APPENDIX E

DERIVATION OF THE PRIMARY FACTOR DEMAND

Producer in each production sector combines labor and capital in order to produce the value added product or the effective input combination of primary factors such that total cost of production is the least which can be shown by the problem:

$$\underset{L_i, K_i}{Min} \quad W_i.L_i + R_i.K_i$$

$$st. \quad VA_i = \left(L_i^{-\rho_i} + K_i^{-\rho_i}\right)^{-\frac{1}{\rho_i}} \quad ; \quad for \quad i \in IND$$

where L_i and W_i are the quantity and wage rate of labor

K_i and R_i are the quantity and rental rate of capital

VA_i is the value added product or effective input combination of primary factors used in producing the output of production sector i

$\sigma_i^{LK} = \left(\dfrac{1}{1+\rho_i}\right)$ is an elasticity of substitution between labor and capital in production sector i

From this minimization problem, Lagrangian function can be written as

$$\xi = W_i.L_i + R_i.K_i - \Lambda.\left[\left(L_i^{-\rho_i} + K_i^{-\rho_i}\right)^{-\frac{1}{\rho_i}} - VA_i\right] \qquad \text{(E.1)}$$

Taking derivative of the Lagrangian function in (E.1) with respect to L_i, K_i and Λ yields the following necessary conditions:

$$\xi_{L_i} = 0 \; ; \quad W_i = \Lambda.\left(\frac{1}{\rho_i}\right).\left(L_i^{-\rho_i} + K_i^{-\rho_i}\right)^{-\left(\frac{1+\rho_i}{\rho_i}\right)}.\left(-\rho_i\right).L_i^{-\rho_i-1} \qquad \text{(E.2)}$$

$$\xi_{K_i} = 0 \; ; \quad R_i = \Lambda.\left(\frac{1}{\rho_i}\right).\left(L_i^{-\rho_i} + K_i^{-\rho_i}\right)^{-\left(\frac{1+\rho_i}{\rho_i}\right)}.\left(-\rho_i\right).K_i^{-\rho_i-1} \qquad \text{(E.3)}$$

$$\xi_{\Lambda} = 0 \; ; \quad VA_i = \left(L_i^{-\rho_i} + K_i^{-\rho_i}\right)^{-\frac{1}{\rho_i}} \qquad \text{(E.4)}$$

Divide (E.2) by (E.3), then

$$\frac{W_i}{R_i} = \left(\frac{L_i}{K_i}\right)^{-(1+\rho_i)} \tag{E.5}$$

Rearrange the term in (E.5), then

$$L_i = \left(\frac{W_i}{R_i}\right)^{-\left(\frac{1}{1+\rho_i}\right)} . K_i \tag{E.6}$$

Substitute (E.6) into (E.4), then

$$VA_i = \left[\left(\frac{W_i}{R_i}\right)^{\frac{\rho_i}{1+\rho_i}} .K_i^{-\rho_i} + K_i^{-\rho_i}\right]^{-\frac{1}{\rho_i}}$$

$$VA_i = \left[\left(\frac{W_i}{R_i}\right)^{\frac{\rho_i}{1+\rho_i}} + 1\right]^{-\frac{1}{\rho_i}} . K_i$$

$$K_i = VA_i . \left[\left(\frac{W_i}{R_i}\right)^{\frac{\rho_i}{1+\rho_i}} + 1\right]^{\frac{1}{\rho_i}}$$

$$K_i = VA_i . \left(\frac{W_i + R_i}{R_i}\right)^{\frac{1}{1+\rho_i}}$$

$$K_i = VA_i . \left(\frac{R_i}{W_i + R_i}\right)^{-\left(\frac{1}{1+\rho_i}\right)} \tag{E.7}$$

Substitute $\sigma_i^{LK} = \left(\frac{1}{1+\rho_i}\right)$ into (E.7), then

$$K_i = VA_i . \left(\frac{R_i}{W_i + R_i}\right)^{-\sigma_i^{LK}} \tag{E.8}$$

Substitute K_i in (E.7) into (E.6), then

$$L_i \quad = \quad VA_i . \left(\frac{W_i}{R_i}\right)^{-\left(\frac{1}{1+\rho_i}\right)} . \left(\frac{R_i}{W_i + R_i}\right)^{-\left(\frac{1}{1+\rho_i}\right)}$$

$$L_i \quad = \quad VA_i . \left(\frac{W_i}{W_i + R_i}\right)^{-\left(\frac{1}{1+\rho_i}\right)} \qquad \text{(E.9)}$$

Substitute $\sigma_i^{LK} = \left(\frac{1}{1+\rho_i}\right)$ into (E.9), then

$$L_i \quad = \quad VA_i . \left(\frac{W_i}{W_i + R_i}\right)^{-\sigma_i^{LK}} \qquad \text{(E.10)}$$

Thus, labor and capital demand of the production sector i is given by

$$L_i \quad = \quad VA_i . \left(\frac{W_i}{W_i + R_i}\right)^{-\sigma_i^{LK}} \quad ; \ for \quad i \in IND$$

$$K_i \quad = \quad VA_i . \left(\frac{R_i}{W_i + R_i}\right)^{-\sigma_i^{LK}} \quad ; \ for \quad i \in IND$$