CHAPTER 2 THEORIES

To study tropical cyclone formation, a shallow water model is used to simulate the storm development and bogus wind by an asymmetric wind model is applied to enhance the weak observed wind of the storm.

2.1 Symmetric Wind Model

In the symmetric wind model, wind speed is zero at the center of the storm and increases rapidly to its maximum at the radius of maximum wind and then decreases gradually to zero at large radii. The wind speed can be described by the Holland model (Holland, 1980)

$$V_h(R) = -\frac{fR}{2} + \frac{1}{2}\sqrt{f^2R^2 - 4\frac{b}{\rho}\left(\frac{R_{\max}}{R}\right)^b (p_{env} - p_{center})\exp\left[-\left(\frac{R_{\max}}{R}\right)^b\right]} \qquad (2.1)$$

where $V_h(R)$ is the tangential wind speed at the distance R from the eye of the storm (ms^{-1}), p_{center} is the central pressure of the storm (hectopascal-hPa), p_{env} is the environmental pressure (hPa), $R_{\max}$ is the radius of maximum wind (*m*), R is the radius where wind speed is to be calculated (m), ρ is the air density of fluid which is constant and equal to 1.15 kgm^{-3} and f ($=2\Omega\sin\phi$, where Ω is the angular speed of the earth equal 7.292×10^{-5} $rads^{-1}$ and ϕ is latitude) is the Coriolis parameter.

Harper and Holland (2008) suggest an empirical relation for the parameter b that determines the shape of the radial profile as

$$b = \frac{\rho e (V_{\max} - V_{center})^2}{p_{env} - p_{center}} \qquad \text{for } 1 < b < 2.5$$

The modified Rankine vortex model uses a shape parameter b to adjust the wind speed distribution in the radial direction and requires user to specify the maximum gradient wind speed for a stationary storm V_{max} and R_{max}. For convenience, in this study the maximum wind speed from the gradient wind is used.

In the natural coordinates system, the horizontal momentum equations in form the gradient wind in any situation can be determined by solving the quadratic equation for $V(R)$

$$\frac{V^2}{R} + fV + \frac{1}{\rho}\frac{\partial p}{\partial R} = 0 \qquad (2.2)$$

$$V = -\frac{fR}{2} + \left(\frac{f^2R^2}{4} - \frac{R}{\rho}\frac{\partial p}{\partial R}\right)^{\frac{1}{2}} \qquad (2.3)$$

The air pressure distribution is (Holland 1980)

$$p(R) = p_{center} + \left(p_{env} - p_{center}\right)\exp\left(-\frac{R_{\max}}{R}\right)^{b} \tag{2.4}$$

The pressure gradient from differentiation Eq. (2.4) is given by

$$\frac{\partial p}{\partial R} = \left(p_{env} - p_{center}\right)\left(\frac{b}{R}\right)\left(\frac{R_{\max}}{R}\right)^{b} \exp\left(-\frac{R_{\max}}{R}\right)^{b} \tag{2.5}$$

Substitute Eq. (2.5) into Eq. (2.3)

$$V = \left[\left(p_{env} - p_{center}\right)\left(\frac{b}{\rho}\right)\left(\frac{R_{\max}}{R}\right)^{b} \exp\left(-\frac{R_{\max}}{R}\right)^{b} + \frac{f^2R^2}{4}\right]^{\frac{1}{2}} - \frac{fR}{2} \tag{2.6}$$

At $R = R_{\max}$ the maximum gradient wind speed is given by

$$V = \left[\left(p_{env} - p_{center}\right)\frac{b}{\rho e} + \frac{f^2R_{\max}^2}{4}\right]^{\frac{1}{2}} - \frac{fR_{\max}}{2} \tag{2.7}$$

Since the Coriolis force is small in comparison to the pressure gradient and centrifugal forces near $R_{\max}$, Eq. (2.7) becomes

$$b = \frac{\rho e\left(V_{\max} - V_{center}\right)^2}{p_{env} - p_{center}} \tag{2.8}$$

where V_{center} is the translation speed of the storm.

Eq. (2.8) is the parameter that determines the shape of the radial profile.

An example of a symmetric wind pattern is shown in Figure 2.1.

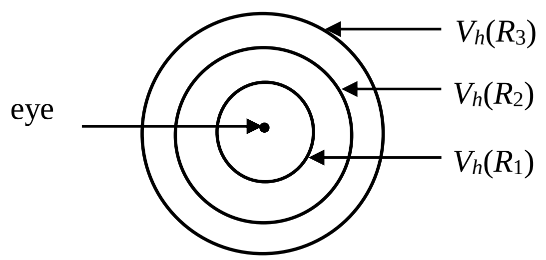
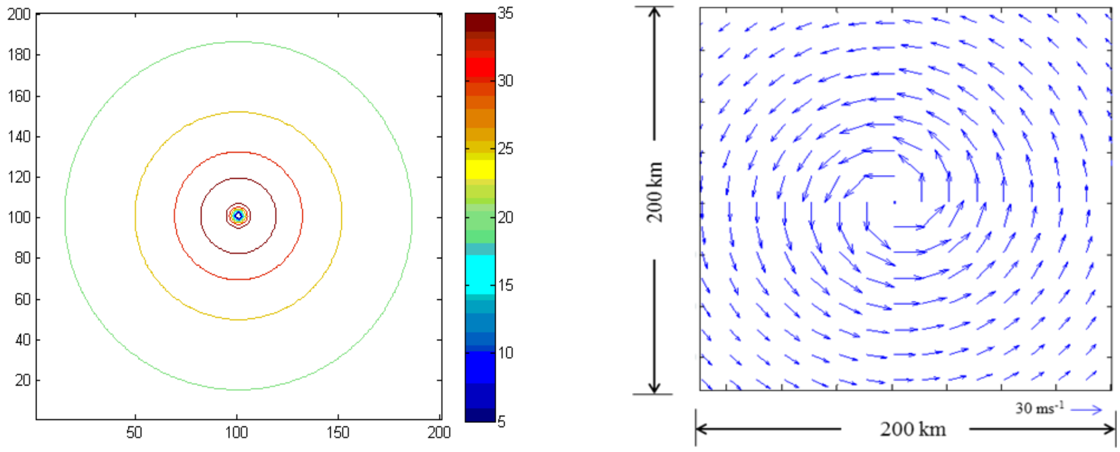


Figure 2.1 An example of symmetric wind speed of tropical cyclone. Each circle represents a contour of wind speed (isotach).

In Figure 2.1, $V_h(R_1) > V_h(R_2) > V_h(R_3)$ are the tangential wind speeds at distance R_1, R_3 and R_2, respectively, where R_1 is the radius of maximum wind.

Figure 2.2 shows schematic diagrams of symmetric wind of a storm from the symmetric wind model with wind speed around the storm and wind vector of the storm in Figure 2.2(a) and Figure 2.2(b), respectively.



(a) (b)

Figure 2.2 (a) Symmetric wind speed (ms^{-1}) and (b) Symmetric wind vector.

2.2 Asymmetric Wind Model

The asymmetric wind around the storm is generated by including the effect of storm center movement. Translation vector of the center is added to the rotation vector of the storm obtained from Holland model. This asymmetric wind model gives wind speed pattern that is close to the real tropical cyclone wind than that of Holland model.

Combining translation speed of tropical cyclone with tangential speed at the distance R, the asymmetric wind speed is obtained,

$$\vec{V}_{asym}(R,\theta) = V_h(R)\vec{u}_\theta + \vec{V}_{center} \tag{2.9}$$

where $\vec{V}_{asym}(R,\theta)$ is the asymmetric wind vector of a moving cyclone at distance R from the center with the angle θ from the reference axis, $V_h(R)$ is the tangential wind speed of symmetric wind model in Eq. (2.1) at distance R from the center, $\vec{V}_{center}$ is the translation vector of the storm center and $\vec{u}_\theta$ is the unitary vector pointing in the direction of rotation in the symmetric wind model.

To show the vectors of the cyclone winds, consider Figure 2.3 at the point A of radius R_1 with angle θ_1, and a point B of radius R_2 with angle θ_2.

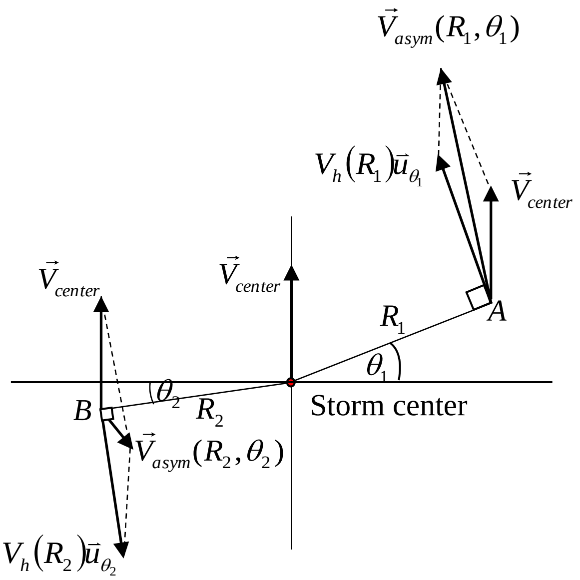


Figure 2.3 Wind vectors associated with the asymmetric wind model (Mouton, 2012).

A is the first position in polar coordinate and defined as (R_1, θ_1), where R_1 is the distance from the center of storm to point *A*, θ_1 is the angle between radius R_1 and the reference axis. *B* is the second position in polar coordinate and defined as (R_2, θ_2), where R_2 is the distance from the center of storm to point *B* and θ_2 is the angle between radius R_2 and reference axis. Noting that $\vec{V}_{asym}(R,\theta)$ is the sum of two vector, $\vec{V}_{center}$ and $V_h(R)\vec{u}_\theta$.

The non-moving model's wind vector is

$$\vec{V}(R,\theta) = V_h(R)\vec{u}_\theta \tag{2.10}$$

The equation of asymmetric wind vector is

$$\vec{V}_{asym}(R,\theta) = V_h(R)\vec{u}_\theta + \vec{V}_{center} \tag{2.11}$$

From Figures 2.4 (a) and (b), and using vector addition and Pythagoras theorem, the asymmetric wind speed of a moving cyclone can be obtained.

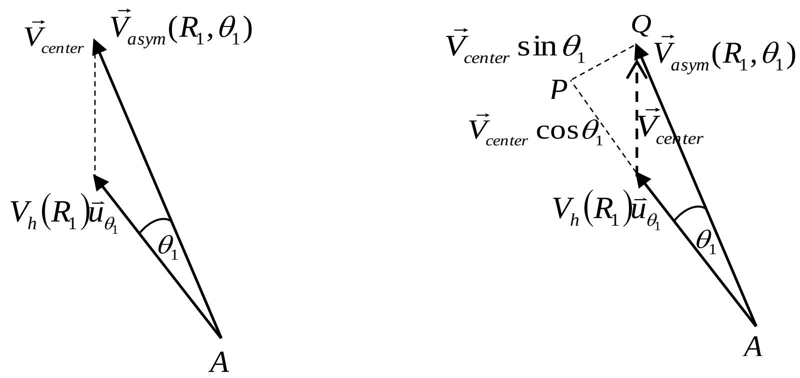


Figure 2.4 Relationships between $\vec{V}_{asym}(R,\theta), V_h(R)$ and $\vec{V}_{center}$.

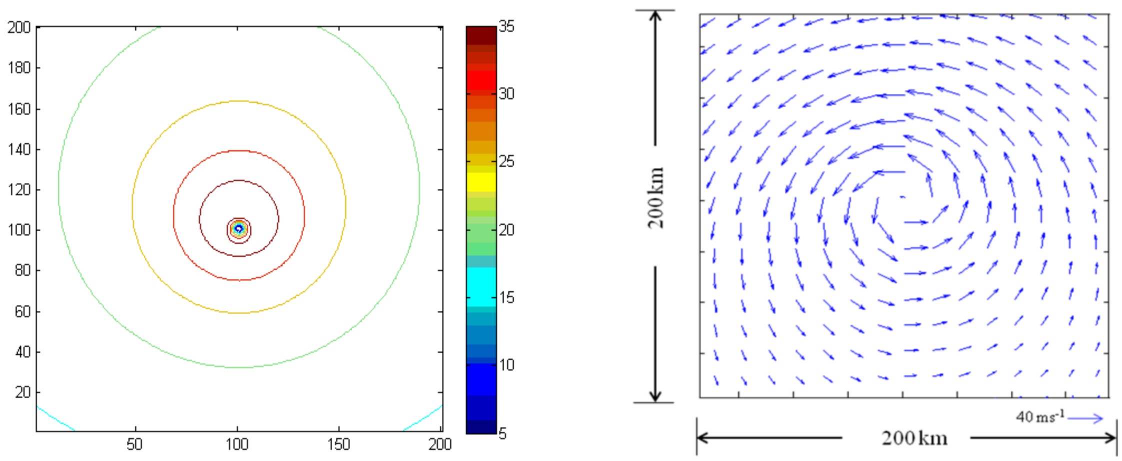
The asymmetric wind model (AWM) is formulated by combining the movement of tropical cyclone to the symmetric wind model. The research by Kongied (2011) presents an asymmetric wind model as

$$V_{asym}(R,\theta)=\left\|\vec{V}_{asym}(R,\theta)\right\|=\sqrt{V_h^2(R)+V_{center}^2-2V_h(R)V_{center}\cos\theta} \qquad (2.12)$$

where $V_{asym}(R, \theta)$ is the asymmetric wind speed of a moving cyclone at the position (R, θ), V_{center} is the translation speed of the storm center and θ is the angle between the line passing through the point of interest and the reference axis.

The main difference between symmetric wind and asymmetric wind is that symmetric wind is a scalar summation, while asymmetric wind is a vector summation of rotation wind vector and translation vector.

Figure 2.5 shows schematic diagrams of asymmetric wind of the storm from asymmetric wind model with wind speed and wind vector of the storm in Figures 2.5(a) and Figure 2.5(b), respectively.



(a) (b)

Figure 2.5 (a) Asymmetric wind speed (ms^{-1}) and (b) Asymmetric wind vector.

2.3 Shallow Water Model

The shallow water equations are a set of hyperbolic partial differential equations that describes the flow below a pressure surface in an inviscid shallow fluid layer in a rotating frame of reference. The equations explain the evolution of a hydrostatic homogeneous (constdcant density), and incompressible fluid in response to gravitational and rotational accelerations. The shallow water model is a simplified version of the Navier-Stokes equations.

The momentum, hydrostatic and continuity equations for a shallow water model can be written as (Holton, 2004)

$$\frac{du}{dt}-fv+\frac{1}{\rho}\frac{\partial p}{\partial x}=0 \qquad (2.13)$$

$$\frac{dv}{dt} + fu + \frac{1}{\rho}\frac{\partial p}{\partial y} = 0 \tag{2.14}$$

$$\frac{\partial p}{\partial z} + \rho g = 0 \tag{2.15}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{2.16}$$

where u is the wind component along x axis, v is the wind component along y axis, w is the vertical wind speed component along z axis, p is the pressure, g is the gravity.

The total time derivative is defined by $\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}$.

Consider a thin layer of fluid above a flat surface. The depth of the fluid (h) is a function of x and y ($h = h(x, y)$) and the mean depth H.

Assume that the pressure at the top of the fluid layer is a constant p_0. Integrating the hydrostatic Eq. (2.15) between the limits z and h to get

$$p = \rho g(h - z) + p_0 \tag{2.17}$$

where h is the height of the interface.

Assume that the pressure at a point is given by the weight of fluid above it (plus p_0). It implies that the horizontal pressure gradient at a depth z is given by

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = g\frac{\partial h}{\partial x} \quad \text{and} \quad \frac{1}{\rho}\frac{\partial p}{\partial y} = g\frac{\partial h}{\partial y} \tag{2.18}$$

The expressions on the right hand sides are independent of the depth z.

Assume that the horizontal velocity (u, v) is initially independent of depth z. Then it can be assumed that the velocity (u, v) is constant throughout the fluid layer. Integrating the continuity equation, Eq. (2.16), through the full depth of the fluid. Since u and v are constant with respect to z, the first two terms give

$$\int_0^h \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dz = h\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \tag{2.19}$$

Because the bottom is flat, the vertical velocity must vanish there. Moreover, the vertical velocity of a fluid particle at the top surface is given by $w(h) = \frac{dh}{dt}$.

Thus, the third term of Eq. (2.16) can be integrated as

$$\int_0^h \left(\frac{\partial w}{\partial z}\right) dz = w(h) - w(0) = \frac{dh}{dt} \tag{2.20}$$

Finally, the result of integrating Eq. (2.16) may be written as

$$\frac{dh}{dt} + h\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \qquad (2.21)$$

The set of shallow water equations are as follows.

Equations of motion,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv + g\frac{\partial \Phi}{\partial x} = 0 \qquad (2.22)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + fu + g\frac{\partial \Phi}{\partial y} = 0 \qquad (2.23)$$

where Φ is the geopotential height

Continuity equation,

$$\frac{\partial \Phi}{\partial t} + u\frac{\partial \Phi}{\partial x} + v\frac{\partial \Phi}{\partial y} + \Phi\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \qquad (2.24)$$

In the Lagrangian framework, the shallow water model can be written as

$$\frac{du}{dt} - fv + g\frac{\partial \Phi}{\partial x} = 0 \qquad (2.25)$$

$$\frac{dv}{dt} + fu + g\frac{\partial \Phi}{\partial y} = 0 \qquad (2.26)$$

$$\frac{d\Phi}{dt} + \Phi\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \qquad (2.27)$$

2.4 Vorticity and Circulation

The circulation, C, about a closed contour in a fluid is defined as the line integral evaluated along the contour of the component of the velocity vector that is locally tangent to the contour. In a horizontal plane

$$C = \oint (u\Delta x + v\Delta y) \qquad (2.28)$$

The vertical component of vorticity is defined as the circulation about a closed contour in the horizontal plane divided by the area enclosed, in the limit where the area approaches zero. Figure 2.6 shows a diagram for a derivation of vorticity.

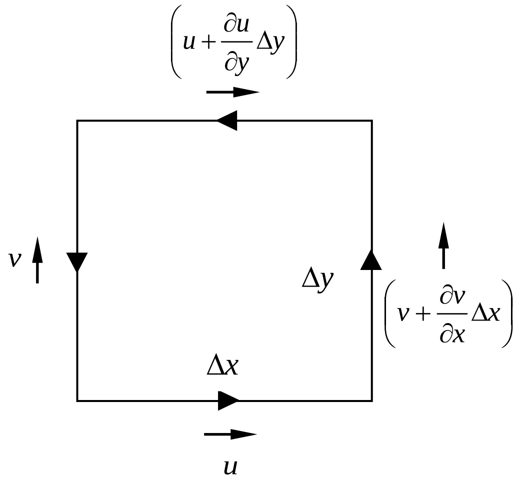


Figure 2.6 Derivation of vorticity in Cartesian coordinates (Holton, 2004).

The circulation for a small area in Figure 2.6 is

$$dC = u\Delta x + \left(v + \frac{\partial v}{\partial x}\Delta x\right)\Delta y - \left(u + \frac{\partial u}{\partial y}\Delta y\right)\Delta x - v\Delta y$$

$$dC = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\Delta x\Delta y \tag{2.29}$$

Dividing through by the area (dA=$\Delta x\Delta y$) of the rectangular element yields and the vertical component of vorticity

$$\zeta = \frac{dC}{dA} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \tag{2.30}$$

In this research the finite difference method is used to compute ζ. By the finite difference method, the domain of solution of the given partial differential equation is first divided into a number of grid points. The wind data are given on a rectangular grid (Figure 2.7).

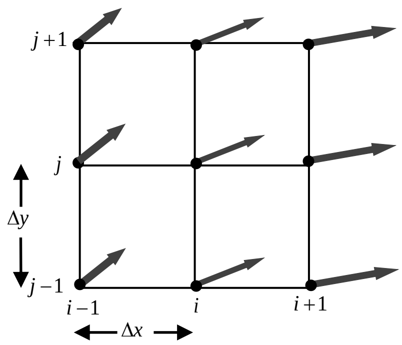


Figure 2.7 Wind vectors on a rectangular grid.

Figure 2.7 shows a part of a rectangular grid with wind vectors. The flow parallel to the sides of the square is very important for the possible rotation or vorticity of the air around point (i, j).

The central-difference approximation at the grid point (i, j) transforms Eq. (2.30) into

$$\zeta_{i,j} = \left[\frac{v_{i+1,j} - v_{i-1,j}}{2\Delta x}\right] - \left[\frac{u_{i,j+1} - u_{i,j-1}}{2\Delta y}\right] \tag{2.31}$$

where Δx and Δy are the grid intervals along x-axis and y-axis, respectively.

The values of $u_{i,j}$ and $v_{i,j}$ in Eq. (2.31) are obtained from $V_{asym}(R, \theta)$ in Eq. (2.12) by coordinates transformation from polar to cartesian coordinates.

The translation vector, the resultant vector and the combination of the tangent wind and the translation vectors are shown in Figure 2.8.

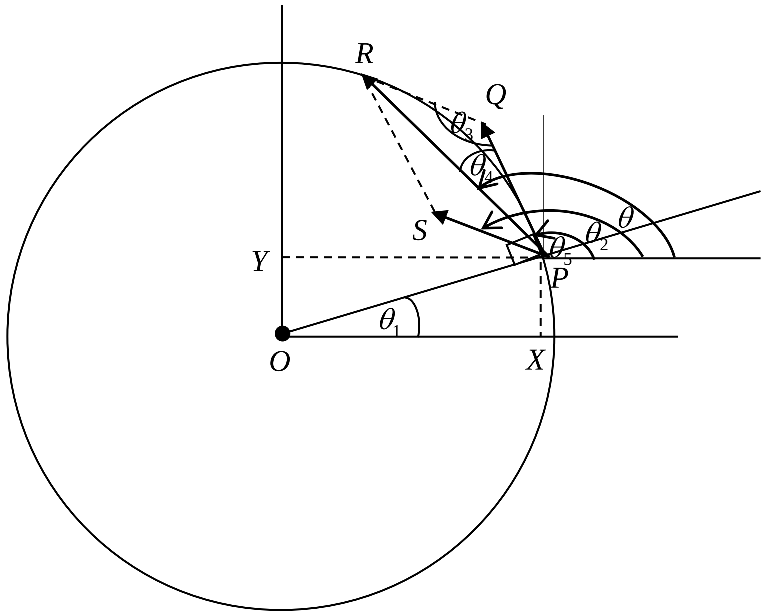


Figure 2.8 Combination of the tangent wind and the translation vectors.

Let O is the storm center (origin of the coordinates), θ is the angle of the resultant vector, θ_1 is the angle of point P with respect to the x-axis, θ_2 is the angle of the translation vector, θ_3 is $P\hat{Q}R$, θ_4 is $Q\hat{P}R$, θ_5 is the angle of the tangent wind vector at point P, ***PQ*** is the tangent wind vector, ***PS*** is the translation vector and ***PR*** is the resultant vector.

In Figure 2.8, the angles θ_1 to θ are defined as follows

$\theta_1 = \tan^{-1}(XP/OX)$

$\theta_2 = \theta_0 + 180^\circ$; $\theta_0 = \tan^{-1}(\Delta lat/\Delta lon)$

$\theta_3 = |180 - \theta_2 + \theta|$ with the conditions $\theta_3 := |360^\circ - \theta_3|$ if $\theta_3 > 180^\circ$

By the law of cosine, consider the triangle PQR and denote $PQ = |\boldsymbol{PQ}| = V_h(R, \theta_5)$, $QR = V_{center}$ and $PR = |\boldsymbol{PR}| = V_{asym}(R, \theta)$ (where $R = OP$).

where V_{center} is the translation speed of the storm center

V_{asym} is the asymmetric wind speed.

$\theta_5 = \theta_1 + 90^o$ with the condition $\theta_5 := \theta_5 - 360^o$ if $\theta_5 > 360^o$

where Δlat = latitude at the initial position of the storm minus latitude of the previous 6-hour position. Similarly, Δlon = longitude at the initial position minus longitude of the previous 6-hour position.

Then the asymmetric wind speed is

$$V_{asym}(R,\theta) = PR = \sqrt{(PQ)^2 + (QR)^2 - 2(PQ)^2(RQ)\cos\theta_3} \tag{2.32}$$

and the angle between the asymmetric wind and tangential wind (θ_4) is

$$\theta_4 = \sin^{-1}\left(\frac{QR\sin\theta_3}{PR}\right) \tag{2.33}$$

Therefore, the direction of asymmetric wind vector, θ, can be calculated by two conditions.

$$\theta = \begin{cases} \theta + \theta_4 & \text{if } \theta_5 < \theta_2 \\ \theta - \theta_4 & \text{if } \theta_5 \geq \theta_2 \end{cases}$$

The x-component (u) and y-component (v) of the asymmetric wind can be calculated by Eqs. (2.34a) and (2.34b).

$$u = PR\cos\theta \tag{2.34a}$$

$$v = PR\sin\theta \tag{2.34b}$$

2.5 Wind Direction and Degree

In meteorology, wind direction is the direction in which the wind is coming from, as shown in Figure 2.9 and Table 2.1.

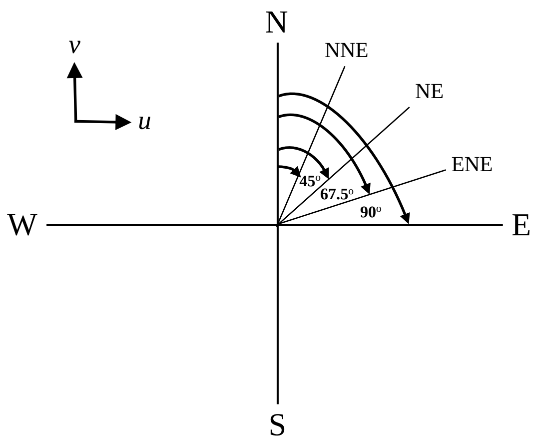


Figure 2.9 Wind cardinal and degree directions.

Table 2.1 Meteorological wind direction and degree.

Cardinal Direction	Degree
North (N)	0.0
North northeast (NNE)	22.5
Northeast (NE)	45.0
East northeast (ENE)	67.5
East (E)	90.0

In this research, northeast monsoon wind is defined as wind with direction between 22.5 to 67.5 degree.

The horizontal wind vector ($\boldsymbol{V_H}$) is represented by the bold black line in the Figure 2.10 below; $\boldsymbol{i}$ and $\boldsymbol{j}$ represent unit vectors towards east and north, respectively (The Natural Environment Research Council, 2013).

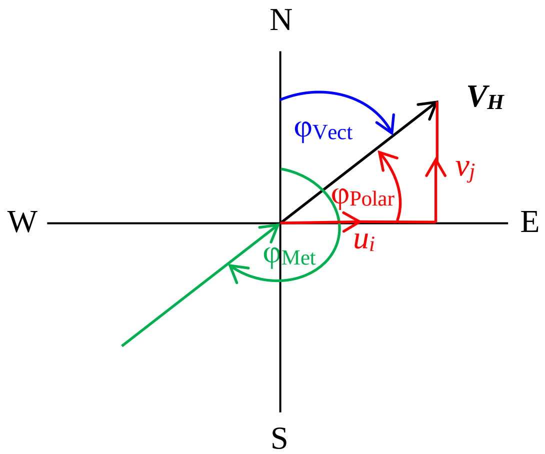


Figure 2.10 Wind vector and meteorological wind direction (The Natural Environment Research Council, 2013).

In Figure 2.10, φ_{Vect} is the wind vector angle. It increases clockwise from north when viewed from above, φ_{Met} is the meteorological wind direction angle. It also increases clockwise from north when viewed from above and φ_{Polar} which is the wind vector in polar angle in two-dimensions. It increases anticlockwise from east (x-axis), this in the opposite sense to the wind vector and the meteorological wind direction, and from a different origin.

The use of trigonometric functions assumes that angles are expressed in units of radians, φ(rad), rather than degrees. Directions are converted from units of degrees to radians through the relationship:

$$\varphi(\text{rad}) = \frac{\pi}{180} \times \varphi(\text{deg}) \tag{2.35}$$

Transformation of u and v components of wind vector to meteorological wind direction (φ_{Met}) can be done as follows,

$$\varphi_{Vect}(\text{deg}) = \tan^{-1}\left(\frac{v}{u}\right) \times \frac{180}{\pi} \tag{2.36}$$

These directions are typically expressed in units of degrees, $\varphi(deg)$, but can either be in the interval -180° to +180° or 0° to 360°. The wind vector and meteorological wind direction are related by

$$\varphi_{Met}(deg) = \varphi_{Vect}(deg)+180 \tag{2.37}$$

Subtracting 360° where appropriate in order to keep the values within the desired range.