controller and PI controller when heat exchanger networks have noise and time delays.

Figure 4.15 shows the results of controlled variable responses after changing the inlet hot temperature of stream H1 at the time of 1,500 seconds and 3,000 seconds. Both controllers can adjust the target temperatures. The operation of closed loop control of heat exchanger networks with noise and time delay gives the result quite the same as the closed loop control of heat exchanger networks without noise and time delay. The responses of outlet hot temperature of stream H1, outlet cold temperature of stream C1, and outlet cold temperature of stream C2 with PI controller are more oscillate than passive controller. Moreover, the rising time of system with PI controller is more than with the passive controller. However, Figure 4.16 shows split faction of heat exchanger bypasses 1 with passive controller, which is more sensitive or oscillated than the PI controller.

In fact, the close loop control of heat exchanger networks with noise and time delay does not operate in practical because the manipulated variables are oscillation. This problem affects to damage equipment and lost. Therefore, the filter is added into close loop control of heat exchanger networks with noise and time delay in order to reduce oscillation of manipulated variables.

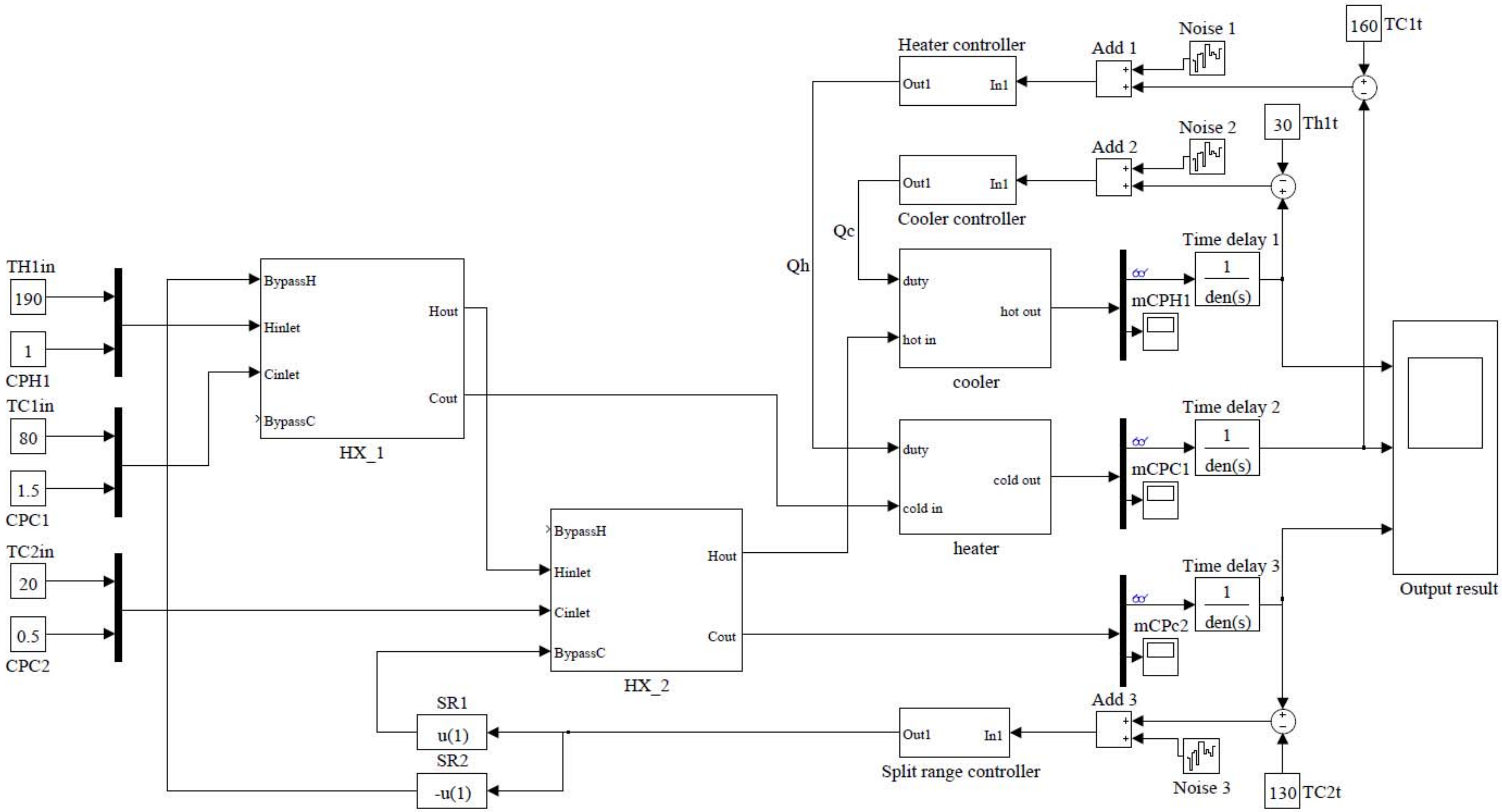

Figure 4.13 The closed loop control of heat exchanger networks (add noise and time delay) with passive controller in case study 1.

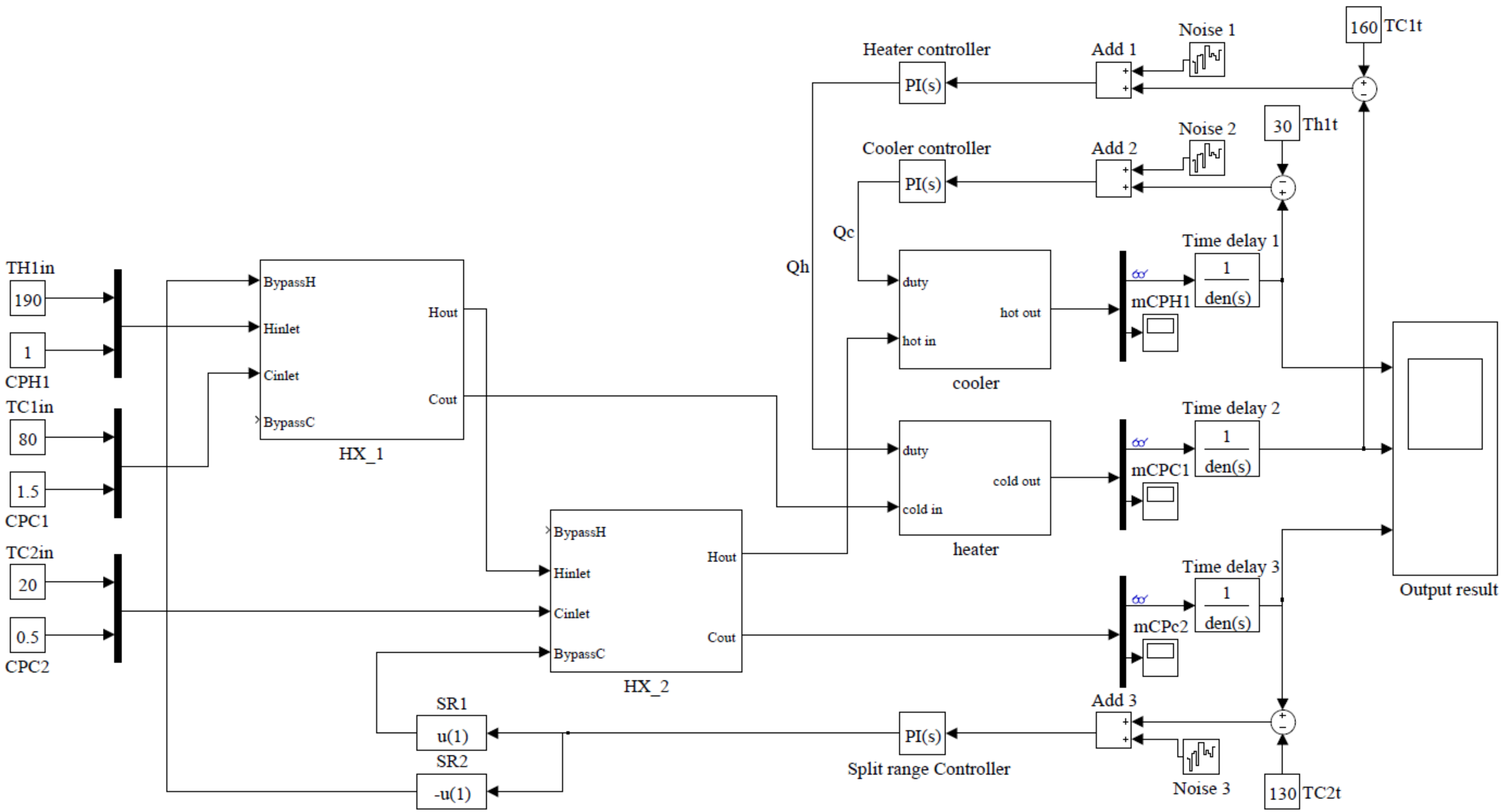

Figure 4.14 The closed loop control of heat exchanger networks (add noise and time delay) with PI controller in case study 1.

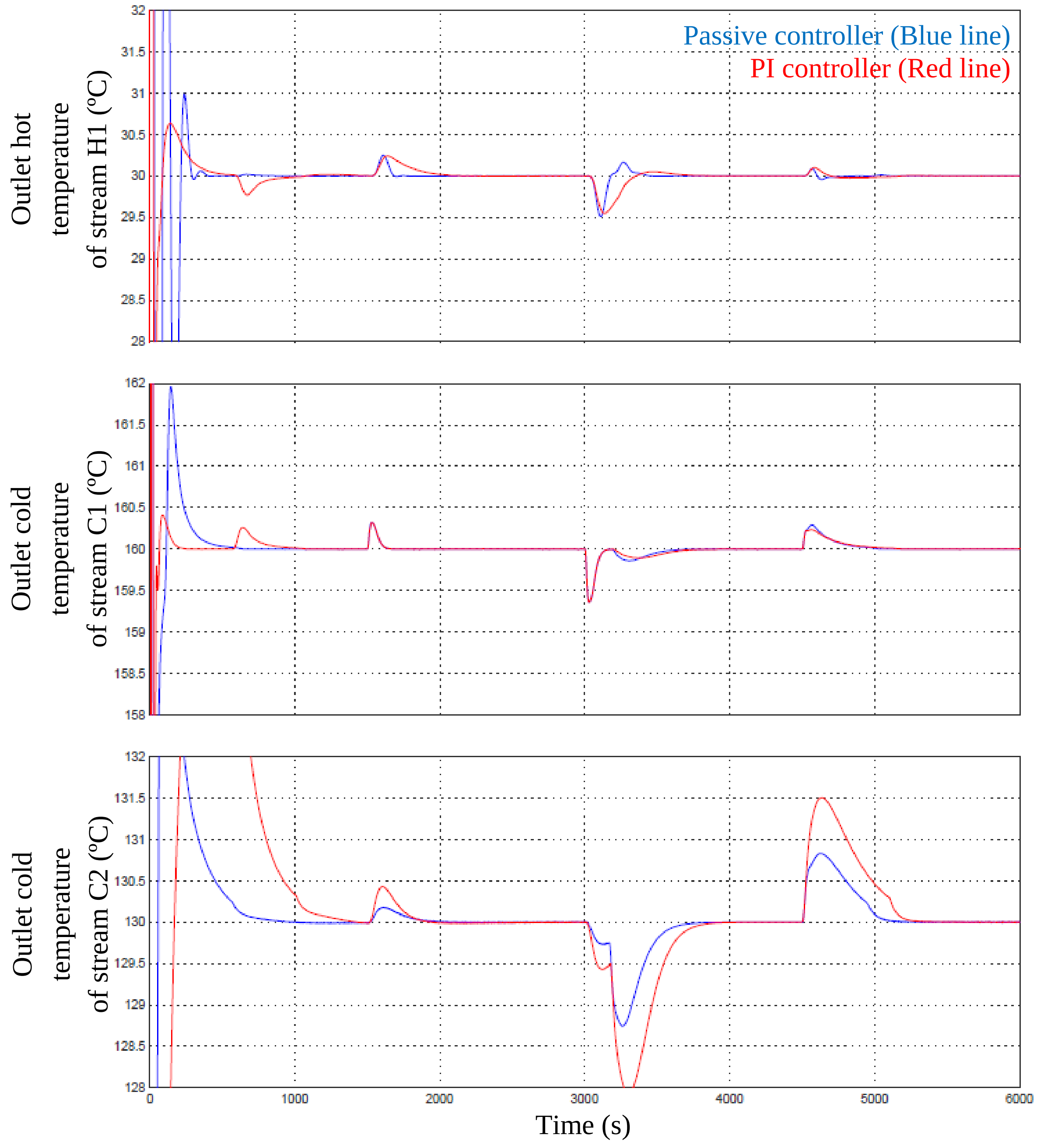

Figure 4.15 The comparison of controlled variable responses between passive controller and PI controller when heat exchanger networks have noise and time delay in case study 1.

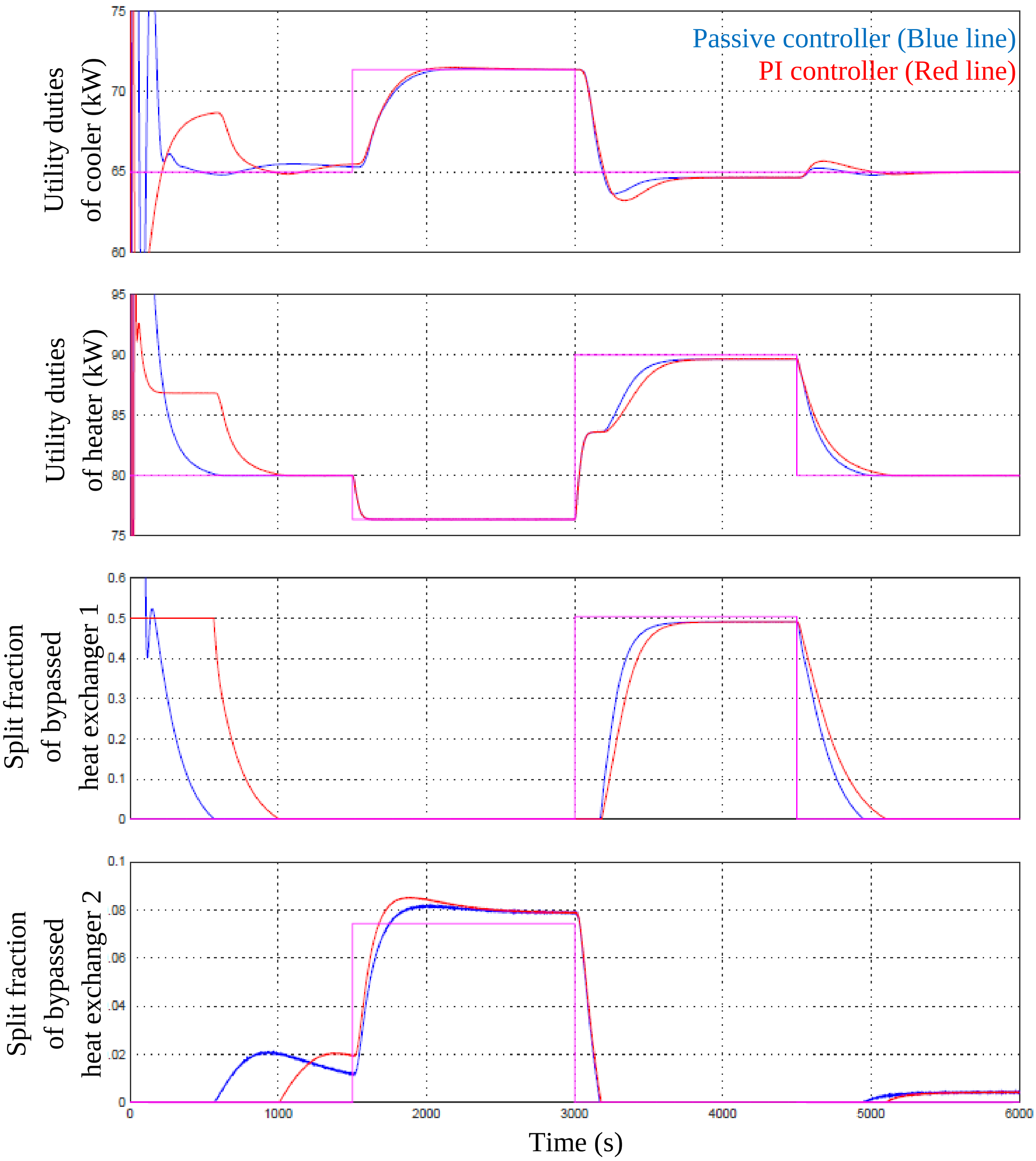

Figure 4.16 The comparison of manipulated variable responses between passive controller and PI controller when heat exchanger networks have noise and time delay in case study 1.

According to case study 1, the passivity concept can practically be applied to split range control of heat exchanger networks. Moreover, the passive controllers can adjust the target temperature. Therefore, the passivity concept will be used in practiced case which is more complex split range control of heat exchanger networks. This case will be included more amount of heat exchanger, number of stream, and number of the region than the existing model in order to check the passivity controller design performance.

4.2 Case study 2

4.2.1 The split range control of heat exchanger networks.

The classical heat exchanger networks from *Biegler* [15] are studied. The network has three heat exchangers and three utility exchangers. Furthermore, six manipulated variables, split fraction of bypassed heat exchangers 1, 2 and 3 (u_{b1}, u_{b2}, and u_{b3}) and utility duties of two cooler and one heater (Q_{c1}, Q_{c2} and Q_h), are available for control of all target temperatures that are outlet temperature of stream H1, outlet temperature of stream H2, outlet temperature of stream C1, and outlet temperature of stream C2. The heat exchanger networks configuration are shown Figure 4.17.

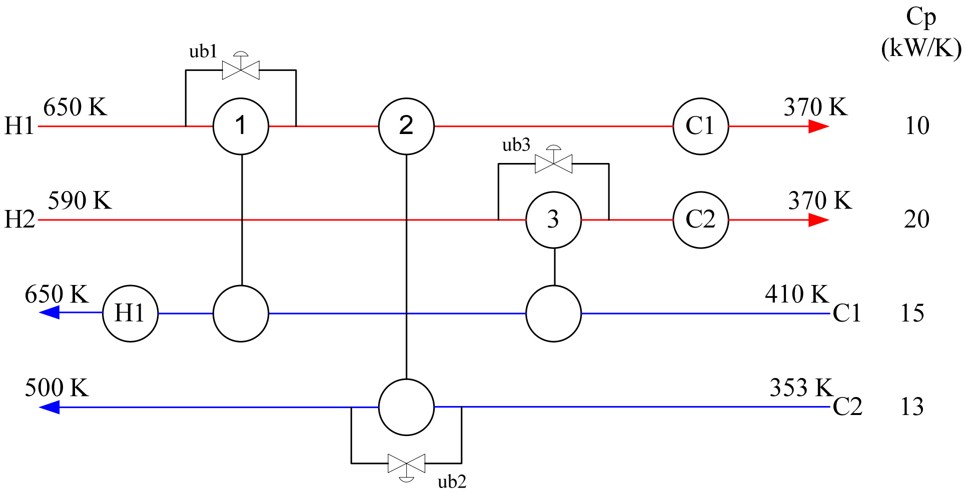

Figure 4.17 The heat exchanger networks in case study 2 [15].

The optimal operation can be achieved by the split range control of heat exchanger networks. The degrees of freedom should be first checked. These are three manipulated variables in the inner and outer heat exchanger networks while there are four for the dimensional spaces spanned by the controlled variables. Therefore, the degree of freedom is 2. Hence, the operation is structurally feasible.

In case study 2, the additional information required is the set of active constraint regions that can be obtained using multi parametric toolbox [16]. Three active constraint regions found are shown in Table 4.3.

Table 4.3 The set of active constraints in case study 2.

region	Manipulated variables					
	Q_{c1}	Q_{c2}	Q_{h1}	u_{b1}	u_{b2}	u_{b3}
1	U	U	U	S_L	U	S_L
2	S_L	U	U	U	U	S_L
3	U	U	S_L	U	U	S_L

U - Unsaturated manipulated variable (inactive constraint),
S_L - Saturated manipulated variable (active constraint) at the lower bound

According to Table 4.3, it demonstrates that the manipulated variables; Q_{c1}, Q_h and u_{b1}, can become the active constraints and implies that these three manipulated variables should be combined as a split range control. On the other hand, the manipulated variables Q_{c2} and u_{b2} cannot be the active constraints and u_{b3} are saturated in both regions. Therefore, they should not be used for any purpose.

4.2.2 Pairing system for split range control.

In case study 2, the integer linear programming will be used to suggest an optimal split range control structure via GAMs. The solutions from split range control structure purpose in terms of the minimization of the sum of relative orders. The result of optimal split range control structure is shown in Figure 4.18. There are four control loops as follows:

- First loop: Outlet hot temperature of stream H1 is controlled by switching between split fraction of bypassed heat exchanger 1 and utility duties of cooler 1.
- Second loop: Outlet hot temperature of stream H2 is controlled by utility duties of cooler 2.
- Third loop: Outlet cold temperature of stream C1 is controlled by switching between split fraction of bypassed heat exchanger 1 and utility duties of heater.
- Fourth loop: Outlet cold temperature of stream C2 is controlled by split fraction of bypassed heat exchanger 2.

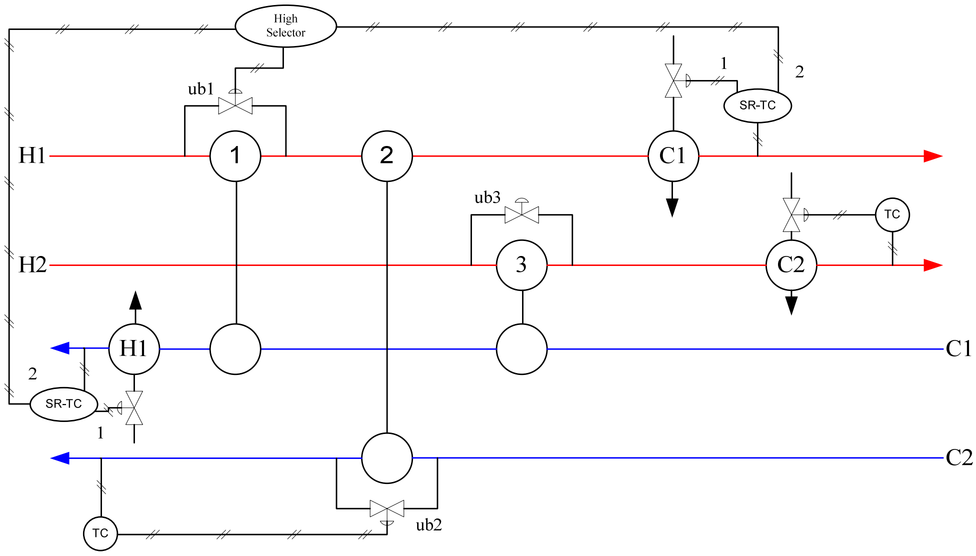

Figure 4.18 The split range control structure of heat exchanger networks in case study 2.

4.2.3 The dynamic model for split range control of heat exchanger networks.

The heat exchanger networks can be classified into 2 sub networks, the inner and outer heat exchanger networks. The outer heat exchanger networks include cooler 1, cooler 2, and heater.

The dynamic models of heat exchanger are considered to transfer the heat between hot side stream and cold side stream with equations 3.1-2. The dynamic model of cooler and heater can be formulated following equations 3.3-4; respectively. The dynamic models are developed based on the controlled, manipulated and disturbance variables of the unit. Since the dynamic models of heat exchanger are the non-linear model, the linearization by using Taylor's series expansion is necessary before generating the state space of heat exchanger. The linear dynamic models are proved in Appendix A.2.

4.2.4 The transfer function based on the passivity method.

The transient responses of heat exchanger networks are studied by using the passivity method. The linear dynamic model is first represented in the state space form. The state space models are proved by rearranging linear dynamic modes and substituting the numerical values at steady state into constant matrix A, B, C, D and E of the state space can be illustrated in Appendix B.2 in order to form the transfer function of the system as follows:

- The state space model of inner heat exchanger networks

$$\begin{bmatrix} \dot{T}_{1H} \\ \dot{T}_{1C} \\ \dot{T}_{2H} \\ \dot{T}_{C2b} \\ \dot{T}_{3H} \\ \dot{T}_{3C} \end{bmatrix} = \begin{bmatrix} -1.671 & 1.647 & 0 & 0 & 0 & 0 \\ 1.647 & -1.683 & 0 & 0 & 0 & 0.03571 \\ 0.02381 & 0 & -4.574 & 4.55 & 0 & 0 \\ 0 & 0 & 4.55 & -4.569 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5.808 & 5.761 \\ 0 & 0 & 0 & 0 & 5.761 & -5.796 \end{bmatrix} \begin{bmatrix} T_{1H} \\ T_{1C} \\ T_{2H} \\ T_{C2b} \\ T_{3H} \\ T_{3C} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 7.335 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} f_{C2} \end{bmatrix}$$

$$+ \begin{bmatrix} 0.02381 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0192 \\ 0 & 0.04762 & 0 & 0 \\ 0 & 0 & 0.03571 & 0 \end{bmatrix} \begin{bmatrix} T_{H1in} \\ T_{H2in} \\ T_{C1in} \\ T_{C2in} \end{bmatrix} \tag{4.22}$$

$$\begin{bmatrix} T_{C2t} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0.6203 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_{1H} \\ T_{1C} \\ T_{2H} \\ T_{C2b} \\ T_{3H} \\ T_{3C} \end{bmatrix} + \begin{bmatrix} -237 \end{bmatrix} \begin{bmatrix} f_{C2} \end{bmatrix} \tag{4.23}$$

- The state space model of cooler 1

$$\begin{bmatrix} \dot{T}_{H1t} \end{bmatrix} = \begin{bmatrix} -0.02381 \end{bmatrix} \begin{bmatrix} T_{H1t} \end{bmatrix} + \begin{bmatrix} -0.002381 \end{bmatrix} \begin{bmatrix} Q_{c1} \end{bmatrix} + \begin{bmatrix} 0.0238 \end{bmatrix} \begin{bmatrix} T_{2H} \end{bmatrix} \tag{4.24}$$

$$\begin{bmatrix} T_{H1t} \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} T_{H1t} \end{bmatrix} \tag{4.25}$$

- The state space model of cooler 2

$$\begin{bmatrix} \dot{T}_{H2t} \end{bmatrix} = \begin{bmatrix} -0.04762 \end{bmatrix} \begin{bmatrix} T_{H2t} \end{bmatrix} + \begin{bmatrix} -0.002381 \end{bmatrix} \begin{bmatrix} Q_{c2} \end{bmatrix} + \begin{bmatrix} 0.0476 \end{bmatrix} \begin{bmatrix} T_{3H} \end{bmatrix} \tag{4.26}$$

$$\begin{bmatrix} T_{H2t} \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} T_{H2t} \end{bmatrix} \tag{4.27}$$

- The state space model of heater

$$\left[\dot{T}_{C1t}\right] = [-0.03571][T_{C1t}] + [0.002381][Q_h] + [0.03571][T_{1C}] \tag{4.28}$$

$$[T_{C1t}] = [1][T_{C1t}] \tag{4.29}$$

The characteristics of heat exchanger networks are analyzed by the process and disturbance transfer functions.

The matrix transfer function can be determined via MATLAB as follows:

- The transfer function of inner heat exchanger networks

Process: $$[T_{C2t}] = \left[\frac{-237s^2 - 2162s - 25.67}{s^2 + 9.143s + 0.1961}\right][f_{C2}] \tag{4.30}$$

Disturbance:

$$[T_{C2t}] = \begin{bmatrix} \dfrac{0.0016\ \mathrm{s} + 0.002693}{s^4 + 12.5s^3 + 30.96s^2 + 1.562s + 0.0194} \\ \dfrac{0.001084}{s^6 + 24.1s^5 + 176.5s^4 + 366.9s^3 + 33.06s^2 + 0.9777s + 0.009346} \\ \dfrac{0.0001412\ \mathrm{s} + 0.0008201}{s^6 + 24.1s^5 + 176.5s^4 + 366.9s^3 + 33.06s^2 + 0.9777s + 0.009346} \\ \dfrac{0.01191\ \mathrm{s} + 0.05447}{s^2 + 9.143s + 0.1961} \end{bmatrix} \begin{bmatrix} T_{H1in} \\ T_{H2in} \\ T_{C1in} \\ T_{C2in} \end{bmatrix} \tag{4.31}$$

- The transfer function of cooler 1

Process: $$[T_{H1t}] = \left[\frac{-0.002381}{\mathrm{s} + 0.02381}\right][Q_{c1}] \tag{4.32}$$

Disturbance: $$[T_{H1t}] = \left[\frac{0.02381}{\mathrm{s} + 0.02381}\right][T_{2H}] \tag{4.33}$$

- The transfer function of cooler 2

Process: $$[T_{H2t}] = \left[\frac{-0.002381}{\mathrm{s} + 0.04762}\right][Q_{c2}] \tag{4.34}$$

Disturbance: $$[T_{H2t}] = \left[\frac{0.04762}{\mathrm{s} + 0.04762}\right][T_{3H}] \tag{4.35}$$

- The transfer function of heater

Process: $$[T_{C1t}] = \left[\frac{0.002381}{\text{s} + 0.03571}\right][Q_h] \quad (4.36)$$

Disturbance: $$[T_{C1t}] = \left[\frac{0.03571}{\text{s} + 0.03571}\right][T_{1C}] \quad (4.37)$$

4.2.5 The passivity index.

After the process and disturbance transfer functions are obtained from MATLAB, the passivity index is obtained for passivity analyze as illustrated in Figure 4.19.

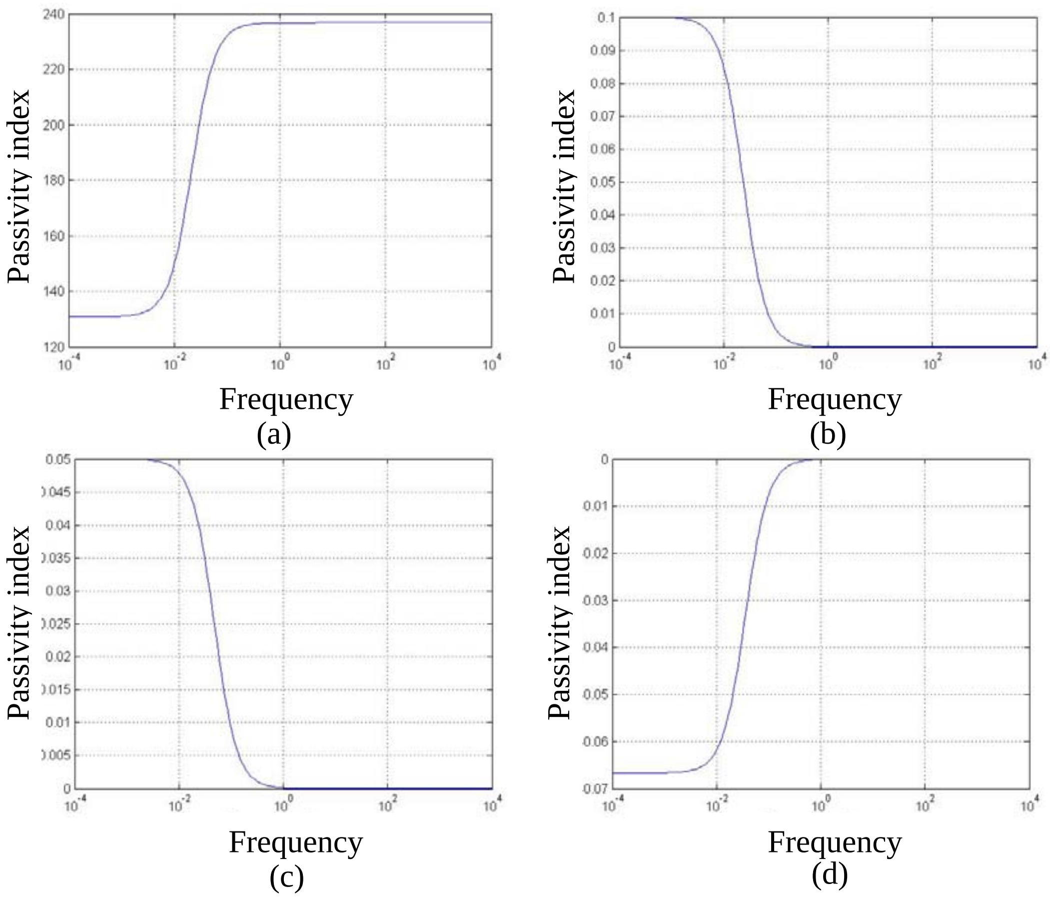

Figure 4.19 The passivity index in case study 2 **(a)** Inner heat exchanger networks, **(b)** cooler 1, **(c)** cooler 2, and **(d)** heater.

Figure 4.19 shows the result of passivity index. The passivity index of heater is a negative value that means the system is the passive system. On the other hand, inner heat exchanger networks, cooler 1, and cooler 2 are non-passive due to positive passivity index. According to the passivity concept, the non-passive processes can be shifted to the passive processes by adding weighting function $w_p(s)$. Their weighting functions are presented as following:

- The weighting function of inner heat exchanger networks

$$w_p(s) = \frac{236.9820 \cdot s \cdot (s + 0.0119)}{(s + 0.0215) \cdot (s)} \tag{4.38}$$

- The weighting function of cooler 1

$$w_p(s) = \frac{0.0015 \cdot s \cdot (s + 1.5814)}{(s) \cdot (s + 0.0235)} \tag{4.39}$$

- The weighting function of cooler 2

$$w_p(s) = \frac{0.0025 \cdot s \cdot (s + 0.9243)}{(s) \cdot (s + 0.0453)} \tag{4.40}$$

The weighing function from equations 4.38-40 is introduced into the transfer functions of inner heat exchanger networks, cooler 1, and cooler 2; respectively, following the $H(s) = G(s) + w_p(s)I$. Then the non-passive cooler shifts to the passive system. There passivity indexes are illustrated in Figure 4.20.

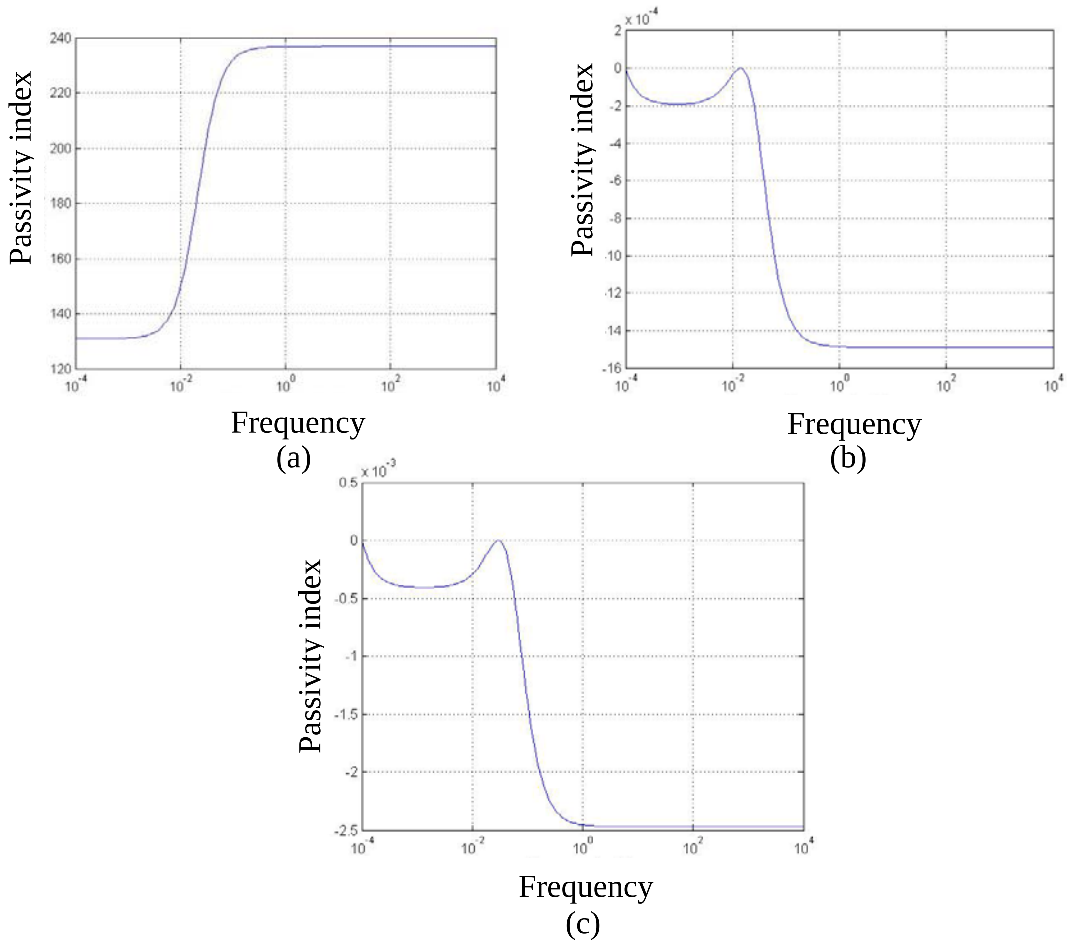

Figure 4.20 The passivity indexes of the systems in case study 2
(a) Inner heat exchanger networks, **(b)** cooler 1 and **(c)** cooler 2.

In principle, the weighting functions cannot be added into the system directly because the system must be the same. Therefore, the weighting functions will be absorbed into the controllers to design the passive controllers.

4.2.6 The passive controller.

By follow the passivity concept, the feedback system comprises of a passive system and strictly passive system is asymptotically stable. In this work, the heat exchanger networks are already shifted to passive system. Therefore, the passive controller for each loop control is designed to make the heat exchanger networks asymptotically stable.

Based on the split range control, the passive controller of the first loop and third loop are used same controller.

The passive controllers with integral action (PI controller) are designed from the optimizations. The results of the $k_{c,i}$ and $\tau_{I,i}$ for each loop control is in Table 4.4.

Table 4.4 The results of the $k_{c,i}$ and $\tau_{I,i}$ for each control loop in case study 2.

Loop No.	$k_{c,i}$	$\tau_{I,i}$ **(second)**
u_{b1}-T_{H1t}	9.5000	14.5522
Q_{c1}-T_{H1t}	9.5000	14.5522
Q_{c2}-T_{H2t}	10.000	1.3802
u_{b1}-T_{C1t}	9.5000	14.5522
Q_{h}-T_{C1t}	7.0980	22.0000
u_{b2}-T_{C2t}	0.0020	30.0000

Since the controllers used in multi control system are PI controller, the values of $k_{c,i}$ and $\tau_{I,i}$ from Table 4.4 are plugged in the simple form of PI controller as follow:

$$k_i = k_{c,i}\left(1+\frac{1}{\tau_{I,i}s}\right)$$

Therefore, k_i of each control loop is shown in the following.

$$k_{u_{b1}-T_{H1t}} = 9.5000\left(1+\frac{1}{14.5522s}\right) \tag{4.41}$$

$$k_{Q_{c1}-T_{H1t}} = 9.5000\left(1+\frac{1}{14.5522s}\right) \tag{4.42}$$

$$k_{Q_{c2}-T_{H2t}} = 10.0000\left(1+\frac{1}{1.3802s}\right) \tag{4.43}$$

$$k_{u_{b1}-T_{C1t}} = 9.5000\left(1+\frac{1}{14.5522s}\right) \tag{4.44}$$

$$k_{Q_h-T_{C1t}} = 7.0980\left(1+\frac{1}{22.0000s}\right) \tag{4.45}$$

$$k_{u_{b2}-T_{C2t}} = 0.0020\left(1+\frac{1}{30.0000s}\right) \tag{4.46}$$

According to the passivity method, the close loop system of inner heat exchanger networks, cooler 1, and cooler 2 will be stabled the weighting function into the controller.

$$k_i^{'}(s) = k_i\ (s)[1-w_p(s)k_i\ (s)]^{-1}$$

These three passive controllers will be used in the system.

$$k_{u_{b1}-T_{H1t}}^{'}(s) = \frac{138.2457s+12.7467}{14.3465s+0.0024} \tag{4.47}$$

$$k_{Q_{c1}-T_{H1t}}^{'}(s) = \frac{138.2457s+12.7467}{14.3465s+0.0024} \tag{4.48}$$

$$k_{Q_{c2}-T_{H2t}}^{'}(s) = \frac{13.8019s+10.6252}{1.3461s+0.0063} \tag{4.49}$$

$$k_{u_{b1}-T_{C1t}}^{'}(s) = \frac{138.2457s+12.7467}{14.3465s+0.0024} \tag{4.50}$$

$$k_{u_{b2}-T_{C2t}}^{'}(s) = \frac{0.06s+0.0033}{15.7811s+0.0032} \tag{4.51}$$

4.2.7 Verification of the heat exchanger networks model.

Opened loop and closed loop control of heat exchanger networks are verified.

1) Opened loop control of heat exchanger networks

The developed heat exchanger networks model is an integration of all possible dynamic equations by including the dynamic models of lump heat exchangers, cooler, and heater. This model is developed for capturing the transient responses of outlet hot and cold temperature. The dynamic model of heat exchanger networks is implemented by MATLAB-SIMULINK.

Figure 4.22 shows the opened loop control of heat exchanger networks that included three heat exchangers and two utility exchangers (two cooler and one heater). There are four inputs (T_{h1in}, T_{h2in}, T_{c1in}, and T_{c2in}) and four outputs (T_{h1t}, T_{h2t}, T_{c1t}, and T_{c2t}) with four disturbances (T_{h1in}, T_{h2in}, T_{c1in}, and T_{c2in}). The disturbance variable of this study is shown in Table 4.5. In this part of study, the profiles of outlet hot temperature of stream

H1, outlet hot temperature of stream H2, outlet cold temperature of stream C1, and outlet cold temperature of stream C2 are shown in Figure 4.21.

Table 4.5 Disturbances and active constraints in case study 2.

Time (s)	Disturbance				Active constraint					
	ΔT_{H1}^{in}	ΔT_{H2}^{in}	ΔT_{C1}^{in}	ΔT_{C2}^{in}	Q_{c1}	Q_{c2}	Q_{h1}	u_{b1}	u_{b2}	u_{b3}
< 1,000	0	0	0	0				S_L		S_L
1,000-2,000	26.52	-23.52	14.52	-11.67	S_L					S_L
> 2,000	25.91	22.91	13.91	-11.06			S_L			S_L

According to Figure 4.21, after the disturbance is introduced in heat at the time of 1,000 seconds, the process will be switched from region 1 to region 2. The outlet hot temperature of stream H1 and the outlet cold temperature of stream C2 are increased suddenly to their new steady state. The outlet hot temperature of stream H2 and the outlet cold temperature of stream C1 are dramatically decreased to their new steady states. Moreover, four outlet temperatures are increased to their new steady states after heat exchanger networks is switched from region 2 to region 3 at the time of 2,000 seconds. Therefore, there is no need to establish the control system. This result shows that the model under identical conditions can predict the system behavior accurately.

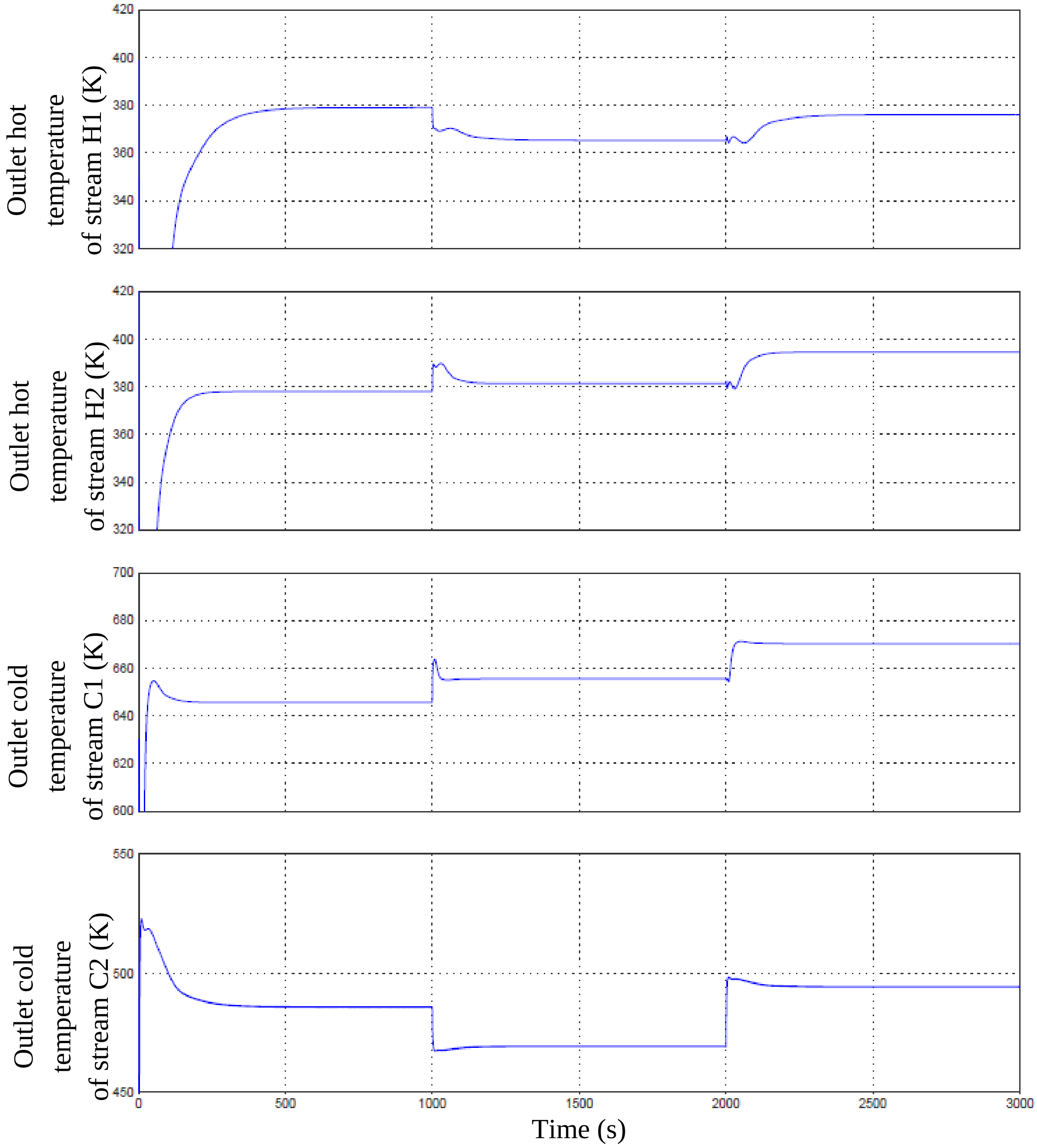

Figure 4.21 The transient response of opened loop control in case study 2.

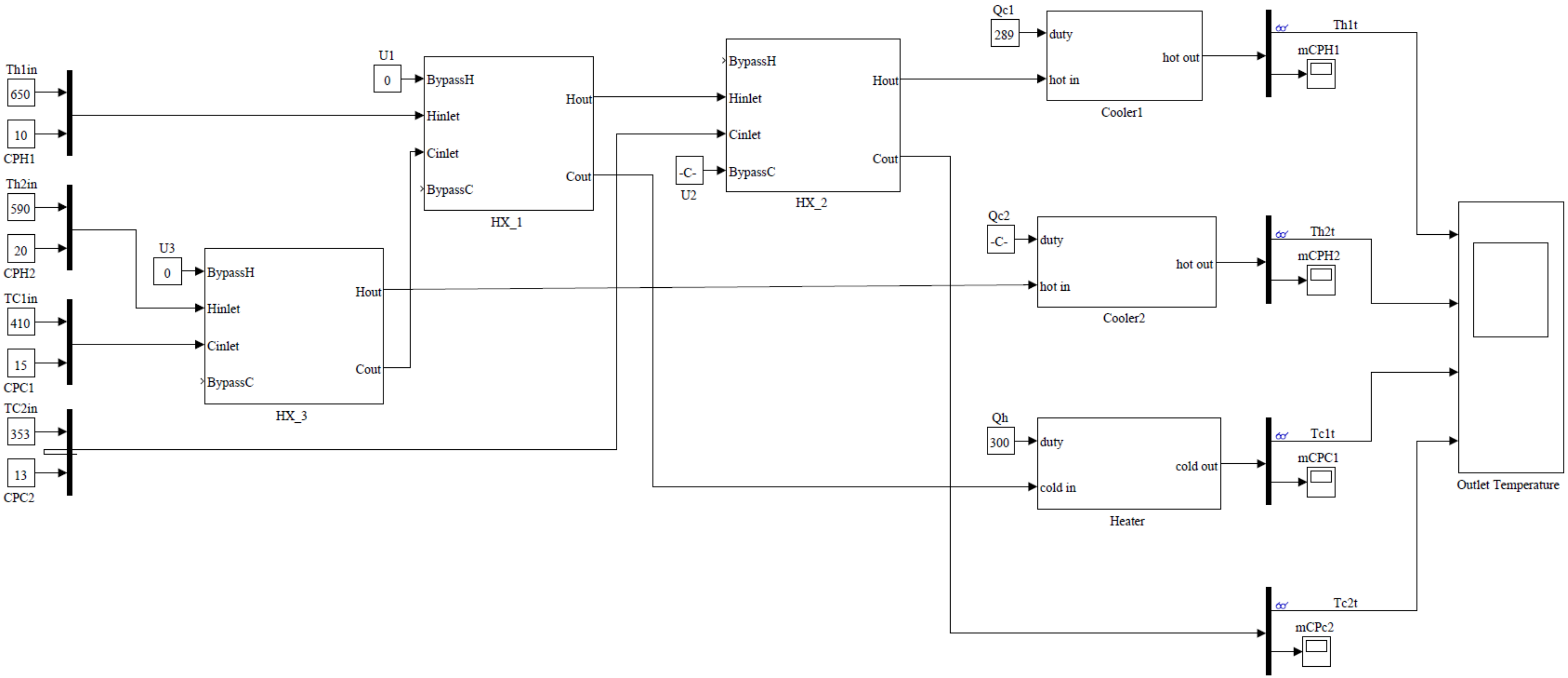

Figure 4.22 The opened loop control of heat exchanger networks in case study 2.

2) Closed loop control of heat exchanger networks

In this case, there are 6 manipulated variables, bypasses of heat exchangers 1, 2, and 3 (u_{b1} and u_{b2}) and three utility duties which are two cooler and one heater (Q_{c1}, Q_{c2} and Q_h), are available for control of all target outlet temperatures. Next, the passive controllers are designed by using the passivity concept. In order to stabilize the closed loop system, the process system ($G(s)$) must be strictly passive and the multi loop controller ($K(s)$) must be passive also. For heat exchanger network, the process renders strictly passive and the controller renders passive by absorbing the minimum phase transfer function $(w_p(s))$, which stabilizes this system. The stability of heat exchanger networks is verified with the disturbance.

Figure 4.23 shows the four control loops of heat exchanger networks with passivity application. First loop is the control outlet hot temperature of stream H1 by manipulating between the utility duties of cooler 1 and the split fraction of bypassed heat exchanger 1. Second loop is the control outlet hot temperature of stream H1 by manipulating the utility duties of cooler 2. Third loop controls the outlet cold temperature of stream C1 by manipulating between the duty utility of the heater and the split fraction of bypassed heat exchanger 1. The last one is to control the outlet cold temperature of stream C2 by manipulating split fraction of bypassed heat exchanger 2.

Figure 4.24 shows the results of the control variable responses after switching between region at the time of 1,000 and 2,000 seconds. When system is disturbed at the time of 1,000 seconds, the system will move the steady state operation to the active constraint region 1. The utility duty of cooler 1 becomes a saturated variable or an active constraint in region 1. The split fraction of bypassed heat exchanger 1 will be operated from saturated in order to control the outlet hot temperature of stream H1 as shown in Figure 4.25. The system will be switched from the active constraint region 1 to the active constraint region 2 after system operate at the time of 2,000 seconds. Figure 4.25 shows the utility duties of heater become the saturated variable or the active constraint in region 2. On the other hand, the utility duties of cooler 1 are operated after 2,000 seconds. All controllers can adjust the target temperature. The responses of all outlet temperature have less oscillation. Moreover, the rising time of system with passive controller is too low. As a result, the split range control of heat exchanger networks that is designed by the passivity method can be adjusted the target temperature and guarantee stable at all region.

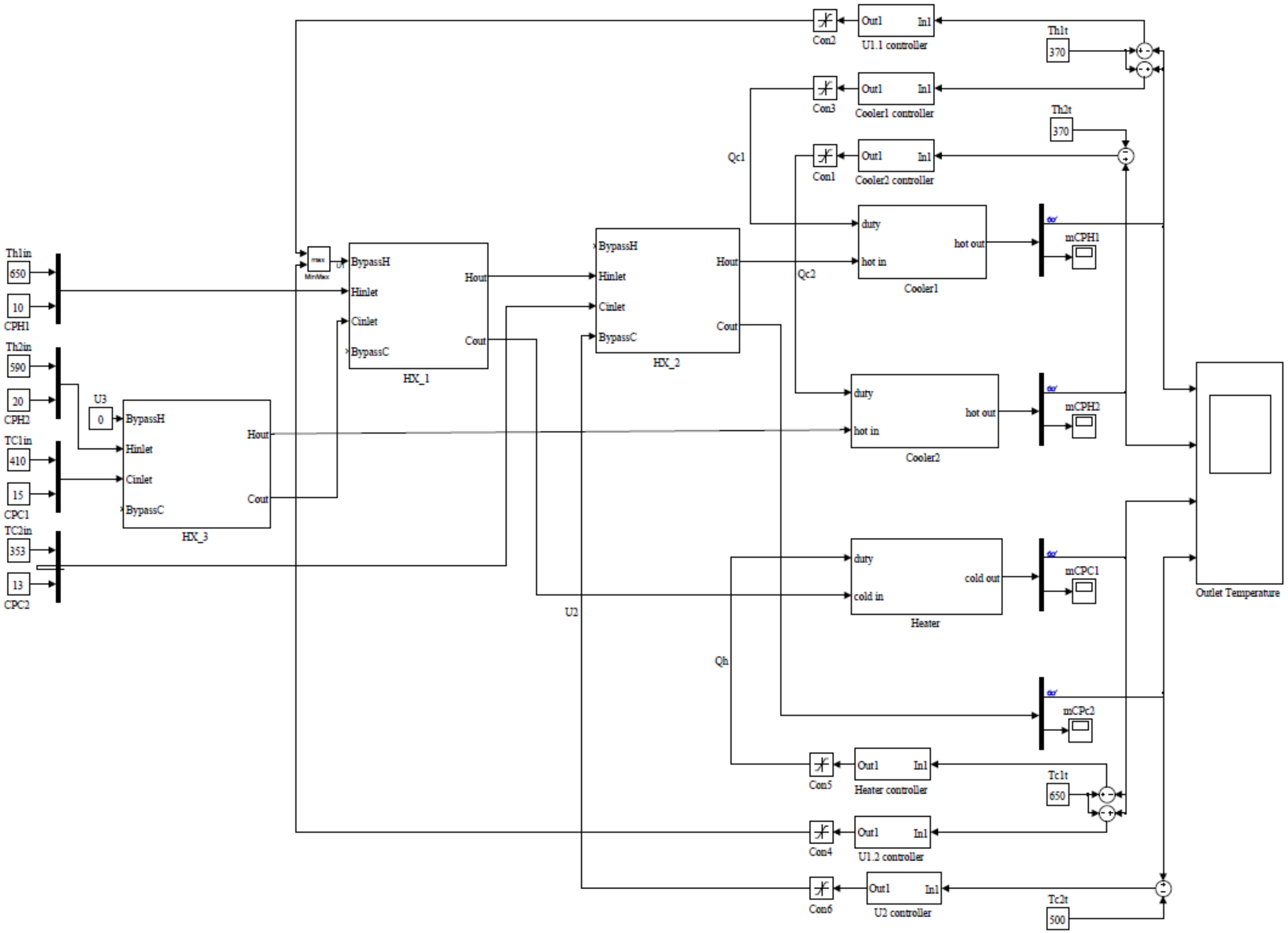

Figure 4.23 The closed loop control of heat exchanger networks with passive controller in case study 2.

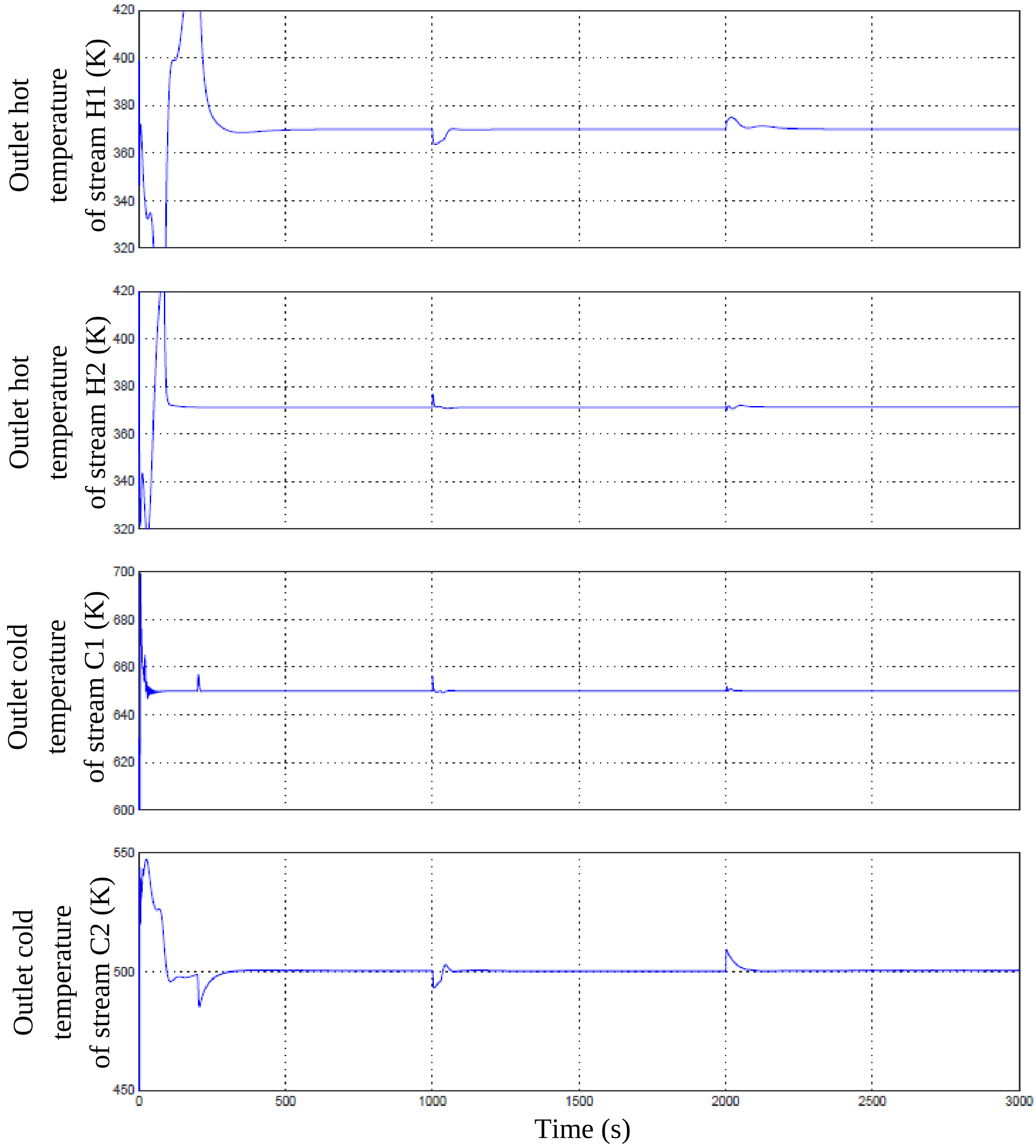

Figure 4.24 The control variable responses of passive controller with load disturbance in case study 2.

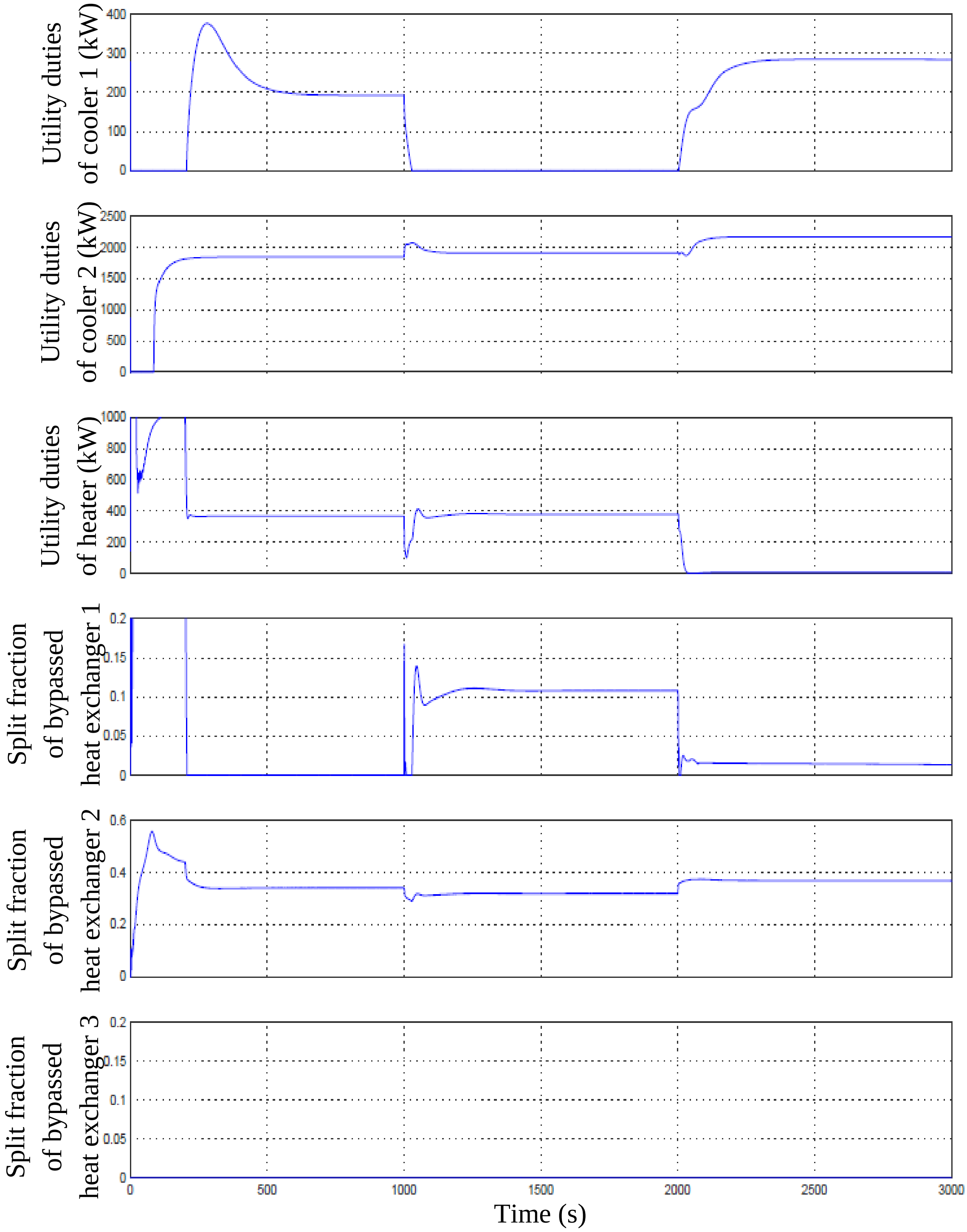

Figure 4.25 The manipulated variable responses of passive controller with load disturbance in case study 2.

The load disturbances are verified for closed loop system and show that the passive controller can apply to more complex split range control of heat exchanger networks. In addition, the set point tracking are considered in order to verify the performance of the designed closed loop system. For this study, the set points are step up 10°C and step down 10°C of all target temperatures (H1, C1, and C2) at the time of 500, 750, 1,250, 1,500, 2,000, 2,250, 2,750 and 3,000 seconds.

Figure 4.26 shows the results of controlled variable responses after step change tracking of outlet hot temperature of stream H1, outlet hot temperature of stream H2, outlet cold temperature of stream C1, and outlet cold temperature of stream C2 at the time of 500, 750, 1,250, 1,500, 2,000, 2,250, 2,750 and 3,000 seconds, respectively. Moreover, Figure 4.27 shows the result of manipulated variables of passive controller.

According to Figure 4.26, the closed loop system of heat exchanger networks from passivity method can control the all target temperature and guarantee the stability. Moreover, all controlled variable responses with passive controller gives the low overshoot and reaches its set point quickly after step up and down 10°C for all set point tracking.

According to Figure 4.27, the manipulated variables do not use the split range control policy when the closed loop system of heat exchanger networks are verified with set point tracking. After set point tracking at the time of 500 seconds, the utility duties of cooler 1, cooler 2 and heater are changed in order to maintain the outlet hot temperature of stream H1, the outlet hot temperature of stream H2 and the outlet cold temperature of stream C1 to a new set point; respectively. On the other hand, the split fraction of bypassed heat exchanger 2 and the utility duties of cooler 1 are changing in order to control only one outlet cold temperature of stream C2 to a new set point after set point tracking at the time of 2,750 seconds.

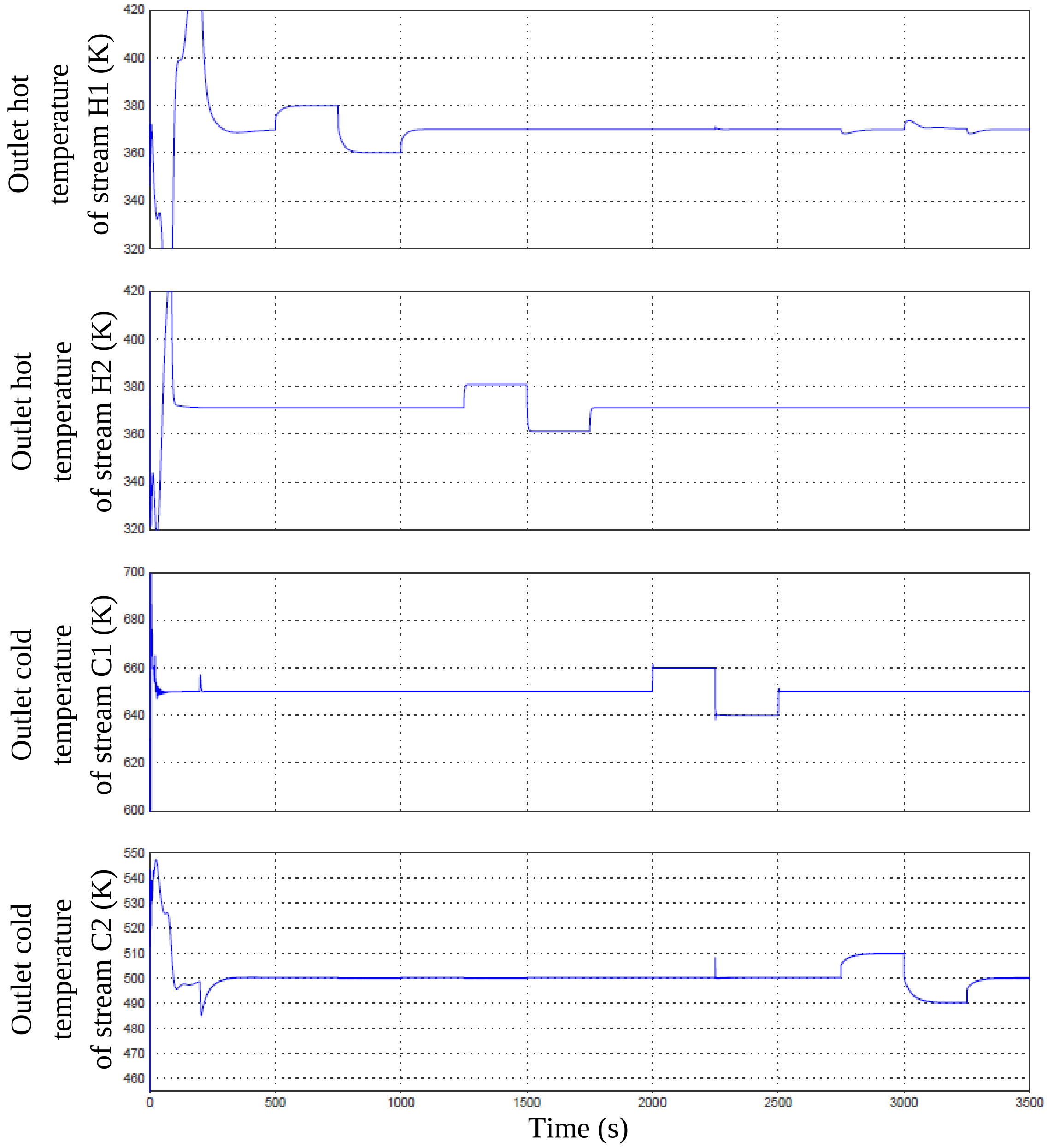

Figure 4.26 The controlled variable responses of passive controller with set point tracking in case study 2.

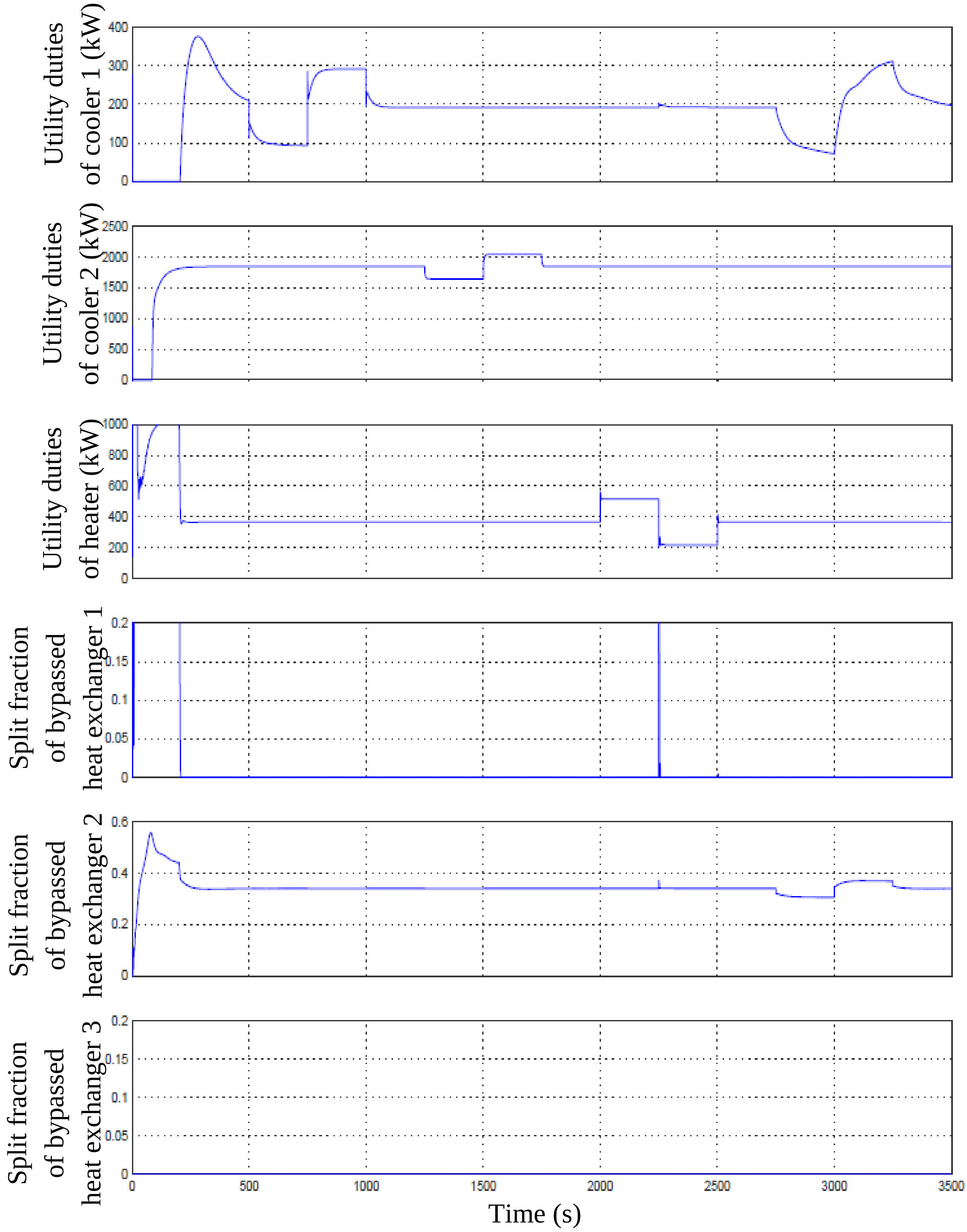

Figure 4.27 The manipulated variable responses of passive controller with set point tracking in case study 2.

CHAPTER 5 CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

This work considers about the control system of split range control in heat exchanger networks. The split range control of heat exchanger networks should be stable for all possible disturbance variations as well as the optimal operation should regulate the temperature at their certain level under the minimum utility cost. The controlled variables of heat exchanger networks using split range are the target temperature controlled by the split fraction of bypassed, utility duties of cooler and heater.

The dynamic models of split range control of heat exchanger networks were generated for the controlled, manipulated and disturbance variables of the unit. Since the dynamic models of heat exchanger networks are the nonlinear model, the linearization by Taylor's series expansion is utilized before generates the state space models. After that the state space models of heat exchanger networks are analyzed in terms of process transfer function and disturbance transfer function. Next, the transfer function was formulated to indicate that the heat exchanger networks were whether passive or not by considering the passivity index. As a result, the split range control of heat exchanger networks are non-passive system. Therefore, the weighting function is added to make heat exchanger networks become strictly passive system. Consequently, the passive controller was designed to make the heat exchanger networks in stable condition.

According to case studies 1 and 2, the passivity concept can practically be applied to split range control of heat exchanger networks. In addition, the passive controllers can adjust the target temperature after load disturbance and set point tracking. In addition, the dynamic responses of the target temperature with passive controllers are less oscillation than PI controller. Moreover, the rising time of system with passive controller is more than PI controller. The passivity concept can practically be used in more complex split range control of heat exchanger networks. This included more amount of heat exchanger, number of stream, and number of operating region. In conclusion, the split range control of heat exchanger networks that is designed by passivity method can be adjusted the target temperature and guarantee the stability for all regions.

5.2 Recommendations

5.2.1 This passivity theorem should be applied with a large scale split range control in heat exchanger networks to guarantee that this approach can handle with highly nonlinear and large interaction system which has not been researched yet.

5.2.2 In this study, the noise and time delay of large scale split range control in heat exchanger networks is not added into the MATLAB-SIMULINK simulation because the target temperature results could not be recorded in this program. Therefore, this system should be applied to other simulation program.

5.2.3 In fact, the close loop control of heat exchanger networks with noise and time delay does not operate in practical because the manipulated variables are oscillation. This problem affects to damage equipment and lost. Therefore, the filter is added into close loop control of heat exchanger networks with noise and time delay in order to reduce oscillation of manipulated variables.

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APPENDIX

APPENDIX A

DYNAMIC MODEL EQUATION

This section shows a derivation of the heat exchanger models used in this work.

A.1 Case study 1

The heat exchanger network from *Glemmestad* [1] as shown in Figure A.1 is studied here. There are two heat exchangers and two utility exchangers. Furthermore, 4 manipulated variables, bypasses of heat exchangers 1 and 2 (u_{b1} and u_{b2}) and utility duties of cooler and heater (Q_c and Q_h), are available for control of all target outlet temperatures that are outlet temperature of stream H1, outlet temperature of stream C1, and outlet temperature of stream C2.

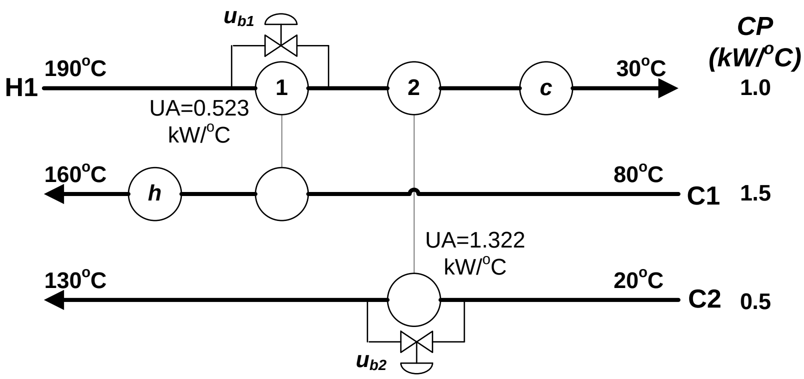

Figure A.1 The heat exchanger networks in case study 1.

In this case study, heat exchanger network can be separated into 2 system, which are inner and outer heat exchanger network.

The dynamic model for inner heat exchanger network:

Since the dynamic models of inner heat exchanger network from equations 3.1-2 are the non-linear model, linearization by using Taylor's series expansion is necessary that are presented as the following:

For T_{1H}:

$$\rho_{H1}V_{H1}C_{PH1}\frac{dT_{1H}}{dt} = \rho_{H1}F_{H1}C_{PH1}(T_{H1in} - T_{1H}) + UA_1(T_{1C} - T_{1H}) \tag{A.1}$$

Where $\tau_{H1} = \rho_{H1}C_{PH1}$ and $\xi_{H1} = \rho_{H1}V_{H1}C_{PH1}$

$$\frac{dT_{1H}}{dt} = \frac{(-\tau_{H1}F_{H1} - UA_1)}{\xi_{H1}}T_{1H} + \frac{UA_1}{\xi_{H1}}T_{1C} + \frac{\tau_{H1}F_{H1}}{\xi_{H1}}T_{H1in} \tag{A.2}$$

At steady state

$$\frac{d\overline{T}_{1H}}{dt} = \frac{(-\tau_{H1}F_{H1} - UA_1)}{\xi_{H1}}\overline{T}_{1H} + \frac{UA_1}{\xi_{H1}}\overline{T}_{1C} + \frac{\tau_{H1}F_{H1}}{\xi_{H1}}\overline{T}_{H1in} = 0 \tag{A.3}$$

Then equation A.3 can be made into the deviation variable form or perturbation form. When $T_{1H} = \overline{T}_{1H} + T_{1H}^P$, $T_{1C} = \overline{T}_{1C} + T_{1C}^P$, and $T_{H1in} = \overline{T}_{H1in} + T_{H1in}^P$.

$$\frac{dT_{1H}^P}{dt} = \frac{(-\tau_{H1}F_{H1} - UA_1)}{\xi_{H1}}T_{1H}^P + \frac{UA_1}{\xi_{H1}}T_{1C}^P + \frac{\tau_{H1}F_{H1}}{\xi_{H1}}T_{H1in}^P \tag{A.4}$$

For T_{1C}:

$$\rho_{C1}V_{C1}C_{PC1}\frac{dT_{1C}}{dt} = \rho_{C1}F_{C1}C_{PC1}(\overline{T}_{C1in} - T_{1C}) + UA_1(T_{1H} - T_{1C}) \tag{A.5}$$

Where $\tau_{C1} = \rho_{C1}C_{PC1}$ and $\xi_{C1} = \rho_{C1}V_{C1}C_{PC1}$

$$\frac{dT_{1C}}{dt} = \frac{UA_1}{\xi_{C1}}T_{1H} + \frac{(-\tau_{C1}F_{C1} - UA_1)}{\xi_{C1}}T_{1C} + \frac{\tau_{C1}F_{C1}}{\xi_{C1}}\overline{T}_{C1in} \tag{A.6}$$

At steady state

$$\frac{d\overline{T}_{1C}}{dt} = \frac{UA_1}{\xi_{C1}}\overline{T}_{1H} + \frac{(-\tau_{C1}F_{C1} - UA_1)}{\xi_{C1}}\overline{T}_{1C} + \frac{\tau_{C1}F_{C1}}{\xi_{C1}}\overline{T}_{C1in} = 0 \tag{A.7}$$

Then equation A.7 can be made into the deviation variable form or perturbation form. When $T_{1C} = \overline{T}_{1C} + T_{1C}^P$ and $T_{1H} = \overline{T}_{1H} + T_{1H}^P$.

$$\frac{dT_{1C}^P}{dt} = \frac{UA_1}{\xi_{C1}}T_{1H}^P + \frac{(-\tau_{C1}F_{C1} - UA_1)}{\xi_{C1}}T_{1C}^P \tag{A.8}$$

For T_{2H}:

$$\rho_{H1}V_{H1}C_{PH1}\frac{dT_{2H}}{dt} = \rho_{H1}F_{H1}C_{PH1}(T_{1H} - T_{2H}) + UA_2(T_{C2b} - T_{2H}) \tag{A.9}$$

Where $\tau_{H1} = \rho_{H1}C_{PH1}$ and $\xi_{H1} = \rho_{H1}V_{H1}C_{PH1}$

$$\frac{dT_{2H}}{dt} = \frac{\tau_{H1}F_{H1}}{\xi_{H1}}T_{1H} + \frac{(-\tau_{H1}F_{H1} - UA_2)}{\xi_{H1}}T_{2H} + \frac{UA_2}{\xi_{H1}}T_{C2b} \tag{A.10}$$

At steady state

$$\frac{d\overline{T}_{2H}}{dt} = \frac{\tau_{H1}F_{H1}}{\xi_{H1}}\overline{T}_{1H} + \frac{(-\tau_{H1}F_{H1} - UA_2)}{\xi_{H1}}\overline{T}_{2H} + \frac{UA_2}{\xi_{H1}}\overline{T}_{C2b} = 0 \qquad \text{(A.11)}$$

Then equation A.11 can be made into the deviation variable form or perturbation form. When $T_{1H} = \overline{T}_{1H} + T_{1H}^P$, $T_{2H} = \overline{T}_{2H} + T_{2H}^P$, and $T_{C2b} = \overline{T}_{C2b} + T_{C2b}^P$.

$$\frac{dT_{2H}^P}{dt} = \frac{\tau_{H1}F_{H1}}{\xi_{H1}}T_{1H}^P + \frac{(-\tau_{H1}F_{H1} - UA_2)}{\xi_{H1}}T_{2H}^P + \frac{UA_2}{\xi_{H1}}T_{C2b}^P \qquad \text{(A.12)}$$

For T_{C2b}:

$$\rho_{C2}V_{C2}C_{PC2}\frac{dT_{C2b}}{dt} = \rho_{C2}F_{C2}C_{PC2}(1 - f_{C2})(\overline{T}_{C2in} - T_{C2b}) + UA_2(T_{2H} - T_{C2b}) \qquad \text{(A.13)}$$

Where $\tau_{C2} = \rho_{C2}C_{PC2}$ and $\xi_{C2} = \rho_{C2}V_{C2}C_{PC2}$

$$\xi_{C2}\frac{dT_{C2b}}{dt} = \tau_{C2}F_{C2}(1 - f_{C2})(\overline{T}_{C2in} - T_{C2b}) + UA_2(T_{2H} - T_{C2b}) \qquad \text{(A.14)}$$

$$\frac{dT_{C2b}}{dt} = \frac{\tau_{C2}F_{C2}}{\xi_{C2}}\overline{T}_{C2in} + \frac{-\tau_{C2}F_{C2}}{\xi_{C2}}T_{C2b} + \frac{-\tau_{C2}F_{C2}}{\xi_{C2}}\overline{T}_{C2in}f_{C2} + \frac{\tau_{C2}F_{C2}}{\xi_{C2}}f_{C2}T_{C2b} + \frac{UA_2}{\xi_{C2}}T_{2H} + \frac{-UA_2}{\xi_{C2}}T_{C2b} \qquad \text{(A.15)}$$

From equation A.15, although most terms on the right are linear, the forth is nonlinear due to T_{C2b} and f_{C2} which are state variables and manipulated variables. Therefore, Taylor's series for linearization as in equation A.16 are presented.

$$\frac{\tau_{C2}F_{C2}}{\xi_{C2}}f_{C2}T_{C2b} = \frac{\tau_{C2}F_{C2}}{\xi_{C2}}\overline{f}_{C2}T_{C2b} + \frac{\tau_{C2}F_{C2}}{\xi_{C2}}\overline{T}_{C2b}f_{C2} + \frac{-\tau_{C2}F_{C2}}{\xi_{C2}}\overline{f}_{C2}\overline{T}_{C2b} \qquad \text{(A.16)}$$

The dynamic model of T_{C2b} become

$$\frac{dT_{C2b}}{dt} = \frac{\tau_{C2}F_{C2}}{\xi_{C2}}\overline{T}_{C2in} + \frac{-\tau_{C2}F_{C2}}{\xi_{C2}}T_{C2b} + \frac{-\tau_{C2}F_{C2}}{\xi_{C2}}\overline{T}_{C2in}f_{C2} + \frac{\tau_{C2}F_{C2}}{\xi_{C2}}\overline{f}_{C2}T_{C2b} + \frac{\tau_{C2}F_{C2}}{\xi_{C2}}\overline{T}_{C2b}f_{C2}$$
$$- \frac{-\tau_{C2}F_{C2}}{\xi_{C2}}\overline{f}_{C2}\overline{T}_{C2b} + \frac{UA_2}{\xi_{C2}}T_{2H} + \frac{-UA_2}{\xi_{C2}}T_{C2b} \qquad \text{(A.17)}$$

$$\frac{dT_{C2b}}{dt} = \frac{UA_2}{\xi_{C2}}T_{2H} + \frac{(-\tau_{C2}F_{C2} + \tau_{C2}F_{C2}\overline{f}_{C2} - UA_2)}{\xi_{C2}}T_{C2b} + \frac{(-\tau_{C2}F_{C2}\overline{T}_{C2in} + \tau_{C2}F_{C2}\overline{T}_{C2b})}{\xi_{C2}}f_{C2}$$
$$\frac{\tau_{C2}F_{C2}}{\xi_{C2}}\overline{T}_{C2in} + \frac{-\tau_{C2}F_{C2}}{\xi_{C2}}\overline{f}_{C2}\overline{T}_{C2b} \qquad \text{(A.18)}$$

At steady state

$$\frac{d\overline{T}_{C2b}}{dt}=\frac{UA_2}{\xi_{C2}}\overline{T}_{2H}+\frac{(-\tau_{C2}F_{C2}+\tau_{C2}F_{C2}\overline{f}_{C2}-UA_2)}{\xi_{C2}}\overline{T}_{C2b}+\frac{(-\tau_{C2}F_{C2}\overline{T}_{C2in}+\tau_{C2}F_{C2}\overline{T}_{C2b})}{\xi_{C2}}\overline{f}_{C2}$$
$$+\frac{\tau_{C2}F_{C2}}{\xi_{C2}}\overline{T}_{C2in}+\frac{-\tau_{C2}F_{C2}}{\xi_{C2}}\overline{f}_{C2}\overline{T}_{C2b} \qquad \text{(A.19)}$$

Then equation A.19 can be made into the deviation variable form or perturbation form. When $T_{C2b}=\overline{T}_{C2b}+T^P_{C2b}$, $T_{2H}=\overline{T}_{2H}+T^P_{2H}$, and $f_{C2}=\overline{f}_{C2}+f^P_{C2}$.

$$\frac{dT^P_{C2b}}{dt}=\frac{UA_2}{\xi_{C2}}T^P_{2H}+\frac{(-\tau_{C2}F_{C2}+\tau_{C2}F_{C2}\overline{f}_{C2}-UA_2)}{\xi_{C2}}T^P_{C2b}+\frac{(-\tau_{C2}F_{C2}\overline{T}_{C2in}+\tau_{C2}F_{C2}\overline{T}_{C2b})}{\xi_{C2}}f^P_{C2} \qquad \text{(A.20)}$$

For cold streams of heat exchanger with a single bypass on cold side equations. An output equation in the form of the state space in case of time invariant system shows below.

$$y=Cx+Du$$

The output equation of cold stream for a single heat exchanger with a single bypass referred from (Mathisen, 1994; Glemmestad, 1999) is presented in equation A.21.

$$T_{C2t}=(1-f_{C2})T_{C2b}+f_{C2}\overline{T}_{C2in} \qquad \text{(A.21)}$$

$$T_{C2t}=T_{C2b}-f_{C2}T_{C2b}+f_{C2}\overline{T}_{C2in} \qquad \text{(A.22)}$$

Since the second term on the right hand side is nonlinear, this term has to be linearized using Taylor’s series. The following equation shows the nonlinear term in equation A.23 is linearized.

$$T_{C2t}=T_{C2b}-\overline{f}_{C2}T_{C2b}-\overline{T}_{C2b}f_{C2}+\overline{f}_{C2}\overline{T}_{C2b}+f_{C2}\overline{T}_{C2in} \qquad \text{(A.23)}$$

$$T_{C2t}=(1-\overline{f}_{C2})T_{C2b}+(\overline{T}_{C2in}-\overline{T}_{C2b})f_{C2}+\overline{f}_{C2}\overline{T}_{C2b} \qquad \text{(A.24)}$$

At steady state

$$\overline{T}_{C2t}=(1-\overline{f}_{C2})\overline{T}_{C2b}+(\overline{T}_{C2in}-\overline{T}_{C2b})\overline{f}_{C2}+\overline{f}_{C2}\overline{T}_{C2b} \qquad \text{(A.25)}$$

Then equation A.25 can be made into the deviation variable form or perturbation form. When $T_{C2t}=\overline{T}_{C2t}+T^P_{C2t}$ and $f_{C2}=\overline{f}_{C2}+f^P_{C2}$.

$$T^P_{C2t}=(1-\overline{f}_{C2})T^P_{C2b}+(\overline{T}_{C2in}-\overline{T}_{C2b})f^P_{C2} \qquad \text{(A.26)}$$

The dynamic model for outlet heat exchanger network at cooler:

Since the dynamic models of cooler from equations 3.3 are the non linear model, linearization by using Taylor's series expansion is necessary that are presented as the following:

For T_{H1t}:

$$\rho_{H1}V_{H1}C_{PH1}\frac{dT_{H1t}}{dt} = \rho_{H1}F_{H1}C_{PH1}(T_{2H} - T_{H1t}) - Q_c \tag{A.27}$$

Where $\tau_{H1} = \rho_{H1}C_{PH1}$ and $\xi_{H1} = \rho_{H1}V_{H1}C_{PH1}$

$$\frac{dT_{H1t}}{dt} = \frac{\tau_{H1}F_{H1}}{\xi_{H1}}T_{2H} + \frac{-1}{\xi_{H1}}Q_c + \frac{-\tau_{H1}F_{H1}}{\xi_{H1}}T_{H1t} \tag{A.28}$$

At steady state

$$\frac{d\overline{T}_{H1t}}{dt} = \frac{\tau_{H1}F_{H1}}{\xi_{H1}}\overline{T}_{2H} + \frac{-1}{\xi_{H1}}\overline{Q}_c + \frac{-\tau_{H1}F_{H1}}{\xi_{H1}}\overline{T}_{H1t} = 0 \tag{A.29}$$

Then equation A.29 can be made into the deviation variable form or perturbation form. When $T_{2H} = \overline{T}_{2H} + T_{2H}^P$, $Q_c = \overline{Q}_c + Q_c^P$, and $T_{H1t} = \overline{T}_{H1t} + T_{H1t}^P$.

$$\frac{dT_{H1t}^P}{dt} = \frac{\tau_{H1}F_{H1}}{\xi_{H1}}T_{2H}^P + \frac{-1}{\xi_{H1}}Q_c^P + \frac{-\tau_{H1}F_{H1}}{\xi_{H1}}T_{H1t}^P \tag{A.30}$$

The dynamic model for outlet heat exchanger network at heater:

Since the dynamic models of heater from Equations 3.4 are the non linear model, linearization by using Taylor's series expansion is necessary that are presented as the following:

For T_{C1t}:

$$\rho_{C1}V_{C1}C_{PC1}\frac{dT_{C1t}}{dt} = \rho_{C1}F_{C1}C_{PC1}(T_{1C} - T_{C1t}) + Q_h \tag{A.31}$$

Where $\tau_{C1} = \rho_{C1}C_{PC1}$ and $\xi_{C1} = \rho_{C1}V_{C1}C_{PC1}$

$$\frac{dT_{C1t}}{dt} = \frac{\tau_{C1}F_{C1}}{\xi_{C1}}T_{1C} + \frac{+1}{\xi_{C1}}Q_h + \frac{-\tau_{C1}F_{C1}}{\xi_{C1}}T_{C1t} \tag{A.32}$$

At steady state

$$\frac{d\overline{T}_{C1t}}{dt} = \frac{\tau_{C1}F_{C1}}{\xi_{C1}}\overline{T}_{1C} + \frac{1}{\xi_{C1}}\overline{Q}_h + \frac{-\tau_{C1}F_{C1}}{\xi_{C1}}\overline{T}_{C1t} = 0 \tag{A.33}$$

Then equation A.33 can be made into the deviation variable form or perturbation form. When $T_{1C} = \overline{T}_{1C} + T_{1C}^P$, $Q_h = \overline{Q}_h + Q_h^P$, and $T_{C1t} = \overline{T}_{C1t} + T_{C1t}^P$.

$$\frac{dT_{C1t}^P}{dt} = \frac{-\tau_{C1}F_{C1}}{\xi_{C1}}T_{C1t}^P + \frac{1}{\xi_{C1}}Q_h^P + \frac{\tau_{C1}F_{C1}}{\xi_{C1}}T_{1C}^P \tag{A.34}$$

A.2 Case study 2

The heat exchanger network from *Biegler* [15] is studied here. There are three heat exchangers and three utility exchangers. Furthermore, 6 manipulated variables, bypasses of heat exchangers 1, 2 and 3 (u_{b1}, u_{b2}, and u_{b3}) and utility duties of two cooler and heater (Q_{c1}, Q_{c2} and Q_h), are available for control of all target outlet temperatures that are outlet temperature of stream H1, outlet temperature of stream H2, outlet temperature of stream C1, and outlet temperature of stream C2. The heat exchanger network configuration is shown Figure A.2.

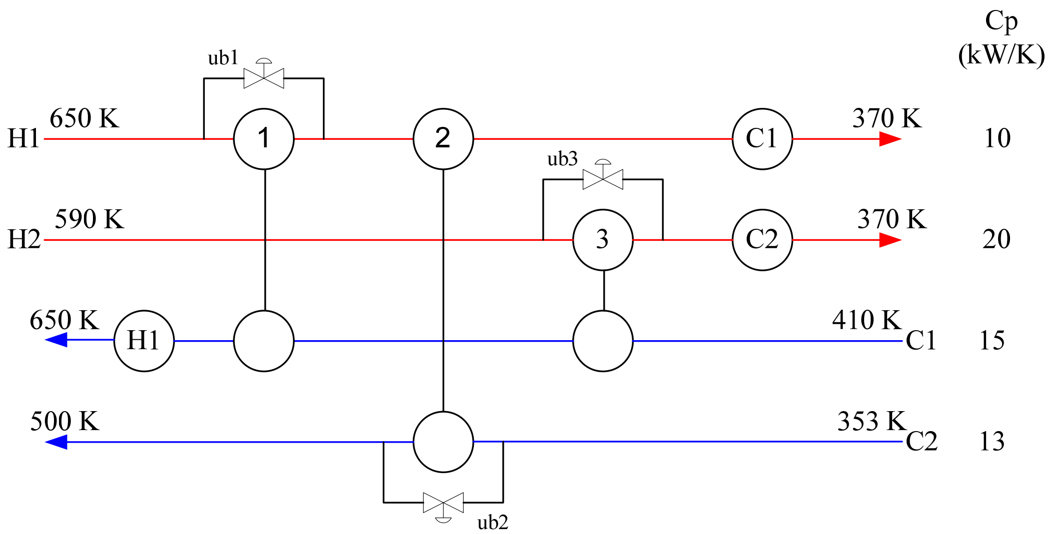

Figure A.2 The heat exchanger networks in case study 2.

In this case study, heat exchanger network can be separated into 2 system, which are inner and outer heat exchanger network.

The dynamic model for inner heat exchanger network:

Since the dynamic models of inner heat exchanger network from equations 3.1-2 are the non linear model, linearization by using Taylor's series expansion is necessary that are presented as the following:

For T_{1H}:

$$\rho_{H1}V_{H1}C_{PH1}\frac{dT_{1H}}{dt}=\rho_{H1}F_{H1}C_{PH1}(T_{H1in}-T_{1H})+UA_1(T_{1C}-T_{1H}) \tag{A.35}$$

Where $\tau_{H1}=\rho_{H1}C_{PH1}$ and $\xi_{H1}=\rho_{H1}V_{H1}C_{PH1}$

$$\frac{dT_{1H}}{dt}=\frac{(-\tau_{H1}F_{H1}-UA_1)}{\xi_{H1}}T_{1H}+\frac{UA_1}{\xi_{H1}}T_{1C}+\frac{\tau_{H1}F_{H1}}{\xi_{H1}}T_{H1in} \tag{A.36}$$

At steady state

$$\frac{d\overline{T}_{1H}}{dt}=\frac{(-\tau_{H1}F_{H1}-UA_1)}{\xi_{H1}}\overline{T}_{1H}+\frac{UA_1}{\xi_{H1}}\overline{T}_{1C}+\frac{\tau_{H1}F_{H1}}{\xi_{H1}}\overline{T}_{H1in}=0 \tag{A.37}$$

Then equation A.37 can be made into the deviation variable form or perturbation form. When $T_{1H}=\overline{T}_{1H}+T_{1H}^{P}$, $T_{1C}=\overline{T}_{1C}+T_{1C}^{P}$, and $T_{H1in}=\overline{T}_{H1in}+T_{H1in}^{P}$.

$$\frac{dT_{1H}^{P}}{dt}=\frac{(-\tau_{H1}F_{H1}-UA_1)}{\xi_{H1}}T_{1H}^{P}+\frac{UA_1}{\xi_{H1}}T_{1C}^{P}+\frac{\tau_{H1}F_{H1}}{\xi_{H1}}T_{H1in}^{P} \tag{A.38}$$

For T_{1C}:

$$\rho_{C1}V_{C1}C_{PC1}\frac{dT_{1C}}{dt}=\rho_{C1}F_{C1}C_{PC1}(T_{3C}-T_{1C})+UA_1(T_{1H}-T_{1C}) \tag{A.39}$$

Where $\tau_{C1}=\rho_{C1}C_{PC1}$ and $\xi_{C1}=\rho_{C1}V_{C1}C_{PC1}$

$$\frac{dT_{1C}}{dt}=\frac{UA_1}{\xi_{C1}}T_{1H}+\frac{(-\tau_{C1}F_{C1}-UA_1)}{\xi_{C1}}T_{1C}+\frac{\tau_{C1}F_{C1}}{\xi_{C1}}T_{3C} \tag{A.40}$$

At steady state

$$\frac{d\overline{T}_{1C}}{dt}=\frac{UA_1}{\xi_{C1}}\overline{T}_{1H}+\frac{(-\tau_{C1}F_{C1}-UA_1)}{\xi_{C1}}\overline{T}_{1C}+\frac{\tau_{C1}F_{C1}}{\xi_{C1}}\overline{T}_{3C}=0 \tag{A.41}$$

Then equation A.41 can be made into the deviation variable form or perturbation form. When $T_{1C}=\overline{T}_{1C}+T_{1C}^{P}, T_{3C}=\overline{T}_{3C}+T_{3C}^{P}$, and $T_{1H}=\overline{T}_{1H}+T_{1H}^{P}$.

$$\frac{dT_{1C}^{P}}{dt}=\frac{UA_1}{\xi_{C1}}T_{1H}^{P}+\frac{(-\tau_{C1}F_{C1}-UA_1)}{\xi_{C1}}T_{1C}^{P}+\frac{\tau_{C1}F_{C1}}{\xi_{C1}}T_{3C}^{P} \tag{A.42}$$

For T_{2H}:

$$\rho_{H1}V_{H1}C_{PH1}\frac{dT_{2H}}{dt}=\rho_{H1}F_{H1}C_{PH1}(T_{1H}-T_{2H})+UA_2(T_{C2b}-T_{2H}) \quad (A.43)$$

Where $\tau_{H1}=\rho_{H1}C_{PH1}$ and $\xi_{H1}=\rho_{H1}V_{H1}C_{PH1}$

$$\frac{dT_{2H}}{dt}=\frac{\tau_{H1}F_{H1}}{\xi_{H1}}T_{1H}+\frac{(-\tau_{H1}F_{H1}-UA_2)}{\xi_{H1}}T_{2H}+\frac{UA_2}{\xi_{H1}}T_{C2b} \quad (A.44)$$

At steady state

$$\frac{d\overline{T}_{2H}}{dt}=\frac{\tau_{H1}F_{H1}}{\xi_{H1}}\overline{T}_{1H}+\frac{(-\tau_{H1}F_{H1}-UA_2)}{\xi_{H1}}\overline{T}_{2H}+\frac{UA_2}{\xi_{H1}}\overline{T}_{C2b}=0 \quad (A.45)$$

Then equation A.45 can be made into the deviation variable form or perturbation form. When $T_{1H}=\overline{T}_{1H}+T^P_{1H}$, $T_{2H}=\overline{T}_{2H}+T^P_{2H}$, and $T_{C2b}=\overline{T}_{C2b}+T^P_{C2b}$.

$$\frac{dT^P_{2H}}{dt}=\frac{\tau_{H1}F_{H1}}{\xi_{H1}}T^P_{1H}+\frac{(-\tau_{H1}F_{H1}-UA_2)}{\xi_{H1}}T^P_{2H}+\frac{UA_2}{\xi_{H1}}T^P_{C2b} \quad (A.46)$$

For T_{C2b}:

$$\rho_{C2}V_{C2}C_{PC2}\frac{dT_{C2b}}{dt}=\rho_{C2}F_{C2}C_{PC2}(1-f_{C2})(T_{C2in}-T_{C2b})+UA_2(T_{2H}-T_{C2b}) \quad (A.47)$$

Where $\tau_{C2}=\rho_{C2}C_{PC2}$ and $\xi_{C2}=\rho_{C2}V_{C2}C_{PC2}$

$$\xi_{C2}\frac{dT_{C2b}}{dt}=\tau_{C2}F_{C2}(1-f_{C2})(T_{C2in}-T_{C2b})+UA_2(T_{2H}-T_{C2b}) \quad (A.48)$$

$$\frac{dT_{C2b}}{dt}=\frac{\tau_{C2}F_{C2}}{\xi_{C2}}\overline{T}_{C2in}+\frac{-\tau_{C2}F_{C2}}{\xi_{C2}}T_{C2b}+\frac{-\tau_{C2}F_{C2}}{\xi_{C2}}T_{C2in}f_{C2}+\frac{\tau_{C2}F_{C2}}{\xi_{C2}}f_{C2}T_{C2b}+\frac{UA_2}{\xi_{C2}}T_{2H}+\frac{-UA_2}{\xi_{C2}}T_{C2b} \quad (A.49)$$

From equation A.49, although most terms on the right are linear, the forth is non linear due to T_{C2b} and f_{C2} which are state variables and manipulated variables. Therefore, Taylor's series for linearization as in equation A.50 are presented.

$$\frac{\tau_{C2}F_{C2}}{\xi_{C2}}f_{C2}T_{C2b}=\frac{\tau_{C2}F_{C2}}{\xi_{C2}}\overline{f}_{C2}T_{C2b}+\frac{\tau_{C2}F_{C2}}{\xi_{C2}}\overline{T}_{C2b}f_{C2}+\frac{-\tau_{C2}F_{C2}}{\xi_{C2}}\overline{f}_{C2}\overline{T}_{C2b} \quad (A.50)$$

$$\frac{-\tau_{C2}F_{C2}}{\xi_{C2}}T_{C2in}f_{C2}=\frac{-\tau_{C2}F_{C2}}{\xi_{C2}}\overline{f}_{C2}T_{C2in}+\frac{-\tau_{C2}F_{C2}}{\xi_{C2}}\overline{T}_{C2in}f_{C2}+\frac{\tau_{C2}F_{C2}}{\xi_{C2}}\overline{f}_{C2}\overline{T}_{C2in} \quad (A.51)$$

The dynamic model of T_{C2b} become

$$\frac{dT_{C2b}}{dt}=\frac{\tau_{C2}F_{C2}}{\xi_{C2}}\overline{T}_{C2in}+\frac{-\tau_{C2}F_{C2}}{\xi_{C2}}T_{C2b}+\frac{-\tau_{C2}F_{C2}}{\xi_{C2}}\overline{f}_{C2}T_{C2in}+\frac{-\tau_{C2}F_{C2}}{\xi_{C2}}\overline{T}_{C2in}f_{C2}+\frac{\tau_{C2}F_{C2}}{\xi_{C2}}\overline{f}_{C2}\overline{T}_{C2in}$$
$$+\frac{\tau_{C2}F_{C2}}{\xi_{C2}}\overline{f}_{C2}T_{C2b}+\frac{\tau_{C2}F_{C2}}{\xi_{C2}}\overline{T}_{C2b}f_{C2}-\frac{-\tau_{C2}F_{C2}}{\xi_{C2}}\overline{f}_{C2}\overline{T}_{C2b}+\frac{UA_2}{\xi_{C2}}T_{2H}+\frac{-UA_2}{\xi_{C2}}T_{C2b} \quad \text{(A.52)}$$

$$\frac{dT_{C2b}}{dt}=\frac{UA_2}{\xi_{C2}}T_{2H}+\frac{(-\tau_{C2}F_{C2}+\tau_{C2}F_{C2}\overline{f}_{C2}-UA_2)}{\xi_{C2}}T_{C2b}+\frac{(-\tau_{C2}F_{C2}\overline{T}_{C2in}+\tau_{C2}F_{C2}\overline{T}_{C2b})}{\xi_{C2}}f_{C2}$$
$$\frac{\tau_{C2}F_{C2}-\tau_{C2}F_{C2}\overline{f}_{C2}}{\xi_{C2}}T_{C2in}+\frac{-\tau_{C2}F_{C2}}{\xi_{C2}}\overline{f}_{C2}\overline{T}_{C2b}+\frac{\tau_{C2}F_{C2}}{\xi_{C2}}\overline{f}_{C2}\overline{T}_{C2in} \quad \text{(A.53)}$$

At steady state

$$\frac{d\overline{T}_{C2b}}{dt}=\frac{UA_2}{\xi_{C2}}\overline{T}_{2H}+\frac{(-\tau_{C2}F_{C2}+\tau_{C2}F_{C2}\overline{f}_{C2}-UA_2)}{\xi_{C2}}\overline{T}_{C2b}+\frac{(-\tau_{C2}F_{C2}\overline{T}_{C2in}+\tau_{C2}F_{C2}\overline{T}_{C2b})}{\xi_{C2}}\overline{f}_{C2}$$
$$+\frac{\tau_{C2}F_{C2}-\tau_{C2}F_{C2}\overline{f}_{C2}}{\xi_{C2}}\overline{T}_{C2in}+\frac{-\tau_{C2}F_{C2}}{\xi_{C2}}\overline{f}_{C2}\overline{T}_{C2b}+\frac{\tau_{C2}F_{C2}}{\xi_{C2}}\overline{f}_{C2}\overline{T}_{C2in} \quad \text{(A.54)}$$

Then equation A.54 can be made into the deviation variable form or perturbation form. When $T_{C2b}=\overline{T}_{C2b}+T^P_{C2b}$, $T_{2H}=\overline{T}_{2H}+T^P_{2H}$, and $f_{C2}=\overline{f}_{C2}+f^P_{C2}$.

$$\frac{dT^P_{C2b}}{dt}=\frac{UA_2}{\xi_{C2}}T^P_{2H}+\frac{(-\tau_{C2}F_{C2}+\tau_{C2}F_{C2}\overline{f}_{C2}-UA_2)}{\xi_{C2}}T^P_{C2b}+\frac{(-\tau_{C2}F_{C2}\overline{T}_{C2in}+\tau_{C2}F_{C2}\overline{T}_{C2b})}{\xi_{C2}}f^P_{C2}$$
$$+\frac{\tau_{C2}F_{C2}-\tau_{C2}F_{C2}\overline{f}_{C2}}{\xi_{C2}}T^P_{C2in} \quad \text{(A.55)}$$

For T_{3H}:

$$\rho_{H2}V_{H2}C_{PH2}\frac{dT_{3H}}{dt}=\rho_{H2}F_{H2}C_{PH2}(T_{H2in}-T_{3H})+UA_3(T_{3C}-T_{3H}) \quad \text{(A.56)}$$

Where $\tau_{H2}=\rho_{H2}C_{PH2}$ and $\xi_{H2}=\rho_{H2}V_{H2}C_{PH2}$

$$\frac{dT_{3H}}{dt}=\frac{(-\tau_{H2}F_{H2}-UA_3)}{\xi_{H2}}T_{3H}+\frac{UA_3}{\xi_{H2}}T_{3C}+\frac{\tau_{H2}F_{H2}}{\xi_{H2}}T_{H2in} \quad \text{(A.57)}$$

At steady state

$$\frac{d\overline{T}_{3H}}{dt}=\frac{(-\tau_{H2}F_{H2}-UA_3)}{\xi_{H2}}\overline{T}_{3H}+\frac{UA_3}{\xi_{H2}}\overline{T}_{3C}+\frac{\tau_{H2}F_{H2}}{\xi_{H2}}\overline{T}_{H2in}=0 \quad \text{(A.58)}$$

Then equation A.58 can be made into the deviation variable form or perturbation form. When $T_{3H} = \overline{T}_{3H} + T_{3H}^P$, $T_{3C} = \overline{T}_{3C} + T_{3C}^P$, and $T_{H2in} = \overline{T}_{H2in} + T_{H2in}^P$.

$$\frac{dT_{3H}^P}{dt} = \frac{(-\tau_{H2}F_{H2} - UA_3)}{\xi_{H2}} T_{3H}^P + \frac{UA_3}{\xi_{H2}} T_{3C}^P + \frac{\tau_{H2}F_{H2}}{\xi_{H2}} T_{H2in}^P \tag{A.59}$$

For T_{3C}:

$$\rho_{C1}V_{C1}C_{PC1}\frac{dT_{3C}}{dt} = \rho_{C1}F_{C1}C_{PC1}(T_{C1in} - T_{3C}) + UA_3(T_{3H} - T_{3C}) \tag{A.60}$$

Where $\tau_{C1} = \rho_{C1}C_{PC1}$ and $\xi_{C1} = \rho_{C1}V_{C1}C_{PC1}$

$$\frac{dT_{3C}}{dt} = \frac{UA_3}{\xi_{C1}} T_{3H} + \frac{(-\tau_{C1}F_{C1} - UA_3)}{\xi_{C1}} T_{3C} + \frac{\tau_{C1}F_{C1}}{\xi_{C1}} T_{C1in} \tag{A.61}$$

At steady state

$$\frac{d\overline{T}_{3C}}{dt} = \frac{UA_3}{\xi_{C1}} \overline{T}_{3H} + \frac{(-\tau_{C1}F_{C1} - UA_3)}{\xi_{C1}} \overline{T}_{3C} + \frac{\tau_{C1}F_{C1}}{\xi_{C1}} \overline{T}_{C1in} = 0 \tag{A.62}$$

Then equation A.62 can be made into the deviation variable form or perturbation form. When $T_{3C} = \overline{T}_{3C} + T_{3C}^P$, $T_{C1in} = \overline{T}_{C1in} + T_{C1in}^P$, and $T_{3H} = \overline{T}_{3H} + T_{3H}^P$.

$$\frac{dT_{3C}^P}{dt} = \frac{UA_3}{\xi_{C1}} T_{3H}^P + \frac{(-\tau_{C1}F_{C1} - UA_3)}{\xi_{C1}} T_{3C}^P + \frac{\tau_{C1}F_{C1}}{\xi_{C1}} T_{C1in}^P \tag{A.63}$$

For cold streams of heat exchanger with a single bypass on cold side equations. An output equation in the form of state space in case of time invariant system shows below.

$$y = Cx + Du$$

The output equation of cold stream for a single heat exchanger with a single bypass referred from (Mathisen and Glemmestad) is presented in Equation.

$$T_{C2t} = (1 - f_{C2})T_{C2b} + f_{C2}\overline{T}_{C2in} \tag{A.64}$$

$$T_{C2t} = T_{C2b} - f_{C2}T_{C2b} + f_{C2}\overline{T}_{C2in} \tag{A.65}$$

Since the second term on the right hand side is nonlinear, this term has to be linearized using Taylor's series. The following equation shows the nonlinear term in equation A.66-67 is linearized.

$$T_{C2t} = T_{C2b} - \overline{f}_{C2}T_{C2b} - \overline{T}_{C2b}f_{C2} + \overline{f}_{C2}\overline{T}_{C2b} + f_{C2}\overline{T}_{C2in} \tag{A.66}$$

$$T_{C2t} = (1-\overline{f}_{C2})T_{C2b} + (\overline{T}_{C2in} - \overline{T}_{C2b})f_{C2} + \overline{f}_{C2}\overline{T}_{C2b} \quad \text{(A.67)}$$

At steady state

$$\overline{T}_{C2t} = (1-\overline{f}_{C2})\overline{T}_{C2b} + (\overline{T}_{C2in} - \overline{T}_{C2b})\overline{f}_{C2} + \overline{f}_{C2}\overline{T}_{C2b} \quad \text{(A.68)}$$

Then equation A.68 can be made into the deviation variable form or perturbation form. When $T_{C2t} = \overline{T}_{C2t} + T^P_{C2t}$ and $f_{C2} = \overline{f}_{C2} + f^P_{C2}$.

$$T^P_{C2t} = (1-\overline{f}_{C2})T^P_{C2b} + (\overline{T}_{C2in} - \overline{T}_{C2b})f^P_{C2} \quad \text{(A.69)}$$

The dynamic model for outlet heat exchanger network at cooler 1:

Since the dynamic models of cooler 1 from equation 3.3 are the non linear model, linearization by using Taylor's series expansion is necessary that are presented as the following:

For T_{H1t}

$$\rho_{H1}V_{H1}C_{PH1}\frac{dT_{H1t}}{dt} = \rho_{H1}F_{H1}C_{PH1}(T_{2H} - T_{H1t}) - Q_{c1} \quad \text{(A.70)}$$

Where $\tau_{H1} = \rho_{H1}C_{PH1}$ and $\xi_{H1} = \rho_{H1}V_{H1}C_{PH1}$

$$\frac{dT_{H1t}}{dt} = \frac{\tau_{H1}F_{H1}}{\xi_{H1}}T_{2H} + \frac{-1}{\xi_{H1}}Q_{c1} + \frac{-\tau_{H1}F_{H1}}{\xi_{H1}}T_{H1t} \quad \text{(A.71)}$$

At steady state

$$\frac{d\overline{T}_{H1t}}{dt} = \frac{\tau_{H1}F_{H1}}{\xi_{H1}}\overline{T}_{2H} + \frac{-1}{\xi_{H1}}\overline{Q}_{c1} + \frac{-\tau_{H1}F_{H1}}{\xi_{H1}}\overline{T}_{H1t} = 0 \quad \text{(A.72)}$$

Then equation A.72 can be made into the deviation variable form or perturbation form. When $T_{2H} = \overline{T}_{2H} + T^P_{2H}$, $Q_{c1} = \overline{Q}_{c1} + Q^P_{c1}$, and $T_{H1t} = \overline{T}_{H1t} + T^P_{H1t}$.

$$\frac{dT^P_{H1t}}{dt} = \frac{\tau_{H1}F_{H1}}{\xi_{H1}}T^P_{2H} + \frac{-1}{\xi_{H1}}Q^P_{c1} + \frac{-\tau_{H1}F_{H1}}{\xi_{H1}}T^P_{H1t} \quad \text{(A.73)}$$

The dynamic model for outlet heat exchanger network at cooler 2:

Since the dynamic models of cooler 2 from equation 3.3 are the non linear model, linearization by using Taylor's series expansion is necessary that are presented as the following:

For T_{H2t}

$$\rho_{H2}V_{H2}C_{PH2}\frac{dT_{H2t}}{dt} = \rho_{H2}F_{H2}C_{PH2}(T_{3H} - T_{H2t}) - Q_{c2} \tag{A.74}$$

Where $\tau_{H2} = \rho_{H2}C_{PH2}$ and $\xi_{H2} = \rho_{H2}V_{H2}C_{PH2}$

$$\frac{dT_{H2t}}{dt} = \frac{\tau_{H2}F_{H2}}{\xi_{H2}}T_{3H} + \frac{-1}{\xi_{H2}}Q_{c2} + \frac{-\tau_{H2}F_{H2}}{\xi_{H2}}T_{H2t} \tag{A.75}$$

At steady state

$$\frac{d\overline{T}_{H2t}}{dt} = \frac{\tau_{H2}F_{H2}}{\xi_{H2}}\overline{T}_{3H} + \frac{-1}{\xi_{H2}}\overline{Q}_{c2} + \frac{-\tau_{H2}F_{H2}}{\xi_{H2}}\overline{T}_{H2t} = 0 \tag{A.76}$$

Then equation A.76 can be made into the deviation variable form or perturbation form. When $T_{3H} = \overline{T}_{3H} + T_{3H}^{P}$, $Q_{c2} = \overline{Q}_{c2} + Q_{c2}^{P}$, and $T_{H2t} = \overline{T}_{H2t} + T_{H2t}^{P}$.

$$\frac{dT_{H2t}^{P}}{dt} = \frac{\tau_{H2}F_{H2}}{\xi_{H2}}T_{3H}^{P} + \frac{-1}{\xi_{H2}}Q_{c2}^{P} + \frac{-\tau_{H2}F_{H2}}{\xi_{H2}}T_{H2t}^{P} \tag{A.77}$$

The dynamic model for outlet heat exchanger network at heater:

Since the dynamic models of heater from equation 3.4 are the non linear model, linearization by using Taylor's series expansion is necessary that are presented as the following:

For T_{C1t}:

$$\rho_{C1}V_{C1}C_{PC1}\frac{dT_{C1t}}{dt} = \rho_{C1}F_{C1}C_{PC1}(T_{1C} - T_{C1t}) + Q_{h} \tag{A.78}$$

Where $\tau_{C1} = \rho_{C1}C_{PC1}$ and $\xi_{C1} = \rho_{C1}V_{C1}C_{PC1}$

$$\frac{dT_{C1t}}{dt} = \frac{\tau_{C1}F_{C1}}{\xi_{C1}}T_{1C} + \frac{+1}{\xi_{C1}}Q_{h} + \frac{-\tau_{C1}F_{C1}}{\xi_{C1}}T_{C1t} \tag{A.79}$$

At steady state

$$\frac{d\overline{T}_{C1t}}{dt} = \frac{\tau_{C1}F_{C1}}{\xi_{C1}}\overline{T}_{1C} + \frac{1}{\xi_{C1}}\overline{Q}_h + \frac{-\tau_{C1}F_{C1}}{\xi_{C1}}\overline{T}_{C1t} = 0 \tag{A.80}$$

Then equation A.80 can be made into the deviation variable form or perturbation form. When $T_{1C} = \overline{T}_{1C} + T_{1C}^P$, $Q_h = \overline{Q}_h + Q_h^P$, and $T_{C1t} = \overline{T}_{C1t} + T_{C1t}^P$.

$$\frac{dT_{C1t}^P}{dt} = \frac{-\tau_{C1}F_{C1}}{\xi_{C1}}T_{C1t}^P + \frac{1}{\xi_{C1}}Q_h^P + \frac{\tau_{C1}F_{C1}}{\xi_{C1}}T_{1C}^P \tag{A.81}$$

APPENDIX B

PROPERTIES OF CONSTANT VALUE

B.1 Case study 1

B.1.1Constant value

Constant densities of the two fluids (ρ_i) = 1,000 kg/m^3

Constant specific heat capacities of the two fluids ($C_{V,i}$) = 4.2 kJ/kg·ºC

Constant and flow independent heat transfer coefficient ($V_{h,i}$,$V_{c,i}$) = 0.1 m^2

B.1.2 Normal operating value

Inlet hot temperature of stream H1 (T_{h1in}) = 190 ºC
Inlet cold temperature of stream C1 (T_{c1in}) = 80 ºC
Inlet cold temperature of stream C2 (T_{c2in}) = 20 ºC
Outlet hot temperature of stream H1 (T_{h1t}) = 30 ºC
Outlet cold temperature of stream C1 (T_{c1t}) = 160 ºC
Outlet cold temperature of stream C2 (T_{c2t}) = 130 ºC
Heat convection of stream H1 per temperature ($mC_{p, h1}$) = 1 kW/ ºC
Heat convection of stream C1 per temperature ($mC_{p, c1}$) = 1.5 kW/ ºC
Heat convection of stream C2 per temperature ($mC_{p, c2}$) = 0.5 kW/ ºC
Overall heat transfer coefficient per heat exchanger area 1 (UA_1) = 0.523 kW/ ºC
Overall heat transfer coefficient per heat exchanger area 2 (UA_2) = 1.322 kW/ ºC

B.1.3 Steady state value

Split fraction of bypassed heat exchanger 1 (u_{h1}) = 0
Split fraction of bypassed heat exchanger 2 (u_{c2}) = 0.00012
Duty utility of cooler (Q_c) = 65 kW
Duty utility of heater (Q_h) = 80 kW
Outlet temperature from heat exchanger 1 (T_{h1b}) = 151.19ºC
Outlet temperature from heat exchanger 2 (T_{c2b}) = 130.89ºC

B.2 Case study 2

B.2.1Constant value

Constant densities of the two fluids (ρ_i) = 1,000 kg/m^3

Constant specific heat capacities of the two fluids ($C_{V,i}$) = 4.2 kJ/kg·K

Constant and flow independent heat transfer coefficient ($V_{h,i}$,$V_{c,i}$) = 0.1 m^2

B.2.2 Normal operating value

Inlet hot temperature of stream H1 (T_{h1in}) = 650 K
Inlet hot temperature of stream H2 (T_{h2in}) = 590 K
Inlet cold temperature of stream C1 (T_{c1in}) = 410 K

Inlet cold temperature of stream C2 (T_{c2in})	= 353	K
Outlet hot temperature of stream H1 (T_{h1t})	= 370	K
Outlet hot temperature of stream H2 (T_{h2t})	= 370	K
Outlet cold temperature of stream C1 (T_{c1t})	= 650	K
Outlet cold temperature of stream C2 (T_{c2t})	= 500	K
Heat convection of stream H1 per temperature ($mC_{p,\ h1}$)	= 10	kW/ K
Heat convection of stream H2 per temperature ($mC_{p,\ h2}$)	= 20	kW/ K
Heat convection of stream C1 per temperature ($mC_{p,\ c1}$)	= 15	kW/ K
Heat convection of stream C2 per temperature ($mC_{p,\ c2}$)	= 13	kW/ K

Overall heat transfer coefficient per heat exchanger area 1 (UA_1) =691.84 kW/ K
Overall heat transfer coefficient per heat exchanger area 2 (UA_2) =1,911 kW/ K
Overall heat transfer coefficient per heat exchanger area 3 (UA_3) =2,419.49 kW/ K

B.2.3 Steady state value

Split fraction of bypassed heat exchanger 1 (u_{h1})	= 0	
Split fraction of bypassed heat exchanger 2 (u_{c2})	= 0.3797	
Split fraction of bypassed heat exchanger 3 (u_{h3})	= 0	
Duty utility of cooler 1 (Q_{c1})	= 289	kW
Duty utility of cooler 2 (Q_{c2})	= 1,700	kW
Duty utility of heater (Q_h)	= 300	kW
Outlet temperature from heat exchanger 2 (T_{c2b})	= 589.98	K

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