APPENDIX F
DERIVATION OF THE HOUSEHOLD CONSUMPTION DEMAND

Household demand for each type of commodity can be found from the problem that a household tries to maximize utility from consuming commodities over the set COM under the budget constraint which can be shown by the problem:

$$\underset{X_c^{(2)}}{Max} \quad U \;=\; \prod_{c\in COM} \left(X_c^{(2)} - \Theta_c \right)^{\beta_c}$$

$$st. \qquad C^H \;=\; \sum_{c\in COM} P_c . X_c^{(2)}$$

where U is the household utility function

$X_c^{(2)}$ is the household consumption on composite commodity c

P_c is the price of composite commodity c

C^H is the total household consumption expenditure

Θ_c is the minimum or subsistence level of composite commodity c

β_c is the marginal budget share of commodity c

Transforming the Stone-Geary utility function with logarithmic function (strictly increasing function) yields

$$\ln U \;=\; \sum_{c\in COM} \beta_c . \ln\left(X_c^{(2)} - \Theta_c \right)$$

Let $Z_c \;=\; \left(X_c^{(2)} - \Theta_c \right)$, then the above problem becomes

$$\underset{Z_c}{Max} \quad \ln U \;=\; \sum_{c\in COM} \beta_c . \ln\left(Z_c \right)$$

$$st. \qquad \sum_{c\in COM} P_c . Z_c \;=\; C^H - \sum_{c\in COM} P_c . \Theta_c$$

From this maximization problem, Lagrangian function is written as

$$\xi \;=\; \sum_{c\in COM} \beta_c . \ln\left(Z_c \right) \;-\; \lambda . \left[\sum_{c\in COM} P_c . Z_c - C^H + \sum_{c\in COM} P_c . \Theta_c \right] \qquad \text{(F.1)}$$

Taking derivative the Lagrangian function in (F.1) with respect to Z_c and λ yields the following necessary conditions:

$$\xi_{Z_c} = 0 \;; \quad \frac{\beta_c}{Z_c} = \lambda . P_c \tag{F.2}$$

$$\xi_{\lambda} = 0 \;; \quad \sum_{c \in COM} P_c . Z_c = C^H - \sum_{c \in COM} P_c . \Theta_c \tag{F.3}$$

Rewrite the terms in (F.2), then

$$P_c . Z_c = \frac{\beta_c}{\lambda} \tag{F.4}$$

From (F.3) and (F.4), then

$$\frac{\sum_{c \in COM} \beta_c}{\lambda} = C^H - \sum_{c \in COM} P_c . \Theta_c \tag{F.5}$$

By Engel aggregation, $\sum_{c \in COM} \beta_c = 1$, then (F.5) becomes

$$\frac{1}{\lambda} = C^H - \sum_{c \in COM} P_c . \Theta_c \tag{F.6}$$

Substitute (F.6) into (F.4), then

$$P_c . Z_c = \beta_c . \left(C^H - \sum_{c \in COM} P_c . \Theta_c \right) \tag{F.7}$$

Substitute $Z_c = \left(X_c^{(2)} - \Theta_c \right)$ into (F.7), then

$$P_c . \left(X_c^{(2)} - \Theta_c \right) = \beta_c . \left(C^H - \sum_{k \in COM} P_k . \Theta_k \right)$$

Therefore, the household demand for commodity c is given by

$$X_c^{(2)} = \frac{\beta_c . \left(C^H - \sum_{k \in COM} P_k . \Theta_k \right)}{P_c} + \Theta_c \;; \quad for \;\; c \in COM \tag{F.8}$$