APPENDIX D

DERIVATION OF THE PERCENTAGE CHANGE OF VARIABLE

According to Dixon et al. (1992), in deriving the percentage change forms of variables in the model, three rules are applied as follows.

Let the capital letter (X , Y , Z) be the variable and small letter (x , y , z) be the percentage change of that variable where α and β are constant parameters.

- **The product rule**

$$Z = \beta.\prod_{i=1}^{n} X_i \tag{D.1}$$

Taking logarithmic function on equation (D.1), thus, yields

$$\ln Z = \ln\beta + \sum_{i=1}^{n} \ln X_i \tag{D.2}$$

Taking total differentiation of (D.2) yields

$$\frac{1}{Z}.dZ = \sum_{i=1}^{n} \frac{1}{X_i}.dX_i$$

or $$z = \sum_{i=1}^{n} x_i$$

- **The power rule**

$$Z = \beta.\prod_{i=1}^{n} X_i^{\alpha_i} \tag{D.3}$$

Taking logarithmic function on (D.3), thus, yields

$$\ln Z = \ln\beta + \sum_{i=1}^{n} \alpha_i.\ln X_i \tag{D.4}$$

Taking total differentiation of (D.4) yields

$$\frac{1}{Z}.dZ = \sum_{i=1}^{n} \alpha_i.\frac{1}{X_i}.dX_i$$

or $$z = \sum_{i=1}^{n} \alpha_i . x_i$$

- **The sum rule**

$$Z = \sum_{i=1}^{n} X_i . Y_i \qquad \text{(D.5)}$$

Taking logarithmic function on (D.5), thus, yields

$$\ln Z = \ln\left(\sum_{i=1}^{n} X_i . Y_i\right) \qquad \text{(D.6)}$$

Taking total differentiation of (D.6) yields

$$\frac{1}{Z}.dZ = \frac{1}{\sum_{i=1}^{n} X_i . Y_i}.\left[d\left(\sum_{i=1}^{n} X_i . Y_i\right)\right]$$

$$\frac{1}{Z}.dZ = \frac{1}{\sum_{i=1}^{n} X_i . Y_i}.\left[d\left(X_1 . Y_1\right) + ... + d\left(X_n . Y_n\right)\right]$$

$$\frac{1}{Z}.dZ = \frac{1}{\sum_{i=1}^{n} X_i . Y_i}.\left[\left(X_1 . dY_1 + Y_1 . dX_1\right) + ... + \left(X_n . dY_n + Y_n . dX_n\right)\right]$$

$$z = \frac{1}{\sum_{i=1}^{n} X_i . Y_i}.\left[X_1 Y_1 .\left(\frac{dY_1}{Y_1} + \frac{dX_1}{X_1}\right) + ... + X_n Y_n .\left(\frac{dY_n}{Y_n} + \frac{dX_n}{X_n}\right)\right]$$

$$z = \frac{X_1 Y_1}{\sum_{i=1}^{n} X_i . Y_i}.\left(x_1 + y_1\right) + ... + \frac{X_n Y_n}{\sum_{i=1}^{n} X_i . Y_i}.\left(x_n + y_n\right)$$

or $$z = \sum_{i=1}^{n} S_i .\left(x_i + y_i\right) \quad \text{where} \quad S_i = \frac{X_i Y_i}{\sum_{i=1}^{n} X_i Y_i}$$