OUTPUT-FEEDBACK H_∞ FUZZY CONTROLLER DESIGN FOR SPEED CONTROL OF BRUSHLESS DC MOTOR

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Abstract

This paper proposes a design of the output-feedback H_∞ fuzzy controller for speed control of Brushless DC (BLDC) motors. This type of motors has played an important role for high safety industrial applications. BLDC motors have advantages in torque, speed, efficiency, and reliability. However, the existing conventional controllers for BLDC motors still have not provided good enough speed and suffered from disturbance and nonlinearity. To overcome these problems, an output-feedback H_∞ fuzzy controller based on an LMI approach is designed for such a system. Finally, the validity of the designed approach is demonstrated via simulations. The results show the improvement in performances, i.e., transient response and steady state error that certainly guarantee the stability of the nonlinear fuzzy system.

หัวข้อโครงงานศึกษาวิจัย	การออกแบบตัวควบแบบเอชอินฟินิตี้ฟัซซี่เอาท์พุทป้อนกลับ สำหรับควบคุมความเร็วของมอเตอร์กระแสไฟฟ้าตรงไร้แปลงถ่าน
หน่วยกิตทั้งหมด	6
ผู้เขียน	นางสาวสาวิตรี นาสวัสดิ์
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สาขาวิชา	วิศวกรรมไฟฟ้าและสารสนเทศ
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ปีการศึกษา	2557

บทคัดย่อ

งานวิจัยนี้นำเสนอ การออกแบบการป้อนกลับเอาท์พุท เอชอินฟินิตี้ ด้วยตัวควบคุมแบบฟัซซี่ สำหรับการควบคุมความเร็วของมอเตอร์กระแสไฟฟ้าไร้แปรงถ่าน มอเตอร์ชนิดนี้เป็นที่นิยมใช้ในงานด้านอุตสาหกรรมที่ต้องการความปลอดภัยสูง มอเตอร์ไฟฟ้ากระแสตรงไร้แปลงถ่านมีข้อได้เปรียบในเรื่องของความเร็วรอบ แรงบิด และประสิทธิภาพที่สูง อย่างไรก็ตามตัวควบคุมแบบอื่นที่นำมาใช้กับมอเตอร์ไร้แปลงถ่านนี้ ยังตอบสนองเรื่องความเร็วยังไม่ดีพอภายใต้ข้อจำกัดบางอย่าง เช่น ภาวะโหลดและการไม่เป็นเชิงเส้นของระบบ และเพื่อแก้ไขปัญหาเหล่านี้ที่เกิดขึ้น การออกแบบการป้อนกลับเอาท์พุท เอชอินฟินิตี้ ฟัซซี่ คอนโทรลเลอร์ที่ได้รวมวิธีการของฟัซซี่ คอนโทรล และวิธีการของ เอชอินฟินิตี้ คอนโทรล เข้าไว้ด้วยกันบนพื้นฐานวิธีการของ แอลเอ็มไอ สุดท้าย วิธีการออกแบบแสดงให้เห็นถึงตัวอย่างแบบจำลอง ผลลัพธ์จากการจำลองแสดงให้เห็นว่าระบบมีประสิทธิภาพและมีการตอบสนองได้เร็วขึ้น

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LIST OF SYMBOLS

SYMBOL	DEFINITION
emf	electromotive force
$\emptyset$	magnet flux, Wb
$E_a(t)$	back emf for phase A, V
$E_b(t)$	back emf for phase B, V
$E_c(t)$	back emf for phase C, V
I	instantaneous current, A
$i_a(t)$	phase A current, A
$i_b(t)$	phase B current, A
$i_c(t)$	phase C current, A
$i_d(t)$	steady-state d-axis current flowing into the motional emf in rotor reference frames, A
$i_q(t)$	steady-state q-axis current flowing into the motional emf in rotor reference frames, A
J	rotor moment of inertia, kg-m^2
B	flux density, N.m1(rad/s)
R	stator resistance, Ω
L, M	stator self and mutual inductances in motor, H
$L_q,\ L_d$	quadrature and direct axis stator self-inductances in rotor reference frames, H
P	number of pole
$T_e(t)$	electromagnetic torque, N.m
$T_{(i_q i_d)}(t)$	electromagnetic torque in qd axis, N.m
$T_L(t)$	load torque, N.m
$V_a(t), V_b(t), V_c(t)$	stator voltages, V
$V_{ab}(t)$	line-to-line voltages between a and b phase, V
$V_{bc}(t)$	line-to-line voltages between b and c phase, V
$V_{ca}(t)$	line-to-line voltages between c and a phase, V
v_q, v_d	qd axis voltages, V
θ_r	angular position, rad

LIST OF SYMBOLS (Cont.)

SYMBOL	DEFINITION
λ_m	mutual air gap flux linkages, V-s
$\omega_m(t)$	rotor mechanical speed, rad/

CHAPTER 1 INTRODUCTION

1.1 Overview

Brushless DC (BLDC) motor plays an important role in many industrial sectors from a small scale, yet very critical application to a large scale automotive application. BLDC provides better performance than conventional motor especially in speed, torque, efficiency, and reliability. Hence, it can replace other motors in a wide range of application. To overcome nonlinearity and disturbances problems, this research proposes a new design of the output feedback H_∞ fuzzy controller for controlling speed of brushless DC motor. The results show improvement steady state error and transient performance, including a stability criterion in terms of Lyapunov method which can guarantee the stability of the nonlinear fuzzy system. Finally, the effectiveness of this approach is demonstrated via simulation. [1]

1.1.1 Brushless DC Motor

Nowadays, the BLDC motor is widely used in many industrial sectors from a small scale, yet very critical application to a large scale automotive applications e.g. Air condition, cars, remote-control, medical etc. It is better to use BLDC motor over the conventional DC motor with brushes because of its stable speed, low maintenance requirement, long life expectation, low noise floor, easy to control, light weight, and compact design.

Figure 1.1 Brushless DC motor [2]

For electric motors, there are two different categories: DC (Direct Current) and AC (Alternating Current). Within these categories, there are numerous ways to design a motor and the classification of these motors has become less rigorous and many other types of motor have appeared, such as DC motors, brushless DC motors, AC synchronous motors, AC induction motors and stepping motors.

Recently, brushless direct current (BLDC) motors have been recently utilized in many applications e.g. air conditioners, remote-controlled, car and medical applications. It is better to use BLDC motor over the conventional DC motor with brushes because of its stable and high speeds, low maintenance requirement, long life expectation, low noise floor, easy to control, light weight, greater output power and compact design. It is easy to control speed of BLDC in a wide range and their torque-speed characteristic has, historically, been easier to tailor than that of all AC motor categories. BLDC motors are also major contributors to the accuracy and repeatable performance in wire and ball bonders, microlithography and micropositioning systems. The general trend of falling cost of electronic products and the increased use of microprocessors to control the performance of machines and application are leading to increased interest in the use of BLDC motors.

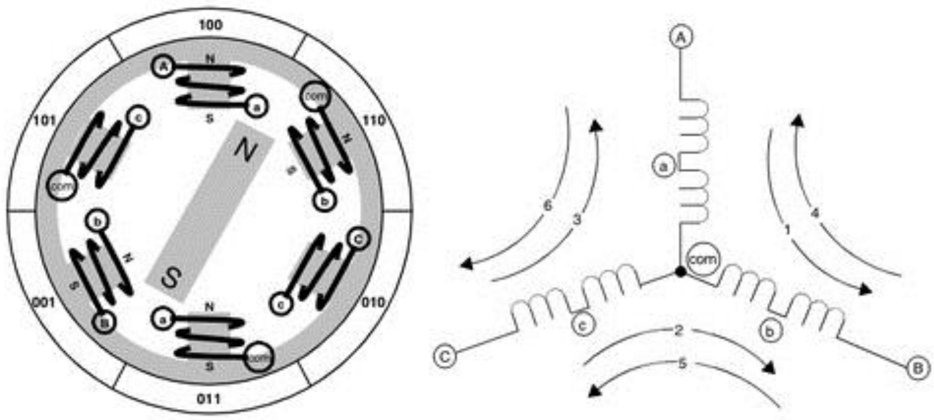


Figure 1.2 BLDC motor diagrams [3]

Figure 1.2 is application to show both electrical and mechanical diagrams of a three phase BLDC motor. The purpose of the controller is to ensure that the motor operates efficiently under various conditions. BLDC motors have provided the advantages of over brushed DC motors in terms of variable speed operation but without the drawbacks of brushes. The most obvious advantage of the brushless configuration is the elimination of brush maintenance and sparking. Several of these are:

- High dynamic response
- High efficiency
- Long operating life
- Better heat dissipation
- Higher speed ranges
- Low noise floor
- Low maintenance requirement
- Easy to control
- Light weight
- Compact design

In addition, the ratio of torque delivered in BLDC motors in relation to motor size is higher than that in non-BLDC motors, making BLDC motors an excellent match for space and weight sensitive applications [1],[4].

1.1.1.1 History of brushless DC Motor

Before there were brushless DC motors, there had been brush DC motors. In mid-1950s, H. D. Brailsford introduced the first DC motors to be called “brushless.” Then brushless DC motors made the scene in 1962, by T.G. Wilson and P.H. Trickey unveiled and they called “a DC machine with solid state commutation.” This motor became a popular choice for special applications such as computer disk drives, robotics and in aircraft because of low humidity. That all changed in the 1980s, when permanent magnet material became readily available, the advent of high-energy magnetic-material and high power and high voltage transistors. The end of the 1980s, Robert E. Lordo of the POWERTEC Industrial Corporation unveiled the first large brushless DC motor, which had at least ten times the power of the earlier brushless DC motors.

1.1.1.2 Comparing Brushless DC motor to Brush DC motor

The operation of BLDC motor compared to brush DC motors and induction motors is sufficiently different. Brushless DC Motor uses a permanent-magnet external rotor, three phases of driving coils, one or more Hall Effect devices to sense the position of the rotor, and the associated drive electronics. On the other hand, a brushed Motor has a rotating set of wound wire coils called an armature which acts as the electromagnet with two poles.

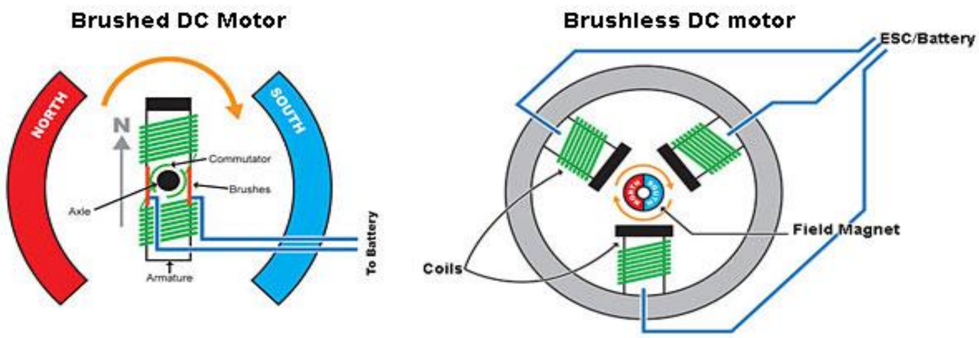


Figure 1.3 Brush DC motor and Brushless DC motor) [5]

1.1.2 Fuzzy Logic Control

Fuzzy logic control (FLC) is best utilized in managing complex ill-defined processes, controlling nonlinear, and time-varying, fuzzy that can be controlled by a skilled human operator without much knowledge of their underlying dynamics. Fuzzy logic was first proposed by an American professor, Lotfi A. Zadeh in 1965, in the University of California, Berkeley, USA in 1965 when he presented his seminal paper on "fuzzy set". Nowadays, fuzzy logic control has been proposed as an alternative to the traditional control techniques with many successful applications. The basic idea behind is to incorporate the "expert experience" of a human operator in the design of the controller. Fuzzy control rules are characterized by a collection of fuzzy IF-THEN rules in which the preconditions and consequents involve linguistic variables.

A fuzzy model is a nonlinear in nature and is suitable for expressing nonlinear dynamic properties. It uses fuzzy rules, which are linguistic if-then statements involving fuzzy sets, fuzzy logic, and fuzzy inference. Fuzzy rules play a key role in representing expert control/modeling knowledge and experience and in linking the input variables of fuzzy controllers/models to output variable. There are two major types of fuzzy rules exist, namely, Mamdani fuzzy rules and Takagi-Sugeno fuzzy rules. The Takagi-Sugeno (TS) fuzzy model [16] has attracted the most attention. Takagi-Sugeno (TS) fuzzy controllers are close to gain scheduling mechanism approaches. This approach makes it possible to introduce the design requirements, such as stability and other performances, in the synthesis procedure of fuzzy control systems.

1.1.3 H_∞ Methods

H_∞ (H-infinity) methods are used in control theory to synthesize controllers achieving stabilization with guaranteed performance. These methods were introduced into control theory in the late 1970s to early 1980s by George Zames (sensitivity minimization), J. William Helton (broadband matching), and Allen Tannenbaum. Throughout the decades of 1980 and 1990, H_∞ control method had a significant impact in the development of control systems; nowadays the technique has become fully grown and it is applied to industrial problems. To use H_∞ methods, the control designer expresses the control problem as a mathematical optimization problem, then finds the controller solution. H_∞ techniques have the advantage over classical control techniques in which the techniques are readily applicable to problems involving multivariate systems with cross-coupling between channels; disadvantages of H_∞ techniques include the high level of mathematical understanding needed to apply them successfully and the need for a reasonably best model of the system to be controlled. The problem formulation is important, since any synthesized controller will be “optimal” in the formulated sense.

The H_∞ name derives from the fact that mathematically the problem may be set in the H_∞ space which consists of all bounded functions that are analytic in the right-half complex plane. The H_∞ norm is the maximum singular value of the function. H_∞ techniques can be interpreted as a maximum gain in any direction and at any frequency. H_∞ method is also used to minimize the closed-loop impact of a perturbation: depending on the problem formulation, the impact will be in terms of either stabilization or performance.

1.2 Problem Statement

Since the brushless DC (BLDC) motor is multivariable and nonlinear system, it is difficult to find an appropriate controller that meets the required specifications. The existing controller, which uses a PID or conventional, has disturbance and the nonlinear problems. The PID controllers have the advantages of simple structure and low cost controller. However, PID controller has poor performance when applied to BLDC motor system that containing nonlinear variable which it is not good enough to control the speed. To overcome these problems, the output feedback control design technique is proposed to be used in the H_∞ fuzzy controller in order to improve the speed performance of BLDC motor.

1.3 Literature Review

1.3.1 Brushless DC Motor

Pillay and Krishnan [6] presented advantage existing in transforming the *abc* equations of the sinusoidal PMSM to the *d*, *q* coordinate model and a third order state space model for its speed control application. Simulation results showed that the torque was held constant, the electric torque of motor increased when the load torque change. Reference phase currents were transformed from the desired currents in *dq* coordinates, which in turn were detennined by the speed error and torque command.

Lee and Kwok [7] designed the variable structure controller with the state space model of the BLDC motor. The switching effort is reduced when the system state deviation from the switching surface. The simulation results the system performance with respect to the sensitivity to the parameter variation in torque constant has a more significant effect on the performance than the motor

Hu et al. [8] presented Nonlinear Tracking Controllers for Brushless DC Motors. This system will develop nonlinear controllers designed to yield high-performance position tracking control of a brushless DC (BLDC) motor driving a mechanical load. The results verify a nonlinear model-based controller can exhibit greatly improved position tracking performance over that of a standard linear controller.

Pelczewski and Kunz [9] developed model of the BLDC motor and designed and optimal controller for position control of BLDC motor. Instead of using a *dq* coordinate model, a 4^{th} order state space model was used for the BLDC motor dynamics including rotor mechanical dynamics and stator coil electrical dynamics. The stator current dynamics were simplified as one of the state equations.

Rubaai and Kotaru [10] studied a three-layer feed forward artificial neural network (F ANN) and a dynamic back propagation (DBP) neural network controller for a BLDC motor application. An adaptive online training strategy was proposed for the FANN. It converged much faster than the DBP learning algorithm with a constant learning rate. The stability of the neural network controllers was not discussed.

Ozturkl et al. [11] presented algorithm for power factor correction (PFC) of direct torque control (DTC) brushless dc motor drive based on the input- output voltage which is the dc-link of the BLDC motor drive. The result showed that feed-forward voltage compensation method has been implemented in a single sampling time of the proposed DTC of a BLDC motor drive under two-phase conduction mode in the constant torque region.

Yingfa et al. [12] presented a new kind method of genetic neural network algorithm by optimizing controller. The method optimizes topology and parameters of hidden unit, and increases efficiency of learning algorithm. The results show a controller can improve the performance of BLDCM speed regulation system in the aspects of control precision, robust and dynamic characteristics.

Abidin and Hassan [13] studied the comparative between PI, fuzzy and hybrid PI-fuzzy controller for speed control of brushless dc (BLDC) motor. The PI controller mainly supports steady-state accuracy and cancels disturbance effects when load torque change while the fuzzy controller acts in the case of sufficiently large reference input changes especially during start-up.

Hakan et al. [14] presented a fuzzy logic speed and current controller for brushless DC motor (BLDCM). This work will result in high efficiency, high torque density, and small size of Brushless DC Motors (BLDCMs). BLDCMs are nonlinear. So, Fuzzy

Logic Controllers (FLCs) was used to control the speed and current. The inner loop has a fuzzy logic current controller; the outer loop has a fuzzy logic speed.

1.3.2 Fuzzy Control Applied to Brushless DC Motor

Mirtalaei et al. [15] presented a novel sensorless control method of brushless DC (BLDC) motors based on a simple fuzzy logic observer. This technique uses a fuzzy back-EMF observer with fuzzy function approximation 9 rules.

Rajan [16] implemented using reconfigurable Vertex II Pro development board. This solution uses fuzzy logic technique for a BLDC motor for variable frequency and variable duty ratio operation. The fuzzy logic ranges defined using the linguistic terms as low, medium and high. The input to the fuzzy logic controller is speed and current which are fed from the analog-to-digital converter

Stinean, et al. [17] proposed four hybrid Takagi Sugeno fuzzy controllers which consist of two hybrid PI neuro fuzzy controllers and two adaptive sliding mode fuzzy controllers, for speed and position control of brush less direct current motor drives.

Xuanfeng and Xignyan [18] proposed a novel P-fuzzy self-adaptive PID intelligent method for controlling the speed of servo BLDC motor system. Dual-mode control system technique is based on basic fuzzy control and P control. The simulation results show that system to speed up response time, almost no overshoot.

1.3.3 Fuzzy Controller H_∞ Method and LMI Approach

Lu and Doyle [19] proposed H_∞ control problem for linear state-space systems which can be reduced to corresponding augmented stabilization problems d in terms of structured Linear Matrix Inequalities (LMIs). This method defines μ or Q stability, performance and pursues the controller synthesis problem using the notion.

Takagi and Sugeno [20] presented the method of identification of a system using its input-output data with a mathematical tool to build a fuzzy model of a system where fuzzy implications and reasoning. The obtained fuzzy model is applied to the control of the converter. It is divided into two parts: structure identification and parameter identification.

Huaizhong and Minyue [21] studied robust H_∞ filtering problem that can be solved using linear matrix inequality (LMI) techniques, which are numerically efficient owing to recent advances in convex optimization. There are three approaches to filtering: algebraic riccati equation approach, polynomial equation approach and interpolation approach.

Nguang and Shi [22] developed a technique for designing an H_∞ fuzzy output feedback control law which globally stabilizes this class of nonlinear systems. The convex LMIs can then be solved in a computationally efficient manner using a convex optimization technique such as the interior point method. The results given the precise variables of the H_∞ fuzzy output, feedback controller are allowed to be different from the premise variables of the Takagi-Sugeno suzzy model of the plant.

Assawinchaichote et al. [23] developed a state-feedback controller that guarantees the L_2-gain of the mapping from the exogenous input noise to the regulated output being less than some prescribed value and the closed-loop system. A fuzzy dynamic model proposed by Takagi and Sugeno is described by IF-THEN rules.

1.4 Research Objectives

1.4.1 To develop speed output feedback H_∞ fuzzy controller of the BLDC motor, using MATLAB simulation.

1.4.2 To improve speed performance of BLDC motor such reducing overshoot, steady state error and fast setting time.

1.5 Scope of Research

1.5.1 Simulate the output feedback H_∞ Fuzzy controller for BLDC motor using Matlab/Simulink software.

1.5.2 Exists two fuzzy rules.

1.5.3 Simulate under disturbance and no-disturbance conditions.

1.5.4 Compare speed control between H_∞ Fuzzy controller and PID controller.

1.5.5 Use motor parameters: APPENDIX A

1.6 Research Procedure

1.6.1 Study the related previous article and research.

1.6.2 Collect the information for development and solving the problem of previous research.

1.6.3 Propose method or technique for the controlling speed of BLDC motor.

1.6.4 Adjust the research from result to follow the objectives.

1.6.5 Analyze the result.

1.6.6 Conclude simulation and propose future development.

1.7 Expected Benefits

1.7.1 To improve the efficiency of speed control of BLDC motor.

1.7.2 To apply the output feedback H_∞ fuzzy controller.

1.8 Research Concept

There are many different controller designs in BLDC motor. In this research, we propose an output feedback H_∞ fuzzy controller for speed control of brushless DC motor in order to overcome nonlinearity and disturbances as shown in Figure 1.4. The advantage of fuzzy logic is the ability deal with this problem by varying the linguistic rule or input variable. The nonlinear H_∞ control problem can be stated as follows: given a dynamic system with the exogenous input noise and the measured output, find controller such that $\mathcal{L}_2$-gain of the mapping from exogenous input noise to the regulated output is minimized or no larger than a prescribed level.

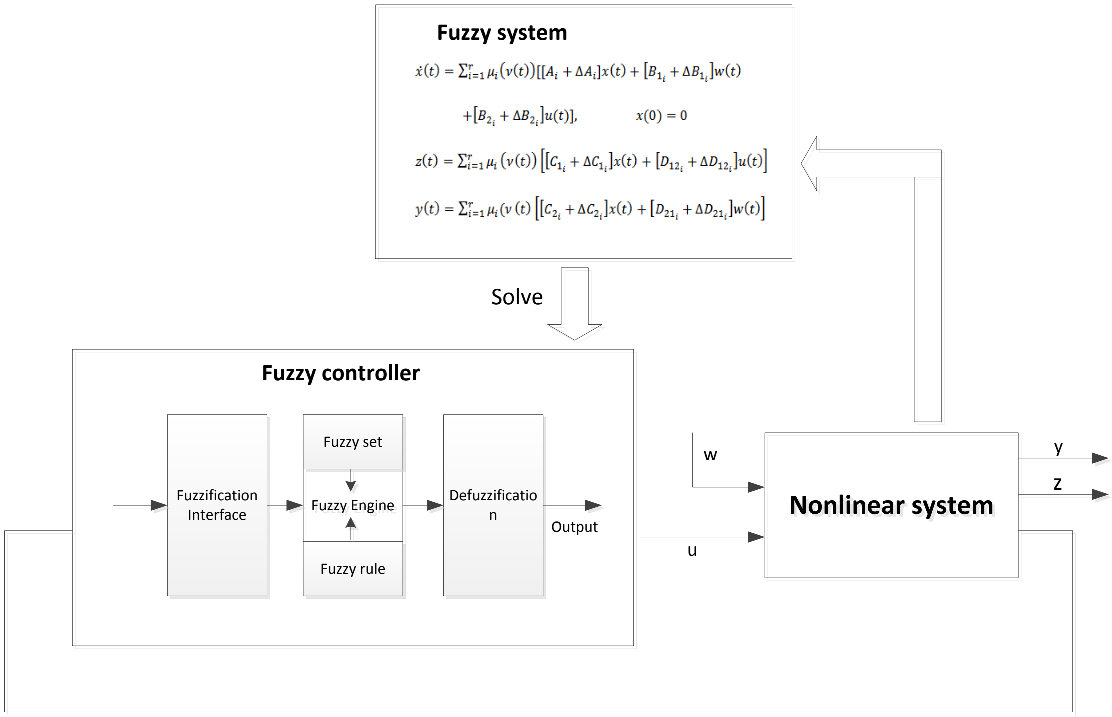


Figure 1.4 output feedback H_∞ fuzzy controller

CHAPTER 2 BRUSHLESS DC MOTOR SYSTEM

2.1 Basic Principle of Brushless DC motor

Brushless DC (BLDC) motors are referred to many aliases: brushless permanent magnet, permanent magnet ac motors, permanent magnet synchronous motors and etc. The confusion arises since a brushless dc motor does not directly operate from a dc voltage source.

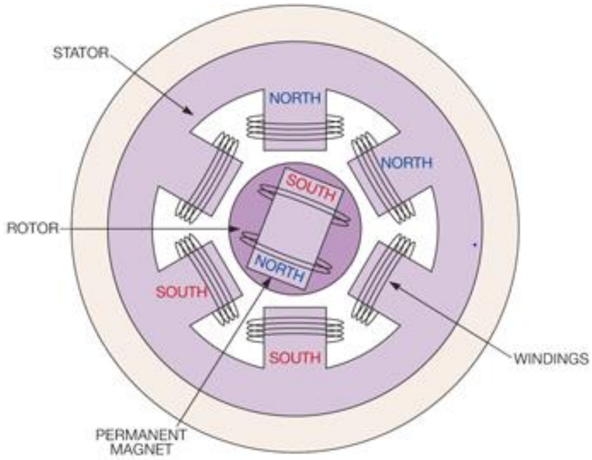


Figure 2.1 Three phase BLDC motor with two rotor poles and six stator slots. [24]

A BLDC has a rotor with permanent magnets and a stator with windings. The permanent magnet is essentially a dc motor turned inside out and a spinning armature on the inside. The rotors have two poles i.e., north and south poles as shown in Figure 2.1. There are six stator slots where the stator of a BLDC motor consists of stacked steel laminations with windings placed in the slots that are axially cut along the inner periphery.

There are two basic configurations for brushless DC motors structure: Inner-Rotor Motors and Outer-Rotor Motors. The outer-rotor motors have much more magnetic material than that of the inner-rotor. This means they are capable of more flux when the identical material is used in both structures [2]. BLDC motor design is based on a square and sine waveform motor distribution of the air gap flux density and winding density of stator phases. In BLDC motor, the coil is wound on the stator, the rotor has surface-mounted permanent magnets. The stator windings can generate magnetic field when powered, which will attract or repel the permanent magnet (rotor).

2.2 Electrical Dynamics

The dynamics of the BLDC motor is described by a set of mathematical differential equation. For a three phase BLDC motor, the basic circuit equations of the stator windings, in terms of motor electrical constants, are shown in [1].

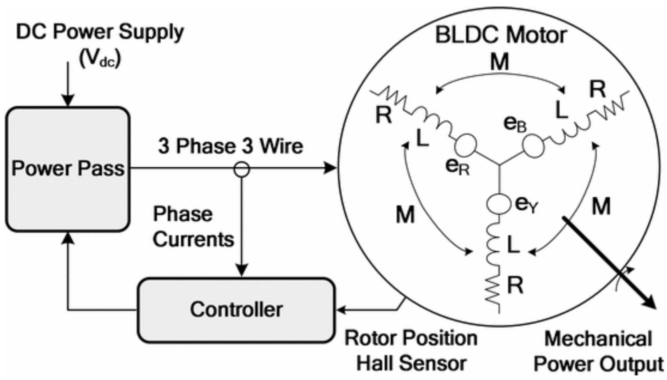


Figure 2.2 The basic elements of a conventional BLDC motor drive system [7]

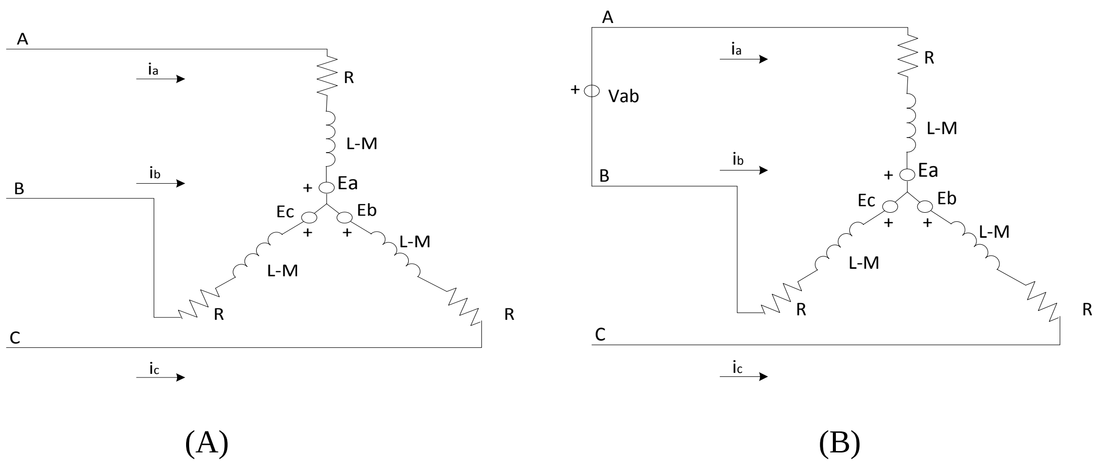


Figure 2.3 Equivalent circuit of three-phase BLDC motor (A) opened circuit, (B) V_{ab} Voltage is applied across two phases

If voltage V_{ab} is applied to the terminals A and B (Figure 2.3 B), there will be a current i_a and as rotor start to rotate, back EMF E_a and E_b will have certain value. The voltage equation for Figure 2.3 will be:

$$V_{ab}(t) = 2Ri_a(t) + 2(L-M)\frac{d}{dt}i_a(t) + (e_a(t) - e_b(t))$$

$$V_{bc}(t) = 2Ri_b(t) + 2(L-M)\frac{d}{dt}i_b(t) + \left(e_b(t) - e_c(t)\right) \quad (2.1)$$

$$V_{ca}(t) = 2Ri_c(t) + 2(L-M)\frac{d}{dt}i_c(t) + (e_c(t) - e_a(t))$$

where

R	is stator resistance
L	is stator inductance
M	is stator mutual inductance
$i_a(t), i_b(t), i_c$	is phase current
$e_a(t), e_b(t), e_c(t)$	is back EMF for phase A, B and C
$V_{ab}(t), V_{bc}(t), V_{ca}(t)$	is line to line voltages phase A, B and C

Per phase equation would be:

$$V_{an}(t) = Ri_a(t) + 2(L-M)\frac{d}{dt}i_a(t) + e_a(t) \quad (2.2)$$

Where $V_{an}(t)$ is per phase and back EMF value can be presented as a function of rotor speed and machine constant K_e as

$$e_a(t) = k_e(t)w(t) \quad (2.3)$$

The coupled circuit equations of the stator windings in terms of motor electrical constants can be written as follows

$$V_{ab}(t) = R_a i_a(t) + \frac{d}{dt}\left(L_{aa}i_a(t) + L_{ba}i_a(t) + L_{ca}i_c(t)\right) + e_a(t)$$

$$V_{bc}(t) = R_b i_b(t) + \frac{d}{dt}\left(L_{ab}i_b(t) + L_{bb}i_b(t) + L_{cb}i_c(t)\right) + e_b(t) \quad (2.4)$$

$$V_{ca}(t) = R_c i_c(t) + \frac{d}{dt}\left(L_{ac}i_c(t) + L_{bc}i_c(t) + L_{cc}i_c(t)\right) + e_c(t)$$

where

$$R_a = R_b = R_c = R$$

$$L_{aa} = L_{bb} = L_{cc} = L_s$$

$$L_{ba} = L_{ab} = L_{ca} = L_{ac} = L_{bc} = L_{cb} = M$$

Substituting above values (2.4) become

$$\begin{bmatrix} V_{an}(t) \\ V_{bn}(t) \\ V_{cn}(t) \end{bmatrix} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix} \begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix} + \begin{bmatrix} L & M & M \\ M & L & M \\ M & M & L \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix} + \begin{bmatrix} e_a(t) \\ e_b(t) \\ e_c(t) \end{bmatrix} \quad (2.5)$$

Since

$$i_a(t) + i_b(t) + i_c(t) = 0$$

Therefore

$$Mi_b(t) + Mi_c(t) = -Mi_a(t)$$

where

L is stator self-inductance

M is mutual inductance

Hence

$$\begin{bmatrix} V_{an}(t) \\ V_{bn}(t) \\ V_{cn}(t) \end{bmatrix} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix} \begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix} + \begin{bmatrix} L-M & M & M \\ M & L-M & M \\ M & M & L-M \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix} + \begin{bmatrix} e_a(t) \\ e_b(t) \\ e_c(t) \end{bmatrix} \quad (2.6)$$

The brushless dc motor and electromagnetic torque and on characteristics can be analyzed by a mathematical model. The motor three-phase winding state equation is shown below

$$\begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix} = \begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L-m} & 0 & 0 \\ 0 & \frac{1}{L-M} & 0 \\ 0 & 0 & \frac{1}{L-M} \end{bmatrix} \begin{bmatrix} V_a(t) \\ V_b(t) \\ V_c(t) \end{bmatrix} - \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix} \begin{bmatrix} i_a(t) \\ i_b(t) \\ ic(t) \end{bmatrix} - \begin{bmatrix} e_a(t) \\ e_b(t) \\ e_c(t) \end{bmatrix} \quad (2.7)$$

The relationship between the back-emfs sinusoidal against the function of rotor speed is as follows.

$$\begin{bmatrix} e_a(t) \\ e_b(t) \\ e_c(t) \end{bmatrix} = \omega_r(t)\lambda_m \begin{bmatrix} sin(\theta_r) \\ sin\left(\theta_r - \frac{2\pi}{3}\right) \\ sin\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix} \tag{2.8}$$

where

$\omega_r(t)$	is motor angular velocity
λ_m	is mutual flux due to magnet
θ_r	is electrical angular position

2.3 Mechanical Dynamics

In addition to voltage equation of BLDC, the torque produced by BLDC motor as steady-state operation, electromagnetic torque will be counter-balanced by load torque, inertia torque and friction torque

$$T_e(t) = \frac{e_a(t)i_a(t)+e_b(t)i_b(t)+e_c(t)i_c(t)}{\omega_r} \tag{2.9}$$

Hence

$$T_e(t) = \lambda_m[i_a(t) \quad i_b(t) \quad i_c(t)] \begin{bmatrix} \cos(\theta_r) \\ \cos(\theta_r - \frac{2\pi}{3}) \\ \cos(\theta_r + \frac{2\pi}{3} \end{bmatrix} \tag{2.10}$$

Hence

$$T_e = \omega_r(t)B + J\frac{d}{dt}\omega_r(t) + T_L \tag{2.11}$$

where

$T_L(t)$	is load torque
J	is load inertia
B	is viscous friction coefficient

The equation of motion is:

$$\dot{\omega}_r(t) = \frac{B}{J}\omega_r(t) + \frac{(T_e(t)-T_L(t))}{J} \tag{2.12}$$

2.4 DQ Frame

The axes convention is shown in Figure 2.4. The phase and the permanent magnet fluxes are aligned when rotor angle θ_r is zero. The block supports a second rotor axis definition in which rotor angle is defined as the angle between the phase magnetic axis and the rotor q axis.

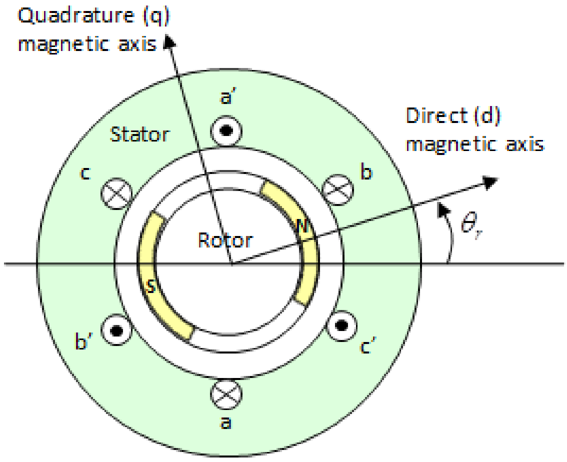


Figure 2.4 The motor construction with a single pole-pair on the rotor [26]

θ_r is angle by which d-axis leads the magnetic axis of phase a winding, electrical (rad).

ω_r is rotor angular velocity, electrical (rad/sec).

$d - axis$ is the direct *(d)* axis, centered magnetically in the center of the north pole.

$q - axis$ is the quadrature *(q)* axis, 90 electrical degrees ahead of $d - axis$.

Recall that the BLDC motor is fed by an inverter such that every stator winding is connected to the positive supply, and the other two stators are connected to ground. That is to say, the motor is controlled only by the magnitude of the supply voltage and it is not able to manipulate each individual stator voltage independently. By solving (2.7), one may obtain a set of control voltages in the stator abc variables, but a practical realization of it is doubtful. In order to obtain a single accessible control point, the rotor frame qd variables are obtained from the abc variables via the Park's transform as below.

$$\begin{bmatrix} v_a(t) \\ v_b\,(t) \\ v_b(t) \end{bmatrix} = C^{-1}\begin{bmatrix} v_q \\ v_d \end{bmatrix}, \quad \begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix} = C^{-1}\begin{bmatrix} i_q \\ i_d \end{bmatrix} \tag{2.13}$$

$$C^{-1}\begin{bmatrix} \cos(\theta_r) & \sin(\theta_r) \\ \cos(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r - \frac{2\pi}{3}) \\ \cos(\theta_r + \frac{2\pi}{3}) & \sin(\theta_r + \frac{2\pi}{3}) \end{bmatrix} \tag{2.14}$$

The qd transformation can be applied to the reference a-b-c voltage to obtain the reference V_{out} in the *qd* plane.

$$\begin{bmatrix} v_q \\ v_d \end{bmatrix} = C\begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix}, \quad \begin{bmatrix} i_q \\ i_d \end{bmatrix} = C\begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix} \tag{2.15}$$

$$C = \frac{2}{3}\begin{bmatrix} \cos(\theta_r) & \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r + \frac{2\pi}{3}) \\ \sin(\theta_r) & \sin(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r + \frac{2\pi}{3}) \end{bmatrix} \tag{2.16}$$

where

v_q, v_d is qd – axis voltages

i_q, i_d is qd – axis current

$\frac{d}{dt}\theta_r$ is ω_r

A set voltage is obtained as [3]

$$\begin{bmatrix} v_q \\ v_d \end{bmatrix} = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}\begin{bmatrix} i_q \\ i_d \end{bmatrix} + \begin{bmatrix} \omega_r L i_d + \lambda_m \omega_r \\ -\omega_r L i_q \end{bmatrix} \tag{2.17}$$

where

L_q, L_d is qd – axis inductance

R is qd – axis resistance

The electromagnetic torque can be expressed in terms of mechanical parameters as [7]

$$T_{(i_q i_d)}(t) = \frac{3P\left[\lambda_m i_q(t) + \left(L_q - L_d\right) i_d(t) i_d(t)\right]}{2} \tag{2.18}$$

where

$T_{(i_q i_d)}(t)$ is electromagnetic torque, N.m

P is number of poles

CHAPTER 3 THE THEORY OF FUZZY MODEL AND CONTROL

3.1 Fundamentals of Fuzzy Logic Controller

A fuzzy model is a nonlinear model in nature and is suitable for expressing nonlinear dynamic properties. It uses fuzzy rules, which are linguistic if-then statements involving fuzzy sets, fuzzy logic, and fuzzy inference. Fuzzy rules play a key role in representing expert control/modeling knowledge and experience and in linking the input variables of fuzzy controllers/models to output variable. There are three types of fuzzy rules including Mamdani fuzzy rule, Takagi-Sugeno (TS), and Standard Additive Model (SAM). The different fuzzy models, the Takagi-Sugeno (TS) fuzzy model has attracted the most attention. The TS fuzzy are reasonable for modeling the dynamics of nonlinear system. On other hand, the disadvantage of fuzzy system is no learning process own.

To implement fuzzy logic technique to a real application, it requires the following three steps:

1. Fuzzification – convert classical data or crisp data into fuzzy data or Membership Functions (MFs).
2. Fuzzy Inference Process – combine membership functions with the control rules to derive the fuzzy output.
3. Defuzzification – use different methods to calculate each associated output and put them into a table, the lookup table. Pick up the output from the lookup table based on the current input during an application.

Fuzzy logic controller can cope with the above situations. Precisely, fuzzy logic is an extension of a multi-valued logic system. Boolean logic is based on the two-value logic system. In a two-value logic system, a proposition is either true or false whereas in a multi-valued logic system, a proposition can also have any intermediate truth value apart from either being true or false, as the case may be. Fuzzy logic maps a data or an element to an infinite truth value set ranging between 0 and 1; 0 is FALSE and 1 is TRUE (100%). A very simple example is the statement such as, "I am 80% confident that I would get a distinction." The confidence is not (100%) true or totally false (0%).

For any set A, the characteristic function of A is defined by

$$A(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

So, characteristic function $A: X \to 0,1$ induce a constraint with well-defined boundaries on the elements of the universe X that can be assigned to a set A. The fundamental idea of fuzzy set is to relax this requirement by admitting intermediate values of class membership. Therefore, we may assign intermediate values between 0 and 1 to quantify our perception on how compatible the values are with the class, with 0 meaning incompatibility (complete exclusion) and I compatibility (complete membership). Membership values thus express the degrees to which each element of a universe is compatible with the properties distinctive to the class. Intermediate membership values means that no natural threshold exists and that elements of a universe can be a member of a class and at the same time belong to other classes with different degrees. Gradual, less strict membership degrees are the essence of fuzzy sets.

Fuzzy models have received particular attention in the area of nonlinear modeling. The Takagi-Sugeno(TS) fuzzy models, due to their capability to approximate any nonlinear behavior. The performance of TS fuzzy model depends on its complexity (number of fuzzy rules) on type of membership function and also on antecedent variables and consequent repressors.

3.2 Fuzzy Structural System

Fuzzy inference systems also known as fuzzy rule-based systems or fuzzy models are schematically shown in Figure 3.1. They are composed of five conventional blocks: a fuzzification interface which transforms the crisp inputs into degrees of match with linguistic values, a defuzzification interface which transform the fuzzy results of the inference into a crisp output, a rule base containing a number of fuzzy if-then rules, a database which defines the membership functions of the fuzzy sets used in the fuzzy rules the fuzzy inference engine unit which performs the inference operations on the rules.

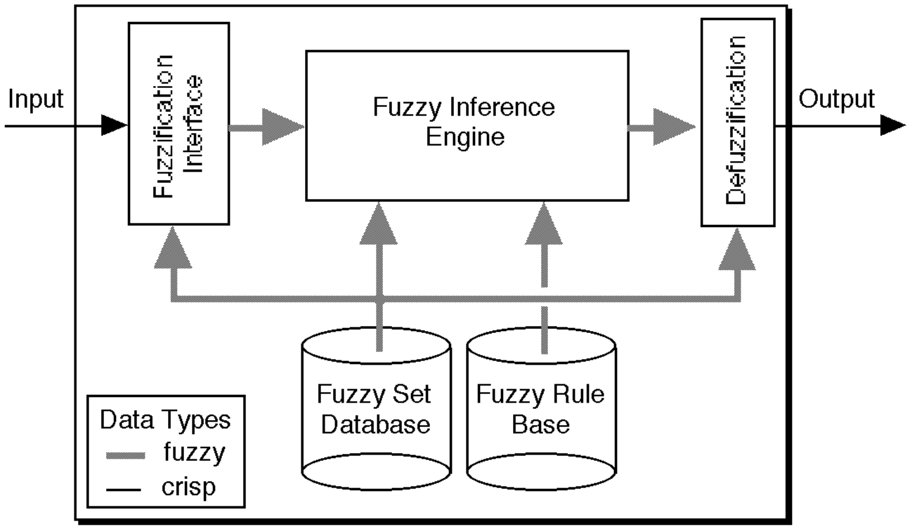


Figure 3.1 Fuzzy system structure [27]

3.2.1 Fuzzification

The fuzzification comprises the process of transforming crisp values into grades of membership for linguistic terms of fuzzy sets. The membership function (MF) is used to associate a grade to each linguistic term, for example: if we take x as a variable and low, normal and high as trapezoidal, triangle and trapezoidal MFs, respectively is shown in Figure 3.2.

- The MF low will be defined by three points: $(x_1, x_2\ , x_3,)$. However, in order to have a real trapezoid, we need four points on the left of x1 (any negative one, e.g x_0).
- Following the same reasoning, the MF high will be defined by four points: $(x_3, x_4\ , x_5, x_6)$ ($x_6$any positive > x_5, being x_5 the higher possible value for x)
- Finally, the MF normal (like any other triangular MF) will be defined by three points: $(x_2, x_3\ , x_4\ )$

How the fuzzification step works; first, for a given value of x, for example x_n which can belong to one or more MF we calculate the y value for each of the MF/s to which x_n belong. This y value has to be between 0 and 1. For example: consider three MF: low, normal and high and given value of x_n, then the degrees of membership to each MF (y values) for x_n can be, for example: 0.6 for the MF low and 0.4 for the MF normal is show in Figure 3.2. Likewise, we can fuzzificate all the values of any variable. Any of the values will belong to at least one MF with a certain degree of membership.

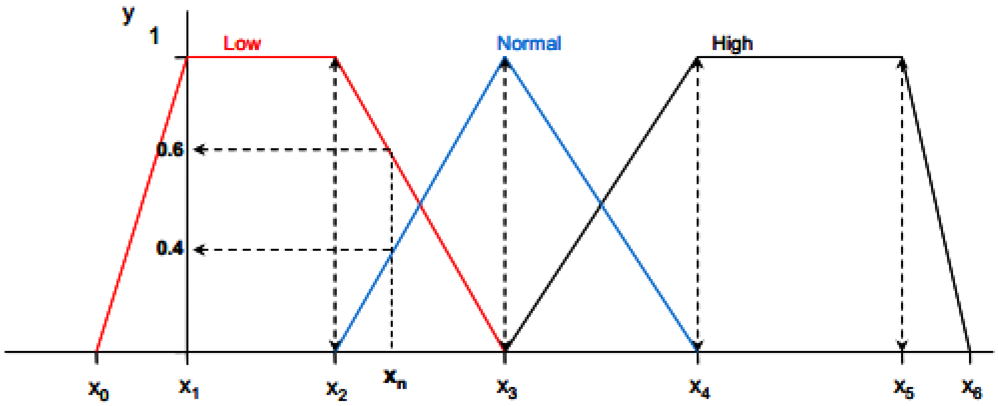


Figure 3.2 the three membership function (MF).

3.2.2 Defuzzyfication

The process of producing a quantifiable result in fuzzy logic is called defuzzification, given fuzzy sets and corresponding membership degrees. It is typically needed in fuzzy control systems. These will have a number of rules that transform a number of variables into a fuzzy result. Thus, the result is described in terms of membership in fuzzy sets.

3.2.3 Fuzzy Rule based definition

Once the input and output variables and MF are defined, and also have to design the rule base composed of expert IF -THEN rules. These rules transform the input variables to an output that will tell us the risk of operational problems, this problem have to be defined with MF, usually low, normal and high risk. Depending on the number of MF for the input and output variables, we will be able to define more or less potential rules. The easier case is a rule-based concerning only one input and one output variable. The format of fuzzy rule is IF ... THEN: suppose *z* is an output variable and *x* is an input variables. Then, a possible fuzzy logic rule is stated mentioned below.

"IF *x* is low OR normal AND IF *y* is high THEN *z* is normal"

3.3 TS Fuzzy Model

For the TS fuzzy controller, we need TS fuzzy model for nonlinear system. Therefore, the construction of fuzzy model represents an important and basic procedure. Figure 3.3 shows the mode-based fuzzy control design. In general, there are two approaches for constructing fuzzy models:

1. First Identification (fuzzy modeling) using input-output data.
2. Derivation from a set of given nonlinear system equations.

There has been an extensive literature on fuzzy modeling using input-output data following Takagi's, Sugeno's and Kang's excellent work [8], [10]. The procedure mainly consists of two parts: structure identification and parameter identification. The identification approach to fuzzy modeling is suitable for plants that are unable or too difficult to be represented by analytical and/or physical models. On the other hand, nonlinear dynamic models for mechanical systems can be readily obtained, for examples, the Lagrange method and the Newton-Euler method. This section focuses on derivation of a fuzzy model from given nonlinear dynamical models for approach. This approach utilizes the idea of "sector nonlinearity," "local approximation," or a combination of them to construct fuzzy models.

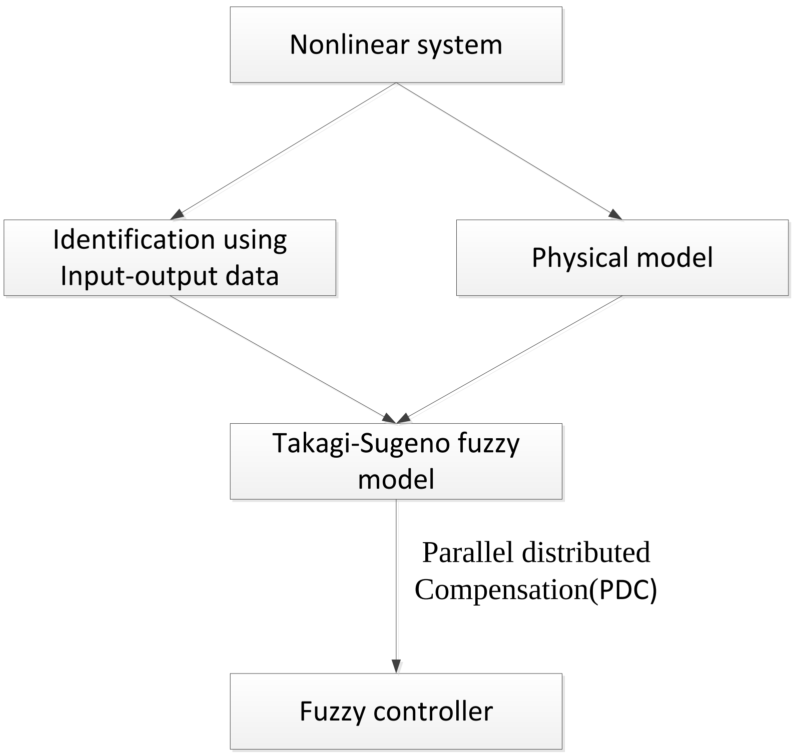
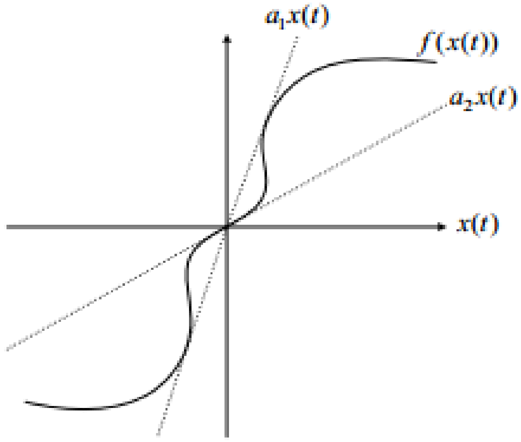
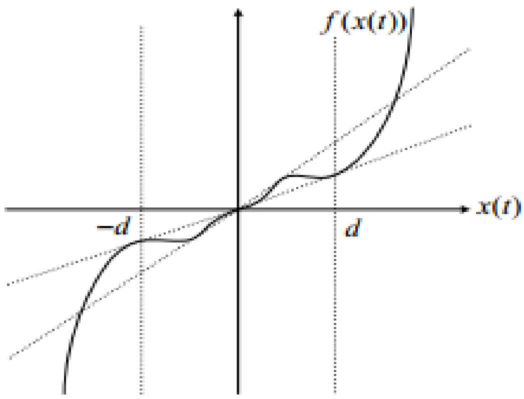


Figure 3.3 Fuzzy control design model

The idea of using sector nonlinearity in fuzzy model construction first appeared in [30]. Sector nonlinearity is based on the following idea.



(a)



(b)

Figure 3.4 (a) Global sector nonlinearity (b) Local sector nonlinearity [30]

Figure 3.4 (a) and Figure 3.4 (b) show the concrete steps to construct fuzzy models. Consider a simple nonlinear system. $\dot{x}(t) = f(x(t))$, where $f(0) = 0$. This system is find the global sector such that $\dot{x}(t) = f\big(x(t)\big) \in [a_1\ a_2]x(t)$. The idea of using sector nonlinear in fuzzy model construction first appeared in [36]. The global sector nonlinearity approach is shown in Figure 3.4 (a). This figure guarantees an exact fuzzy model construction. However, it is sometimes difficult to find global sector for general nonlinear systems. In this case, we consider local sector nonlinearity. This is reasonable as variables of physical systems are always bounded. The local sector nonlinearity is show in Figure 3.4 (b), where two lines become the local sectors under $-d < x(t) <$

d. The fuzzy model exactly represents the nonlinear system in that "local" region, $-d < x(t) < d$.

The system is nonlinear, scalar SISO systems of the form:

$$\frac{dx}{dt} = A(u)x + B(u) \tag{3.1}$$

Here, x is the system state, u is the control input, and A and B scalar functions. The TS approach is to associate with the i^{th} operating region, a set of n fuzzy sets centered at the i^{th} operating point, and of characterizing the system in term of a Taylor series expansion as above but using linguistic rules.

The i^{th} rule of this fuzzy model is of the following form:
Plant Rule i:

IF $x_1(t)$ is M_{i_1} ... and $x_n(t)$ is M_{i_1}

THEN $x(t) = Ax(t) + Bu(t)$

$i = 1,2,\ldots,r$ where r is the number of rules, and M_{ij} are linguistic terms such as high, low slightly, defined in terns of fuzzy sets over the domains of definition of the corresponding variables. The rules in effect suggest what form the plant model takes (in terms of ($A + B$), depending on the region of operation and in particular depending on the value of inputs. This model will be called an affine TS model.

CHAPTER 4 THE THEORY OF H_∞ FUZZY CONTROL

4.1 Linear matrix inequalities

A linear matrix inequality is an expression of the form

$$M(p) = M_0 + p_1 M_1 + \ldots + p_n M_n < 0 \tag{4.1}$$

where $M_0,\ M_1, \ldots, M_n$ are symmetric matrices, $p = [p_1 p_2 \ldots . p_n]^T$ is a vector of real numbers called the decision variables, and the matrix inequality $M(p) < 0$ means that the left-hand side is negative definite.

An important property of LMI is that the set of all solution p is convex.

Linear matrix inequalities can be used as contraints for minimization problem.

$$\min c^T\ p\ subject\ to\ M(p) < 0 \tag{4.2}$$

where the element of the vector cast function are weights on the individual decision variable. This problem is convex and can be solved by efficient, polynomial-time interior-point method.

The condition that the poles of a system are located within a given region in the complex plane can be formulated as an LMI constraint. Here is simple example: the homogenous system

$$\dot{x}(t) = Ax(t) \tag{4.3}$$

is stable if and only if the matrix A has all eigenvalues in the left half plane, which in turn is true if and only if there exists a positive definite, symmetric matrix P that satisfies:

$$PA^T + AP < 0 \tag{4.4}$$

where P is variable, $A \epsilon\ R^{nxn}$ is given and $P = P^T$ is the variable. Let $p_1, \ldots, p_n$ be a basis for symmetric nxn matices, where $m = n\,(n + 1)/2$. Then take $F_0 = 0$ and $F_i = A^T p_i + p_i A$. Yet, it is more convenient and efficient to describe it in its natural form (4.4). The requirement $P > 0,\ A^T P + PA < 0$ is what we now call a Lyapunov inequality on P.

The design of a state feedback for a state-space model is straight forward. From the Lyapunov equation, the problem of controller design can be stated as: Find a matrix $P > 0$ and a matrix $K \epsilon R^{nxn}$ (p number of inputs) such that

$$(A + BK)^T P + P(A + BK) < 0 \quad (4.5)$$

where K is the state feedback.

The equation is not LMI since there are product of unknowns, P and K. The obtain an LMI is to create the new variables $Q = P^{-1}$ and $Y = KP^{-1}$ and to pre and post multiply (4.4) by P^{-1}

$$QA^T + AQ + Y^T B^T + BY < 0 \quad (4.6)$$

This equation is clearly an LMI and can be solved by an LMI algorithm. The feedback K is computed from the solution of this LMI by taking $K = Y Q^{-1}$.

4.2 H_∞Performance

While taking into account external disturbance acting on the process to be controlled as:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t)B_w w(t) \\ z(t) &= Cx(t) + D_{21} w(t) \\ y(t) &= C_2 x(t) + D_{21} u(t) \end{aligned} \quad (4.7)$$

Here $x(t), u(t)$ and $z(t)$ denote the state, the input control and output control vector, respectively. The vector $w(t)$ is the disturbance vector. The matrices A, B, C, D and Bw are matrices of appropriate dimensions.

Consider again the linear model (4.7) with A stable and let $G(s)$ denote the transfer function matrix from w to z. The H_∞-norm of $G(s)$is defined as

$$\|G\|_\infty = \sup_{\omega} \sigma_{max}\big(G(j\omega)\big),$$

where $\sigma_{max}(G)$ denote the largest singular value of G. For practical purposes, the supermum “sup” of the function is the same as its maximum value; (it may approach its maximum as a limit). The H_∞-norm can be interpreted as the maximum transfer function matrix gain on $j\omega$ axis.

An equivalent definition of the H_∞-norm is

$$\|G\|_\infty^2 = sup \frac{\int_0^\infty z^T(t)z(t)dt}{\int_0^\infty w^T(t)w(t)dt}$$

where it is assumed that $x(0) = 0$. Therefore, $\|G\|_\infty$ is the maximum possible gain in signal energy. This fact can be used to express constraints on the H_∞-norm in terms of linear matrix inequalities. From the above, it follows that $\|G\|_\infty < \gamma$ is equivalent to

$$\int_0^\infty (z^T(t)z(t) - \gamma^2 w^T(t)w(t))dt < 0$$

CHAPTER 5 SPEED CONTROL DESIGN FOR BLDC MOTOR

BLDC motors do not use brushes for commutation. They use an electronically controlled commutation system instead of a mechanical commutation system. BLDC motors require good controller to support function. Currently, the most used conventional controller is PID controller, however PID controller may not be good controller since the PID controller shows an unsatisfied performance in the applications of BLDC motor system that containing nonlinear variables which are suffered from larger overshoot, oscillation and slower response.

In this research, we propose speed control of BLDC with H_∞ fuzzy controller based on LMI approach since it provides advantages to the system such as more stability, fast response and not overshoot.

There are several methods of controller and this chapter will present the PID controller and H_∞ fuzzy controller based on LMI approach.

5.1 PID Controller

The PID controller is a common form of feedback. A PID controller consists the three proportional terms (P), integral (I), and derivative (D). Its behavior can be roughly interpreted as the sum of three-term actions:

- The P term gives a rapid control response and a possible steady state error;
- The I term eliminates the steady state error; and
- The D term improves the behavior of the control system during transients.

Figure 5.1 shows general block diagram of parallel PID control system. So, we can combine the three different control actions in a PID controller. It is obtained by

$$u(t) = K_p e(t) + K_i \int e(t)dt + K_d \frac{de(t)}{dt} \tag{5.1}$$

whose transfer function is

$$\frac{U(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s_{T_d s}}\right) = K_p\left(\frac{T_i T_d s^2 + T_i s + 1}{T_i s}\right) \quad (5.2)$$

where K_p is proportional gain, K_i is integral gain, K_d is derivative gain, and e is error.

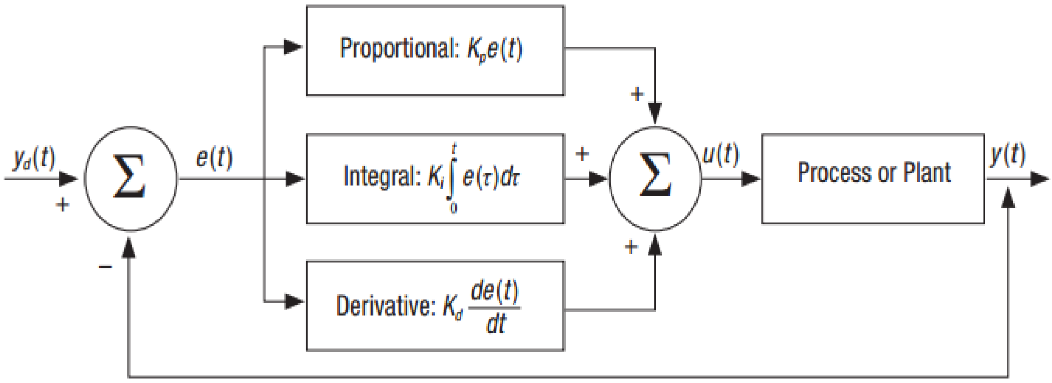


Figure 5.1 Block diagram structure of parallel PID control system [31]

Ziegler Nichols closed-loop method

The Ziegler-Nichols closed-loop tuning method allows using the ultimate gain value. Ziegler-Nichols is a simple method of tuning PID controllers and can be refined to give better approximations of the controller. The Ziegler-Nichols closed-loop tuning method is limited to tuning processes that cannot run in an open-loop environment. The specifications of the drive application are usually available in terms of percentage overshoot and settling time. The PID parameters are chosen so as to place the poles at appropriate locations to get the desired response. The ultimate period is the time required to complete one full oscillation while the system is at steady state. These two parameters, K_u and P_u, are used to find the loop-tuning constants of the controller (P, PI, or PID). To find the values of these parameters and to calculate the tuning constants, the following step can be used. First, remove integral and derivative action. Set integral time (T_i) to 999 or its largest value and set the derivative controller (T_d) to zero. Then, create a small disturbance in the loop by changing the set point. Adjust the proportional, increasing and/or decreasing the gain until the oscillations have constant amplitude. Record the gain value (K_u) and period of oscillation (P_u). Finally, plug these values into the Ziegler-Nichols closed-loop equations and determine the necessary settings for the controller. The control law settings are then obtained from Table 5.l.

Table 5.1 Ziegler-Nichols parameters [31]

Controller	K_p	T_i	T_d
P	$0.5K_u$	-	-
PI	$0.45K_u$	$0.8P_u$	-
PID	$0.59K_u$	$0.5P_u$	$0.125P_u$

5.2 H_∞ Fuzzy Controller Based on LMI

TS fuzzy system with parametric uncertainties is as follows [32]:

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i\big(\nu(t)\big)[[A_i + \Delta A_i]x(t) + [B_{1_i} + \Delta B_{1_i}]w(t)$$
$$+[B_{2_i} + \Delta B_{2_i}]u(t)], \qquad x(0) = 0$$
$$z(t) = \sum_{i=1}^{r} \mu_i\big(\nu(t)\big)\Big[[C_{1_i} + \Delta C_{1_i}]x(t) + [D_{12_i} + \Delta D_{12_i}]u(t)\Big] \tag{5.3}$$
$$y(t) = \sum_{i=1}^{r} \mu_i(\nu\,(t)\Big[[C_{2_i} + \Delta C_{2_i}]x(t) + [D_{21_i} + \Delta D_{21_i}]w(t)\Big]$$

where $\nu(t) = [\nu_1(t) \ldots \nu_\vartheta(t)]$ is the premise variable vector that may depend on states in many cases, $\mu_i\big(\nu(t)\big)$ denotes the normalized time-varying fuzzy weighting function for each rule (i.e., $\mu_i\big(\nu(t)\big) \geq 0$ and $\sum_{i=1}^{r} \mu_i\big(\nu(t)\big) = 1$), ϑ is the number of fuzzy sets, $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the input , $w(t) \in \Re^p$ is disturbance which belongs to $\mathcal{L}_2[0,\infty]$, $y(t) \in \Re^\ell$ is the measurement, $z(t) \in \Re^s$ is the controlled output, the matrices $A_i, B_{1_i}, B_{2_i}, C_{1_i}, C_{1_i}, D_{12_i}\ and\ D_{21_i}$ are of appropriate dimensions, and r is the number of IF-THEN rules. The matrices ΔA_i, $\Delta B_{1_i}, \Delta B_{2_i}, \Delta C_{1_i}$, $\Delta C_{2_i}, \Delta D_{12_i}$, and ΔD_{12_i} represent the uncertainties in the system.

Assumption 1

$$\Delta A_i = F(x(t),t)H_{1_i},$$
$$\Delta B_{1_i} = F(x(t),t)H_{2_i}, \quad \Delta B_{2_i} = F(x(t),t)H_{3_i},$$
$$\Delta C_{1_i} = F(x(t),t)H_{4_i}, \qquad \Delta C_{2_i} = F(x(t),t)H_{5_i},$$
$$\Delta D_{12_i} = F(x(t),t)H_{6_i}, \quad \text{and} \qquad \Delta D_{21_i} = F(x(t),t)H_{7_i}$$

where $H_{j_i}, j = 1,2,\dots,7$ are known matrix function which characterize the structure of the uncertainties. Furthermore, the following inequality holds:

$$\|F(x(t),t)\| \leq \rho \quad (5.4)$$

for any known positive constant ρ

Next, let us recall the following definition:

Definition 1: Suppose γ is a given positive number. The system from (5.3) is said to have an $\mathcal{L}_2$ gain less than or equal to γ if

$$\int_0^T z^T(t)z(t)dt \leq \gamma^2 \left[\int_0^{T_f} w^T(t)w(t)dt\right],\ x(0) = 0 \quad (5.5)$$

for all $T_f \geq 0$ and $w(t) \in L_2[0, T_f]$

Note that for the symmetric block matrices, we use (*) as ellipsis for the terms that are induced by symmetry.

5.3 Robust H_∞ output feedback control design

To design a full order dynamic H_∞ fuzzy output feedback controller, the following equation is used [37].

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\,\mu_j[\hat{A}_{ij}\hat{x}(t) + \hat{B}_i y(t)] \quad (5.6)$$

$$u(t) = \sum_{i=1}^{r}\mu_i\hat{C}_i\,\hat{x}(t)$$

Before presenting our next result, the following lemma is recalled.

Lemma 1: *Consider the system (5.3), given a prescribed H_∞ performance γ and a positive constant δ, if there exists a matrix $P = P^T$ satisfying the following linear matrix inequalities:*

$$P > 0 \quad (5.7)$$

$$\begin{pmatrix} \begin{pmatrix} A_{cl}^{ij}P \\ +P(A_{cl}^{ij})^T \end{pmatrix} & (*)^T & (*)^T \\ (B_{cl}^{ij})^T & -\gamma^2 I & (*)^T \\ C_{cl}^{ij}P & 0 & -I \end{pmatrix} \quad < \quad 0, \tag{5.8}$$

where $i, j = 1,2,\ldots, r$

$$A_{cl}^{ij} = \begin{bmatrix} A_i & B_{2_i}\hat{C}_j \\ \hat{B}_i C_{2_j} & \hat{A}_{ij} \end{bmatrix},$$

$$B_{cl}^{ij} = \begin{bmatrix} \tilde{B}_{1_i} \\ \hat{B}_i \tilde{D}_{21_j} \end{bmatrix},$$

$$and \quad C_{cl}^{ij} = \begin{bmatrix} \tilde{C}_{1_i} & \tilde{D}_{12_i} & \hat{C}_j \end{bmatrix}$$

with

$$\tilde{B}_{1_i} = [\delta I \quad I \quad \delta I \quad 0 \quad B_{1_i} \quad 0],$$

$$\tilde{C}_{1_i} = \begin{bmatrix} \frac{\gamma\rho}{\delta} H_{1_i}^T & 0 & \frac{\gamma\rho}{\delta} H_{5_i}^T & \sqrt{2}\lambda\rho H_{4_i}^T & \sqrt{2}\lambda C_{1_i}^T \end{bmatrix}^T,$$

$$\tilde{D}_{12_i} = \begin{bmatrix} 0 & \frac{\gamma\rho}{\delta} H_{3_i}^T & 0 & \sqrt{2}\lambda\rho H_{6_i}^T & \sqrt{2}\lambda D_{12_i}^T \end{bmatrix}^T$$

$$\tilde{D}_{21_i} = [0 \quad 0 \quad 0 \quad \delta I \quad D_{21_i} \quad I]$$

$$\lambda = \left(1 + \rho^2 \sum_{i=1}^{r} \sum_{j=1}^{r} \left[\| H_{2_i}^T H_{2_j} \| + \| H_{7_i}^T H_{7_j} \|\right]\right)^{\frac{1}{2}}$$

then the inequality (5.5) is guaranteed.

Proof: The state space form of the fuzzy system model (5.3) with the controller (5.6) is given by

$$\dot{\breve{x}}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \, \mu_j [A_{cl}^{ij} \breve{x}(t) + B_{cl}^{ij} \tilde{w}(t)] \tag{5.9}$$

$$\breve{z}(t) = \sum_{i=1}^{r} \mu_i \mu_j C_{cl}^{ij} \, \breve{x}(t)$$

where $\breve{x}(t) = [x^T(t) \quad \hat{x}^T(t)]^T$ and the matrix functions A_{cl}^{ij}, B_{cl}^{ij} and C_{cl}^{ij} are defined in Lemma 1 and the disturbance is

$$\widetilde{w}(t) = \begin{bmatrix} \frac{1}{\delta}F(x(t),t)H_{1_i}, x(t) \\ F(x(t),t)H_{2_i}w(t) \\ \frac{1}{\delta}F(x(t),t)H_{3_i}, \hat{C}_j\hat{x}(t) \\ \frac{1}{\delta}F(x(t),t)H_{5_i}, x(t) \\ w(t) \\ F(x(t),t)H_{7_i}, w(t) \end{bmatrix} \tag{5.10}$$

Let's choose a Lyapunov function

$$V(\breve{x}(t)) = \breve{x}^T(t)Q\breve{x}(t) \tag{5.11}$$

where $Q = P^{-1}$. Differentiate $V(\breve{x}(t))$ along the closed-loop system (5.9) yields

$$\dot{V}(\breve{x}(t)) = \dot{\breve{x}}(t)Q\breve{x}(t) + \breve{x}^T(t)Q\dot{\breve{x}}(t)$$

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\,\mu_j\Big(\breve{x}^T(t)\big(A_{cl}^{ij}\big)^T Q\breve{x}(t) + \breve{x}^T(t)QA_{cl}^{ij}\breve{x}^T(t) +$$

$$\widetilde{w}^T(t)\big(B_{cl}^{ij}\big)^T Q\breve{x}(t) + \breve{x}^T QB_{cl}^{ij}\widetilde{w}(t)\Big) \tag{5.12}$$

Add and subtract $-\breve{z}^T(t)\breve{z}(t) + \gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\,\mu_j\mu_m\mu_n\,[\widetilde{w}^T(t)\widetilde{w}(t)]$ to and from (5.12)

$$\dot{V}(\breve{x}(t)) = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\,\mu_j\mu_m\mu_n\,[\breve{x}^T(t) \quad \widetilde{w}(t)]$$

$$\left(\begin{matrix} \begin{pmatrix} \big(A_{cl}^{ij}\big)^T Q + QA_{cl}^{ij} \\ +\big(C_{cl}^{ij}\big)^T C_{cl}^{mn} \end{pmatrix} & (*)^T \\ QB_{cl}^{ij} & -\gamma^2 I \end{matrix}\right)\begin{bmatrix}\breve{x}(t) \\ \widetilde{w}(t)\end{bmatrix} \tag{5.13}$$

$$-\breve{z}^T(t)\breve{z}(t) + \gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\,\mu_j\mu_m\mu_n\,[\widetilde{w}^T(t)\widetilde{w}(t)]$$

Now suppose there exists a matrix $P > 0$ such that (5.8) holds, i.e.,

$$\begin{pmatrix} A_{cl}^{ij}P + P\big(A_{cl}^{ij}\big)^T & (*)^T & (*)^T \\ \big(B_{cl}^{ij}\big)^T & -\gamma^2 I & (*)^T \\ C_{cl}^{ij} & 0 & -I \end{pmatrix} < 0 \tag{5.14}$$

Pre and post multiply (5.14) by $\begin{pmatrix} Q & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}$ yields

$$\begin{pmatrix} \left(A_{cl}^{ij}\right)^T Q + QA_{cl}^{ij} & (*)^T & (*)^T \\ \left(B_{cl}^{ij}\right)^T Q & -\gamma^2 I & (*)^T \\ C_{cl}^{ij} & 0 & -I \end{pmatrix} < 0 \tag{5.15}$$

The Schurz complement of (5.15) is

$$\begin{pmatrix} \left(A_{cl}^{ij}\right)^T Q + QA_{cl}^{ij} + (C_{cl}^{ij})^T C_{cl}^{ij} & (*)^T \\ \left(B_{cl}^{ij}\right)^T & -\gamma^2 I \end{pmatrix} < 0 \tag{5.16}$$

Since (5.16) is less than zero and the fact that $\mu_i \geq 0$ and $\sum_{n=1}^{r} \mu_i = 1$, then (5.13) becomes

$$\dot{V}\left(\breve{x}(t)\right) \leq -\,\breve{z}^T(t)\breve{z}(t) + \gamma^2 \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{m=1}^{r} \sum_{n=1}^{r} \mu_i\, \mu_j \mu_m \mu_n [\widetilde{w}^T(t)\widetilde{w}(t)] \tag{5.18}$$

Integrate both sides of (5.18) yields

$$\int_0^{T_f} \dot{V}\left(\breve{x}(t)\right) dt \leq \int_0^{T_f} (-\,\breve{z}^T(t)\breve{z}(t) + \gamma^2 \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{m=1}^{r} \sum_{n=1}^{r} \mu_i\, \mu_j \mu_m \mu_n [\widetilde{w}^T(t)\widetilde{w}(t)])\, dt$$

$$V\left(\breve{x}(T_f)\right) - V\left(\breve{x}(0)\right) \leq \int_0^{T_f} (-\,\breve{z}^T(t)\breve{z}(t) + \gamma^2 \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{m=1}^{r} \sum_{n=1}^{r} \mu_i\, \mu_j \mu_m \mu_n [\widetilde{w}^T(t)\widetilde{w}(t)])\, dt$$

Assuming that $\breve{x}(0) = 0$ and $V\left(\breve{x}(T_f)\right) > 0$, we have

$$\int_0^{T_f} \breve{z}^T(t)\breve{z}(t)\, dt \;\leq\; \gamma^2 \left[\int_0^{T_f} \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{m=1}^{r} \sum_{n=1}^{r} \mu_i\, \mu_j \mu_m \mu_n [\widetilde{w}^T(t)\widetilde{w}(t)]\right] dt \tag{5.19}$$

or

$$\int_0^{T_f} z^T(t) z(t)\, dt \;\leq\; \gamma^2 \int_0^{T_f} w^T(t) w(t)\, dt \tag{5.20}$$

Hence, the inequality (5.5) is guaranteed. ■

Knowing that the controller's premise variable is the same as the plant's premise variable, the left hand of (5.8) can be re-expressed as following:

$$A_{cl}^{ij}P + P\left(A_{cl}^{ij}\right)^T + \gamma^{-2}B_{cl}^{ij}\left(B_{cl}^{ij}\right)^T + P\left(C_{cl}^{ij}\right)^T\left(C_{cl}^{ij}\right)^T P \qquad (5.21)$$

Before providing LMI-based sufficient condition for the system (5.3) to have an H_∞ performance, let us partition the matrix P as follows:

$$P = \begin{bmatrix} X & Y^{-1} - X \\ Y^{-1} - X & X - Y^{-1} \end{bmatrix} \qquad (5.22)$$

where $X \in \Re^{nxn}$ and $Y \in \Re^{nxn}$. Utilizing the partition above, we define the new controller's input and output matrices as

$$\mathcal{B}_i \triangleq [Y^{-1} - X]\hat{B}_i$$

$$\mathcal{C}_i \triangleq \hat{C}_i Y \qquad (5.23)$$

Using the changes of variable, we have the following theorem.

Theorem 1: Consider the system (4.3), given a prescribed H_∞ performance $\gamma > 0$ and a positive constant δ, if there exist matrices $X = X^T,\ Y = Y^T,\ \mathcal{B}_i\ and\ \mathcal{C}_i,\ i = 1,2,\dots,r$, that satisfy the following linear matrix inequalities:

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0 \qquad (5.24)$$

$$X > 0 \qquad (5.25)$$

$$Y > 0 \qquad (5.26)$$

$$\Psi_{11_{ii}} < 0,\ i = 1,2,\dots,r \qquad (5.27)$$

$$\Psi_{22_{ii}} < 0,\ i = 1,2,..,r \qquad (5.28)$$

$$\Psi_{11_{ij}} + \Psi_{11_{ji}} < 0,\ i < j \leq r \qquad (5.29)$$

$$\Psi_{22_{ij}} + \Psi_{22_{ji}} < 0,\ i < j \leq r \qquad (5.30)$$

where

$$\Psi_{11_{ij}} = \begin{pmatrix} (A_iY + YA_i^T + B_{2_i}\mathcal{C}_j + \mathcal{C}_i^T B_{2_j}^T + \gamma^{-2}\tilde{B}_{1_i}\tilde{B}_{1_j}^T) & (*)^T \\ [Y\tilde{C}_{1_i}^T + \mathcal{C}_i^T\widetilde{D}_{12_j}^T]^T & -I \end{pmatrix} \tag{5.31}$$

$$\Psi_{22_{ij}} = \begin{pmatrix} (A_i^T X + XA_i + \mathcal{B}_i C_{2_j} + C_{2_i}^T\mathcal{B}_j^T + \tilde{C}_{1_i}^T\tilde{B}_{1_j}) & (*)^T \\ [X\tilde{B}_{1_i} + \mathcal{B}_i\widetilde{D}_{21_j}]^T & -\gamma^2 I \end{pmatrix} \tag{5.32}$$

with

$$\tilde{B}_{1_i} = [\delta I \quad I \quad \delta I \quad 0 \quad B_{1_i} \quad 0]$$

$$\tilde{C}_{1_i} = \left[\tfrac{\gamma\rho}{\delta}H_{1_i}^T \quad 0 \quad \tfrac{\gamma\rho}{\delta}H_{5_i}^T \quad \sqrt{2}\lambda\rho H_{4_i}^T \quad \sqrt{2}\lambda C_{1_i}^T\right]^T$$

$$\widetilde{D}_{12_i} = \left[0 \quad \tfrac{\gamma\rho}{\delta}H_{3_i}^T \quad 0 \quad \sqrt{2}\lambda\rho H_{6_i}^T \quad \sqrt{2}\lambda D_{12_i}^T\right]^T$$

$$\widetilde{D}_{21_i} = [0 \quad 0 \quad 0 \quad \delta I \quad D_{21_i} \quad I]$$

$$\lambda = (1 + \rho^2 \textstyle\sum_{i=1}^r \sum_{j=1}^r \left[\| H_{2_i}^T H_{2_j} \| + \| H_{7_i}^T H_{7_j} \|\right]$$

then the prescribed H_∞ performance $\gamma > 0$ is guaranteed. Furthermore, a suitable controller i_d of the form (5.6) with

$$\begin{aligned} \hat{A}_{ij} &= [Y^{-1} - X]^{-1}\mathcal{M}_{ij}Y^{-1} \\ \hat{B}_i &= [Y^{-1} - X]^{-1}\mathcal{B}_i \\ \hat{C}_i &= \mathcal{C}_i Y^{-1} \end{aligned} \tag{5.33}$$

where

$$\begin{aligned} \mathcal{M}_{ij} = {} & -A_i^T - XA_iY - XB_{2_i}\hat{C}_jY \\ & -[Y^{-1} - X]\hat{B}_i\hat{C}_{2_j}Y - \tilde{C}_{1_i}^T\left[\tilde{C}_{1_j}Y + \widetilde{D}_{12_j}\hat{C}_jY\right] \\ & -\gamma^{-2}\{X\tilde{B}_{1_i} + [Y^{-1} - X]\hat{B}_i\widetilde{D}_{21_i}\}\tilde{B}_{1_j}^T \end{aligned} \tag{5.34}$$

Proof: Suppose there exist X and Y such that the inequalities (5.24) and (5.25)-(5.26) hold. The inequality (5.24) implies that matrix P defined in (5.21) is a positive definite matrix. Using the partition (5.22), the controller (5.23) and multiplying (5.21) to the left by $\begin{bmatrix} Y & I \\ Y & 0 \end{bmatrix}$ and to the right by $\begin{bmatrix} Y & Y \\ I & 0 \end{bmatrix}$, we have

$$\begin{bmatrix} \Phi_{11_{ij}} & 0 \\ 0 & \Phi_{22_{ij}} \end{bmatrix} \tag{5.35}$$

where

$$\Phi_{11_{ij}} = A_i Y + Y A_i^T + B_{2_i} \mathcal{C}_j + \mathcal{C}_i^T B_{2_j}^T + \gamma^{-2} \tilde{B}_{1_i} \tilde{B}_{1_j}^T$$
$$+ \left[Y \tilde{C}_{1_i}^T + \mathcal{C}_i^T \widetilde{D}_{12_j}^T \right] \left[Y \tilde{C}_{1_i}^T + \mathcal{C}_i^T \widetilde{D}_{12_j}^T \right]$$
(5.36)

$$\Phi_{22_{ij}} = A_i^T X + X A_i + \mathcal{B}_i C_{2_j} + C_{2_i}^T \mathcal{B}_j^T + \tilde{C}_{1_i}^T \tilde{C}_{1_j}$$
$$+\gamma^{-2} \left[X \tilde{B}_{1_i} + \mathcal{B}_i \widetilde{D}_{21_j} \right] \left[X \tilde{B}_{1_i} + \mathcal{B}_i \widetilde{D}_{21_j} \right]^T \tag{5.37}$$

Note that $\Phi_{11_{ij}}$ and $\Phi_{22_{ij}}$ are the Schur complements of $\Psi_{11_{ij}}$ and $\Psi_{22_{ij}}$. Using (5.27)-(5.30), we have (5.35) less than zero. Hence, by Theorem 1, we learn that the inequality (5.5) holds. ■

CHAPTER 6 NUMERICAL SIMULATION RESULT

6.1 Mathematical Model of BLDC Motor

Model of BLDC motor can be developed in the similar manner as a three-phase synchronous machine. Since there is a permanent magnet mounted on the rotor, some dynamic characteristics are different. Flux linkage from the rotor depends upon the magnetic material. Therefore, saturation of magnetic flux linkage is typical for this kind of motor. As any typical three-phase motors, one structure of the BLDC motor is fed by a three-phase voltage source. The source is not necessarily to be sinusoidal. Square wave or other wave-shape can be applied as long as the peak voltage does not exceed the maximum voltage limit of the motor [33].

The electromagnetic torque of motor is obtained by the following equation:

$$T_e = \frac{3}{2}\frac{P}{2}[\lambda_m i_q + (L_d - L_q)i_d i_q \tag{6.1}$$

where

T_e = electromagnetic torque, N.m

P = number of poles

The relation between electrical torque, speed and position is follows:

$$T_e(t) = J\left(\frac{2}{P}\right)\frac{d}{dt}\dot{\omega}_r(t) + B\left(\frac{2}{P}\right)\omega_r(t) + T_L(t) \tag{6.2}$$

$$\omega_r(t) = \frac{d}{dt}\theta_r(t) \tag{6.3}$$

$$\theta_m(t) = \theta_r(t)\left(\frac{2}{P}\right) \tag{6.4}$$

$$\omega_m(t) = \omega_r(t)\left(\frac{2}{P}\right) \tag{6.5}$$

where

J is load inertia, Kg-m^2

B is viscous friction coefficient, N.m/(rad/s)

$T_L(t)$	is load torque, N.m
$\theta_m(t)$	is angle between the mutual flux and stator current phases, rad
$\omega_m(t)$	is rotor mechanical speed, rad/s

$$\dot{\omega}_r(t) = -\frac{B}{J}\omega_r(t) + \frac{3}{2}\frac{P}{J}\lambda_m i_q(t) - \frac{P}{J}T_L(t) \tag{6.6}$$

The flux linkage for voltage on dq axis of stator and rotor is obtained by using the following equations:

$$V_q(t) = Ri_q(t) + \frac{d}{dt}\lambda_q(t) - \omega_r(t)\lambda_d(t) \tag{6.7}$$

$$V_d(t) = Ri_d(t) + \frac{d}{dt}\lambda_d(t) - \omega_r(t)\lambda_q(t) \tag{6.8}$$

$$\lambda_q(t) = L_q(t)i_q(t) \tag{6.9}$$

$$\lambda_d(t) = L_d(t)i_d(t) + \lambda_m \tag{6.10}$$

where

$V_q(t),\ V_d(t)$	is qd axis voltages, V
$i_q(t),\ i_d(t)$	is qd axis current, A
$L_q,\ L_d$	is qd axis inductance, H
λ_m	is mutual air gap flux linkages, V-s

From voltage equation qd frame can be written as current equation as

$$i_q(t) = -\frac{\lambda_m}{L_q}\omega_r(t) - \frac{R}{L_q}i_q(t) - \omega_r(t)i_d(t)\frac{L_d}{L_q} + \frac{1}{L_q}V_q(t) \tag{6.11}$$

$$i_d(t) = \omega_r(t)i_q(t)\frac{L_q}{L_d} - \frac{R}{L_d}i_d(t) + \frac{1}{L_d}V_d(t) \tag{6.12}$$

From the above definitions and values, a set of equation in the matrix can be generated as

$$\dot{x}(t) = Ax(t) + Bu(t) + w(t) \tag{6.13}$$

$$z(t) = C_1x(t) + D_{12}u(t) \tag{6.14}$$

$$y(t) = C_2x(t) \tag{6.15}$$

Equation (6.13) is defined as the state equation of the system. It describes the behavior of the system. The matrices A and B are called the state matrix and input matrix, equation (6.14), which describe the relationship between the states and the measured output equation (6.15).

Combining with the equation of motion, one has the system equation in terms of qd variable in matrix form:

$$\begin{bmatrix} \dot{\omega}_r(t) \\ i_q(t) \\ i_d(t) \end{bmatrix} = \begin{bmatrix} -\frac{B}{J} & \frac{3}{2}\frac{P}{J}\lambda_m & 0 \\ -\frac{\lambda_m}{L_q} & -\frac{R}{L_q} & -\omega_r(t)i_d(t) \\ \omega_r(t)i_q(t) & 0 & -\frac{R}{L_d} \end{bmatrix} \begin{bmatrix} \omega_r(t) \\ i_q(t) \\ i_d(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{L_q} & 0 \\ 0 & \frac{1}{L_d} \end{bmatrix} \begin{bmatrix} \nu_q(t) \\ \nu_d(t) \end{bmatrix} + \begin{bmatrix} -\frac{P}{J}T_L(t) \\ 0 \\ 0 \end{bmatrix} \quad (6.16)$$

6.2 Numerical Simulation

The matrix from (6.13) – (6.15) can be rewritten and described by following state equations:

$$\dot{x}_1(t) = -\frac{B}{J}x_1(t) + \frac{3}{2}\frac{P}{J}\lambda_m x_2(t) - \frac{P}{J}w_3(t)$$

$$\dot{x}_2(t) = -\frac{\lambda_m}{L_q}x_1(t) - \frac{R}{Lq}x_2(t) - \frac{L_d}{L_q}x_1(t)x_3(t) + \frac{1}{L_q}\, u_1(t)$$

$$\dot{x}_3(t) = \frac{L_q}{L_d}x_1(t)x_2(t) - \frac{R}{L_d}x_3(t) + \frac{1}{L_d}\, u_2(t) \quad (6.17)$$

$$z(t) = x_1(t)$$

$$y(t) = x_1(t)$$

where $x_1(t) = \omega_r(t), x_2(t) = i_q(t), x_3(t) = i_d\ (t),\ w_1(t), w_2(t),\ and\ w_3(t)$ are the disturbance factor of torque and $z(t)$ is the controlled output, $u(t)$ is the controlled input, and $y(t)$ is the measured output.

It is found that the currents and speed in dynamic model of BLDC motor from (6.17) are highly nonlinear. Simultaneously, it is also possible for the load torque to change. Thus, the nonlinearity and uncertainties including the load torque disturbance have to be taken into account [35]. The nonlinear system plant can be approximated by TS fuzzy rules. Let's choose the membership functions of the fuzzy sets as in Figure 6.2. The membership function can be written as

$$M_1(x_1(t)) = \frac{-x_1+N_2}{N_2-N_1} \text{ and } M_2(x_1(t) = \frac{x_1+N_2}{N_2-N_1} \quad (6.18)$$

The TS fuzzy plant model can be obtained as:

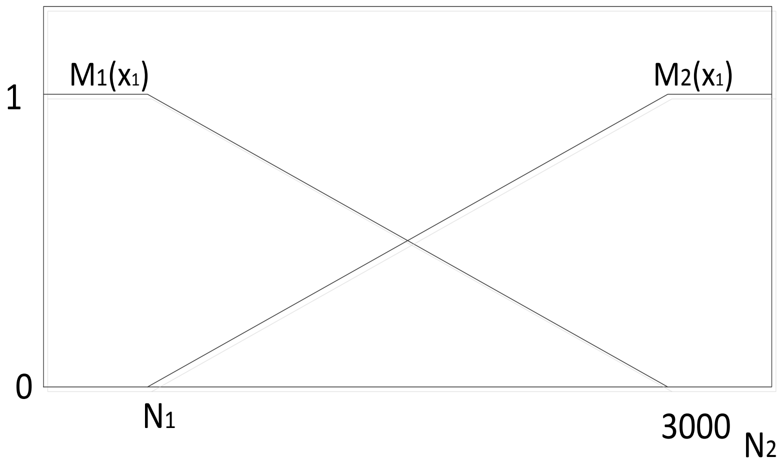


Figure 6.1 Membership function for two fuzzy set.

Plant Rule 1:

IF $x_1(t) \in [N_1 \quad N_2]$ is $M_1(x_1(t))$ THEN

$$\dot{x}(t) = \begin{bmatrix} -\frac{B}{J} & \frac{3}{2}\frac{P}{J}\lambda_m & 0 \\ -\frac{\lambda_m}{L_q} & -\frac{R}{L_q} & -N_1 \\ \omega_r(t)i_q(t) & N_1 & -\frac{R}{L_d} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{L_q} & 0 \\ 0 & \frac{1}{L_d} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & -\frac{P}{J} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_1(t) \\ w_2(t) \\ w_3(t) \end{bmatrix} \tag{6.19}$$

$$z(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

can be rewritten as

$$\dot{x}(t) = A_1 x(t) + B_1 u(t) + B_w w(t)$$

$$z(t) = C_1 x(t) + D_{12} u(t)$$

$$y(t) = C_2 x(t)$$

Plant Rule 2:

IF $x_1(t) \in [N_1 \quad N_2]$ is $M_1(x_1(t))$ THEN

$$\dot{x}(t) = \begin{bmatrix} -\frac{B}{J} & \frac{3}{2}\frac{P}{J}\lambda_m & 0 \\ -\frac{\lambda_m}{L_q} & -\frac{R}{L_q} & -N_2 \\ \omega_r(t)i_q(t) & N_2 & -\frac{R}{L_d} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{L_q} & 0 \\ 0 & \frac{1}{L_d} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & -\frac{P}{J} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_1(t) \\ w_2(t) \\ w_3(t) \end{bmatrix} \tag{6.20}$$

$$z(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

can be rewritten as

$$\dot{x}(t) = A_2 x(t) + B_2 u(t) + B_w w(t)$$

$$z(t) = C_1 x(t) + D_{12} u(t)$$

$$y(t) = C_2 x(t)$$

where

$$A_1 = \begin{bmatrix} -0.1758 & 312.41 & 0 \\ -171.23 & 2.8949 & -N_1 \\ 0 & N_1 & -2.8949 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} -0.1758 & 312.41 & 0 \\ -171.23 & 2.8949 & -N_2 \\ 0 & N_2 & -2.8949 \end{bmatrix},$$

$$B_i = \begin{bmatrix} 0 & 0 \\ 567.8 & 0 \\ 0 & 567.8 \end{bmatrix},$$

$$B_w = \begin{bmatrix} 0 & 0 & 666.67 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$C_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$D_{12} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Using the LMI optimization algorithm and Theorem 1 with variable set as $\gamma = 1, \delta = 1,$ and $\rho = 1$, we obtain a positive symmetric matrix P as

$$\hat{A}_{11} = \begin{bmatrix} -1.5003 & 0 & 0 \\ 0.525 & -1 & 0.993 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\hat{A}_{12} = \begin{bmatrix} 0.005 & 0 & 0 \\ 0.525 & -2 & 0.872 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\hat{A}_{21} = \begin{bmatrix} -0.974 & 0 & 0 \\ 0.825 & -1.5 & 0.0123 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\hat{A}_{12} = \begin{bmatrix} 0.5703 & 0 & 0 \\ 0.925 & -1 & 0.4567 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\hat{B}_1 = \begin{bmatrix} 0 & -6.76 & 4.3 \\ 0.0003 & 0 & 0.345 \\ 00.00987 & -0.475 & 0.5 \end{bmatrix},$$

$$\hat{B}_2 = \begin{bmatrix} 0 & -9.3001 & -0.8 \\ 0.9 & -0.0876 & 0.645 \\ 00.0987 & -5.475 & 0.97 \end{bmatrix},$$

$$\hat{C}_1 = \begin{bmatrix} 0.225 & -0.5 & 0.987 \\ -4.876 & 0.00311 & -0.5 \end{bmatrix},$$

$$\hat{C}_2 = \begin{bmatrix} 0.356 & -0.03 & 1 \\ 0.457 & 0.00411 & -0.005 \end{bmatrix}$$

6.3 Matlab/Simulink BLDC motor Model

The aforementioned modeling was combined to create Matlab/Simulink model as in shown Figure 6.2. Inputs to the motor are i_q, i_d, ω_r and the load torque. Outputs are created.

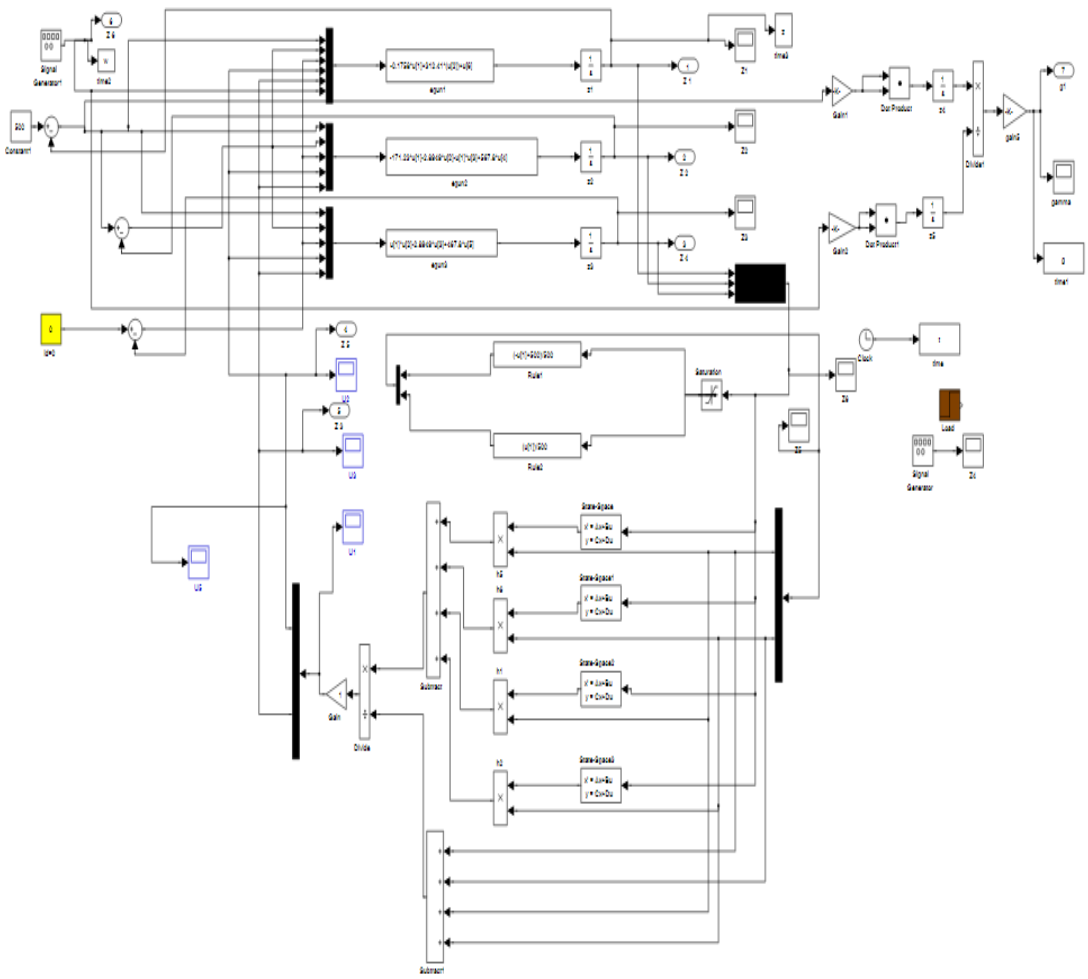


Figure 6.2 Simulation diagram of H_∞ output feedback fuzzy controller speed control BLDC motor.

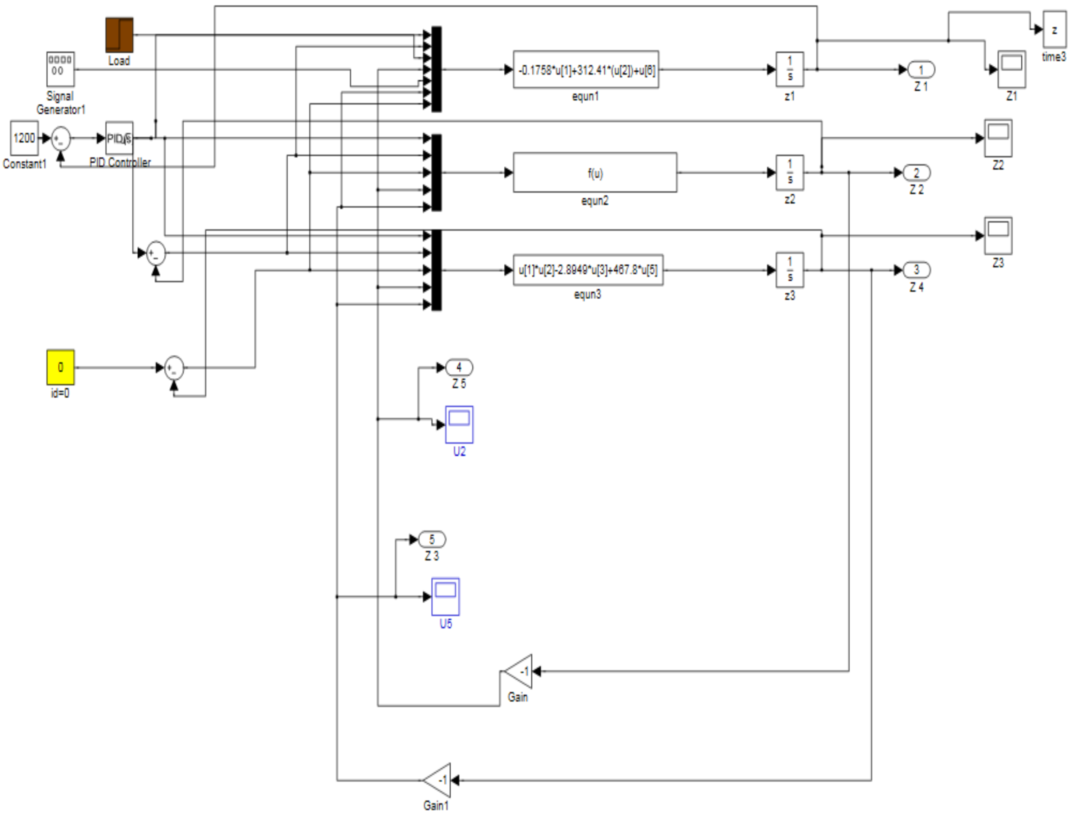


Figure 6.3 Simulation diagram of *P*ID controller speed control BLDC motor

6.4 The Simulation Results

6.4.1 Closed-loop control system using H_∞ fuzzy output feedback controller and PID control

Case 1: At a BLDC motor speed of 800 rpm, the simulation results are as follows:

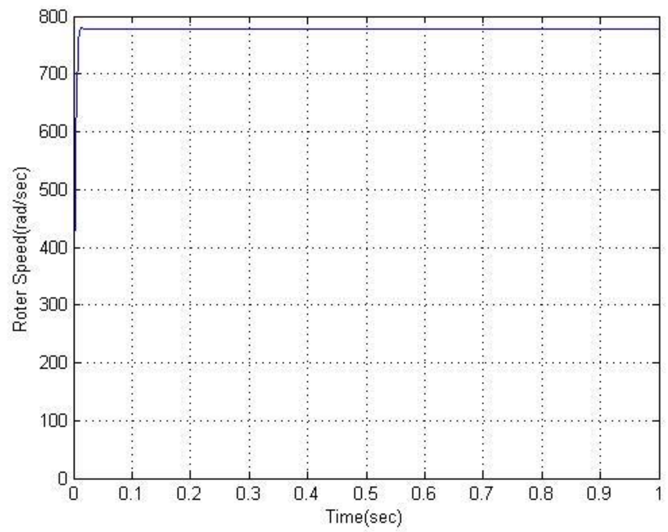


Figure 6.4 Rotation speed simulation using H_∞fuzzy output feedback controller at 800 rpm

The speed response of Figure 6.4 shows that the settling time of the motor with fuzzy controller is about 0.01 second and the overshoot is eliminated by H_∞ fuzzy controller.

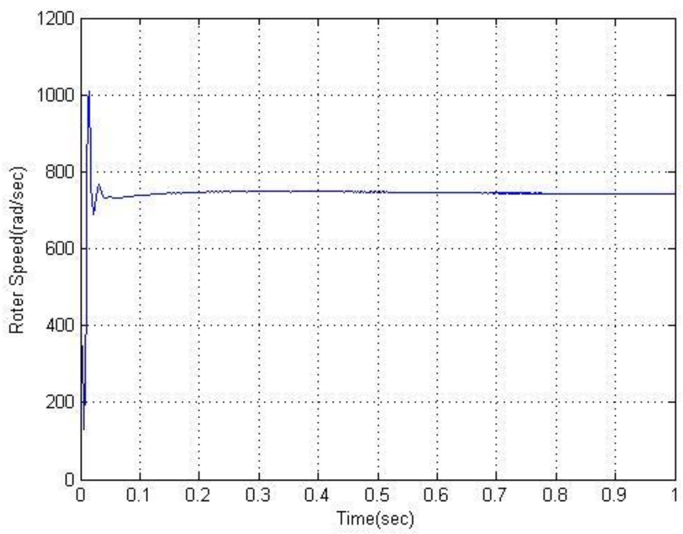


Figure 6.5 Rotation speed simulation using PID controller at 800 rpm

Substituting the values of the motor parameters and using Ziegler Nichols method, the tuning parameters are determined as $K_p = 0.02$, $K_i = 0.3$ and K_d= 0.125. From Figure 6.5, we obtain 26% overshoot and 0.1-second settling time.

Case 2: At a BLDC motor speed of 1200 rpm, the simulation results are as follows:

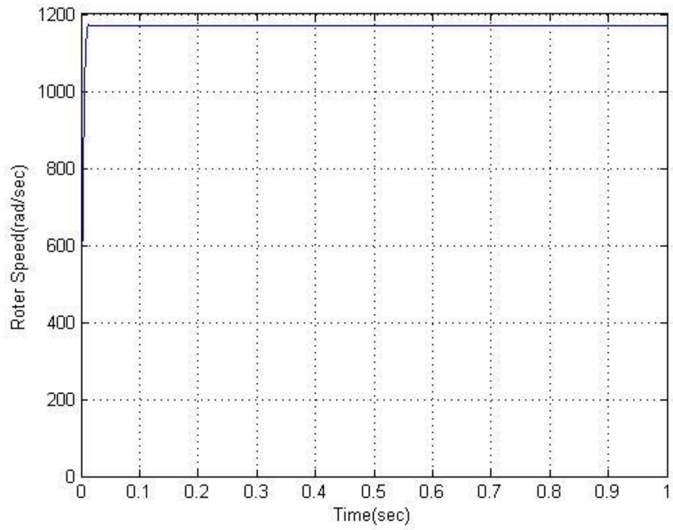


Figure 6.6 Rotation speed simulation using H_∞ fuzzy output feedback controller at 1200 rpm

The speed response of Figure 6.6 shows that the settling time of the motor with fuzzy controller is about 0.01 second and the overshoot is eliminated by H_∞ fuzzy controller.

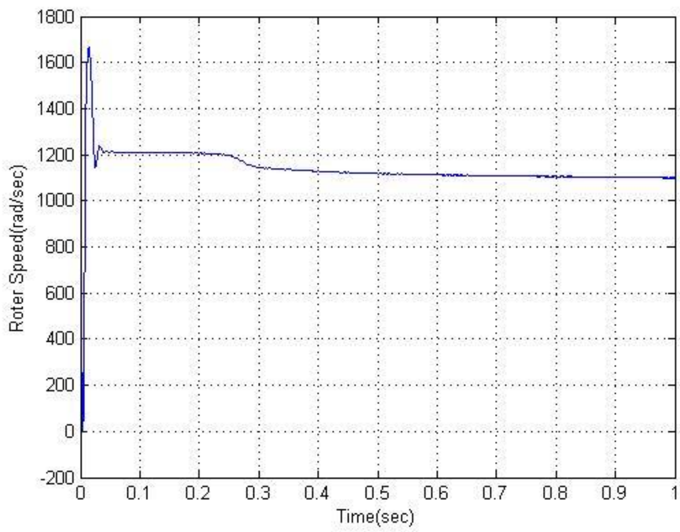


Figure 6.7 Rotation speed simulation using PID controller at 1200 rpm

Substituting the values of the motor parameters and using Ziegler Nichols method, the tuning parameters are determined as $K_p = 0.02$, $K_i = 0.4$ and K_d= 0.125. From Figure 6.7, we obtain 34% overshoot and 0.3-second settling time.

Case 3: At a BLDC motor speed of 1500 rpm, the simulation results are as follows:

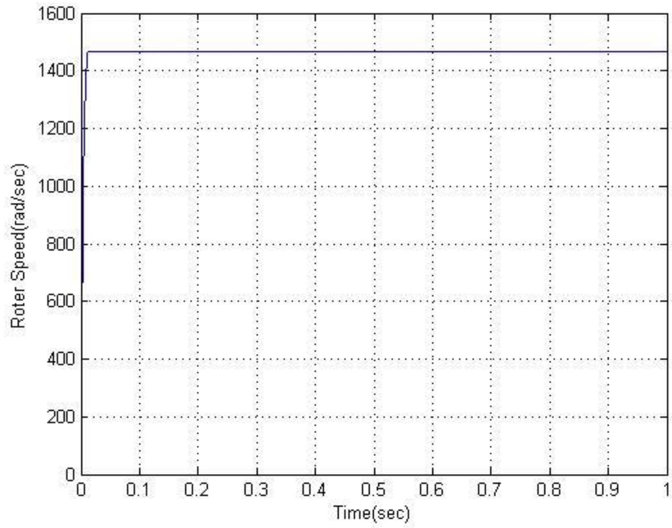


Figure 6.8 Rotation speed simulation using H_∞ fuzzy output feedback controller at 1500 rpm

The speed response of Figure 6.8 shows that the settling time of the motor with fuzzy controller is about 0.01 second and the overshoot is eliminated by H_∞ fuzzy controller.

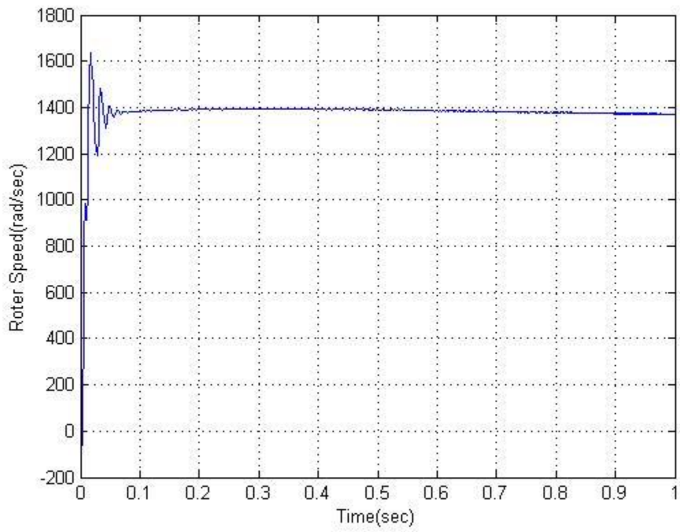


Figure 6.9 Rotation speed simulation using PID controller at 1500 rpm

Substituting the values of the motor parameters and using Ziegler Nichols method, the tuning parameters are determined as $K_p = 0.04$, $K_i = 0.4$ and K_d= 0.125. From Figure 6.9, we obtain 15% overshoot and 0.07-second settling time.

Case 4: At a BLDC motor speed of 1800 rpm, the simulation results are as follows:

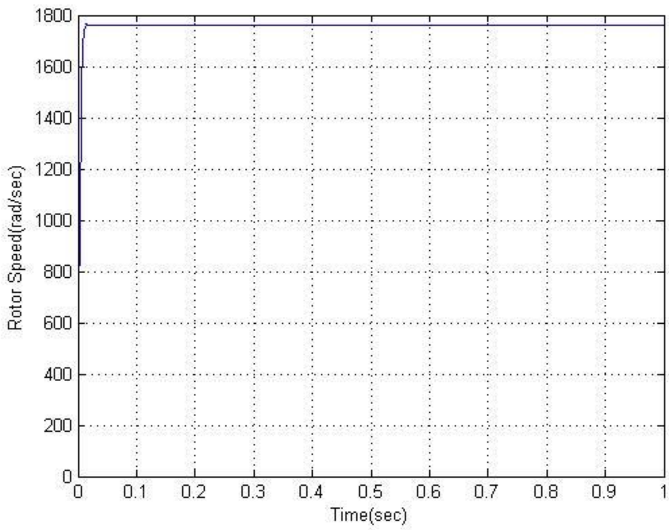


Figure 6.10 Rotation speed simulation using H_∞ fuzzy output feedback controller at 1800 rpm

The speed response of Figure 6.10 shows that the settling time of the motor with fuzzy controller is about 0.01 second and the overshoot is eliminated by H_∞ fuzzy controller.

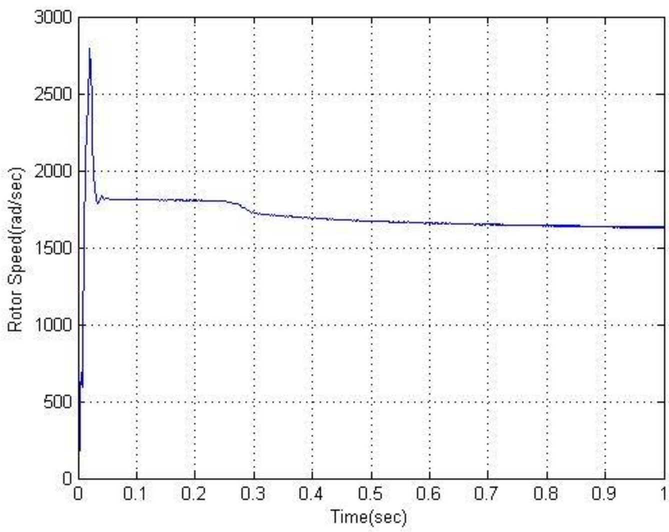


Figure 6.11 Rotation speed simulation using PID controller at 1800 rpm

Substituting the values of the motor parameters and using Ziegler Nichols method, the tuning parameters are determined as $K_p = 0.04$, $K_i = 0.3$ and K_d= 0.125. From Figure 6.11, we obtain 53% overshoot and 0.3-second settling time.

Case 5: At a BLDC motor speed of 2000 rpm, the simulation results are as follows:

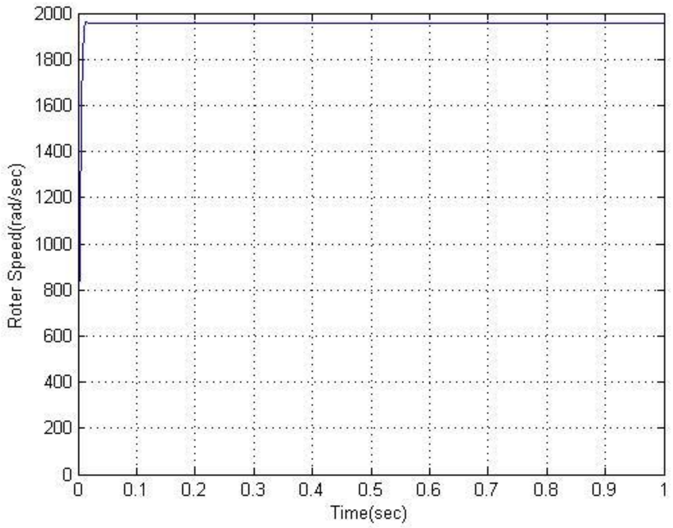


Figure 6.12 Rotation speed simulation using H_∞ fuzzy output feedback controller at 2000 rpm

From the speed response of Figure 6.12 shows that the settling time of the motor with fuzzy controller is about 0.01 second and the overshoot is eliminated by H_∞ fuzzy controller.

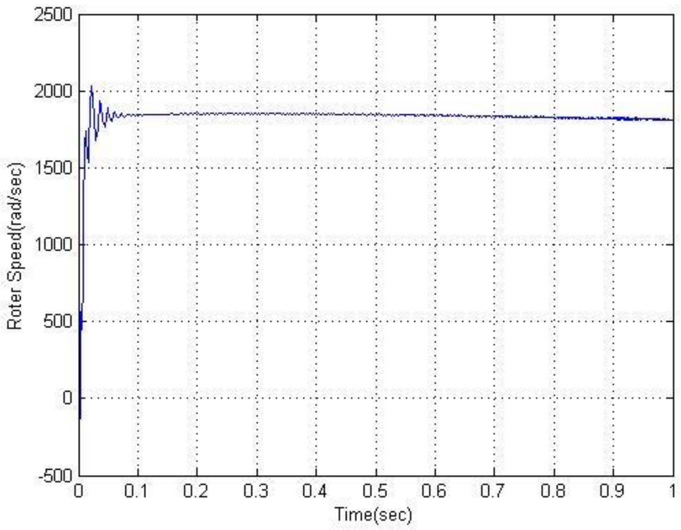


Figure 6.13 Rotation speed simulation using PID controller at 2000 rpm

Substituting the values of the motor parameters and using Ziegler Nichols method, the tuning parameters are determined as $K_p = 0.04$, $K_i = 0.4$ and K_d= 0.125. From Figure 6.13, we obtain 6% overshoot and 0.08-second settling time.

Case 6: At a BLDC motor speed of 2500 rpm, the simulation results are as follows:

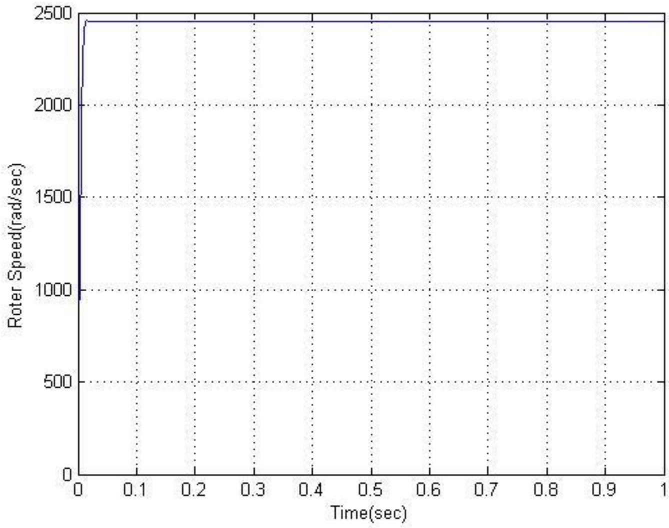


Figure 6.14 Rotation speed simulation using H_∞fuzzy output feedback controller at 2500 rpm

From the speed response of Figure 6.14 shows that the settling time of the motor with fuzzy controller is about 0.01 second and the overshoot is eliminated by H_∞ fuzzy controller.

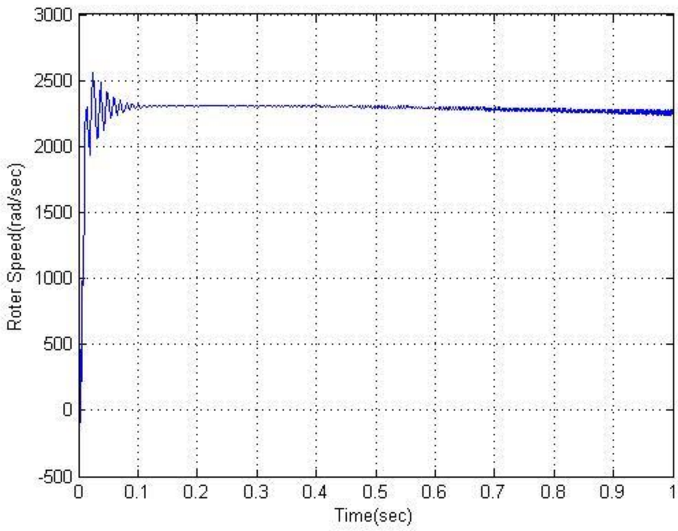


Figure 6.15 Rotation speed simulation using PID controller at 2500 rpm

Substituting the values of the motor parameters and using Ziegler Nichols method, the tuning parameters are determined as $K_p = 0.03$, $K_i = 0.4$ and K_d= 0.125. From Figure 6.15, we obtain 10% overshoot and 0.1-second settling time.

Table 6.1 Performance analysis of H_∞ fuzzy and PID controller.

Controller	RPM	Settling Time (sec)	% Overshoot
H_∞ fuzzy	800	0.01	-
PID	800	0.1	26
H_∞ fuzzy	1200	0.01	-
PID	1200	0.3	34
H_∞ fuzzy	1500	0.01	-
PID	1500	0.07	15
H_∞ fuzzy	1800	0.01	-
PID	1800	0.3	53
H_∞ fuzzy	2000	0.01	-
PID	2000	0.08	6
H_∞ fuzzy	2500	0.01	-
PID	2500	0.1	10

In order to verify the validity of proposed controller, conventional PID controller is compared with H_∞ fuzzy output-feedback controller. Table 6.1 shows the performance analysis of H_∞ fuzzy output-feedback controller and PID controller. The simulation results are at 800, 1200, 1500, 1800, 2000, and 2500 rpm. The setting times are obtained as 0.01, 0.01, 0.01, 0.01, 0.01, and 0.01 s, respectively, and the overshoot is eliminated with H_∞ fuzzy output-feedback controller. The conventional PID controller; the settling times are 0.1, 0.3, 0.07, 0.3, 0.08, and 0.1 s, respectively, and the percentage overshoot are 26%, 34%, 15%, 53%, 6%, and 10%, respectively.

CHAPTER 7 CONCLUSIONS

Literature review of techniques leads to the research including the results of preliminary simulations. The simulations show that the result of H_∞fuzzy output feedback controller can be applied to control the speed of the BLDC motor which the model could come in handy when the motor parameters are to be determined instantaneously with very vague information about the target.

The performance section shows the analysis of a BLDC motor system. There are two types of methods that are used to analyze the speed of the BLDC motor system which H_∞ fuzzy output-feedback controller and PID controller. By comparison with both controllers, it is observed that H_∞fuzzy output feedback controller gives much better dynamic response for the system. It is found that the system responds fast and no overshoot, including a stability criterion in terms of Lyapunov method can guarantee the stability of the nonlinear fuzzy system. Simulation results show that the proposed method provides a high performance control system for the BLDC motor.

Because H_∞ method can deal with multi-input and multi-output problems as well as disturbance and model error problems, the advantage of fuzzy logic is the ability to tune certain variables easily by varying the linguistic rules or input variables and LMI approach can be solved very efficiently using the convex optimization techniques.

As a result the meaningful contributions of this proposed method can be summarized as:

- enable a direct design (non-iterative process) of TS model-based fuzzy controller
- overcome the speed control dependent nonlinearity of BLDC motor system with guaranteed stability, no overshoot, and improved steady state error.

Therefore, to design a fuzzy controller, the control engineer must gather information on how an artificial decision maker would act in the closed-loop control system. Sometimes, this information can come from a human decision maker who performs the control task, understand the plant dynamics and write down a set of rules about how to control the system. Prior knowledge is required. If a case is missed, the controller would not work properly. Furthermore, it is tedious to develop fuzzy rules and membership functions and fuzzy outputs that can be interpreted in a number of ways, making analysis difficult. In addition, it requires lot of data and expertise to develop a fuzzy system. Fuzzy system requires more fine tuning and simulation before it begins operation.

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APPENDIX A

MOTOR PARAMETERS

MOTOR PARAMETERS

No. of pole pair, *P*	2
Inductance, L, L_q, L_d	0.0018 H
Stator resistance, *R*	0.5 Ω
Motor inertia, *J*	0.003 kg.m^2
Mutual air gap flux linkages, λ_m	0.311 volts/rad/sec
Friction coefficient, *B*	0.0008 N.m/rad/sec.

APPENDIX B

M-FILE MATLAB SOURCE CODE

M-FILE MATLAB SOURCE CODE

```
n=3;
setlmis([]);
X=lmivar(1,[n 1]);
Y=lmivar(1,[n 1]);
Ci1=lmivar(2,[2 n]);
Ci2=lmivar(2,[2 n]);
Bi1=lmivar(2,[3 n]);
Bi2=lmivar(2,[3 n]);

A1=[-0.1758 312.41 0;-171.23 -2.8949 0;0 0 -2.8949];
A2=[-0.1758 312.41 0;-171.23 -2.8949 -3000;0 -3000 -2.8949];
BL1=[0 0;567.8 0;0 567.8];
BL2=[0 0;567.8 0;0 567.8];

C3=[1 0 0;0 0 0;0 0 0];
D12=[0 0 0];
D21=[0 0 0];
Bw=[0 0 666.67;0 0 0;0 0 0];
E=[-1 0 0;0 -1 0;0 0 -1];
L=[-1];
F=[0 0 1];

r=0.1;
S=0.1;
Lamda=0.1;
I=eye(3,3);
O=S*I;
I1=eye(15,15);
I2=eye(18,18);
M=zeros(3,3);
C1T=sqrt(2)*Lamda*C3;
C1h=[M M M M C1T]';
B1h=[O I O M Bw M];
B=(r^-2)*B1h*B1h';
U=C1h'*C1h;
rI=(r*r)*I2;
lmiterm([-1 1 1 X],1,1);
lmiterm([-1 2 1 0],I);
```

```
lmiterm([-1 2 2 Y],1,1);

lmiterm([-2 1 1 X],1,1);

lmiterm([-3 1 1 Y],1,1);

lmiterm([4 1 1 Y],1,A1,'s');
lmiterm([4 1 1 Ci1],BL1,1);
lmiterm([4 1 1 -Ci1],1,BL1');
lmiterm([4 1 1 0],B);
lmiterm([4 2 1 -Y],C1h,1);
lmiterm([4 2 2 0],-I1);

lmiterm([5 1 1 X],1,A1,'s');
lmiterm([5 1 1 Bi1],C3,1);
lmiterm([5 1 1 -Bi1],1,C3');
lmiterm([5 1 1 0],U);
lmiterm([5 2 1 -X],B1h',1);
lmiterm([5 2 2 0],-rI)

lmiterm([6 1 1 Y],1,A1,'s');
lmiterm([6 1 1 Ci2],BL1,1);
lmiterm([6 1 1 -Ci1],1,BL2');
lmiterm([6 1 1 0],B);
lmiterm([6 1 1 Y],1,A1,'s');
lmiterm([6 1 1 Ci1],BL2,1);
lmiterm([6 1 1 -Ci2],1,BL1');
lmiterm([6 1 1 0],B);
lmiterm([6 2 1 -Y],C1h,1);
lmiterm([6 2 1 -Y],C1h,1);
lmiterm([6 2 2 0],-I1);
lmiterm([6 2 2 0],-I1);

lmiterm([7 1 1 X],1,A1,'s');
lmiterm([7 1 1 Bi1],C3,1);
lmiterm([7 1 1 -Bi1],1,C3');
lmiterm([7 1 1 0],U);
lmiterm([7 1 1 X],1,A1,'s');
lmiterm([7 1 1 Bi1],C3,1);
lmiterm([7 1 1 -Bi1],1,C3');
```

```
lmiterm([7 1 1 0],U);
lmiterm([7 2 1 -X],B1h',1);
lmiterm([7 2 1 -X],B1h',1);
lmiterm([7 2 2 0],-rI);
lmiterm([7 2 2 0],-rI);

LMIsimple=getlmis;
[tmin,xfeasp]=feasp(LMIsimple)
x=dec2mat(LMIsimple,xfeasp,X)
y=dec2mat(LMIsimple,xfeasp,Y)

B1=dec2mat(LMIsimple,xfeasp,Bi1)
B2=dec2mat(LMIsimple,xfeasp,Bi2)
C1=dec2mat(LMIsimple,xfeasp,Ci1)
C2=dec2mat(LMIsimple,xfeasp,Ci2)

Bh1=(inv(inv(y)-x))*B1
Bh2=(inv(inv(y)-x))*B2
Ch1=C1*inv(y)
Ch2=C2*inv(y)

M11=-A1'-(x*A1*y)-(x*BL2*Ch1*y)-(inv(y)-x)*(Bh1*C3*y)-C1h'*(C1h*y)-
(r^-2)*(x*B1h)*B1h'
M12=-A1'-(x*A1*y)-(x*BL2*Ch2*y)-(inv(y)-x)*(Bh1*C3*y)-C1h'*(C1h*y)-
(r^-2)*(x*B1h)*B1h'
M21=-A2'-(x*A2*y)-(x*BL2*Ch1*y)-(inv(y)-x)*(Bh2*C3*y)-C1h'*(C1h*y)-
(r^-2)*(x*B1h)*B1h'
M22=-A2'-(x*A2*y)-(x*BL2*Ch2*y)-(inv(y)-x)*(Bh2*C3*y)-C1h'*(C1h*y)-
(r^-2)*(x*B1h)*B1h'

Ah11=(inv(inv(y)-x))*M11*(inv(y))
Ah12=(inv(inv(y)-x))*M12*(inv(y))
Ah21=(inv(inv(y)-x))*M21*(inv(y))
Ah22=(inv(inv(y)-x))*M22*(inv(y))
```

CURRICULUM VITAE

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PUBLICATION	Sawitree Nasawat and Wudhichai Assawinchaichote, 2014, "Output Feedback H_∞ Fuzzy Controller for Speed Control of Brushless DC Motor", **International Science, Social Sciences, Engineering and Energy Conference,** Prajaktra Design Hotel, Udon Thani, Thailand, 17-19 December.