APPENDIX A

A.1. Hill's 1948 yield criterion

The r-value based anisotropic parameters

The anisotropic parameters can be also calculated from the r-values. Equation 2.7 was re-written as

$$f(\sigma) = \phi(\sigma) - \bar{\sigma}^2 = 0$$

$$= (G+H)\sigma_{xx}{}^2 + (F+H)\sigma_{yy}{}^2 - 2H\sigma_{xx}\sigma_{yy} + 2N\sigma_{xy}^2 - \bar{\sigma}^2 \quad \text{(A-1)}$$

Using the associate flow rule, the plastic strain increments are

$$d\varepsilon_{xx} = d\lambda \frac{\partial f(\sigma)}{\partial \sigma_{xx}} = 2d\lambda\left[(G+H)\sigma_{xx} - H\sigma_{yy}\right] \quad \text{(A-2)}$$

$$d\varepsilon_{yy} = d\lambda \frac{\partial f(\sigma)}{\partial \sigma_{yy}} = 2d\lambda\left[(H+F)\sigma_{yy} - H\sigma_{xx}\right] \quad \text{(A-3)}$$

$$d\varepsilon_{xy} = d\lambda \frac{\partial f(\sigma)}{\partial \sigma_{xy}} = 2Nd\lambda\sigma_{xy} \quad \text{(A-4)}$$

where λ indicates the proportionality factor. Using the three uniaxial tension tests at 0°, 45°, and 90° from the rolling direction, Equation (A-2) to Equation (A-4) give the following equations.

For the uniaxial tension at 0° from the rolling direction,

$$d\varepsilon_{xx}^0 = 2d\lambda(G+H)\sigma_0 \quad \text{(A-5)}$$

$$d\varepsilon_{yy}^0 = -2d\lambda H\sigma_0 \quad \text{(A-6)}$$

$$d\varepsilon_{yy}^0 = 0 \quad \text{(A-7)}$$

For the uniaxil tension at from the rolling direction,

$$d\varepsilon_{xx}^{90} = -2d\lambda H\sigma_{90} \quad \text{(A-8)}$$

$$d\varepsilon_{yy}^{90} = 2d\lambda(H+F)\sigma_{90} \quad \text{(A-9)}$$

$$d\varepsilon_{xy}^0 = 0 \quad \text{(A-10)}$$

and for the uniaxial tension at 45° from the rolling direction,

$$d\varepsilon_{xx}^{45} = d\lambda G\sigma_{45} \tag{A-11}$$

$$d\varepsilon_{yy}^{45} = d\lambda F\sigma_{45} \tag{A-12}$$

$$d\varepsilon_{xy}^{45} = d\lambda N\sigma_{45} \tag{A-13}$$

In case of uniaxial tension at 45° from the rolling direction, it is necessary to transform the values $d\varepsilon_{xx}^{45}, d\varepsilon_{yy}^{45}$, $d\varepsilon_{xy}^{45}$ and into the values in the sample coordinates (X' for the tensile direction, y' for the width direction) because the r-values are usually measured in the sample coordinate system. Then,

$$d\varepsilon'_{xx} = d\varepsilon_{xx}^{45}\cos^2\frac{\pi}{4} + d\varepsilon_{yy}^{45}\sin^2\frac{\pi}{4} + 2d\varepsilon_{yy}^{45}\sin\frac{\pi}{4}\cos\frac{\pi}{4}$$

$$= \frac{G+F+2N}{2}d\lambda\sigma_{45} \tag{A-14}$$

$$d\varepsilon'_{yy} = d\varepsilon_{xx}^{45}\sin^2\frac{\pi}{4} + d\varepsilon_{yy}^{45}\cos^2\frac{\pi}{4} - 2d\varepsilon_{yy}^{45}\sin\frac{\pi}{4}\cos\frac{\pi}{4}$$

$$= \frac{G+F+2N}{2}d\lambda\sigma_{45} \tag{A-15}$$

The three r-value can be expressed as

$$r_0 = \frac{d\varepsilon_{yy}^{0}}{d\varepsilon_{zz}^{0}} = -\frac{d\varepsilon_{yy}^{0}}{d\varepsilon_{xx}^{0} + d\varepsilon_{yy}^{0}} = -\frac{-2d\lambda H\sigma_0}{2d\lambda(G+H)\sigma_0 - 2d\lambda H\sigma_0} = \frac{H}{G} \tag{A-16}$$

$$r_{90} = \frac{d\varepsilon_{xx}^{90}}{d\varepsilon_{zz}^{90}} = -\frac{d\varepsilon_{xx}^{90}}{d\varepsilon_{xx}^{90} + d\varepsilon_{yy}^{90}} = -\frac{-2d\lambda H\sigma_{90}}{-2d\lambda(G+H)\sigma_{90} + 2d\lambda(H+F)\sigma_{90}} = \frac{H}{F} \tag{A-17}$$

$$r_{45} = \frac{d\varepsilon'_{yy}}{d\varepsilon'_{zz}} = -\frac{d\varepsilon'_{yy}}{d\varepsilon'_{xx} + d\varepsilon'_{yy}}$$

$$= -\frac{\left(\frac{G+F-2N}{2}d\lambda\sigma_{45}\right)}{\left(\frac{G+F+2N}{2}d\lambda\sigma_{45}\right) + \left(\frac{G+F-2N}{2}d\lambda\sigma_{45}\right)} = \frac{2N-G-F}{2(G+F)} \tag{A-18}$$

Solutions of the above simultaneous equations are

$$F = \frac{r_0}{r_{90}\left(1+r_{90}\right)}\left(\frac{\overline{\sigma}}{\sigma_0}\right)^2 \tag{A-19}$$

$$G = \frac{1}{\left(1+r_0\right)}\left(\frac{\overline{\sigma}}{\sigma_0}\right)^2 \tag{A-20}$$

$$H = \frac{r_0}{\left(1+r_0\right)}\left(\frac{\overline{\sigma}}{\sigma_0}\right)^2 \tag{A-21}$$

$$N = \frac{\left(r_0+r_{90}\right)\left(1+2r_{45}\right)}{2r_{90}\left(1+r_0\right)}\left(\frac{\overline{\sigma}}{\sigma_0}\right)^2 \tag{A-22}$$

Finally, Hill's 1948 yield function characterized by the r-value can be written as

$$f\left(\sigma\right) = \frac{1}{r_0}\sigma_{xx}^2 + \frac{1}{r_{90}}\sigma_{yy}^2 + \left(\sigma_{xx}-\sigma_{yy}\right)^2 + \left(\frac{1}{r_0}+\frac{1}{r_{90}}\right)\left(2r_{45}+1\right)\tau_{xy}^2 - \left(1+\frac{1}{r_0}\right)\sigma_0^2 = 0 \tag{A-23}$$

A.2. Yld2000-2d

To identify the coefficients describing the material anisotropy in the Yld2000-2d yield function, four tests, such as uniaxial tensile tests at 0°, 45°, and 90° from the rolling direction and the balanced biaxial tensile test are necessary to provide $\sigma_0, \sigma_{90}, \sigma_{45}, \sigma_b, r_0, r_{90}, r_{45}$ and r_b. The value r_b denotes the incremental strain ratio of the transverse direction to the rolling direction and defined as

$$r_b = d\varepsilon_{yy} / d\varepsilon_{xx} \tag{A-24}$$

This ratio can be obtained by the compression of the circular disk specimen in the sheet normal direction, which will be explained in the next section.

The eight coefficients, α_k, are computed from the non-linear equations using the Newton-Raphson method or the downhill simplex method. As shown in Equation (A-25) and Equation (A-26), to calculate the coefficients α_k to α_k, two equations, one for the yield stress and the other for the r-value, should be solved for the two uniaxial stress states ($0^\circ, 90^\circ$) and for balanced biaxial tension. Each stress state can be expressed in terms of the deviatoric components, $s_x = \gamma\sigma$ and $s_y = \delta\sigma$

$$F = \phi - 2\left(\frac{\bar{\sigma}}{\sigma_0}\right)^2 = 0 \qquad \text{(A-25)}$$

$$G = q_x \frac{\partial \phi}{\partial s_{xx}} - q_y \frac{\partial \phi}{\partial s_{xx}} = 0 \qquad \text{(A-26)}$$

The yield criterion ϕ can be written as

$$\phi = \left|\alpha_1 \gamma - \alpha_2 \delta\right|^a + \left|\alpha_3 \gamma - 2\alpha_4 \delta\right|^a + \left|2\alpha_5 \gamma - \alpha_6 \delta\right|^a = 2\left(\frac{\bar{\sigma}}{\sigma_0}\right)^a \qquad \text{(A-27)}$$

Where $\gamma, \delta, q_x,$ and q_x

	γ	δ	q_x	q_y
0° tension	2/3	-1/3	$1\text{-}r_0$	$2\text{+}r_0$
90° tension	-1/3	2/3	$2\text{+}r_{90}$	$1\text{-}r_{90}$
Balanced biaxial tension	-1/3	-1/3	$1\text{+}2r_b$	$2\text{+}r_b$

Using the experimental data, σ_{45} and r_{45} obtained in the uniaxial tension at $45°$ from the rolling direction, the remaining two coefficients α_7 and α_8 can be obtained following

$$F = \left|\frac{\sqrt{k_2'^2 + 4\alpha_7^2}}{2}\right|^a + \left|\frac{3k_1'' - \sqrt{k_2'' + 4\alpha_8^2}}{4}\right|^a + \left|\frac{3k_1'' + \sqrt{k_2'' + 4\alpha_8^2}}{4}\right|^a - 2\left(\frac{\bar{\sigma}}{\sigma_0}\right)^a = 0 \qquad \text{(A-28)}$$

Where $k_2' = \dfrac{\alpha_1 - \alpha_2}{3}$, $k_1'' = \dfrac{2\alpha_5 + \alpha_6 + \alpha_3 + 2\alpha_4}{9}$, $k_2'' = \dfrac{2\alpha_5 + \alpha_6 - \alpha_3 - 2\alpha_4}{3}$.

The r-value, r_{45} , can characterize the anisotropy using the associated flow rule and is expressed as

$$G = \frac{\partial \phi}{\partial \varepsilon_{xx}} + \frac{\partial \phi}{\partial \varepsilon_{yy}} - \frac{2a\bar{\sigma}^a}{\sigma(1 + r_{45})} = 0 \qquad \text{(A-29)}$$

APPENDIX B

The coefficients which define the material anisotropy for the Yld2000-2d yield function can also be determined by using a least square error method, especially when the number of material data exceed or one less than the number of unknowns (or anisotropic constants) of the yield function. By using additional experimental data from the in-plane biaxial tension tests, the predicted stresses and r-values with trial coefficients can make the error function as

$$error(\alpha_1, \alpha_2, ..., \alpha_8) = \omega_u \sum_i \left(\frac{\sigma_i^{pre}}{\sigma_i^{\exp}} - 1 \right)^2 + \omega_r \sum_j \left(\frac{r_j^{pre}}{r_j^{\exp}} - 1 \right)^2 + \omega_b \sum_k \left(\frac{\sigma_k'^{pre}}{\sigma_k'^{\exp}} - 1 \right)^2 \quad \text{(A-30)}$$

Where the superscripts "pre" and "exp" denote predicted and experimentally measured values, respectively, and σ and r represent yield stress and r-values, respectively. Then, the definition of the variables in the above equations becomes:

σ_i^{pre} : predicted yield stress in uniaxial loading along the direction i

$\sigma_i^{\exp}$: Measured yield stress in uniaxial loading along the direction i

r_j^{pre} : Predicted r-value in uniaxial loading along the direction j.

$r_j^{\exp}$: Measured r-value in uniaxial loading along the direction j.

$\sigma_k'^{pre}$: Predicted yield stress in biaxial loading along the fixed path k.

$\sigma_k'^{\exp}$: Measured yield stress in biaxial loading along the fixed path k.

ω_u , ω_r , ω_b : Weight parameters corresponding to uniaxial tension, r-value and biaxial tension, respectively.

In order to find the minimum error value, the coefficients are approximated through the Newton-Raphson method