CHAPTER 1 INTRODUCTION

1.1 Background

In most process industries, a mass exchanger network (MEN) plays a crucial role in economic and environment consideration. The mass exchanger network is widely used in many applications, for instance, purification, product recovery and becoming more important in hazardous waste minimization due to growing of environment concern. Main operations take place in the mass exchanger are absorption, adsorption, extraction, ion exchange, leaching, and stripping. Generally, the mass exchanger network consists of a set of mass exchangers, which are the direct-contact mass transfer units. The duty of mass exchanger is employment of mass separating agent (MSA) in a lean stream in order to remove a certain component from a rich stream. The schematics of a single mass exchanger and mass exchanger network are depicted in Figures 1.1 and 1.2 respectively.

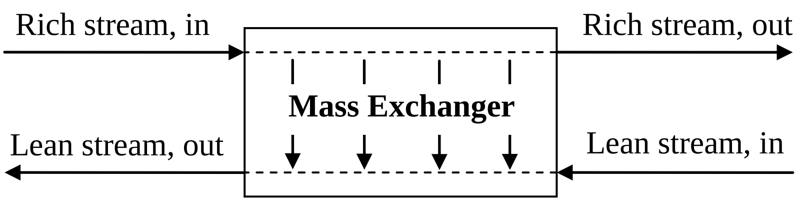


Figure 1.1 Schematic of a single mass exchanger.

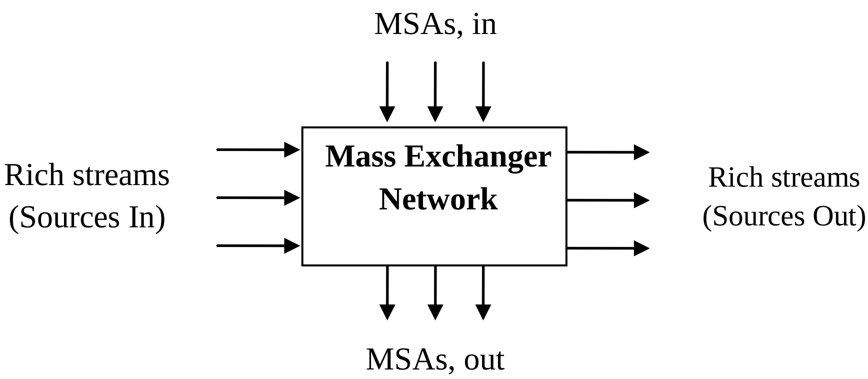


Figure 1.2 Schematic of a mass exchanger network (MEN).

However, most of published researches on the mass exchanger network mainly concentrated on steady state synthesis of optimal structure and operation of mass exchanger network by minimization the total annual cost of the mass exchanger network and the amount of mass separation agent. Therefore, dynamic behavior of the mass exchanger network becomes interest. Consequently, a control point of view of the mass exchanger network is a main motivation in this work.

A way that commonly uses to describe the dynamics behavior of the system is the state space equation which is adopted throughout this work. In addition, one of the techniques to analyze the stability of general processes is the passivity theorem which is the most useful method to achieve both stable and controllable process. Through a lag of dynamic behavior of MEN and benefit of passivity theorem, control point of view of MEN and application of passivity concept are motivation of this work.

Regarding to the motivation as mentioned above, the generic state space equations of a single mass exchanger with recycle is proposed through this work and then extend to mass exchanger network. The selected structure of mass exchanger network is used as a case study in order to illustrate the control point of view based passivity concept.

1.2 Objectives

1. To develop the state space equations of mass exchanger network.

2. To apply the passivity theorem to analyze and design the control system of mass exchanger network.

1.3 Scope of work

1. Develop the state space model (dynamic model) of mass exchanger network in Matlab-Simulink.

2. Design passive controller of mass exchanger network based on passivity theorem.

3. Test of control loop efficiency by both disturbance rejection and set point tracking.

CHAPTER 2 LITERATURE REVIEWS AND THEORIES

This chapter presents the literature reviews and theories which relate to a mass exchanger network (MEN) and a passivity concept.

2.1 Literature reviews

2.1.1 Mass exchanger network (MEN)

El-Halwagi and Manousiouthakis (1989) first introduced the mass exchanger network (MEN). They first applied pinch analysis to MEN and considered the problem of transferring the certain species from a set of rich process streams to a set of lean streams. A two-stage procedure was defined to find cost of MEN. In the first stage, a thermodynamic procedure was used to identify the thermodynamic bottleneck (pinch points), which limits the extent of mass exchanged between rich and lean process streams. A minimum allowable composition difference (ε) between rich and lean streams is introduced. Their work resulted in targets for the minimum amount of MSA, also referred as utilities. In the second stage was a development of the final configuration of the MENs that satisfied the assigned exchange duty at minimum venture cost. However, their methods only applied to a single key component, and the final network is not derived through a systematic procedure.

Yang, et al. (1999) introduced a unified model for the prediction of structural disturbance propagation in mass exchanger network. A unified model structure is introduced to model mass separating agent-based separators, splitters, mixers and a system composed of them in order to evaluate process alternatives and select the most desirable one. However, this work does not any comparison with the dynamic models.

Yan, et al. (2002) proposed disturbance rejection model for mass exchanger network with recycle. The model is extended from disturbance propagation model by Yang, et al. (1999). The structurally of mass exchanger network with recycle is investigated regarding to optimal placement of recycle for disturbance rejection problem.

2.1.2 Passivity theorem (and/or passivity properties)

The concept of passive systems originally arose in the context of electrical circuit theory. In such electrical systems, no energy is generated, e.g., a network consisting of only inductors, resistors and capacitors.

Willems (1972) analyzed the stability of interconnected passive systems. The important theorem, which can be used to determine the input-output stability of passive systems, is the passivity theorem and can be simply stated as: a system formed by the negative feedback of a passive system and a feed-forward strictly passive system with finite gain is asymptotically stable.

Bao, et al. (2000) proposed the passivity-based conditions for closed-loop stability, and also the tuning method for multi-loop PI controllers was developed satisfying the above conditions. This leads to a failure-tolerant design as each control loop can be arbitrarily and independently detuned even switched off, without affecting the closed-loop stability.

Bao, et al. (2002) provided a new approach to stability analysis for multi-loop control systems. The passivity index not only used to check whether the system is passive but also used to decide pairing schemes. The decentralized unconditional stability condition, which implies closed-loop stability of decentralized control systems under control loop failure, was derived.

Kanjanabat (2010) proposed the design of the passivity based controller for the proton exchange membrane fuel cell. The passivity concept is applicable with proton exchange membrane fuel cell and achieves both stable and controllable process.

Chaiwattanapong (2011) analyzed the stability of heat exchanger network by using passivity concept. Passivity theorem as the stability analysis tool of the interconnected systems was studied and implemented in this thesis through both a single bypass heat exchanger and heat exchanger network. The results illustrated that the passivity approach gives better set-point tracking than conventional PI controllers from the simulator and the proposed controllers could capably achieve fault-tolerant control while the other PI controllers had some deficiency and could not be controllable.

2.2 Theories

2.2.1 Overall mass balance equation (Geankoplis, 2003 and Stephanopoulos, 1985)

For most of the processing systems of interest to a chemical engineer, there are only three fundamental quantities: mass, energy and momentum. In deriving the general equation for the overall balance of the property mass, the law of mass conservation can be stated as follows:

$$\begin{bmatrix}\text{rate of mass}\\\text{accumulation}\\\text{within control volume}\end{bmatrix}=\begin{bmatrix}\text{rate of mass}\\\text{input to}\\\text{control volume}\end{bmatrix}-\begin{bmatrix}\text{rate of mass}\\\text{output from}\\\text{control volume}\end{bmatrix}+\begin{bmatrix}\text{rate of mass}\\\text{generation within}\\\text{control volume}\end{bmatrix}-\begin{bmatrix}\text{rate of mass}\\\text{consumption within}\\\text{control volume}\end{bmatrix} \tag{2.1}$$

The mass conservation law is the main concept to understand the behavior of mass exchanger.

2.2.2 State space equation

There are several ways to describe a system of linear differential equations. One of the techniques call state space equation to describe the dynamic behavior of the system is the state space equation. The state space representation is given by the following equations

$$\dot{x} = Ax + Bu + Ex_0 \tag{2.2}$$

$$y = Cx + Du \tag{2.3}$$

where	$x \in X \subset R^n$	:	State vector
	$u \in U \subset R^m$	:	Manipulated vector
	$x_0 \in X_0 \subset R^k$	:	Disturbance vector
	$y \in Y \subset R^j$	:	Controlled vector
	A, B, C, D and E	:	Constant matrix

The representation $x(t) = \Phi(t, t_0, x_0, u)$ is used to denote the state at time t reached from the initial state x_0 at t_0.

Equations 2.2-2.3 are represented in the form of standard state-space concept which is normally used in the modern control system. The feature of the state-space approach is the representation of the processes under examination by systems of first-order differential equations. Since the interest system has the standard state-space representation, the transfer function (matrix) of the system and disturbance are given explicitly by

$$G_p(s) = C(sI - A)^{-1}B + D \tag{2.4}$$

$$G_d(s) = C(sI - A)^{-1}E \tag{2.5}$$

2.2.3 Passivity theorem and related properties

This section introduces the basic concepts of passive systems including a linear passive system, a passivity index, a weighting function and a passive controller design.

1) Linear Passive Systems (Willems, 1972)

A linear time invariant system $\sum: G(s) := C(sI - A)^{-1}B + D$, (G(s) is a n×n transfer function matrix, and *A, B, C, D* are coefficients of a state space) is passive if and if *G(s)* is positive real (*PR*), or equivalently following conditions.

1. $Re[\lambda_i(A)] \leq 0$ for i = 1, …, n.

2. $G(j\omega) + G^*(j\omega) \geq 0$ for all real ω, $j\omega \neq \lambda_i(A)$.

3. The imaginary eigenvalues of *A* are non-repeated, and the residue matrix at those eigenvalues is Hermitian and nonnegative definite.

In addition, *G(s)* is said to be strictly passive or strictly positive real (*SPR*) if :

1. $Re[\lambda_i(A)] < 0$ for i = 1, …, n,

2. $G(j\omega) + G^*(j\omega) > 0$ for all real ω, $j\omega \neq \lambda_i(A)$

For the multi-loop control system as shown in Figure 2.1, if the multivariable process *G(s)* is strictly passive, then the closed loop is stable if the multi-loop controller transfer function, *K(s)*, is passive, regardless of loop interactions.

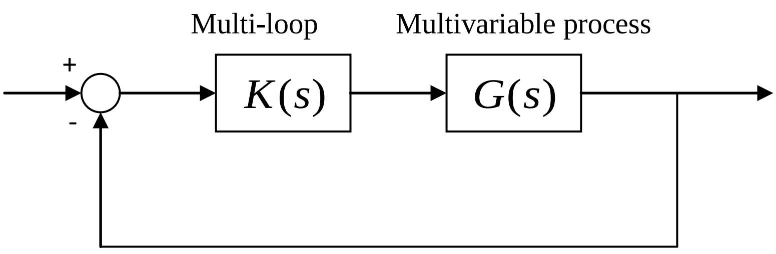


Figure 2.1 Multi-loop control system **(**Bao *et al.*, 2002)

To facilitate future stability analysis, $G^+(s) = G(s)U$ is defined, where *U* is a diagonal matrix with either 1 or –1 along the diagonal. The signs of *U* elements are determined such that the diagonal elements of $G^+(s)$ are positive at steady state, that is $G^+_{ii}(0) \geq 0$, i = 1, …, n, Let denote

$$K^+(s) = U^{-1}K(s) = UK(s) \qquad (2.6)$$

$$G^+(s) = G(s)U \qquad (2.7)$$

2) Passivity Index (ν_F) (Bao, et al., 2000)

The passivity index is defined as:

$$\nu_F(G,\omega) = -\lambda_{min}\left(\frac{1}{2}\left[G(j\omega)+G^*(j\omega)\right]\right) \qquad (2.8)$$

Passive systems are phase bounded. The phase of passive system lies in [-π/2, +π/2], therefore, the small gain condition is not required for a closed-loop system comprising of two passive systems.

Index $\nu(G,\omega)$ indicates how far the system *G(s)* is from being passive and is negative if *G(s)* is passive. Apparently, for a system *G(s)* with its passivity index $\nu(G(s),\omega)$, if a stable and minimum phase transfer function *w(s)* is chosen such that

$$\nu(w(s),\omega) < -\nu(G(s),\omega) \qquad (2.9)$$

Then, the *G(s)+w(s)I* is strictly passive.

3) Weighting function (Bao, et al., 2002)

The passivity index as previously mentioned is used to check whether the system is passive or not. If the system is non passive. The weighting function, *w(s)*, is used to render system to be passive. The weighting function has the following form

$$w(s) = \frac{ks(s+a)}{(s+b)(s+c)} \qquad (2.10)$$

Where, the parameters *a, b, c* and *k*, decision variable, can be obtained from optimization problem as followed:

$$\min_{a,b,c,k} \sum_{i=1}^{m} \left(Re\left(w(j\omega_i)\right) - \upsilon_s\left(G^+(s),\omega_i\right)\right)^2 \qquad (2.11)$$

subject to

$$Re\left(w(j\omega_i)\right) > \upsilon_s\left(G^+(s),\omega_i\right), \quad \forall i=1,\ldots,m \qquad (2.12)$$

4) Passive controller design (Bao, et al., 2002)

To stabilize the closed-loop system and achieve decentralized unconditional stability, the passive controller is designed by satisfying the following conditions:

1. $Re\left\{ \frac{k_i^+\left(j\omega\right)}{1-\nu_s\left(G\left(s\right),\omega\right)k_i^+\left(j\omega\right)} \right\} \geq 0 \qquad \forall \omega \in R, i = 1,...,n$ (2.13)

2. *K(s)* is analytic in *Re(s) > 0* (2.14)

Multi-loop PI controllers can be designed based on the passivity based stability conditions. In order to achieve decentralized unconditional stability of the closed-loop system, a controller tuning method is proposed to minimize the sensitivity function of each loop, subject to the conditions in Equations (2.13) and (2.14). For multi-loop PI controller synthesis, the condition as shown in Equations (2.13) and (2.14) are converted into the following optimization problem:

$$\min_{k_{c,i}^+,\tau_{I,i}} \left(-\gamma_i\right) \tag{2.15}$$

subject to

$$\left| \frac{w_i(j\omega)\gamma_i}{1+G_{ii}^+\left(j\omega\right)k_{c,i}^+\left[1+\frac{1}{\tau_{I,i}\times j\omega}\right]} \right| < 1 \tag{2.16}$$

and

$$\frac{k_{c,i}^+\nu_s\left(\omega\right)}{\left[1-k_{c,i}^+\nu_s\left(\omega\right)\right]\omega^2} \leq \tau_{I,i}^2 \qquad \forall \omega \in R, i = 1,...,n \tag{2.17}$$

For a given stable process *G(s)*, a multi-loop PI controller can be obtained by solving problem as a result the tuning controller parameter as Equation (2.18).

$$k(s) = diag\{k_i(s)\} = diag\left\{ k_{c,i}(1+\frac{1}{\tau_{I,i}s}) \right\} \tag{2.18}$$

If the system is non-passive system, the weighting function; $w(s)$ as Equation (2.10); is applied to stabilize the system. Therefore, the designed equation for passive controller is transform as following

$$k_i^{'}(s) = k_i^+(s)[1-w(s)k_i^+(s)]^{-1} \tag{2.19}$$

CHAPTER 3 METHODOLOGY

The aim of this work is to apply the passivity theorem on mass exchanger network (MEN). The main steps to accomplish this goal are presented as follows.

3.1 Mass exchanger network and passivity concept

This step literatures review and technical survey on a topic of the mass exchanger, the mass exchanger network and passivity concept as shown in chapter 2.

3.2 Development of dynamic model of mass exchanger network based on the controlled and the manipulated variables

The dynamic model of mass exchanger unit is derived in this step where outlet composition of rich and lean streams, recycle valve and inlet composition of rich and lean streams are defined as controlled, manipulated and disturbance variables; respectively. The model is derived based on the assumption of lumped parameter system, both rich and lean streams are well mixed, linear equilibrium relation over the operating range and under constant isothermal and isobaric condition. Finally, the derived model is linearlized and rearranged in a form of state space equation in order to fulfill the requirement of the system that applicable with passivity concept.

3.3 Process transfer function based on the passivity concept

Since the dynamic models of the mass exchanger system is developed in a way that applicable with passivity concept, the numerical value is substitute into the state space equation. Thus, *A, B, C, D and E* matrix of the state space equation is used to determine the process and disturbance transfer function by using Matlab.

3.4 Passivity index and the passive analysis

Passivity index is used to characterize the system weather it passive or not. Even though the non-passivity can be adjust to passive region by introducing the weighting function. After the system shows its passive property, the introduction of DUS for PI controller design is implemented.

3.5 Design of the passive controller

This step, PI passive controller have been designed. The tuning parameter is determined by solving optimization problem as shown in section 2.2.3

3.6 Analyzation and conclusion the results

In this step, all results are analyzed and discussed. The designed passive controllers are tested with disturbance rejection and set point tracking problem. Finally, conclusions of all work have been done and present in chapter 5.

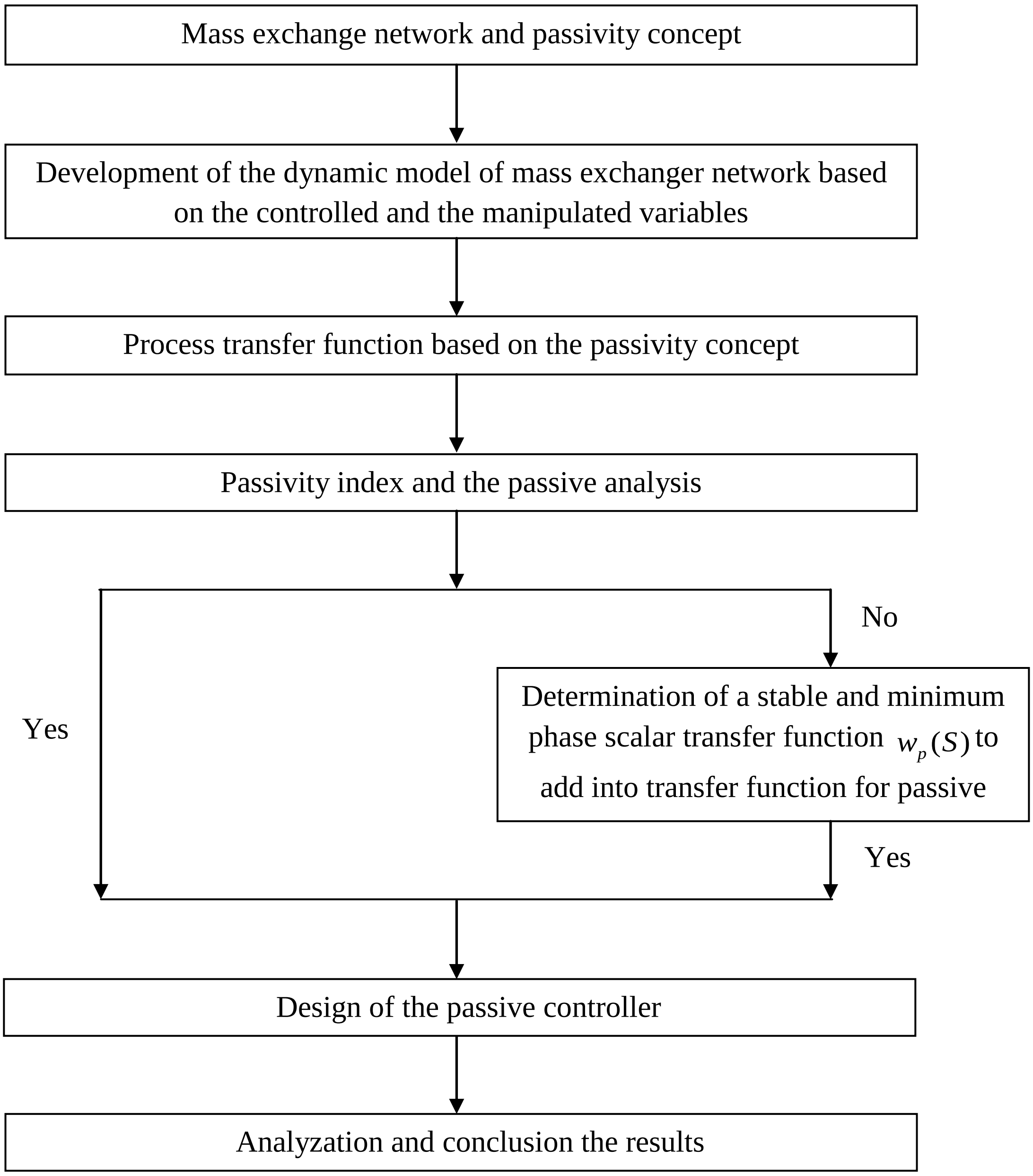


Figure 3.1 Methodology of passivity analysis and controller design for mass exchanger network**.**

CHAPTER 4 RESULTS AND DISCUSSION

4.1 Generic state space equations of a single mass exchanger with recycle

The single mass exchanger with recycle is shown in Figure 4.1. In general, the duty of mass exchanger unit is to transfer the key component from rich stream to lean stream which leads to composition decreasing in the rich stream from y_{in}' to y_{out} and increasing in the lean stream from x_{in}' to x_{out}. The dynamic model of a single mass exchanger is developed based on the controlled (outlet composition of rich and lean stream), manipulated (recycle valve) and disturbance (inlet composition of rich and lean stream) variables of the system and the following assumptions:

1. A mass exchanger model is an approximate lumped parameter system.

2. Both rich and lean streams are well mixed.

3. Equilibrium relation is linear over the operating range and express as $y = mx + c$.

4. Mass of rich and lean streams in the mass exchanger is constant.

5. The mass exchange operates under isothermal and isobaric conditions.

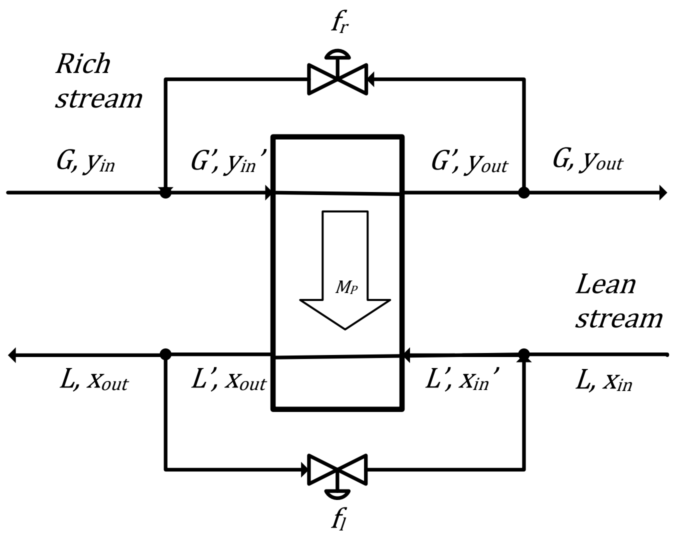


Figure 4.1 A general schematic of mass exchanger unit with recycles

The overall mass transfer (*Mp*) in the unit can be expressed as the following equation:

$$Mp = KA\frac{\left(x_{yin}^{*} - x_{out}\right) - \left(x_{yout}^{*} - x_{in}\right)}{\ln\left(\dfrac{x_{yin}^{*} - x_{out}}{x_{yout}^{*} - x_{in}}\right)} \tag{4.1}$$

To simplify the model, a logarithmic mean is replaced by an arithmetic mean, so the overall mass transfer becomes

$$Mp = KA\frac{\left(x_{yin}^{*} - x_{out}\right) + \left(x_{yout}^{*} - x_{in}\right)}{2} \tag{4.2}$$

A basic mass balance relationship based on above assumption in each stream can be derived as

$$M_G \frac{dy_{out}}{dt} = G' y_{in}' - G' y_{out} - KA\frac{\left(x_{yin}^{*} - x_{out}\right) + (x_{yout}^{*} - x_{in})}{2} \tag{4.3}$$

$$M_L \frac{dx_{out}}{dt} = L' x_{in}' - L' x_{out} + KA\frac{\left(x_{yin}^{*} - x_{out}\right) + (x_{yout}^{*} - x_{in})}{2} \tag{4.4}$$

Where

$$G' = G + f_r G' \tag{4.5}$$

$$L' = L + f_l L' \tag{4.6}$$

$$y_{in}' G' = y_{in} G + y_{out} f_r G' \tag{4.7}$$

$$x_{in}' L' = x_{in} L + x_{out} f_l L' \tag{4.8}$$

Since the dynamic models of mass exchanger unit from Equations (4.3)-(4.4) are the non-linear model, the linearization by Taylor's series expansion is needed before generating the state space of the mass exchanger system. The linear dynamic models of mass exchanger unit are presented as the following:

$$\frac{dy_{out}}{dt} = \frac{G - KA\left(1 - \overline{f}_r\right)}{2aM_G} y_{in} + \frac{-G - KA\left(1 + \overline{f}_r\right)}{2aM_G} y_{out} + \frac{KA\left(1 - \overline{f}_r\right)}{2aM_L} x_{in} + \frac{KA\left(1 + \overline{f}_r\right)}{2M_G} x_{out}$$

$$+ \frac{KA\left(\overline{y}_{in} - \overline{y}_{out}\right)}{2aM_G} f_r + \frac{KA\left(\overline{x}_{out} - \overline{x}_{in}\right)}{2M_G} f_l \tag{4.9}$$

$$\frac{dx_{out}}{dt} = \frac{L - KA\left(1-\overline{f}_l\right)}{2M_L} x_{in} + \frac{-L - KA\left(1+\overline{f}_l\right)}{2M_L} x_{out} + \frac{KA\left(1-\overline{f}_r\right)}{2aM_L} y_{in} + \frac{KA\left(1+\overline{f}_r\right)}{2aM_L} y_{out} + \frac{KA\left(\overline{y}_{out} - \overline{y}_{in}\right)}{2aM_L} f_r + \frac{KA\left(\overline{x}_{in} - \overline{x}_{out}\right)}{2M_L} f_l \qquad (4.10)$$

To study the transient response of mass exchanger unit by the passivity concept, the linear dynamic model of mass exchanger is first represented in the state-space form. Equations (4.9)-(4.10) are arranged in the form of state space in order to find out the transfer function of the system as following:

$$\begin{bmatrix} \dot{y}_{out} \\ \dot{x}_{out} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} y_{out} \\ x_{out} \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} f_r \\ f_l \end{bmatrix} + \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix} \begin{bmatrix} y_{in} \\ x_{in} \end{bmatrix} \qquad (4.11)$$

$$\begin{bmatrix} y_{out} \\ x_{out} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_{out} \\ x_{out} \end{bmatrix} \qquad (4.12)$$

Where

$$A_{11} = \frac{-G - KA\left(1+\overline{f}_r\right)}{2aM_G} \qquad (4.13) \qquad\qquad A_{21} = \frac{KA\left(1+\overline{f}_r\right)}{2aM_L} \qquad (4.14)$$

$$A_{12} = \frac{KA\left(1+\overline{f}_l\right)}{2M_G} \qquad (4.15) \qquad\qquad A_{22} = \frac{-L - KA\left(1+\overline{f}_l\right)}{2M_L} \qquad (4.16)$$

$$B_{11} = \frac{KA\left(\overline{y}_{in} - \overline{y}_{out}\right)}{2aM_G} \qquad (4.17) \qquad\qquad B_{12} = \frac{KA\left(\overline{x}_{out} - \overline{x}_{in}\right)}{2M_G} \qquad (4.18)$$

$$B_{21} = \frac{KA\left(\overline{y}_{out} - \overline{y}_{in}\right)}{2aM_L} \qquad (4.19) \qquad\qquad B_{22} = \frac{KA\left(\overline{x}_{in} - \overline{x}_{out}\right)}{2M_L} \qquad (4.20)$$

$$E_{11} = \frac{G - KA\left(1-\overline{f}_r\right)}{2aM_G} \qquad (4.21) \qquad\qquad E_{12} = \frac{KA\left(1-\overline{f}_l\right)}{2M_G} \qquad (4.22)$$

$$E_{21} = \frac{KA\left(1-\overline{f}_r\right)}{2aM_L} \qquad (4.23) \qquad\qquad E_{22} = \frac{L - KA\left(1-\overline{f}_l\right)}{2M_L} \qquad (4.24)$$

4.2 Example of a single mass exchanger unit

A single mass exchange unit of copper recovery in an etching plant (El-Halwagi and Manousiouthakis, 1990) is used as an example for verify the model and apply passivity concept. The numerical value at steady state is shown in Table 4.1.

Table 4.1 Steady state information of mass exchanger in copper recovery unit.

Parameter	**Unit**	**Value**
Rich stream mass rate	kg/s	0.1
Lean stream mass rate	kg/s	0.0925
Inlet rich stream composition	-	0.06
Inlet lean stream composition	-	0.03
Output rich stream composition	-	0.02307
Output lean stream composition	-	0.07
Rich stream mass accumulation	kg	50*
Lean stream mass accumulation	kg	50*
Recycle fraction	-	0
Overall mass transfer coefficient	kg/s	0.7079**

Remark * = hypothesis value
**=calculated value based on steady state model

After substituting the numerical values at steady-state into A, B, and E matrix, the state space of a single mass exchanger is represented as the following:

$$\begin{bmatrix} \dot{y}_{out} \\ \dot{x}_{out} \end{bmatrix} = \begin{bmatrix} -0.0116 & 0.0071 \\ 0.0096 & -0.0089 \end{bmatrix} \begin{bmatrix} y_{out} \\ x_{out} \end{bmatrix} + \begin{bmatrix} 0.0003565 & 0.0002829 \\ -0.0003565 & -0.0002829 \end{bmatrix} \begin{bmatrix} f_r \\ f_l \end{bmatrix} + \begin{bmatrix} -0.0076 & 0.0071 \\ 0.0096 & -0.0052 \end{bmatrix} \begin{bmatrix} y_{in} \\ x_{in} \end{bmatrix} \tag{4.25}$$

$$\begin{bmatrix} y_{out} \\ x_{out} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_{out} \\ x_{out} \end{bmatrix} \tag{4.26}$$

Thus, the process transfer function can be determined by using MATLAB™ as

$$\begin{bmatrix} \dot{y}_{out} \\ \dot{x}_{out} \end{bmatrix} = \begin{bmatrix} \dfrac{0.0003565\mathrm{S} + 6.595\mathrm{x}10^{-7}}{\mathrm{S}^2 + 0.02057\mathrm{S} + 3.57\mathrm{x}10^{-5}} & \dfrac{0.0002829\mathrm{S} + 5.234\mathrm{x}10^{-7}}{\mathrm{S}^2 + 0.02057\mathrm{S} + 3.57\mathrm{x}10^{-5}} \\ \dfrac{-0.0003565\mathrm{S} - 7.13\mathrm{x}10^{-7}}{\mathrm{S}^2 + 0.02057\mathrm{S} + 3.57\mathrm{x}10^{-5}} & \dfrac{-0.0002829\mathrm{S} - 5.658\mathrm{x}10^{-7}}{\mathrm{S}^2 + 0.02057\mathrm{S} + 3.57\mathrm{x}10^{-5}} \end{bmatrix} \begin{bmatrix} f_r \\ f_l \end{bmatrix} \tag{4.27}$$

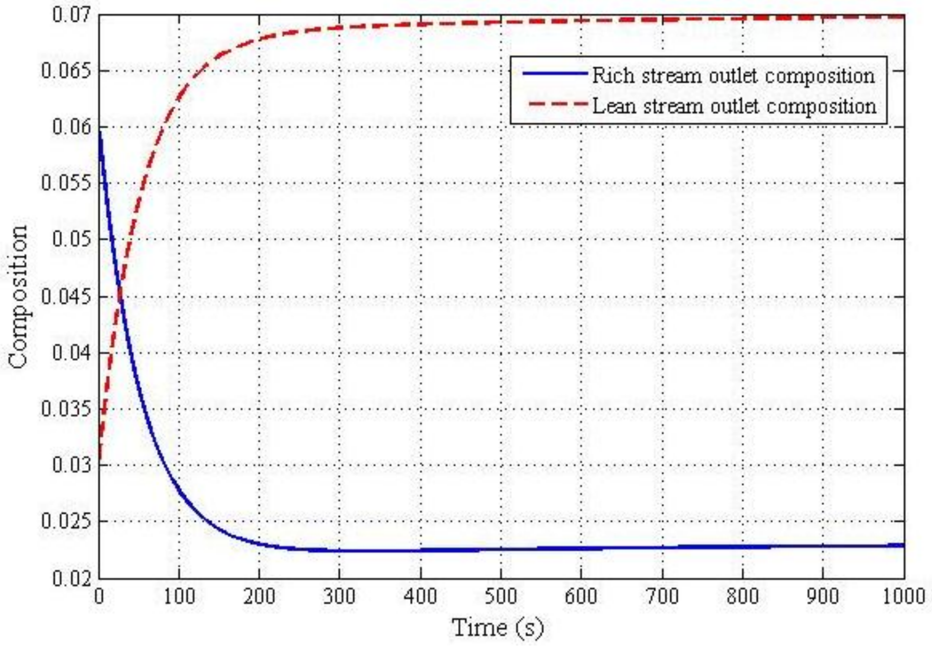


Figure 4.2 Dynamic behavior of copper recovery unit.

The dynamic response of outlet composition of rich and lean stream of copper recovery unit is shown in Figure 4.2. The composition of rich stream and lean stream is initiated at 0.06 and 0.03; respectively. Finally, their concentrations go to 0.02307 and 0.07 which equal to the steady state concentration. Thus, the model is verified and shown its stable response. After the passivity concept is applied to this mass exchanger unit. Passivity index of mass exchanger unit is shown in Figure 4.3

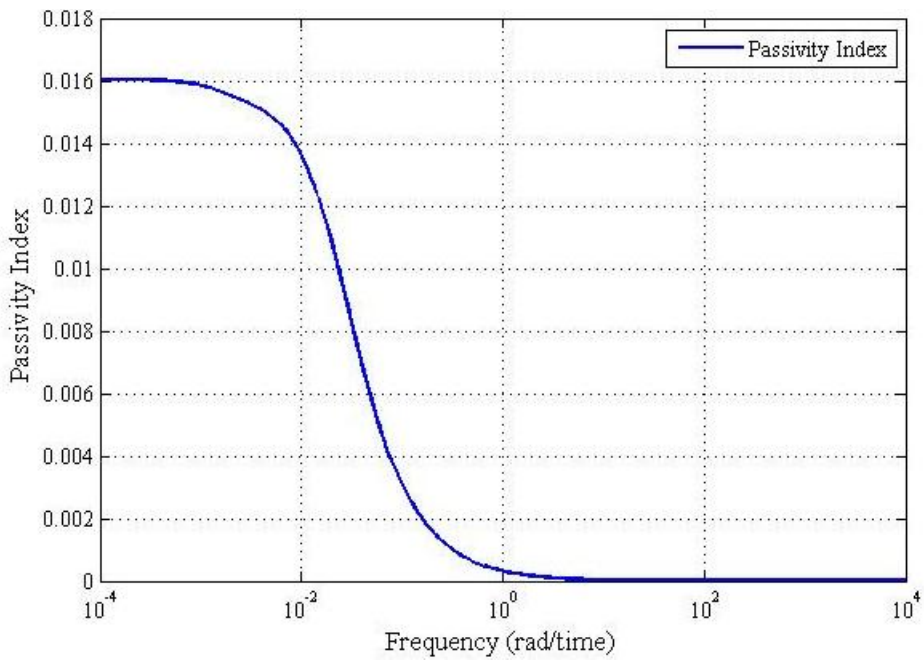


Figure 4.3 Passivity index of copper recovery unit.

From this figure, the passivity index of this system is more than zero along the frequency range 10^{-4}-10^{4} rad/hr; therefore, this system is non-passive. Up to this point, it is clearly that mass exchanger is non-passive.

4.3 State space equations of a mass exchanger network

Consider a five streams mass exchanger network problem which was investigated by El-Halwagi and Manousiouthakis (1989). It was further studied by Yan and Huang (2002) from the disturbance rejection point view and they suggest that the most suitable structure as shown in Figure 4.4. Table 4.2 lists the design data that includes the normal operating information and target concentration of the streams. The control loop connections are also presented in this figure. As normal operation, all valves are half opening and the pinch point is located at the concentration of 0.05 for rich streams *R*1 and *R*2; and 0.05, 0.09, and 0.024 for lean streams *L*1, *L*2, and *L*3; respectively. The equilibrium relations for a key component between a rich stream and each lean stream are given below:

$$y = 0.8x_1 + 0.002 \quad (4.28)$$

$$y = 0.5x_2 \quad (4.29)$$

$$y = 0.2x_3 \quad (4.30)$$

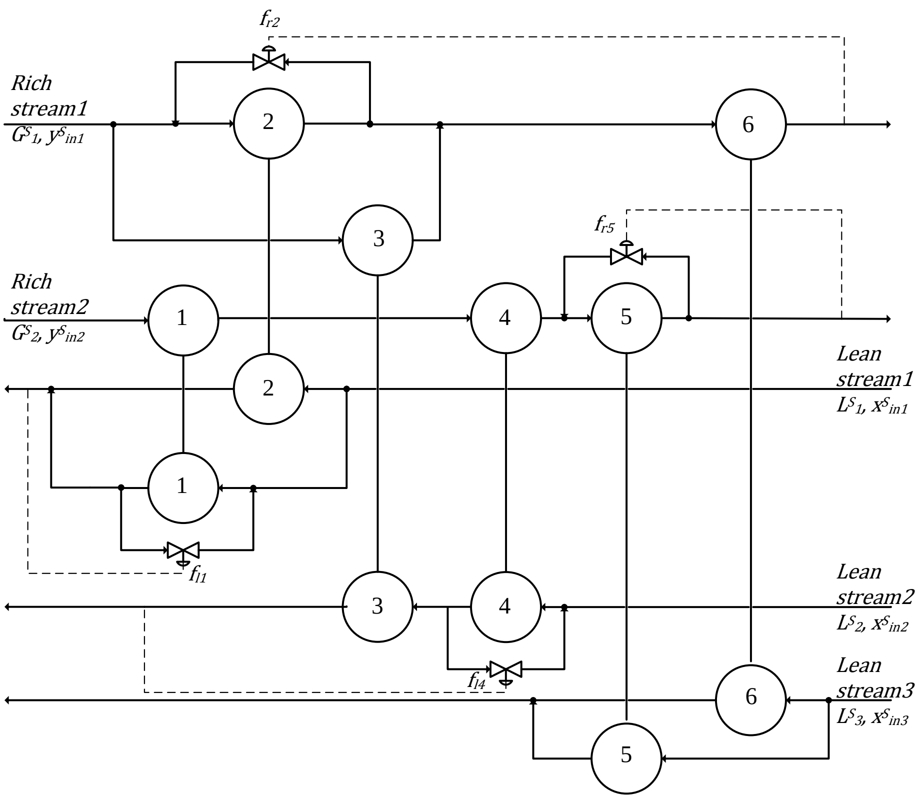


Figure 4.4 Grid diagram of five streams mass exchanger network problem.

Table 4.2 Stream data of five streams mass exchanger network.

Source stream	Mass rate	Source composition	Target composition
G1	1.3	0.115	0.025
G2	1.5	0.1	0.025
L1	2.5	0.05	0.110
L2	0.5	0.035	0.109
L3	0.6	0.01	0.080

The linear dynamic models of the mass exchanger network are derived as presenting in Appendix A1. Therefore, the dynamic model of mass exchanger network as a form of state space equation can be derived where the dimension of metrix A, B, C, D and E are 12x12, 4x12, 12x4, 4x4 and 5x12; respectively. In addition, the elements in A, B, C, D and E matrix are shown in Appendix A2.

Thus, the output stream composition can be determined as Figure 4.5. The composition of rich streams 1 and 2 and lean streams 1 and 2 start at initial point; 0.11, 0.1, 0.05 and 0.035; and finally go to steady concentration; 0.025, 0.025, 0.11 and 0.105 respectively which agree with result of Yan and Huang (2002). Therefore, this model can give the good agreement with the literature.

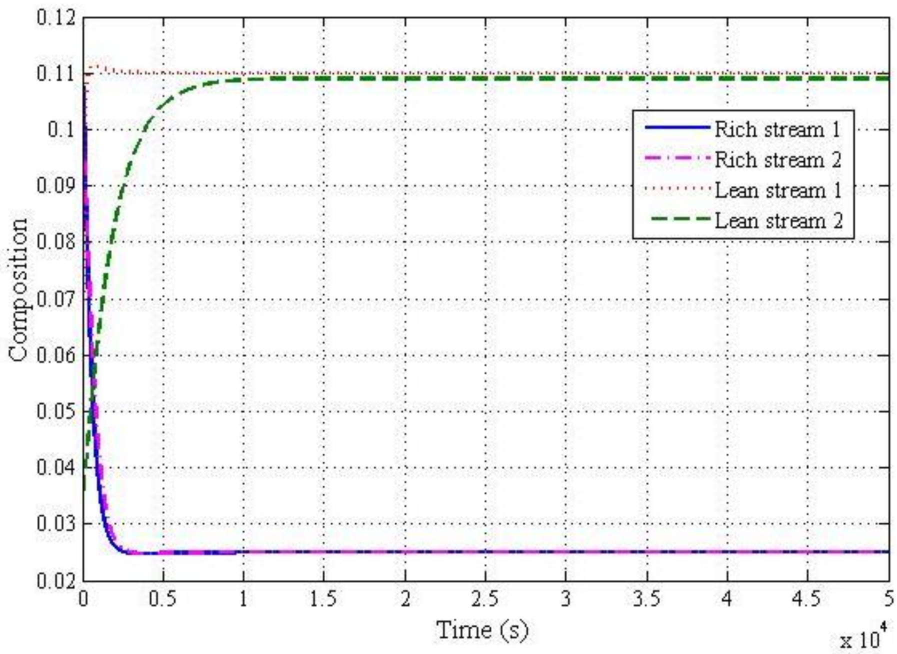


Figure 4.5 Open loop response of five streams mass exchanger network.

4.4 Passive controller design of mass exchanger network

Since the state space equation of the mass exchanger network is derived as section 4.3, the passivity concept can be applied to the system by firstly determining the passivity index. The passivity index of the system is shown in Figure 4.6.

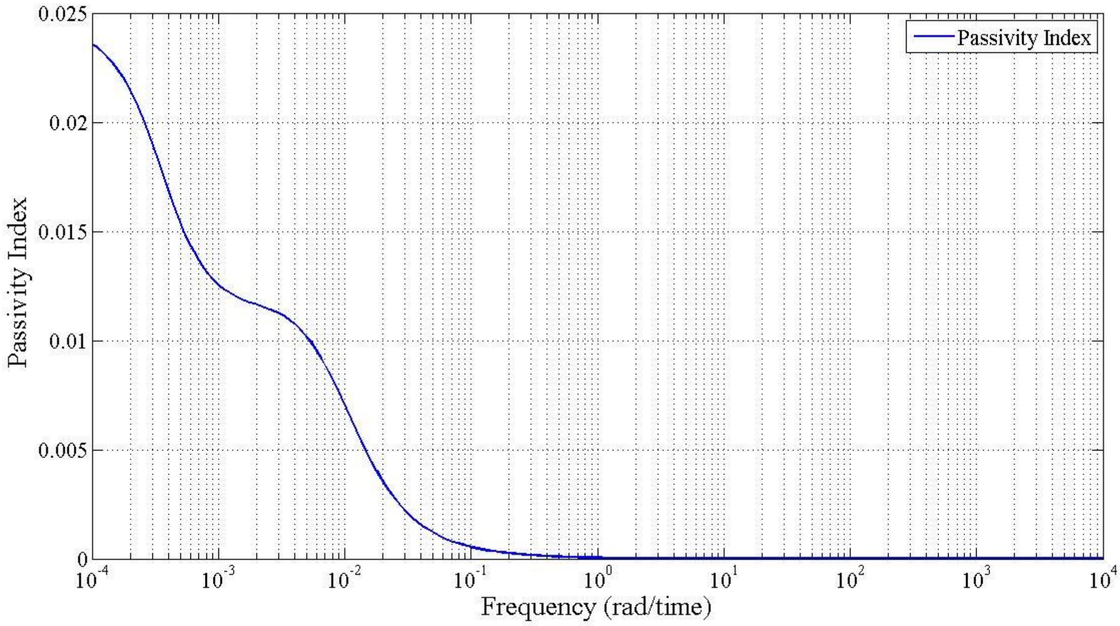


Figure 4.6 Passivity index of five streams mass exchanger network.

According to Figure 4.6, the passivity index of five streams mass exchanger network is more than zero along the frequency range 10^{-4}-10^{4} rad/hr; therefore, this system is non-passive. To shift the system from non-passive to passive system, a stable minimum phase transfer function called weighting function *w*(s) is added into this system. Consequently, the weighting function for this network model is shown in Equation (4.31), and its passivity index is depicted in Figure 4.7.

$$w(s) = \frac{0.0027s(s+0.4393)}{(s+0.001)(s+0.001)} \tag{4.31}$$

After the system is absorbed weighting function, the system show passive property regarding to negative value of passivity index as shown in Figure 4.7. However, weighting function cannot directly add into the system but weighting function will be absorbed into the controller in the passive controller design step instead. The PI tuning parameter for each control loop is obtain by solving optimization problem as Equations (2.14)-(2.16) and PI tuning parameters are presented in Table 4.3.

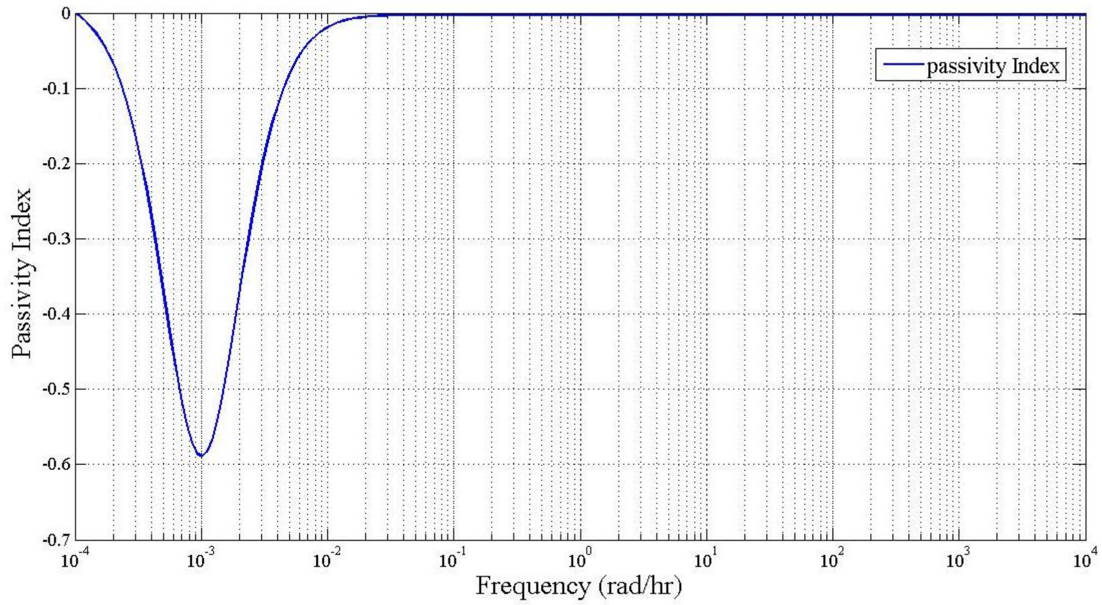


Figure 4.7 Passivity index of five streams mass exchanger network after introduction of the weighting function

Table 4.3 The results of the $k^{+}_{c,i}$ and $\tau_{I,i}$ for each loop control

Loop No.	Control variable	Manipulated variable	$k^{+}_{c,i}$	$\tau_{I,i}$ (second)
1	Output composition of lean stream 1	Lean stream recycle valve of MEX unit 1	0.0075	10
2	Output composition of rich stream 1	Rich stream recycle valve of MEX unit 2	0.0050	10
3	Output composition of lean stream 2	Lean stream recycle valve of MEX unit 4	0.00075	10
4	Output composition of rich stream 2	Rich stream recycle valve of MEX unit 5	0.0075	10

By substituting these tuning parameters from Table 4.3 into Equation (2.17), general form of PI controller k_i^{+} of each loop control are determined and shown as following.

$$k_1^{+} = 0.0075\left(1+\frac{1}{10s}\right) \quad \text{for the control loop1} \quad (4.32)$$

$$k_2^{+} = 0.005\left(1+\frac{1}{10s}\right) \quad \text{for the control loop2} \quad (4.33)$$

$$k_3^+ = 0.00075\left(1+\frac{1}{10s}\right) \qquad \text{for the control loop3} \qquad (4.34)$$

$$k_4^+ = 0.0075\left(1+\frac{1}{10s}\right) \qquad \text{for the control loop4} \qquad (4.35)$$

Since the system is non-passive, the weighting function should be absorbed into the controller and the design equation of controller is modified as shown in Equation (2.19). Therefore, the passive controllers of each loop control are determined as following.

$$k_1^{'}(s) = \frac{(s+0.1)(s+0.001)(s+0.001)}{s(s+0.0019)(s+0.0001)} \qquad (4.36)$$

$$k_2^{'}(s) = \frac{(s+0.1)(s+0.001)(s+0.001)}{s(s+0.0018)(s+0.0002)} \qquad (4.37)$$

$$k_3^{'}(s) = \frac{(s+0.1)(s+0.001)(s+0.001)}{s(s+0.0013)(s+0.0007)} \qquad (4.38)$$

$$k_4^{'}(s) = \frac{(s+0.1)(s+0.001)(s+0.001)}{s(s+0.0019)(s+0.0001)} \qquad (4.39)$$

4.5 Dynamic study of mass exchanger network

Open loop and closed loop model of mass exchanger network is developed by using Matlab-Simulink simulation software. The state space equation as discussed in section 4.3 is used to describe dynamic behavior of the system. In addition, passive controllers; as designed in section 4.4; are also incorporated in closed loop model. The example of five streams mass exchanger network is used as a case study to illustrate a control point of view based passivity concept. The system have 4 control loops where the output composition of lean stream 1, rich stream 1, lean stream 2 and rich stream 2 are controlled by manipulate lean stream recycle valve of MEX unit 1, rich stream recycle valve of MEX unit 2, lean stream recycle valve of MEX unit 4 and rich stream recycle valve of MEX unit 5; respectively. Moreover, input composition of lean stream 1, rich stream 1, lean stream 2 and rich stream 2 are defined as disturbances. The open loop and closed loop models are illustrated in Figures 4.8 and 4.9; respectively.

The designed passive controllers are tested both disturbance rejection and set point tracking problems. Input composition of lean stream 1, rich stream 1, lean stream 2 and rich stream 2 experience most fluctuate as 0.0025, 0.0080, 0.0040 and 0.0060 respectively. The open loop model is stepped test with a single disturbance in each loop and the open loop dynamic response is shown in Figure 4.10.

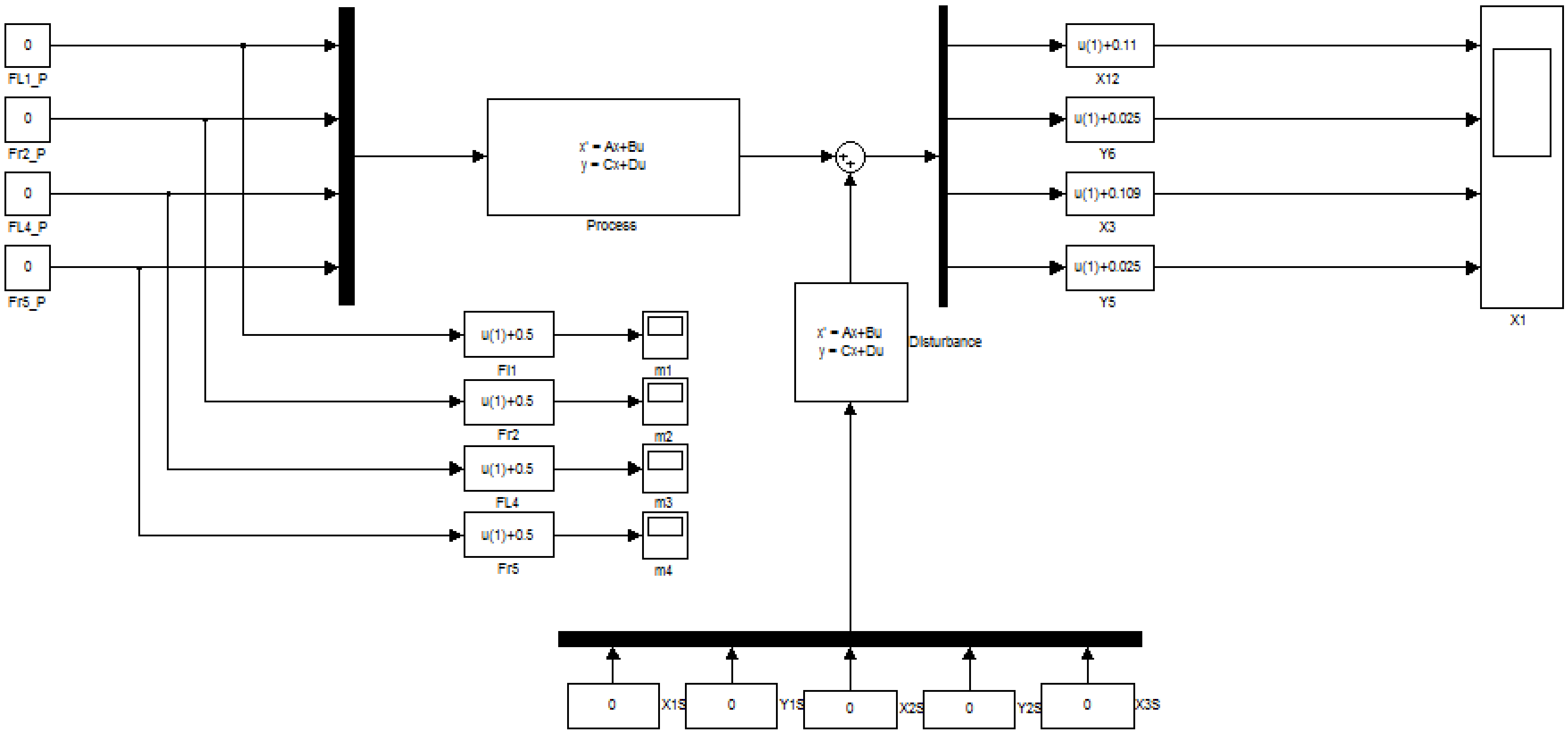


Figure 4.8 Open loop model of five streams mass exchanger network.

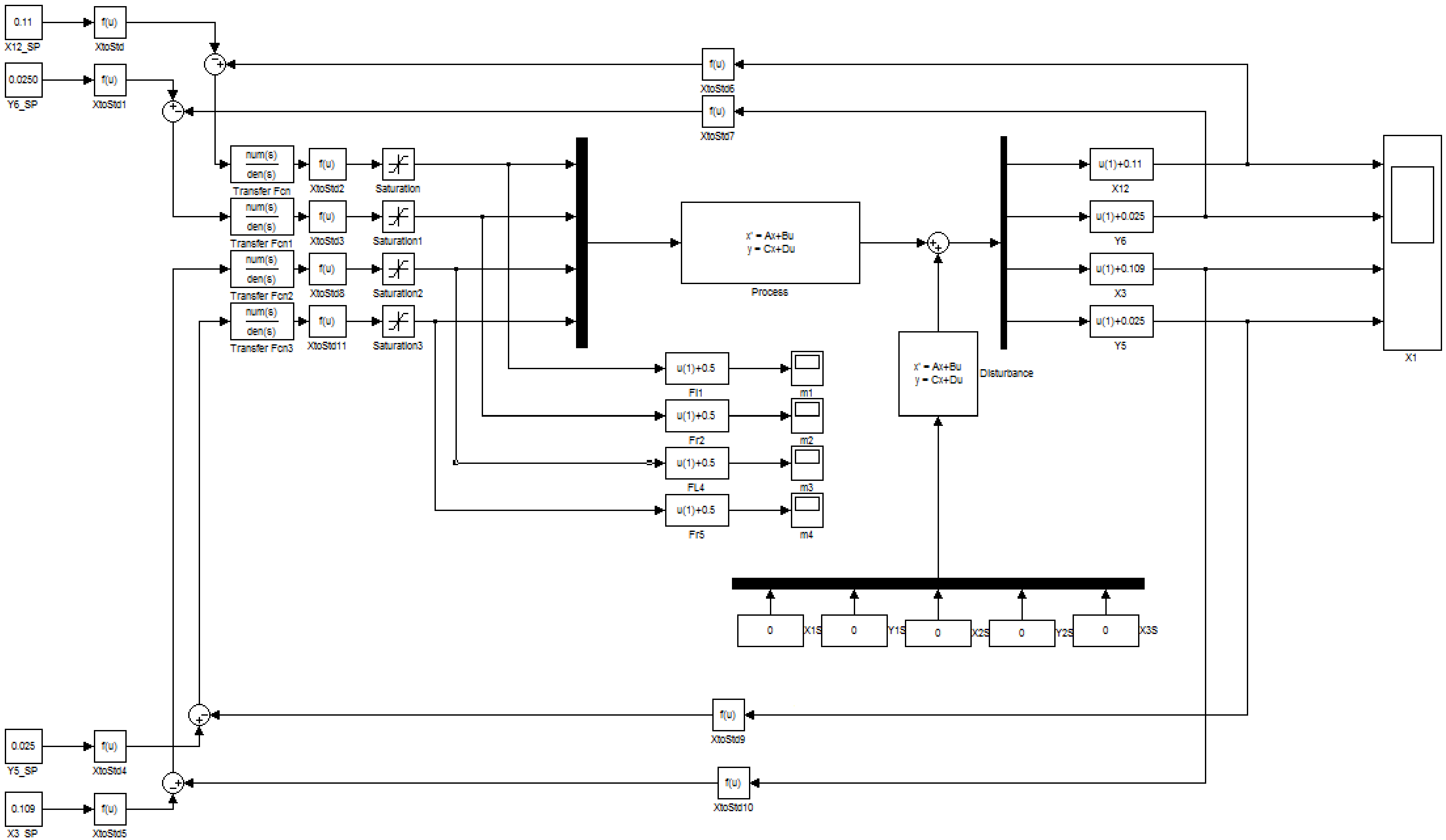


Figure 4.9 Closed loop model of five streams mass exchanger network.

At the beginning stage, the system can operate normally. Then the disturbance is introduced by step up at 50,000 second and stepped down at time 100,000 second. As a result, off spec composition in output stream. Lean stream 1 is first contact with upstream mass exchanger therefore Step change in composition of input lean stream 1 result in changing of other output concentrations as shown in Figure 4.10a.

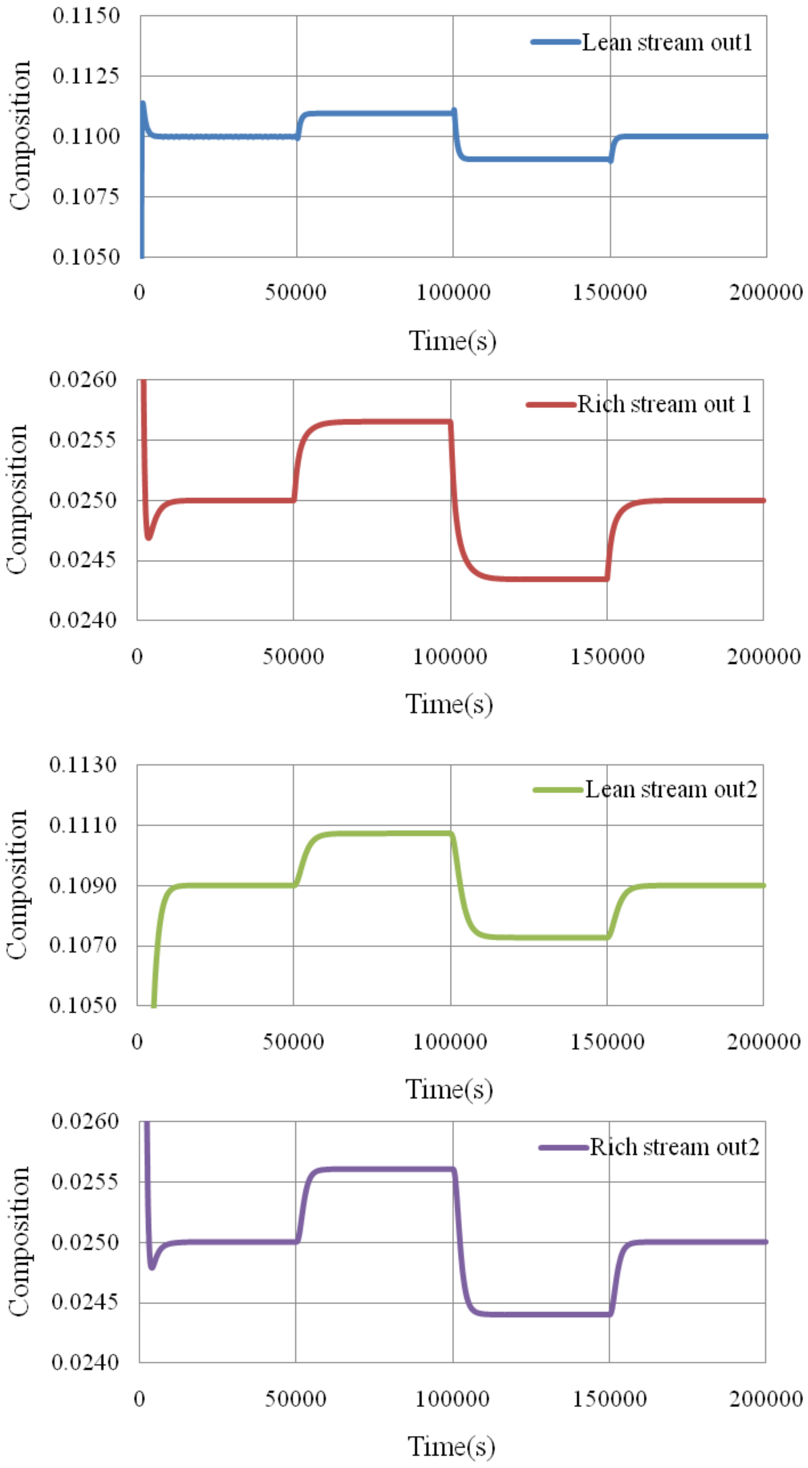


Figure 4.10a Open loop response after step input composition of lean stream 1.

According to Figure 4.10b, lean stream 2 is stepped change and gives result in no deviation in output of lean stream 1 because lean stream 2 is a secondary lean stream that employs to the network after lean stream 1 takes its duty. Step change in input lean stream 2 slightly effect on rich stream 1 because there is less mass transfer load between lean stream 1 and rich stream 1. Lean stream 2 is first contact with MEX unit 4 which is mass exchanger unit between lean stream 2 and rich stream 2 as a result of major concentration deviation of output lean stream 2 and rich stream 2.

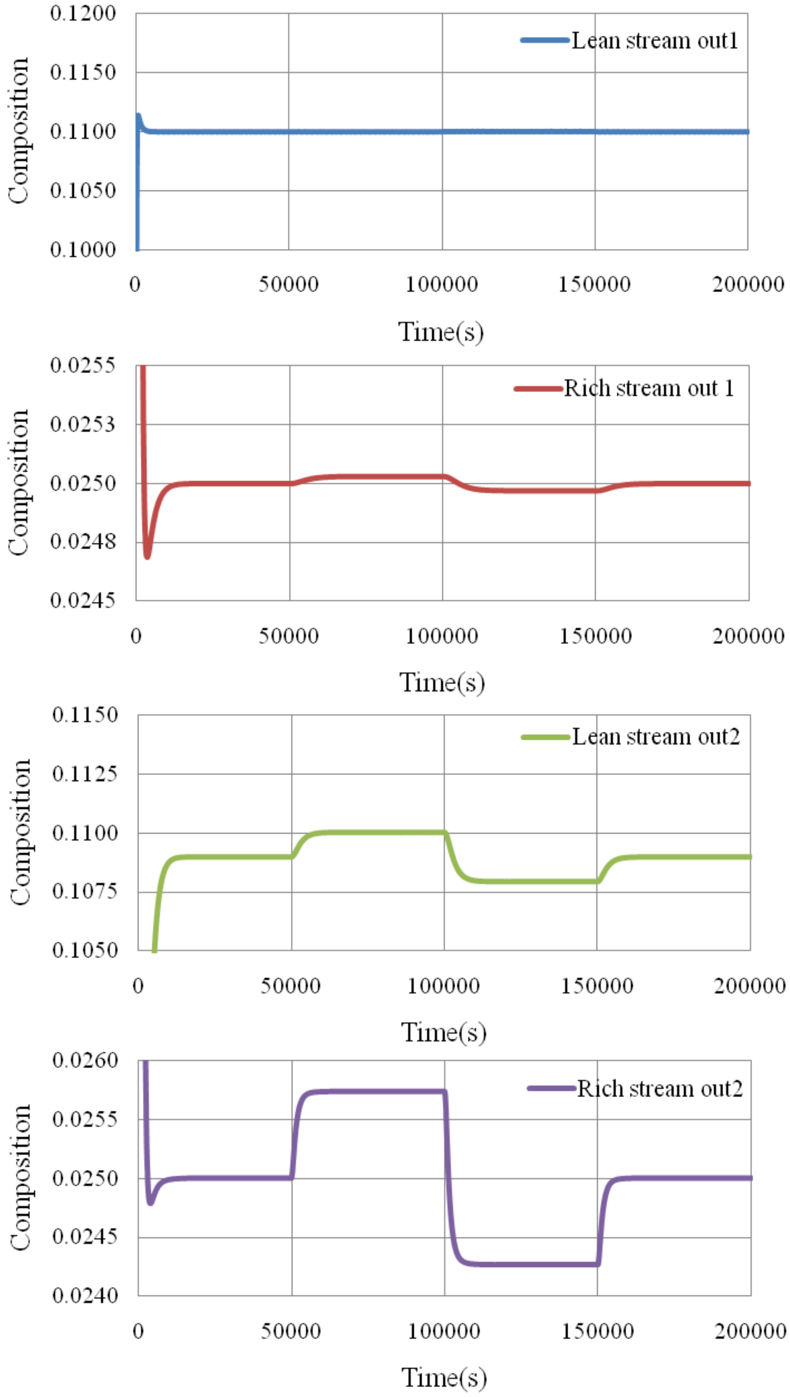


Figure 4.10b Open loop response after step input composition of lean stream 2**.**

In addition, there is no mass exchanger connect between rich stream 1 and 2 therefore step change in rich stream 1 only effect on rich stream 1 and other 2 lean streams and vice-versa for rich stream 2 as shown in Figures 4.10c and 4.10d.

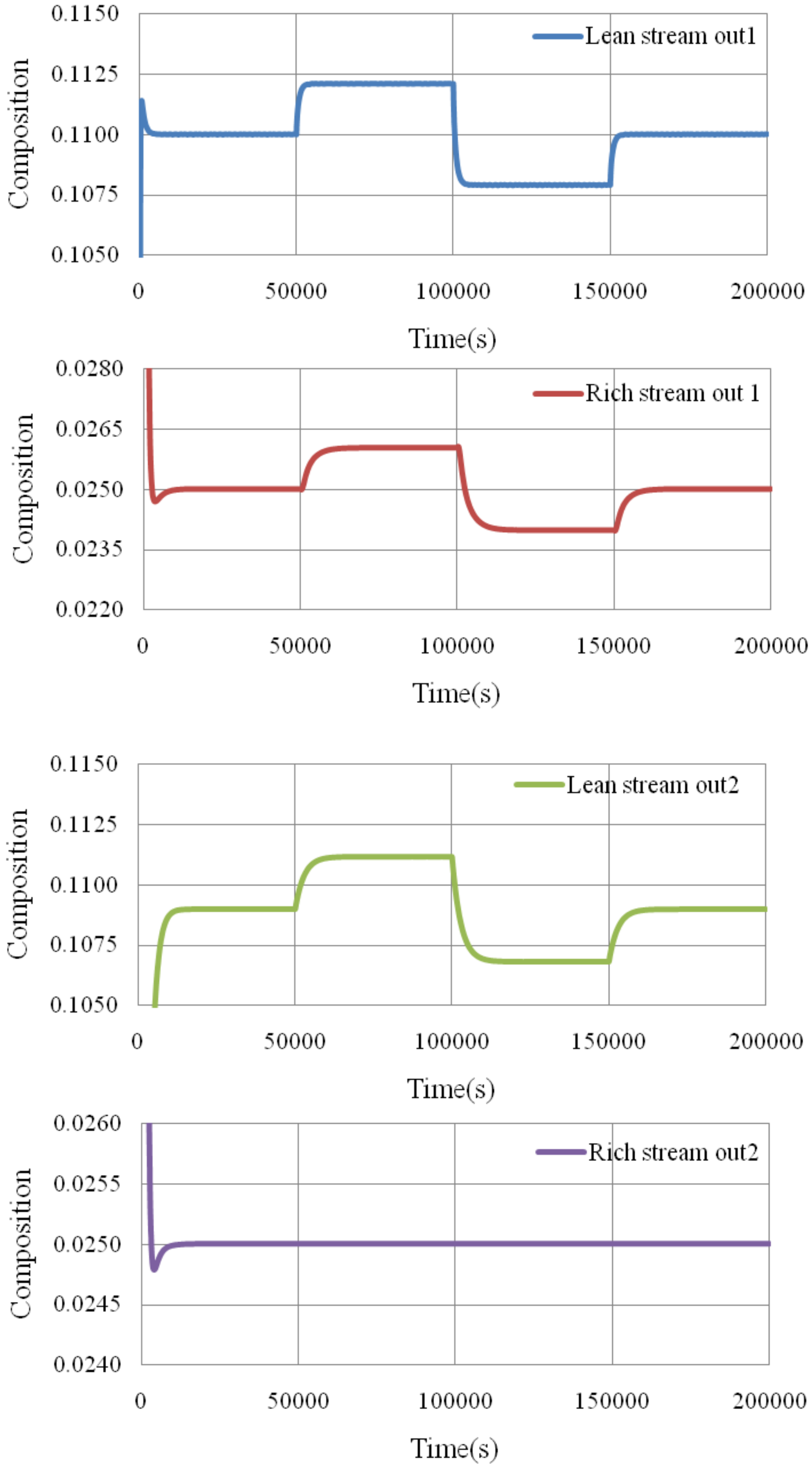


Figure 4.10c Open loop response after step input composition of rich stream 1.

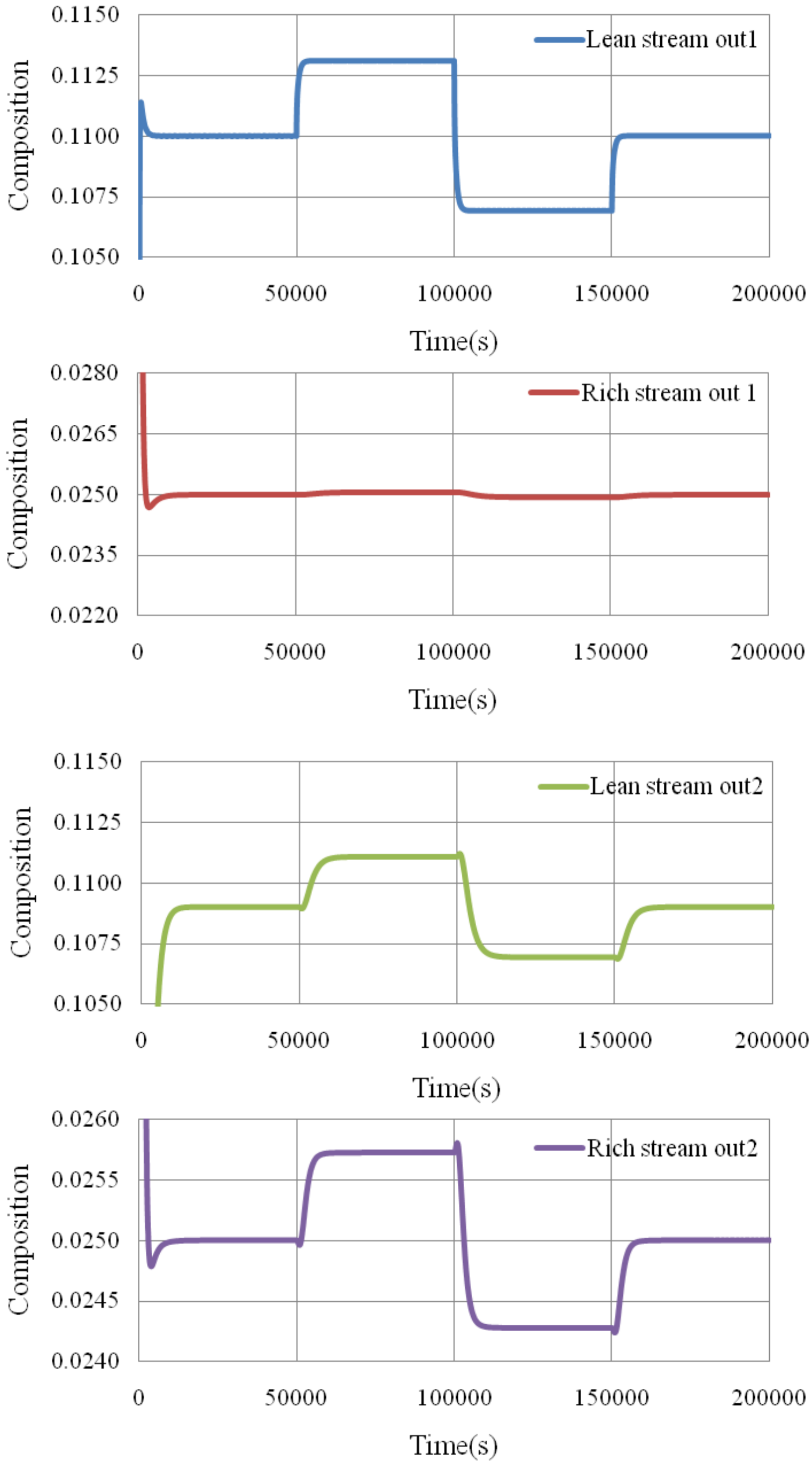


Figure 4.10d Open loop response after step input composition of rich stream 2.

The output composition deviations occur after disturbance is loaded to the system as discuss above. Thus, the passive controllers; as designed in section 4.4; take a role in disturbance rejection. The closed loop output composition and control valve response while single disturbance is applied to the system are shown in Figure 4.11. Since, poles of the controllers close to the origin which make the controllers sensitive. The result shows that all control variables reach their set point without any offset very quickly by manipulating control valve. Therefore, the designed passive controllers are working well under the disturbance rejection problem.

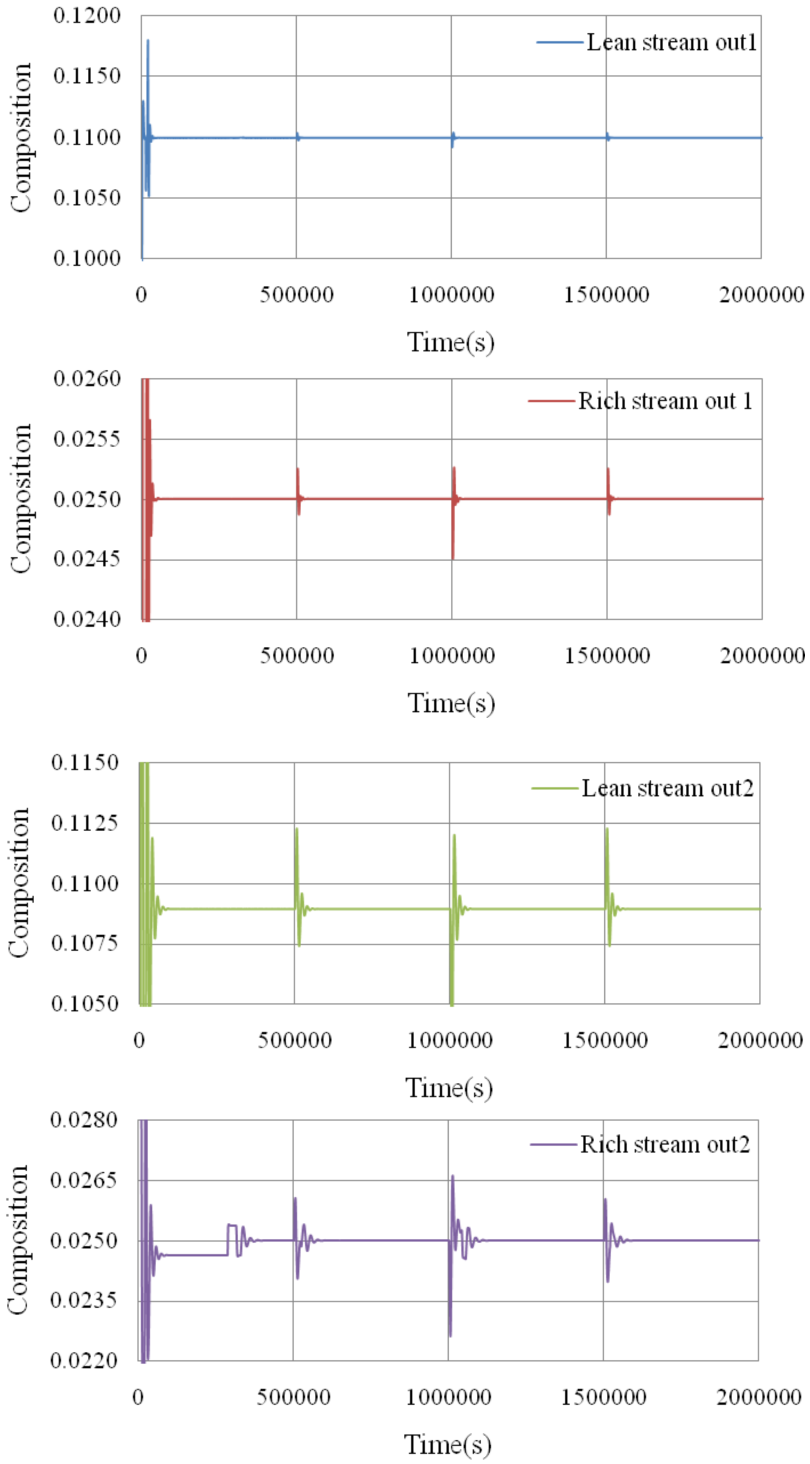


Figure 4.11a Closed loop outlet composition response after step input composition of lean stream 1.

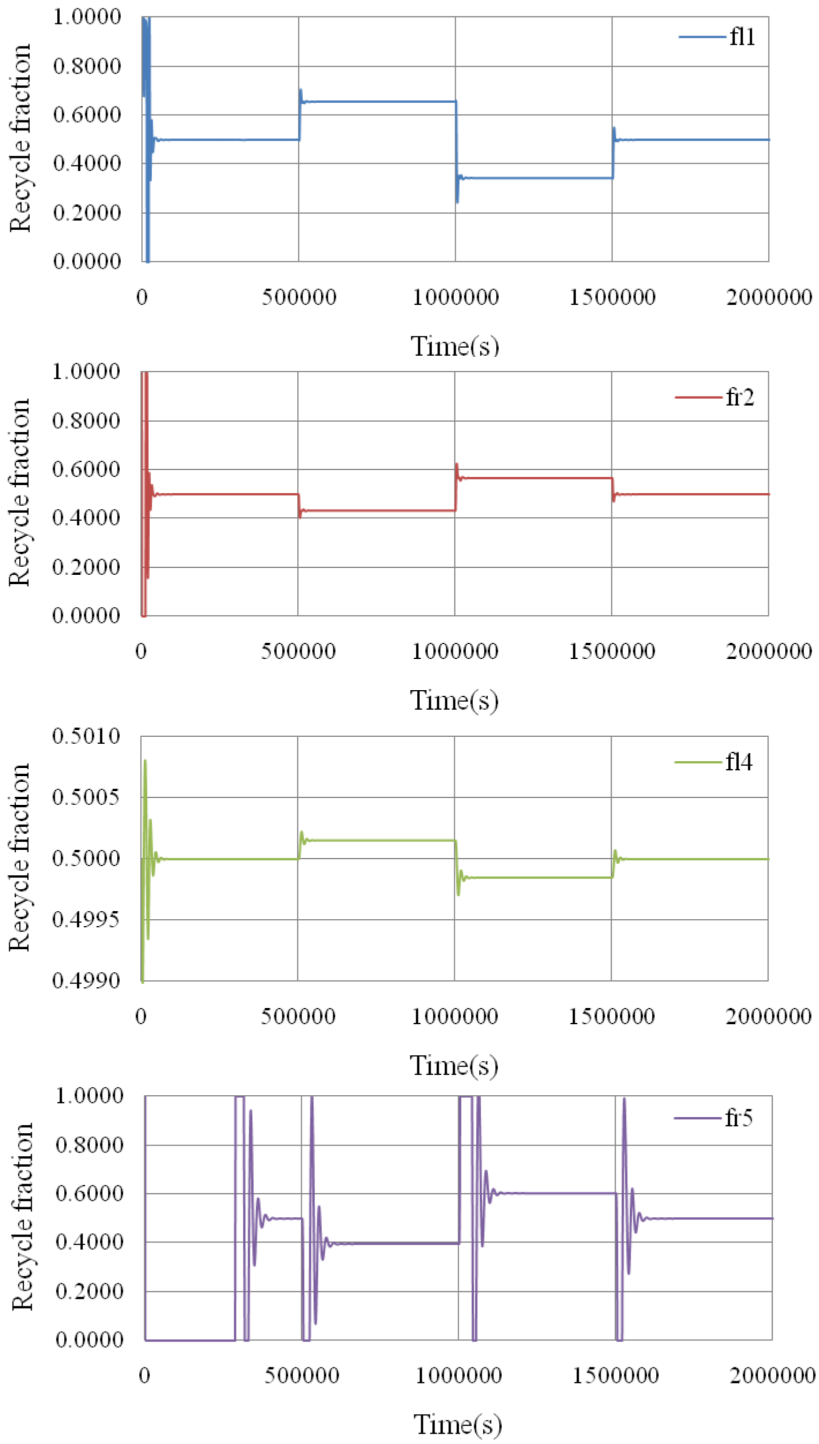


Figure 4.11b Closed loop recycle valve position response after step input composition of lean stream 1.

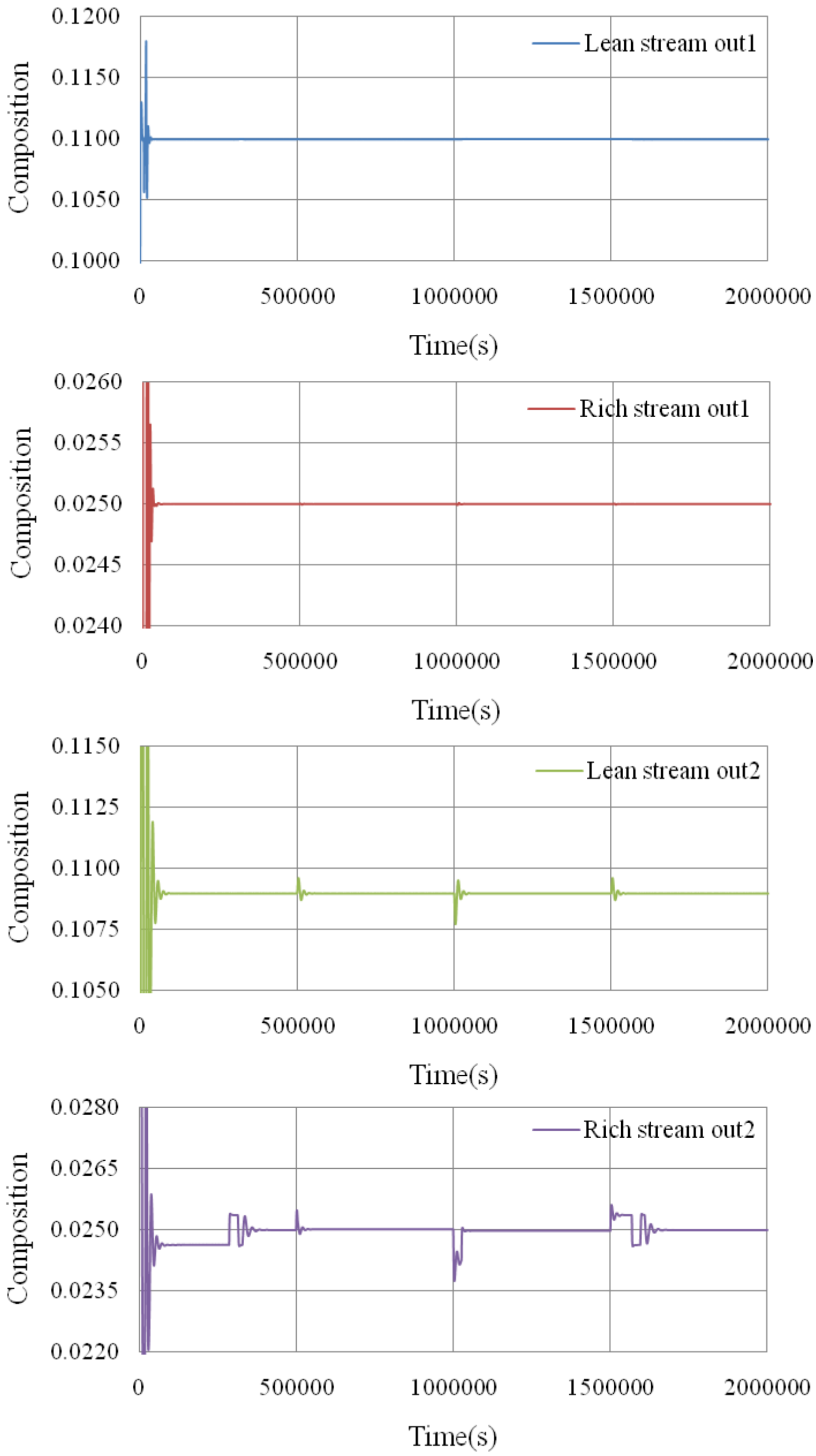


Figure 4.11c Closed loop outlet composition response after step input composition of lean stream 2.

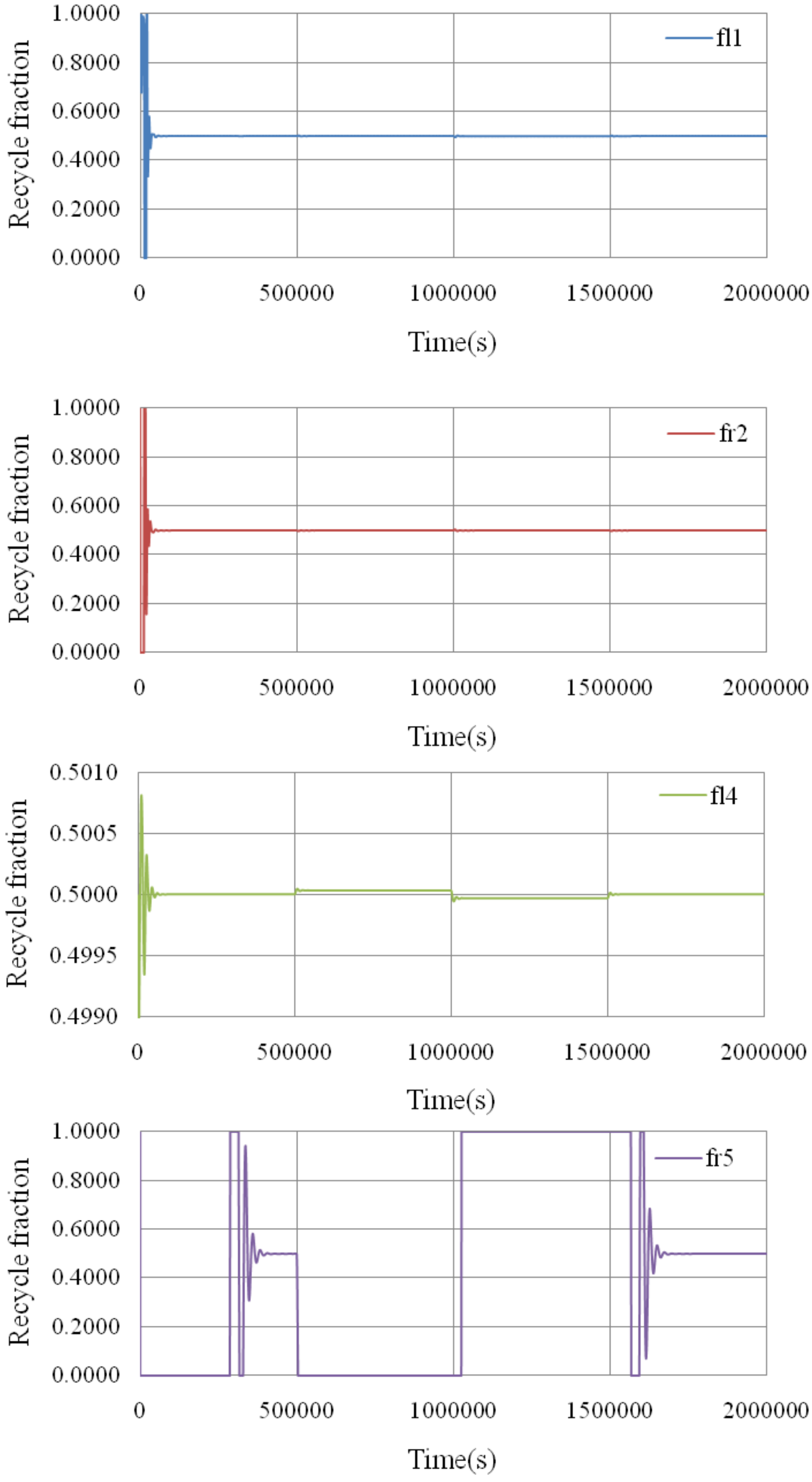


Figure 4.11d Closed loop recycle valve position response after step input composition of lean stream 2.

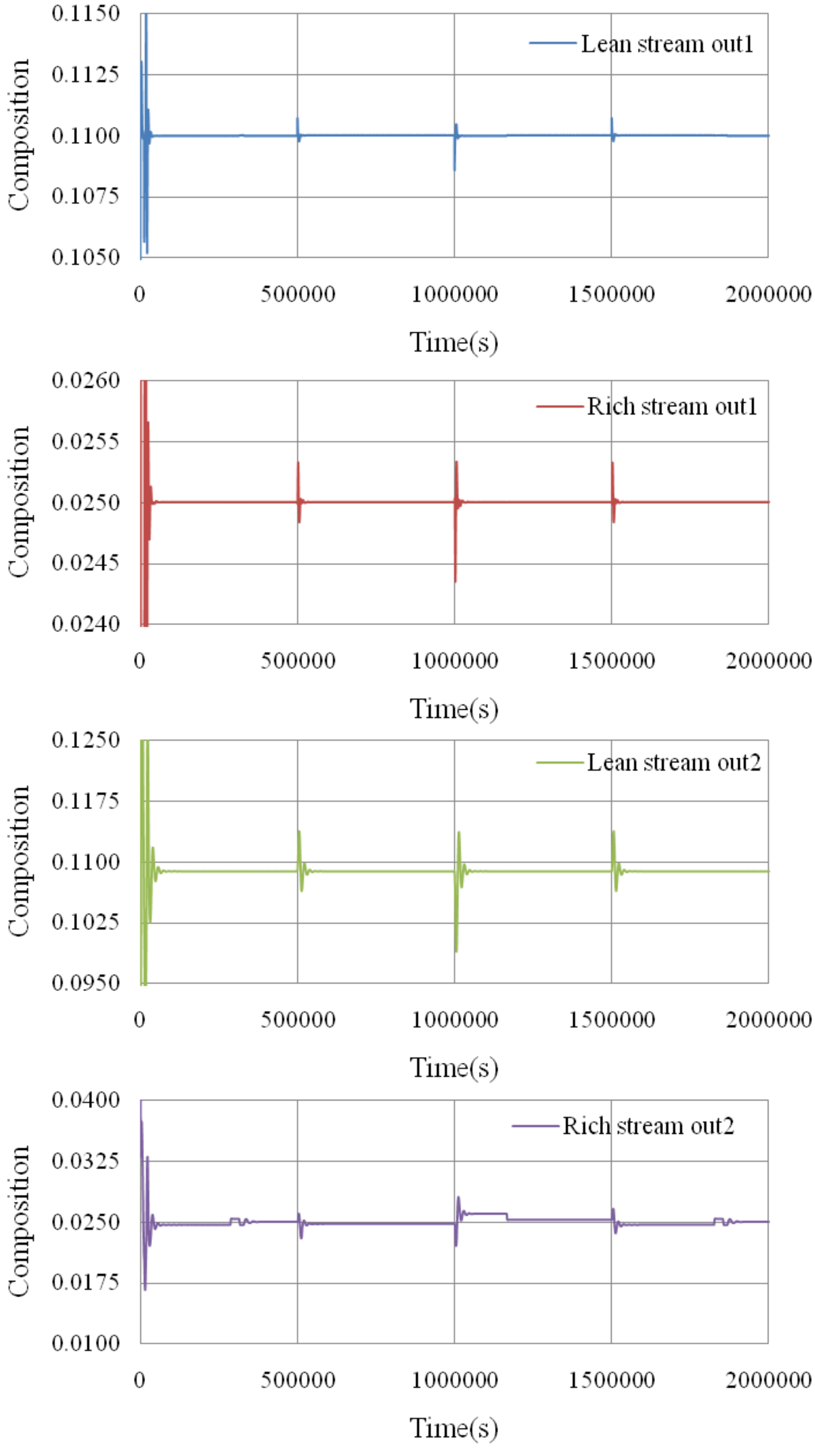


Figure 4.11e Closed loop outlet composition response after step input composition of rich stream 1.

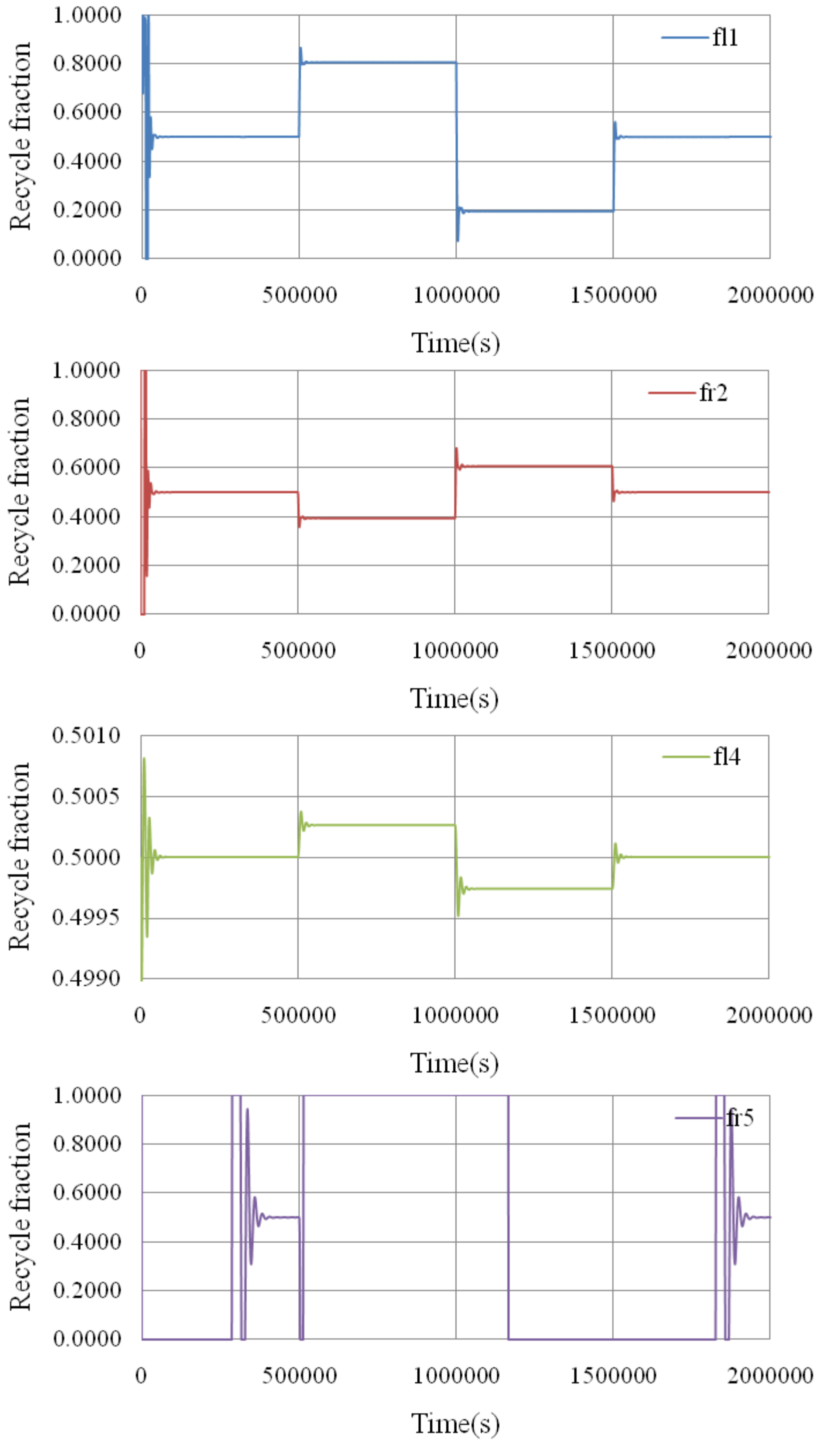


Figure 4.11f Closed loop recycle valve position response after step input composition of rich stream 1.

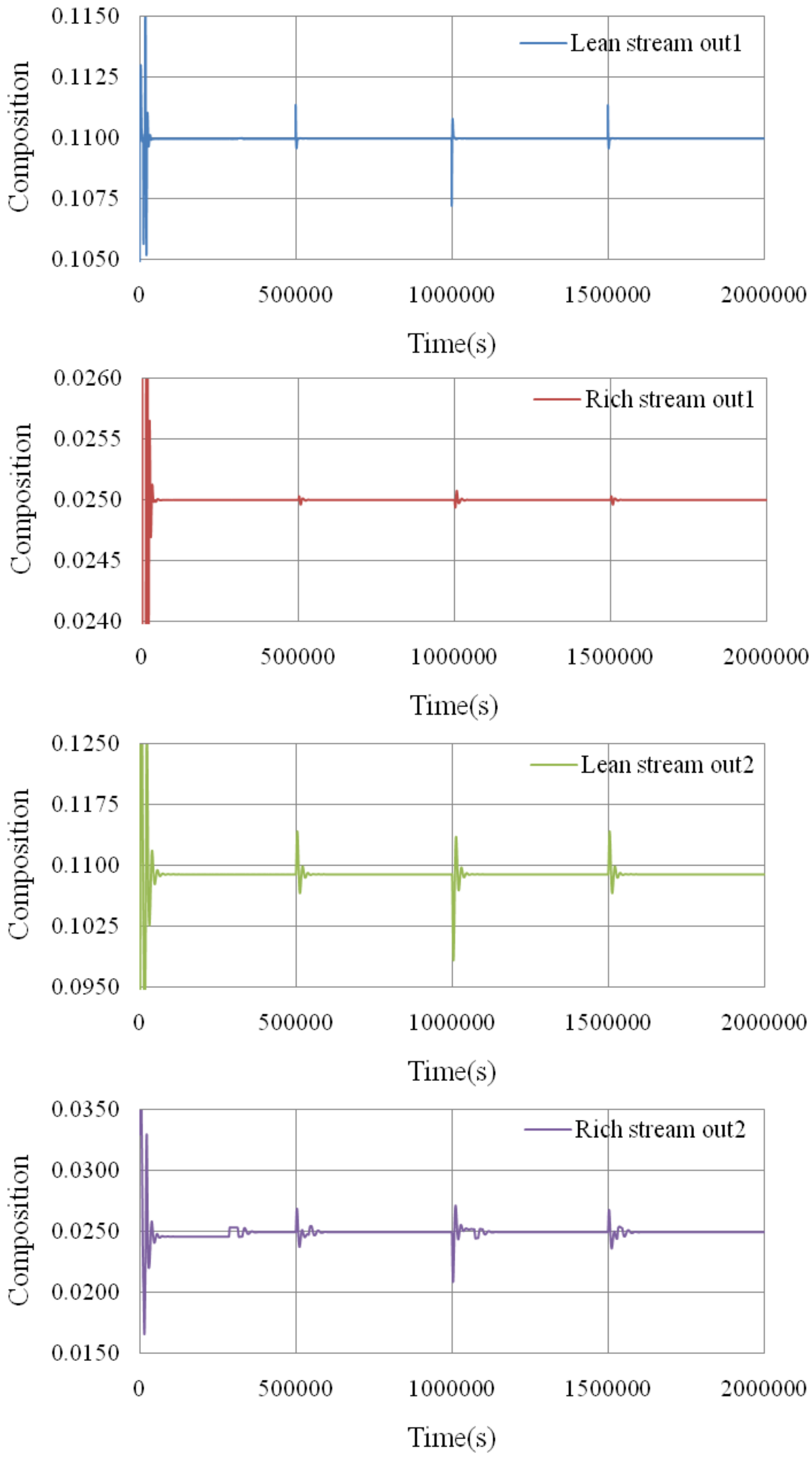


Figure 4.11g Closed loop outlet composition response after step input composition of rich stream 2**.**

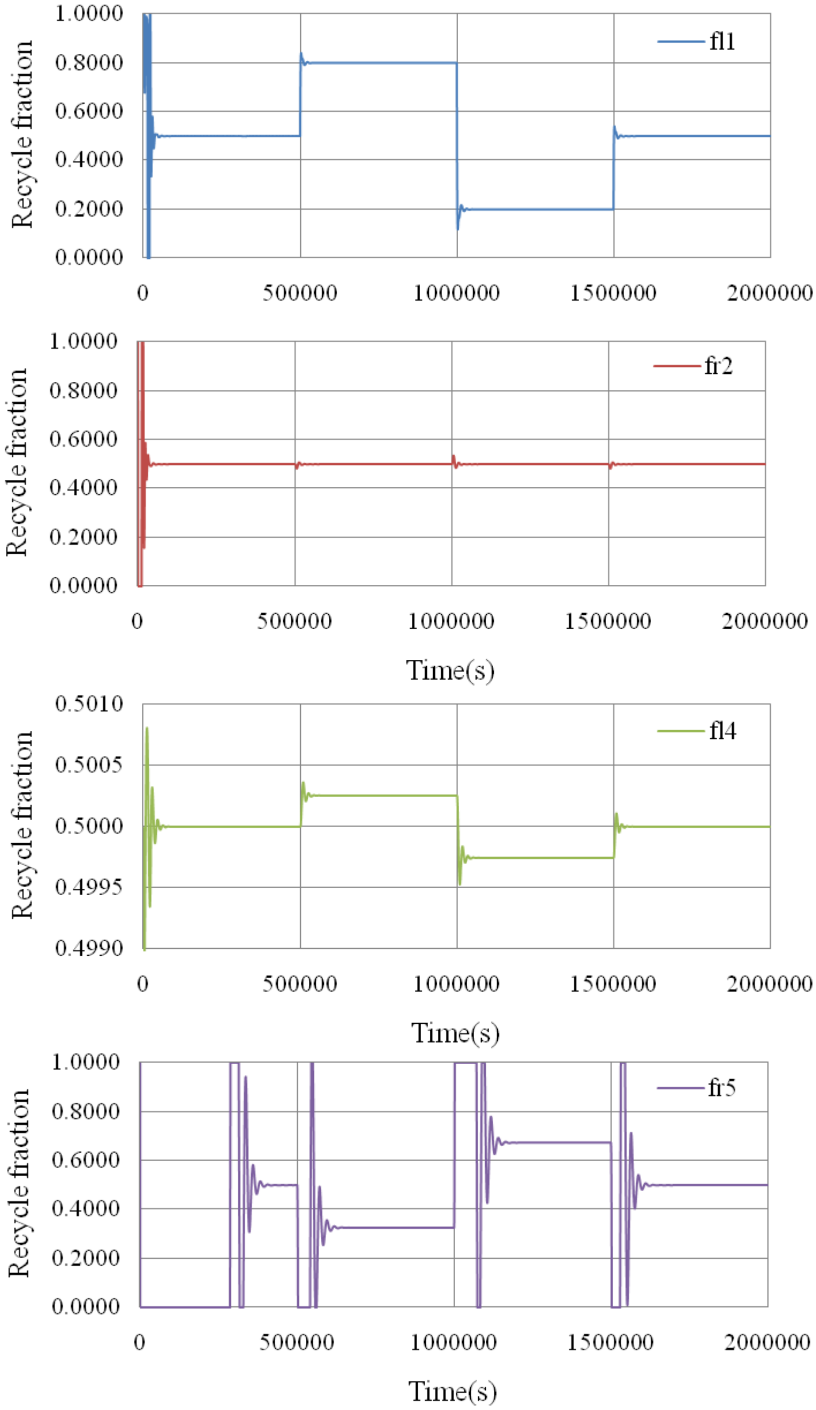


Figure 4.11h Closed loop recycle valve position response after step input composition of rich stream2.

For worst situation, all disturbances are applied to the system simultaneously as a result of deviation of output composition as shown in Figure 4.12. The designed passive controller is applied to the system. The closed loop responses of output composition and recycle valve position are shown in Figures 4.13 and 4.14; respectively. The result show that the valve of controller loop 1 (Lean stream recycle valve of MEX no.1) is saturated lead to off spec of composition of lean stream 1. Thus, the control structure should be redesigned to satisfy disturbance load or split range control should be considered.

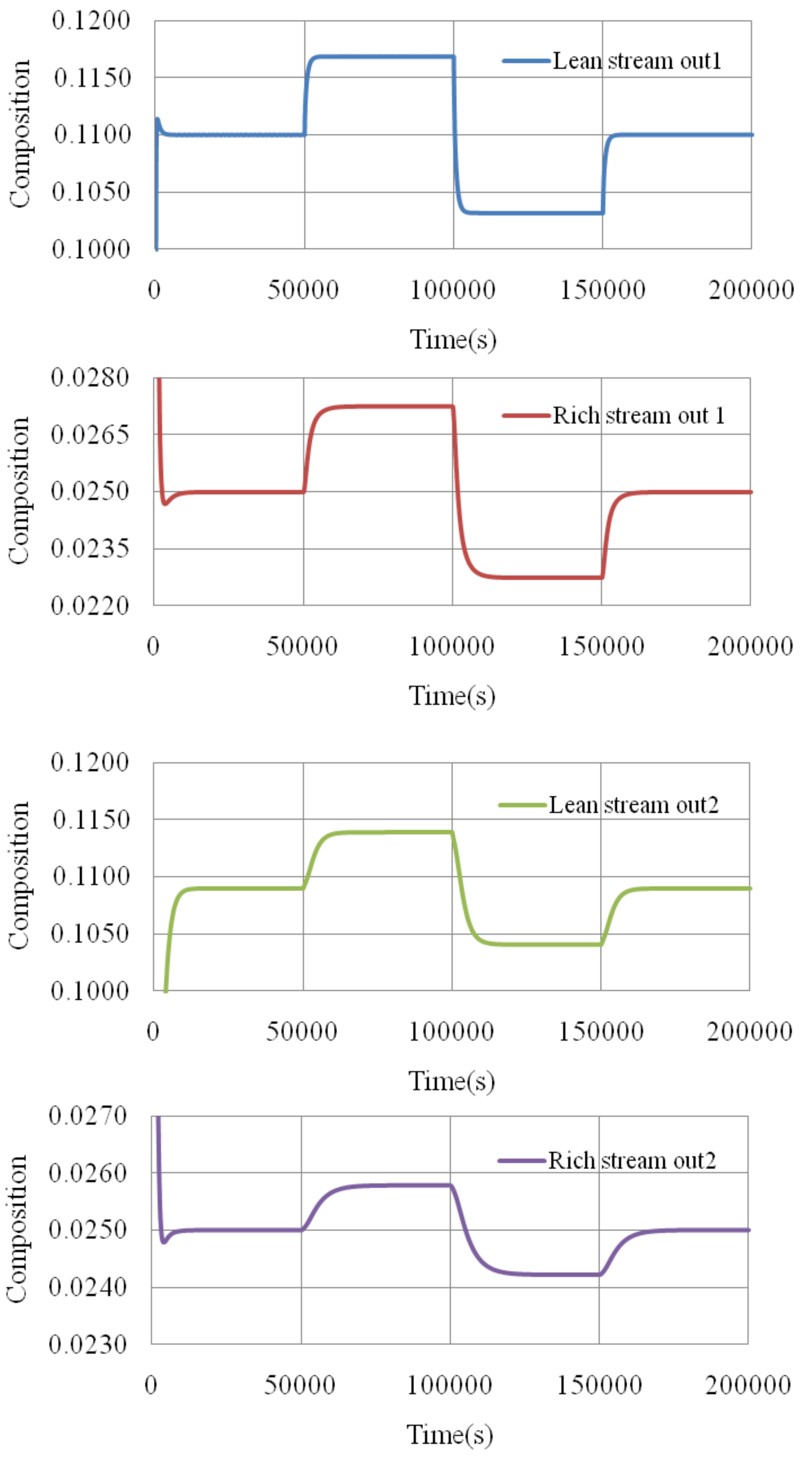


Figure 4.12 Open loop response after step all input composition simultaneously.

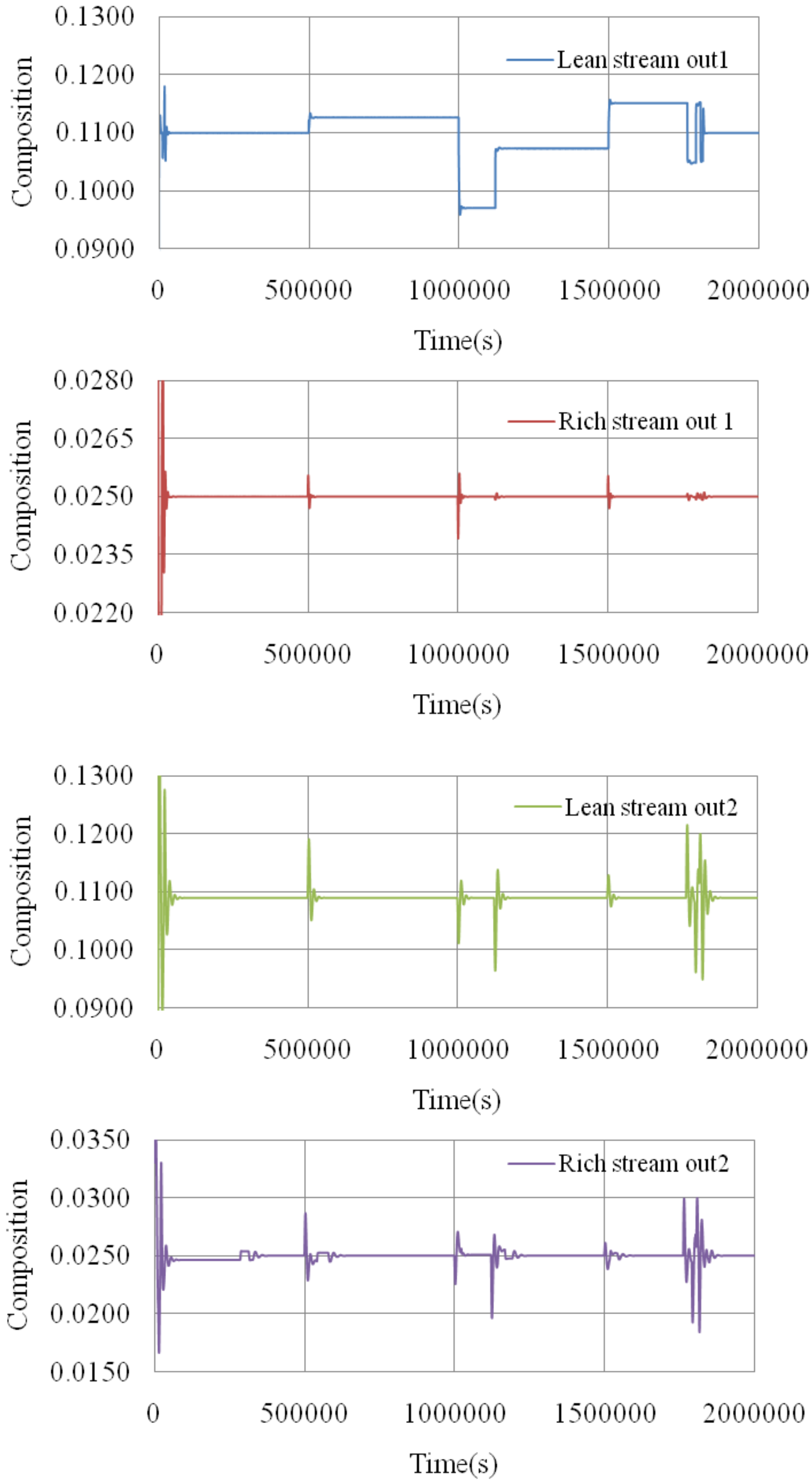


Figure 4.13 Closed loop response after step all input composition simultaneously.

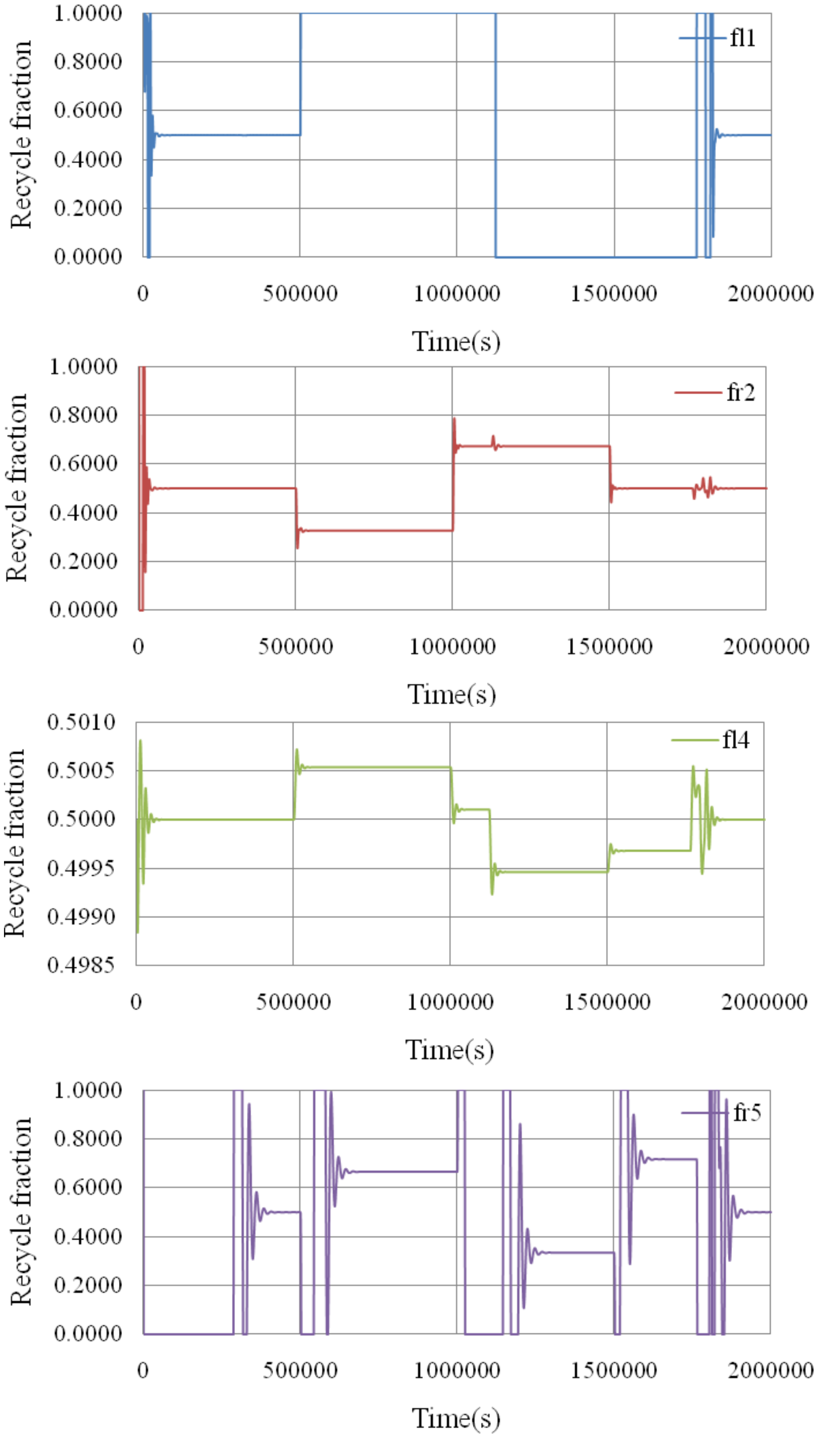
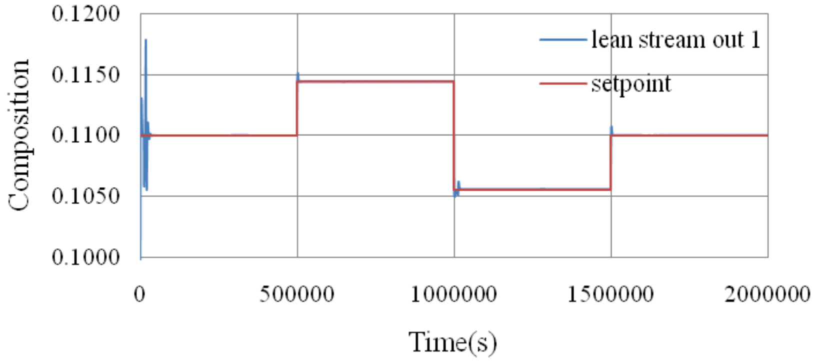
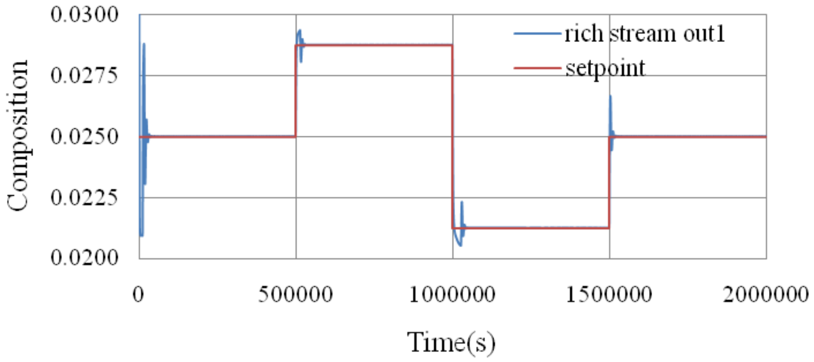


Figure 4.14 Closed loop recycle valve position after step all input compositions.

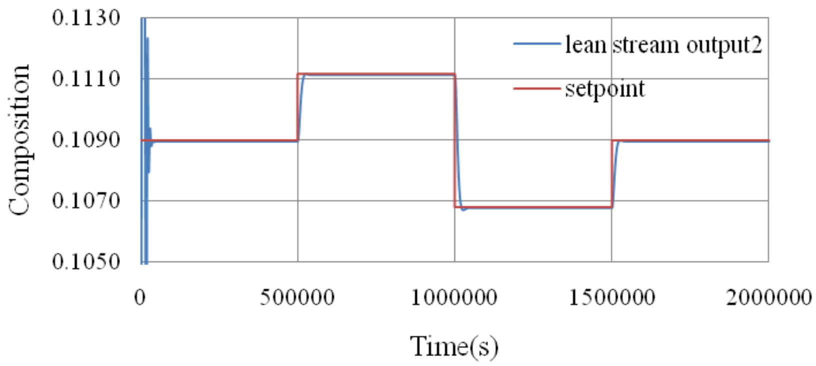
In addition, the designed passive controllers are tested by the set point tracking problem as well. Under no disturbances load to the system, the set point of each control loop is stepped change. The control Loop1, loop2, loop3 and loop4 are capable to track 4%, 10%, 3% and 1.5% deviation of its set point respectively as shown in Figure 4.15.



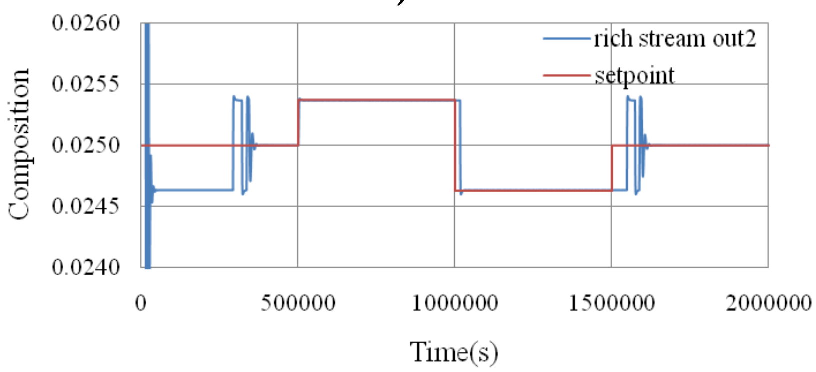
a)



b)



c)



d)

Figure 4.15d Set point tracking of controller; a)loop1, b)loop2, c)loop3 and d)loop4.

The designed PI passive controllers are compared with PI auto tuning controllers. The PI auto tuning parameters are presented in Table 4.4.

Table 4.4 The results of the $k^{+}_{c,i}$ and $\tau_{I,i}$ of auto tuning controller.

Loop No.	Control variable	Manipulated variable	$k^{+}_{c,i}$	$\tau_{I,i}$ (second)
1	Output composition of lean stream 1	Lean stream recycle valve of MEX unit 1	0.18	0.0064
2	Output composition of rich stream 1	Rich stream recycle valve of MEX unit 2	0.44	0.0013
3	Output composition of lean stream 2	Lean stream recycle valve of MEX unit 4	0.30	0.0001
4	Output composition of rich stream 2	Rich stream recycle valve of MEX unit 5	3.35	0.0196

The Step change in lean stream 1 composition is used as a case study in order to illustrate comparison of disturbance rejection performance between passive controllers and auto tuning controllers. Comparison of output composition between passive controller and PI auto tuning is shown in Figure 4.16.

The results show that the auto-tuning PI controllers have less settling time than the designed passive controllers for all control loops due to the reset time of passive controllers is approximately a thousand times higher than auto tuning controllers. The controlled variables return to the set point slowly after a load upset. Correspondingly, the controller gain of the designed passive controllers is less than the controller gain of auto tuning controllers. As a result, passive controllers are more sluggish control response. In the same way, overshoot of output composition response of auto tuning controllers are less than passive controllers for all controller loop. Therefore, the disturbance rejection performance of the auto tuning PI controllers is better than the designed passive controllers for all control loops.

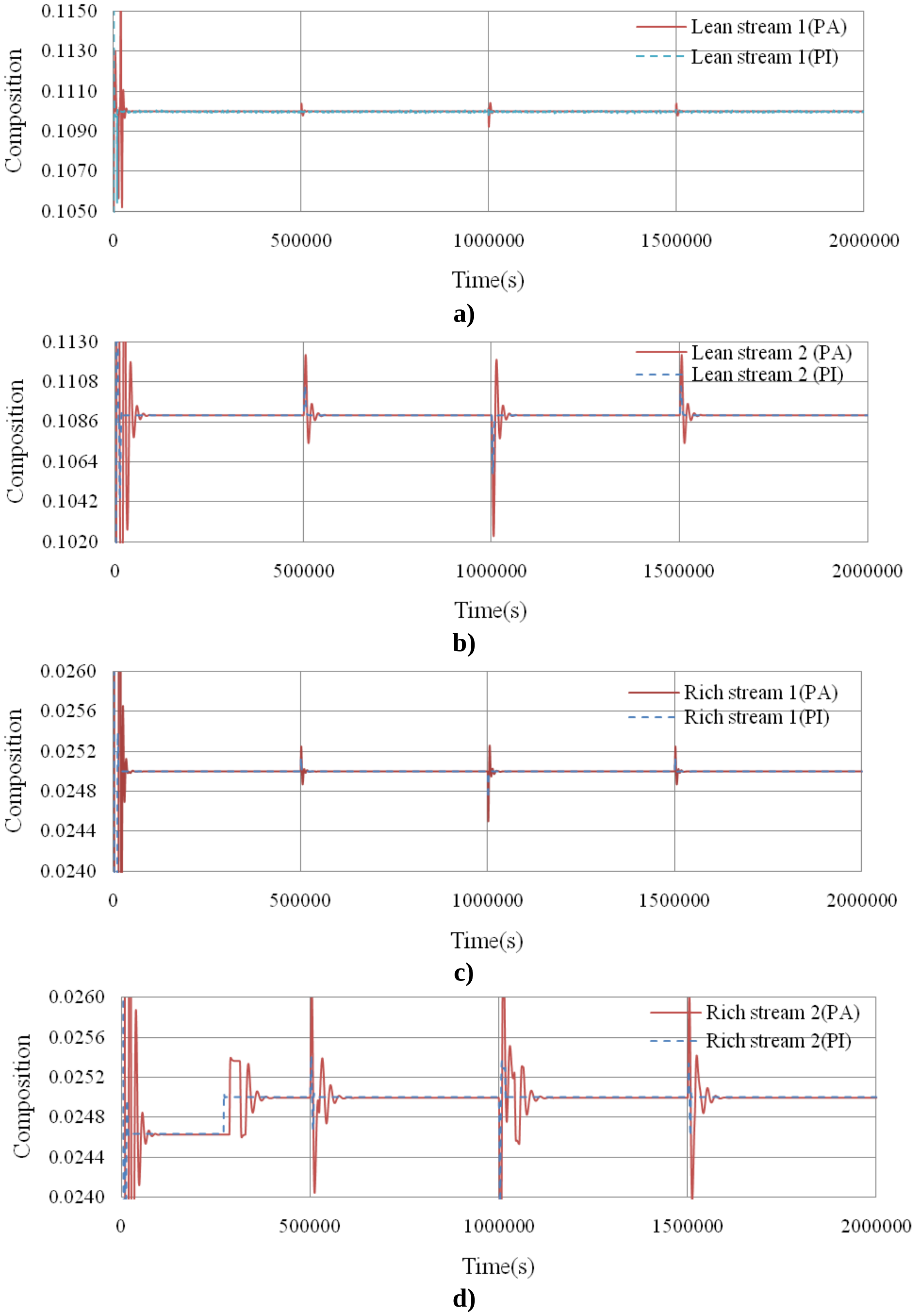


Figure 4.16 Comparison of output composition between passive controller and PI auto tuning for a) control loop1, b) control loop2, c) control loop3 and d) control loop4.

In order to illustrate the set point tracking performance comparison between the designed passive controllers and auto tuning controllers of an output composition response. The set point change of rich stream 1 composition which is controller loop no. 2 is used as case study. After the system approach to first steady state, the set point of outlet composition of rich stream 1 is step change 15%. As a result set point tracking performance comparison of output composition between passive controller and PI auto tuning in controller loop no.2 as shown in Figure 4.17

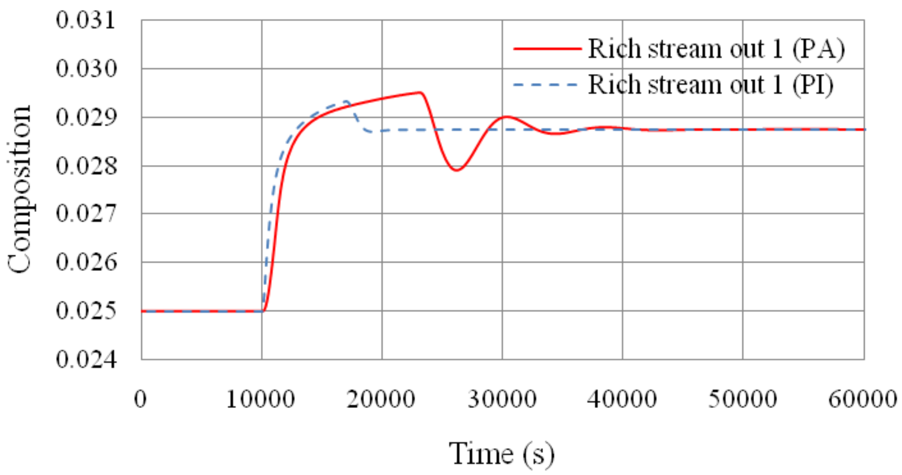


Figure 4.17 Set point tracking performance comparison of output composition between passive controller and PI auto tuning in controller loop no.2.

The result shows that the set point of the designed passive controller is reached at set point approximately 4 times quicker than the auto tuning controller as a result of the controller gain of the auto tuning controller is more than the controller gain of designed passive controller which is the same reasons as discussed in comparison of disturbance rejection performance.

Through this example, passivity concept can be applicable with mass exchanger network. However, it is clearly that the passive controller has less performance than traditional controller design.

CHAPTER 5 CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

This work mainly focuses on a development of state space equations of a mass exchanger network which can be applied to the passivity concept. A mass exchanger model has been developed based on assumptions of the approximate lumped parameter system; both rich and lean stream are well mixed, linear equilibrium relation over the operating range and under constant isothermal and isobaric condition. After applying the passivity concept on the mass exchanger unit, the mass exchanger is characterized as a non-passive system due to positive passivity index.

Then, the passivity concept is extended to the mass exchanger network. Referring to Yan and Huang (2002), the most suitable control structure of five streams mass exchanger network is used as a case study to apply the passivity concept. The mass exchanger network has 4 control loops, where the output composition of lean stream 1, rich stream 1, lean stream 2 and rich stream 2 are controlled by manipulating the lean stream recycle valve of MEX unit 1, rich stream recycle valve of MEX unit 2, lean stream recycle valve of MEX unit 4 and rich stream recycle valve of MEX unit5 respectively; also investigated as non-passive system so weighting function; $w(s) = \dfrac{0.0027s(s+0.4393)}{(s+0.001)(s+0.001)}$ is added to drive the system to passive. Multi-loop passive PI controllers of mass exchanger network are investigated as

$$k_1^{'}(s) = \frac{(s+0.1)(s+0.001)(s+0.001)}{s(s+0.0019)(s+0.0001)}, \qquad k_2^{'}(s) = \frac{(s+0.1)(s+0.001)(s+0.001)}{s(s+0.0018)(s+0.0002)},$$

$$k_3^{'}(s) = \frac{(s+0.1)(s+0.001)(s+0.001)}{s(s+0.0013)(s+0.0007)} \quad \text{and} \quad k_4^{'}(s) = \frac{(s+0.1)(s+0.001)(s+0.001)}{s(s+0.0019)(s+0.0001)}$$

for control loop1, loop2, loop3 and loop4; respectively.

The designed passive control of five streams mass exchanger network is tested with both disturbance rejection and set point tracking problems. For the disturbance rejection problem, input composition of lean stream 1, rich stream 1, lean stream 2 and rich stream 2 is stepped change 0.0025, 0.0080, 0.0040 and 0.0060 deviation respectively. The designed passive controllers are capable of completely eliminating the impact of the disturbance. In addition, the set point tracking capability is investigated. The control loop1, loop2, loop3 and loop4 are capable of tracking 4, 15, 3 and 1.5% change in the set point. However, the designed passive controllers have less performance than the traditional controller design.

5.2 Recommendations

1. A robustness of passive controller should be improved in order to enhance performance of passive controller.

2. In this work, the MENs model concerns about a single contaminant removal. In reality, more than one contaminant might be exchanged in the same mass exchanger. Multi-component mass transfer should be considered.

3. Noise and time delay should be add into the system in order to reflect real situation.

4. This passivity theorem should be applied with a large-scale mass exchanger network to guarantee that this approach can handle with highly nonlinear.

5. The more complex controller design strategy i.e. split range controller design should be considered in order to illustrate that the passivity concept can be applicable for large interaction system.

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Appendix A

Mathematic model

Appendix A1: Mathematic model of five streams mass exchanger network

$$\frac{dy_{out1}}{dt}=\left[\frac{G_1}{M_{G1}}-\frac{KA_1\left(1-\overline{f}_{r1}\right)}{2a_1M_{G1}}\right]y_2^S+\left[-\frac{G_1}{M_{G1}}-\frac{KA_1\left(1+\overline{f}_{r1}\right)}{2a_1M_{G1}}\right]y_{out1}+\left[\frac{KA_1\left(1-\overline{f}_{l1}\right)}{2M_{G1}}\right]x_1^S+\left[\frac{KA_1\left(1+\overline{f}_{l1}\right)}{2M_{G1}}\right]x_{out1}+\left[\frac{KA_1\left(\overline{x}_{out1}-\overline{x}_1^S\right)}{2M_{G1}}\right]f_{l1} \tag{A.1}$$

$$\frac{dx_{out1}}{dt}=\left[\frac{L_1}{M_{L1}}-\frac{KA_1\left(1-\overline{f}_{l1}\right)}{2M_{L1}}\right]x_1^S+\left[-\frac{L_1}{M_{L1}}-\frac{KA_1\left(1+\overline{f}_{l1}\right)}{2M_{L1}}\right]x_{out1}+\left[\frac{KA_1\left(1-\overline{f}_{r1}\right)}{2a_1M_{L1}}\right]y_2^S+\left[\frac{KA_1\left(1+\overline{f}_{r1}\right)}{2a_1M_{L1}}\right]y_{out1}+\left[\frac{KA_1\left(\overline{x}_1^S-\overline{x}_{out1}\right)}{2M_{L1}}\right]f_{l1} \tag{A.2}$$

$$\frac{dy_{out2}}{dt}=\left[\frac{G_2}{M_{G2}}-\frac{KA_2\left(1-\overline{f}_{r2}\right)}{2a_1M_{G2}}\right]y_1^S+\left[-\frac{G_2}{M_{G2}}-\frac{KA_2\left(1+\overline{f}_{r2}\right)}{2a_1M_{G2}}\right]y_{out2}+\left[\frac{KA_2\left(1-\overline{f}_{l2}\right)}{2M_{G2}}\right]x_1^S+\left[\frac{KA_2\left(1+\overline{f}_{l2}\right)}{2M_{G2}}\right]x_{out2}+\left[\frac{KA_2\left(\overline{y}_1^S-\overline{y}_{out2}\right)}{2a_1M_{G2}}\right]f_{r2} \tag{A.3}$$

$$\frac{dx_{out2}}{dt}=\left[\frac{L_2}{M_{L2}}-\frac{KA_2\left(1-\overline{f}_{l2}\right)}{2M_{L2}}\right]x_1^S+\left[-\frac{L_2}{M_{L2}}-\frac{KA_2\left(1+\overline{f}_{l2}\right)}{2M_{L2}}\right]x_{out2}+\left[\frac{KA_2\left(1-\overline{f}_{r2}\right)}{2a_1M_{L2}}\right]y_1^S+\left[\frac{KA_2\left(1+\overline{f}_{r2}\right)}{2a_1M_{L2}}\right]y_{out2}+\left[\frac{KA_2\left(\overline{y}_{out2}-\overline{y}_1^S\right)}{2a_1M_{L2}}\right]f_{r2} \tag{A.4}$$

$$\frac{dy_{out3}}{dt}=\left[\frac{G_3}{M_{G3}}-\frac{KA_3\left(1-\overline{f}_{r3}\right)}{2a_2M_{G3}}\right]y_1^S+\left[-\frac{G_3}{M_{G3}}-\frac{KA_3\left(1+\overline{f}_{r3}\right)}{2a_2M_{G3}}\right]y_{out3}+\left[\frac{KA_3\left(1-\overline{f}_{l3}\right)}{2M_{G3}}\right]x_{out4}+\left[\frac{KA_3\left(1+\overline{f}_{l3}\right)}{2M_{G3}}\right]x_{out3} \tag{A.5}$$

$$\frac{dx_{out3}}{dt}=\left[\frac{L_3}{M_{L3}}-\frac{KA_3\left(1-\overline{f}_{l3}\right)}{2M_{L3}}\right]x_{out4}+\left[-\frac{L_3}{M_{L3}}-\frac{KA_3\left(1+\overline{f}_{l3}\right)}{2M_{L3}}\right]x_{out3}$$
$$+\left[\frac{KA_3\left(1-\overline{f}_{r3}\right)}{2a_2M_{L3}}\right]y_1^S+\left[\frac{KA_3\left(1+\overline{f}_{r3}\right)}{2a_2M_{L3}}\right]y_{out3} \tag{A.6}$$

$$\frac{dy_{out4}}{dt}=\left[\frac{G_4}{M_{G4}}-\frac{KA_4\left(1-\overline{f}_{r4}\right)}{2a_2M_{G4}}\right]y_{out1}+\left[-\frac{G_4}{M_{G4}}-\frac{KA_4\left(1+\overline{f}_{r4}\right)}{2a_2M_{G4}}\right]y_{out4}$$
$$+\left[\frac{KA_4\left(1-\overline{f}_{l4}\right)}{2M_{G4}}\right]x_2^S+\left[\frac{KA_4\left(1+\overline{f}_{l4}\right)}{2M_{G4}}\right]x_{out4}+\left[\frac{KA_4\left(\overline{x}_{out4}-\overline{x}_2^S\right)}{2M_{G4}}\right]f_{l4} \tag{A.7}$$

$$\frac{dx_{out4}}{dt}=\left[\frac{L_4}{M_{L4}}-\frac{KA_4\left(1-\overline{f}_{l4}\right)}{2M_{L4}}\right]x_2^S+\left[-\frac{L_4}{M_{L4}}-\frac{KA_4\left(1+\overline{f}_{l4}\right)}{2M_{L4}}\right]x_{out4}$$
$$+\left[\frac{KA_4\left(1-\overline{f}_{r4}\right)}{2a_2M_{L4}}\right]y_{out1}+\left[\frac{KA_4\left(1+\overline{f}_{r4}\right)}{2a_2M_{L4}}\right]y_{out4}+\left[\frac{KA_4\left(\overline{x}_2^S-\overline{x}_{out4}\right)}{2M_{L4}}\right]f_{l4} \tag{A.8}$$

$$\frac{dy_{out5}}{dt}=\left[\frac{G_5}{M_{G5}}-\frac{KA_5\left(1-\overline{f}_{r5}\right)}{2a_3M_{G5}}\right]y_{out4}+\left[-\frac{G_5}{M_{G5}}-\frac{KA_5\left(1+\overline{f}_{r5}\right)}{2a_3M_{G5}}\right]y_{out5}$$
$$+\left[\frac{KA_5\left(1-\overline{f}_{l5}\right)}{2M_{G5}}\right]x_3^S+\left[\frac{KA_5\left(1+\overline{f}_{l5}\right)}{2M_{G5}}\right]x_{out5}+\left[\frac{KA_5\left(\overline{y}_{out4}-\overline{y}_{out5}\right)}{2a_3M_{G5}}\right]f_{r5} \tag{A.9}$$

$$\frac{dx_{out5}}{dt}=\left[\frac{L_5}{M_{L5}}-\frac{KA_5\left(1-\overline{f}_{l5}\right)}{2M_{L5}}\right]x_3^S+\left[-\frac{L_5}{M_{L5}}-\frac{KA_5\left(1+\overline{f}_{l5}\right)}{2M_{L5}}\right]x_{out5}$$
$$+\left[\frac{KA_5\left(1-\overline{f}_{r5}\right)}{2a_3M_{L5}}\right]y_{out4}+\left[\frac{KA_5\left(1+\overline{f}_{r5}\right)}{2a_3M_{L5}}\right]y_{out5}+\left[\frac{KA_5\left(\overline{y}_{out5}-\overline{y}_{out4}\right)}{2a_3M_{L5}}\right]f_{r5} \tag{A.10}$$

$$\frac{dy_{out6}}{dt}=\left[\frac{G_6}{M_{G6}}-\frac{KA_6\left(1-\overline{f}_{r6}\right)}{2a_3M_{G6}}\right]y_{in6}+\left[-\frac{G_6}{M_{G6}}-\frac{KA_6\left(1+\overline{f}_{r6}\right)}{2a_3M_{G6}}\right]y_{out6}$$
$$+\left[\frac{KA_6\left(1-\overline{f}_{l6}\right)}{2M_{G6}}\right]x_3^S+\left[\frac{KA_6\left(1+\overline{f}_{l6}\right)}{2M_{G6}}\right]x_{out6} \tag{A.11}$$

$$\frac{dx_{out6}}{dt}=\left[\frac{L_6}{M_{L6}}-\frac{KA_6\left(1-\overline{f}_{l6}\right)}{2M_{L6}}\right]x_3^S+\left[-\frac{L_6}{M_{L6}}-\frac{KA_6\left(1+\overline{f}_{l6}\right)}{2M_{L6}}\right]x_{out6}$$
$$+\left[\frac{KA_6\left(1-\overline{f}_{r6}\right)}{2a_3M_{L6}}\right]y_{in6}+\left[\frac{KA_6\left(1+\overline{f}_{r6}\right)}{2a_3M_{L6}}\right]y_{out6} \tag{A.12}$$

Since,

$$y_{in6}=\frac{G_2y_{out2}+G_3y_{out3}}{G_1^s} \tag{A.13}$$

$$\frac{dy_{out6}}{dt}=\left[\left(\frac{G_6}{M_{G6}}-\frac{KA_6\left(1-\overline{f}_{r6}\right)}{2a_3M_{G6}}\right)\frac{G_2}{G_1^s}\right]y_{out2}+\left[\left(\frac{G_6}{M_{G6}}-\frac{KA_6\left(1-\overline{f}_{r6}\right)}{2a_3M_{G6}}\right)\frac{G_3}{G_1^s}\right]y_{out3}$$
$$+\left[-\frac{G_6}{M_{G6}}-\frac{KA_6\left(1+\overline{f}_{r6}\right)}{2a_3M_{G6}}\right]y_{out6}+\left[\frac{KA_6\left(1-\overline{f}_{l6}\right)}{2M_{G6}}\right]x_3^S+\left[\frac{KA_6\left(1+\overline{f}_{l6}\right)}{2M_{G6}}\right]x_{out6} \tag{A.14}$$

$$\frac{dx_{out6}}{dt}=\left[\frac{L_6}{M_{L6}}-\frac{KA_6\left(1-\overline{f}_{l6}\right)}{2M_{L6}}\right]x_3^S+\left[-\frac{L_6}{M_{L6}}-\frac{KA_6\left(1+\overline{f}_{l6}\right)}{2M_{L6}}\right]x_{out6}$$
$$+\left[\frac{KA_6\left(1-\overline{f}_{r6}\right)}{2a_3M_{L6}}\frac{G_2}{G_1^s}\right]y_{out2}+\left[\frac{KA_6\left(1-\overline{f}_{r6}\right)}{2a_3M_{L6}}\frac{G_3}{G_1^s}\right]y_{out3}+\left[\frac{KA_6\left(1+\overline{f}_{r6}\right)}{2a_3M_{L6}}\right]y_{out6} \tag{A.15}$$

Appendix A2: State space equation of five streams mass exchanger network

$$A=\begin{bmatrix}
\mathrm{a}_{0101} & \mathrm{a}_{0102} & \mathrm{a}_{0103} & \mathrm{a}_{0104} & \mathrm{a}_{0105} & \mathrm{a}_{0106} & \mathrm{a}_{0107} & \mathrm{a}_{0108} & \mathrm{a}_{0109} & \mathrm{a}_{0110} & \mathrm{a}_{0111} & \mathrm{a}_{0112} \\
\mathrm{a}_{0201} & \mathrm{a}_{0202} & \mathrm{a}_{0203} & \mathrm{a}_{0204} & \mathrm{a}_{0205} & \mathrm{a}_{0206} & \mathrm{a}_{0207} & \mathrm{a}_{0208} & \mathrm{a}_{0209} & \mathrm{a}_{0210} & \mathrm{a}_{0211} & \mathrm{a}_{0212} \\
\mathrm{a}_{0301} & \mathrm{a}_{0302} & \mathrm{a}_{0303} & \mathrm{a}_{0304} & \mathrm{a}_{0305} & \mathrm{a}_{0306} & \mathrm{a}_{0307} & \mathrm{a}_{0308} & \mathrm{a}_{0309} & \mathrm{a}_{0310} & \mathrm{a}_{0311} & \mathrm{a}_{0312} \\
\mathrm{a}_{0401} & \mathrm{a}_{0402} & \mathrm{a}_{0403} & \mathrm{a}_{0404} & \mathrm{a}_{0405} & \mathrm{a}_{0406} & \mathrm{a}_{0407} & \mathrm{a}_{0408} & \mathrm{a}_{0409} & \mathrm{a}_{0410} & \mathrm{a}_{0411} & \mathrm{a}_{0412} \\
\mathrm{a}_{0501} & \mathrm{a}_{0502} & \mathrm{a}_{0503} & \mathrm{a}_{0504} & \mathrm{a}_{0505} & \mathrm{a}_{0506} & \mathrm{a}_{0507} & \mathrm{a}_{0508} & \mathrm{a}_{0509} & \mathrm{a}_{0510} & \mathrm{a}_{0511} & \mathrm{a}_{0512} \\
\mathrm{a}_{0601} & \mathrm{a}_{0602} & \mathrm{a}_{0603} & \mathrm{a}_{0604} & \mathrm{a}_{0605} & \mathrm{a}_{0606} & \mathrm{a}_{0507} & \mathrm{a}_{0608} & \mathrm{a}_{0609} & \mathrm{a}_{0610} & \mathrm{a}_{0611} & \mathrm{a}_{0612} \\
\mathrm{a}_{0701} & \mathrm{a}_{0702} & \mathrm{a}_{0703} & \mathrm{a}_{0704} & \mathrm{a}_{0705} & \mathrm{a}_{0706} & \mathrm{a}_{0707} & \mathrm{a}_{0708} & \mathrm{a}_{0709} & \mathrm{a}_{0710} & \mathrm{a}_{0711} & \mathrm{a}_{0712} \\
\mathrm{a}_{0801} & \mathrm{a}_{0802} & \mathrm{a}_{0803} & \mathrm{a}_{0804} & \mathrm{a}_{0805} & \mathrm{a}_{0806} & \mathrm{a}_{0807} & \mathrm{a}_{0808} & \mathrm{a}_{0809} & \mathrm{a}_{0810} & \mathrm{a}_{0811} & \mathrm{a}_{0812} \\
\mathrm{a}_{0901} & \mathrm{a}_{0902} & \mathrm{a}_{0903} & \mathrm{a}_{0904} & \mathrm{a}_{0905} & \mathrm{a}_{0906} & \mathrm{a}_{0907} & \mathrm{a}_{0908} & \mathrm{a}_{0909} & \mathrm{a}_{0910} & \mathrm{a}_{0911} & \mathrm{a}_{0912} \\
\mathrm{a}_{1001} & \mathrm{a}_{1002} & \mathrm{a}_{1003} & \mathrm{a}_{1004} & \mathrm{a}_{1005} & \mathrm{a}_{1006} & \mathrm{a}_{1007} & \mathrm{a}_{1008} & \mathrm{a}_{1009} & \mathrm{a}_{1010} & \mathrm{a}_{1011} & \mathrm{a}_{1012} \\
\mathrm{a}_{1101} & \mathrm{a}_{1102} & \mathrm{a}_{1103} & \mathrm{a}_{1104} & \mathrm{a}_{1105} & \mathrm{a}_{1106} & \mathrm{a}_{1107} & \mathrm{a}_{1108} & \mathrm{a}_{1109} & \mathrm{a}_{1110} & \mathrm{a}_{1111} & \mathrm{a}_{1112} \\
\mathrm{a}_{1201} & \mathrm{a}_{1202} & \mathrm{a}_{1203} & \mathrm{a}_{1204} & \mathrm{a}_{1205} & \mathrm{a}_{1206} & \mathrm{a}_{1207} & \mathrm{a}_{1208} & \mathrm{a}_{1209} & \mathrm{a}_{1210} & \mathrm{a}_{1211} & \mathrm{a}_{1212}
\end{bmatrix}$$

$$B=\begin{bmatrix}
b_{0101} & b_{0102} & b_{0103} & b_{0104} \\
b_{0201} & b_{0202} & b_{0203} & b_{0204} \\
b_{0301} & b_{0302} & b_{0303} & b_{0304} \\
b_{0401} & b_{0402} & b_{0403} & b_{0404} \\
b_{0501} & b_{0502} & b_{0503} & b_{0504} \\
b_{0601} & b_{0602} & b_{0603} & b_{0604} \\
b_{0701} & b_{0702} & b_{0703} & b_{0704} \\
b_{0801} & b_{0802} & b_{0803} & b_{0804} \\
b_{0901} & b_{0902} & b_{0903} & b_{0904} \\
b_{1001} & b_{1002} & b_{1003} & b_{1004} \\
b_{1101} & b_{1102} & b_{1103} & b_{1104} \\
b_{1201} & b_{1202} & b_{1203} & b_{1204}
\end{bmatrix}$$

$$C=\begin{bmatrix}
c_{0101} & c_{0102} & c_{0103} & c_{0104} & c_{0105} & c_{0106} & c_{0107} & c_{0108} & c_{0109} & c_{0110} & c_{0111} & c_{0112} \\
c_{0201} & c_{0202} & c_{0203} & c_{0204} & c_{0205} & c_{0206} & c_{0207} & c_{0208} & c_{0209} & c_{0210} & c_{0211} & c_{0212} \\
c_{0301} & c_{0302} & c_{0303} & c_{0304} & c_{0305} & c_{0306} & c_{0307} & c_{0308} & c_{0309} & c_{0310} & c_{0311} & c_{0312} \\
c_{0401} & c_{0402} & c_{0403} & c_{0404} & c_{0405} & c_{0406} & c_{0407} & c_{0408} & c_{0409} & c_{0410} & c_{0411} & c_{0412}
\end{bmatrix}$$

$$D=\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}$$

$$E = \begin{bmatrix} e_{0101} & e_{0102} & e_{0103} & e_{0104} & e_{0105} \\ e_{0201} & e_{0202} & e_{0203} & e_{0204} & e_{0205} \\ e_{0301} & e_{0302} & e_{0303} & e_{0304} & e_{0305} \\ e_{0401} & e_{0402} & e_{0403} & e_{0404} & e_{0405} \\ e_{0501} & e_{0502} & e_{0503} & e_{0504} & e_{0505} \\ e_{0601} & e_{0602} & e_{0603} & e_{0604} & e_{0605} \\ e_{0701} & e_{0702} & e_{0703} & e_{0704} & e_{0705} \\ e_{0801} & e_{0802} & e_{0803} & e_{0804} & e_{0805} \\ e_{0901} & e_{0902} & e_{0903} & e_{0904} & e_{0905} \\ e_{1001} & e_{1002} & e_{1003} & e_{1004} & e_{1005} \\ e_{1101} & e_{1102} & e_{1103} & e_{1104} & e_{1105} \\ e_{1201} & e_{1202} & e_{1203} & e_{1204} & e_{1205} \end{bmatrix}$$

Where, all is zero except following

$$a_{0101} = \left[-\frac{G_1}{M_{G1}} - \frac{KA_1\left(1+\overline{f}_{r1}\right)}{2a_1 M_{G1}} \right] \quad \text{(A.16)} \qquad a_{0102} = \left[\frac{KA_1\left(1+\overline{f}_{l1}\right)}{2M_{G1}} \right] \quad \text{(A.17)}$$

$$a_{0201} = \left[\frac{KA_1\left(1+\overline{f}_{r1}\right)}{2a_1 M_{L1}} \right] \quad \text{(A.18)} \qquad a_{0202} = \left[-\frac{L_1}{M_{L1}} - \frac{KA_1\left(1+\overline{f}_{l1}\right)}{2M_{L1}} \right] \quad \text{(A.19)}$$

$$a_{0303} = \left[-\frac{G_2}{M_{G2}} - \frac{KA_2\left(1+\overline{f}_{r2}\right)}{2a_1 M_{G2}} \right] \quad \text{(A.20)} \qquad a_{0304} = \left[\frac{KA_2\left(1+\overline{f}_{l2}\right)}{2M_{G2}} \right] \quad \text{(A.21)}$$

$$a_{0403} = \left[\frac{KA_2\left(1+\overline{f}_{r2}\right)}{2a_1 M_{L2}} \right] \quad \text{(A.22)} \qquad a_{0404} = \left[-\frac{L_2}{M_{L2}} - \frac{KA_2\left(1+\overline{f}_{l2}\right)}{2M_{L2}} \right] \quad \text{(A.23)}$$

$$a_{0505} = \left[-\frac{G_3}{M_{G3}} - \frac{KA_3\left(1+\overline{f}_{r3}\right)}{2a_2 M_{G3}} \right] \quad \text{(A.24)} \qquad a_{0506} = \left[\frac{KA_3\left(1+\overline{f}_{l3}\right)}{2M_{G3}} \right] \quad \text{(A.25)}$$

$$a_{0508} = \left[\frac{KA_3\left(1-\overline{f}_{l3}\right)}{2M_{G3}} \right] \quad \text{(A.26)} \qquad a_{0605} = \left[\frac{KA_3\left(1+\overline{f}_{r3}\right)}{2a_2 M_{L3}} \right] \quad \text{(A.27)}$$

$$a_{0606} = \left[-\frac{L_3}{M_{L3}} - \frac{KA_3\left(1+\overline{f}_{l3}\right)}{2M_{L3}} \right] \quad \text{(A.28)} \qquad a_{0608} = \left[\frac{L_3}{M_{L3}} - \frac{KA_3\left(1-\overline{f}_{l3}\right)}{2M_{L3}} \right] \quad \text{(A.29)}$$

$$a_{0701}=\left[\frac{G_4}{M_{G4}}-\frac{KA_4\left(1-\overline{f}_{r4}\right)}{2a_2M_{G4}}\right] \quad (A.30) \qquad a_{0707}=\left[-\frac{G_4}{M_{G4}}-\frac{KA_4\left(1+\overline{f}_{r4}\right)}{2a_2M_{G4}}\right] \quad (A.31)$$

$$a_{0708}=\left[\frac{KA_4\left(1+\overline{f}_{l4}\right)}{2M_{G4}}\right] \quad (A.32) \qquad a_{0801}=\left[\frac{KA_4\left(1-\overline{f}_{r4}\right)}{2a_2M_{L4}}\right] \quad (A.33)$$

$$a_{0807}=\left[\frac{KA_4\left(1+\overline{f}_{r4}\right)}{2a_2M_{L4}}\right] \quad (A.34) \qquad a_{0808}=\left[-\frac{L_4}{M_{L4}}-\frac{KA_4\left(1+\overline{f}_{l4}\right)}{2M_{L4}}\right] \quad (A.35)$$

$$a_{0907}=\left[\frac{G_5}{M_{G5}}-\frac{KA_5\left(1-\overline{f}_{r5}\right)}{2a_3M_{G5}}\right] \quad (A.36) \qquad a_{0909}=\left[-\frac{G_5}{M_{G5}}-\frac{KA_5\left(1+\overline{f}_{r5}\right)}{2a_3M_{G5}}\right] \quad (A.37)$$

$$a_{0910}=\left[\frac{KA_5\left(1+\overline{f}_{l5}\right)}{2M_{G5}}\right] \quad (A.38) \qquad a_{1010}=\left[-\frac{L_5}{M_{L5}}-\frac{KA_5\left(1+\overline{f}_{l5}\right)}{2M_{L5}}\right] \quad (A.39)$$

$$a_{1007}=\left[\frac{KA_5\left(1-\overline{f}_{r5}\right)}{2a_3M_{L5}}\right] \quad (A.40) \qquad a_{1009}=\left[\frac{KA_5\left(1+\overline{f}_{r5}\right)}{2a_3M_{L5}}\right] \quad (A.41)$$

$$a_{1103}=\left[\left(\frac{G_6}{M_{G6}}-\frac{KA_6\left(1-\overline{f}_{r6}\right)}{2a_3M_{G6}}\right)\frac{G_2}{G_1^s}\right] \quad (A.42) \qquad a_{1105}=\left[\left(\frac{G_6}{M_{G6}}-\frac{KA_6\left(1-\overline{f}_{r6}\right)}{2a_3M_{G6}}\right)\frac{G_3}{G_1^s}\right] \quad (A.43)$$

$$a_{1111}=\left[-\frac{G_6}{M_{G6}}-\frac{KA_6\left(1+\overline{f}_{r6}\right)}{2a_3M_{G6}}\right] \quad (A.44) \qquad a_{1112}=\left[\frac{KA_6\left(1+\overline{f}_{l6}\right)}{2M_{G6}}\right] \quad (A.45)$$

$$a_{1203}=\left[\frac{KA_6\left(1-\overline{f}_{r6}\right)}{2a_3M_{L6}}\frac{G_2}{G_1^s}\right] \quad (A.46) \qquad a_{1205}=\left[\frac{KA_6\left(1-\overline{f}_{r6}\right)}{2a_3M_{L6}}\frac{G_3}{G_1^s}\right] \quad (A.47)$$

$$a_{1211}=\left[\frac{KA_6\left(1+\overline{f}_{r6}\right)}{2a_3M_{L6}}\right] \quad (A.48) \qquad a_{1212}=\left[-\frac{L_6}{M_{L6}}-\frac{KA_6\left(1+\overline{f}_{l6}\right)}{2M_{L6}}\right] \quad (A.49)$$

$$b_{0101}=\left[\frac{KA_1\left(\overline{x}_{out1}-\overline{x}_1^S\right)}{2M_{G1}}\right] \quad (A.50) \qquad b_{0201}=\left[\frac{KA_1\left(\overline{x}_1^S-\overline{x}_{out1}\right)}{2M_{L1}}\right] \quad (A.51)$$

$$b_{0302}=\left[\frac{KA_2\left(\overline{y}_1^S-\overline{y}_{out2}\right)}{2a_1M_{G2}}\right] \quad (A.52) \qquad b_{0402}=\left[\frac{KA_2\left(\overline{y}_{out2}-\overline{y}_1^S\right)}{2a_1M_{L2}}\right] \quad (A.53)$$

$$b_{0703} = \left[\frac{KA_4 \left(\bar{x}_{out4} - \bar{x}_2^S \right)}{2M_{G4}} \right] \quad \text{(A.54)} \qquad b_{0803} = \left[\frac{KA_4 \left(\bar{x}_2^S - \bar{x}_{out4} \right)}{2M_{L4}} \right] \quad \text{(A.55)}$$

$$b_{0904} = \left[\frac{KA_5 \left(\bar{y}_{out4} - \bar{y}_{out5} \right)}{2a_3 M_{G5}} \right] \quad \text{(A.56)} \qquad b_{1004} = \left[\frac{KA_5 \left(\bar{y}_{out5} - \bar{y}_{out4} \right)}{2a_3 M_{L5}} \right] \quad \text{(A.57)}$$

$$c_{0102} = \left[\frac{L_1}{L_1^S} \right] \quad \text{(A.58)} \qquad c_{0104} = \left[\frac{L_2}{L_1^S} \right] \quad \text{(A.59)}$$

$$c_{0211} = 1 \quad \text{(A.60)} \qquad c_{0306} = 1 \quad \text{(A.61)}$$

$$c_{0409} = 1 \quad \text{(A.62)}$$

$$e_{0101} = \left[\frac{KA_1 \left(1 - \bar{f}_{l1} \right)}{2M_{G1}} \right] \quad \text{(A.63)} \qquad e_{0104} = \left[\frac{G_1}{M_{G1}} - \frac{KA_1 \left(1 - \bar{f}_{r1} \right)}{2a_1 M_{G1}} \right] \quad \text{(A.64)}$$

$$e_{0201} = \left[\frac{L_1}{M_{L1}} - \frac{KA_1 \left(1 - \bar{f}_{l1} \right)}{2M_{L1}} \right] \quad \text{(A.65)} \qquad e_{0204} = \left[\frac{KA_1 \left(1 - \bar{f}_{r1} \right)}{2a_1 M_{L1}} \right] \quad \text{(A.66)}$$

$$e_{0302} = \left[\frac{G_2}{M_{G2}} - \frac{KA_2 \left(1 - \bar{f}_{r2} \right)}{2a_1 M_{G2}} \right] \quad \text{(A.67)} \qquad e_{0301} = \left[\frac{KA_2 \left(1 - \bar{f}_{l2} \right)}{2M_{G2}} \right] \quad \text{(A.68)}$$

$$e_{0401} = \left[\frac{L_2}{M_{L2}} - \frac{KA_2 \left(1 - \bar{f}_{l2} \right)}{2M_{L2}} \right] \quad \text{(A.69)} \qquad e_{0402} = \left[\frac{KA_2 \left(1 - \bar{f}_{r2} \right)}{2a_1 M_{L2}} \right] \quad \text{(A.70)}$$

$$e_{0502} = \left[\frac{G_3}{M_{G3}} - \frac{KA_3 \left(1 - \bar{f}_{r3} \right)}{2a_2 M_{G3}} \right] \quad \text{(A.71)} \qquad e_{0602} = \left[\frac{KA_3 \left(1 - \bar{f}_{r3} \right)}{2a_2 M_{L3}} \right] \quad \text{(A.72)}$$

$$e_{0703} = \left[\frac{KA_4 \left(1 - \bar{f}_{l4} \right)}{2M_{G4}} \right] \quad \text{(A.73)} \qquad e_{0803} = \left[\frac{L_4}{M_{L4}} - \frac{KA_4 \left(1 - \bar{f}_{l4} \right)}{2M_{L4}} \right] \quad \text{(A.74)}$$

$$e_{0905} = \left[\frac{KA_5 \left(1 - \bar{f}_{l5} \right)}{2M_{G5}} \right] \quad \text{(A.75)} \qquad e_{1005} = \left[\frac{L_5}{M_{L5}} - \frac{KA_5 \left(1 - \bar{f}_{l5} \right)}{2M_{L5}} \right] \quad \text{(A.76)}$$

$$e_{1105} = \left[\frac{KA_6 \left(1 - \bar{f}_{l6} \right)}{2M_{G6}} \right] \quad \text{(A.77)} \qquad e_{1205} = \left[\frac{L_6}{M_{L6}} - \frac{KA_6 \left(1 - \bar{f}_{l6} \right)}{2M_{L6}} \right] \quad \text{(A.78)}$$

Appendix B

MATLAB Code

Appendix B1: Dynamic behavior of copper recovery unit

B1.1 Function file

```
function fy = ode(t,y)
fy=zeros(2,1);
L=0.0925; % kg/s
G=0.1; %kg/s
xin=0.03;
xout=0.07;
yin=0.06;
yout=0.02306;
a=0.734;
b=0.001;
fr=0;
fl=0;
frss = 0;
flss = 0;
frp = fr - frss;
flp = fl - flss;
xinss = 0.03;
yinss = 0.06;
xinp = xin - xinss;
yinp = yin - yinss;
MG = 500;
ML = 500;
C1 = ((yin-b)/a) - xout;
C2 = ((yout-b)/a) - xin;
KA = L*(xout-xin)*2/(C1+C2);
eqn1 = '0 = (G - KA*(1-frss)/(2*a))*yin - (G + KA*(1+frss)/(2*a))*y + (KA*(1-
flss)/(2))*xin + (KA*(1+flss)/(2))*x + (KA*b/a)';
eqn2 = '0 = (L - KA*(1-flss)/(2))*xin - (L + KA*(1+flss)/(2))*x + (KA*(1-
frss)/(2*a))*yin + (KA*(1+frss)/(2*a))*y - (KA*b/a)';
xy = solve(eqn1, eqn2, 'x', 'y');
xss = (2*G*KA*yin - 2*G*KA*b + KA*L*xin - G*KA*a*xin + 2*G*L*a*xin +
KA*L*frss*xin + G*KA*a*flss*xin)/(KA*L + G*KA*a + 2*G*L*a + KA*L*frss +
G*KA*a*flss);
yss = (2*KA*L*b - KA*L*yin + G*KA*a*yin + 2*G*L*a*yin + 2*KA*L*a*xin +
KA*L*frss*yin + G*KA*a*flss*yin)/(KA*L + G*KA*a + 2*G*L*a + KA*L*frss +
G*KA*a*flss);
fy(1) = ((-G - KA*(frss+1)/(2*a))*y(1) + (KA*(yin - yss)/(2*a))*frp +
(KA*(1+flss)/(2))*y(2) + (KA*(xss - xin)/(2))*flp)/MG;
fy(2) = ((-L - KA*(flss+1)/(2))*y(2) + (KA*(xin - xss)/(2))*flp +
(KA*(frss+1)/(2*a))*y(1) + (KA*(yss - yin)/(2*a))*frp)/ML;
```

B1.2 Script file

```
clc
clear
```

```
xin=0.03;
xout=0.07;
yin=0.06;
yout=0.02306;
xp = xin - xout;
yp = yin – yout;
simtime = [0:1:1000];
inity = [yp, xp];
[t,y] = ode45('ode', simtime, inity);
Y = Table(:,2) + yout;
X = Table(:,3) + xout;
[t, Y, X]
fraction = [Y,X];
figure(1)
plot(t, fraction)
```

Appendix B2: Passivity concept on mass exchanger unit

```
clc
clear
%% Input
L=0.0925; % kg/s
G=0.1; %kg/s
xin=0.03; xout=0.07;
yin=0.06; yout=0.02306;
a=0.734; b=0.001;
fr=0;
fl=0;
frss = 0;
flss = 0;
frp = fr - frss
flp = fl - flss
MG = 50;
ML = 50;
xinss = 0.03;
yinss = 0.06;
xinp = xin - xinss;
yinp = yin - yinss;
C1 = ((yin-b)/a) - xout;
C2 = ((yout-b)/a) - xin;
KA = G*(yin - yout)*2/(C1+C2)

%%Find steady state condition
 eqn1 = '(G - KA*(1-frss)/(2*a))*yin - (G + KA*(1+frss)/(2*a))*y + (KA*(1-
flss)/(2))*xin + (KA*(1+flss)/(2))*x + (KA*b/a)';
eqn2 = '(L - KA*(1-flss)/(2))*xin - (L + KA*(1+flss)/(2))*x + (KA*(1-frss)/(2*a))*yin
+ (KA*(1+frss)/(2*a))*y - (KA*b/a)';
xy = solve(eqn1, eqn2, 'x', 'y');
disp(xy.x)
disp(xy.y)
xss = (2*G*KA*yin - 2*G*KA*b + KA*L*xin - G*KA*a*xin + 2*G*L*a*xin +
KA*L*frss*xin + G*KA*a*flss*xin)/(KA*L + G*KA*a + 2*G*L*a + KA*L*frss +
G*KA*a*flss)
yss = (2*KA*L*b - KA*L*yin + G*KA*a*yin + 2*G*L*a*yin + 2*KA*L*a*xin +
KA*L*frss*yin + G*KA*a*flss*yin)/(KA*L + G*KA*a + 2*G*L*a + KA*L*frss +
G*KA*a*flss)

%%State space A B C D E
a11 = (-G - KA*(frss+1)/(2*a))/MG;
a12 = (KA*(1+flss)/(2))/MG;
a21 = (KA*(frss+1)/(2*a))/ML;
a22 = (-L - KA*(flss+1)/(2))/ML;

b11 = (KA*(yin - yss)/(2*a*MG));
```

```
b12 = (KA*(xss - xin)/(2*MG));
b21 = (KA*(yss - yin)/(2*a*ML));
b22 = (KA*(xin - xss)/(2*ML));

e11 = (G - KA*(1-frss)/(2*a))/MG;
e12 = (KA*(1-flss))/(2*MG);
e21 = (KA*(1-frss))/(2*a*ML);
e22 = (L - KA*(1-flss)/(2))/ML;

c11 = 1;
c12 = 0;
c21 = 0;
c22 = 1;

d11 = 0;
d12 = 0;
d21 = 0;
d22 = 0;

A = [a11 a12; a21 a22]
B = [b11 b12; b21 b22]
C = [c11 c12; c21 c22]
D = [d11 d12; d21 d22]
E = [e11 e12; e21 e22]
%% Process Transfer function (Gp) system = ss(A,B,C,D)
g = tf(system)
%%Plot Passivity Index(1)
 iw = 200;
w1 = -4;  %Starting Frequency
w2 = 4;   %Ending Frequency
w = logspace(w1,w2,iw)
s = w * sqrt(-1);              %s = jw
I = eye(length (A));           %I = identity matrix
for z=1:length(s)
g= C*((s(z)*I-A)\B)+D;      %g = transfer function
nu1(z)= -min(real(eig(g+g'))/2);  %passivity index
end
figure(1)
semilogx(w,nu1,'g');
xlabel('Frequency (rad/time)');
ylabel('Passivity Index');
legend('Passivity Index');
grid;
```

Appendix B3: Passivity concept on mass exchanger network

```
clc
clear
% Mass load in each MEX
Mp1 = 0.0750;
Mp2 = 0.0750;
Mp3 = 0.0095;
Mp4 = 0.0275;
Mp5 = 0.0100;
Mp6 = 0.0325;
% % Recycle fraction
frss1 = 0;
frss2 = 0.5;
frss3 = 0;
frss4 = 0;
frss5 = 0.5;
frss6 = 0;
flss1 = 0.5;
flss2 = 0;
flss3 = 0;
flss4 = 0.5;
flss5 = 0;
flss6 = 0;
% Source stream mass rate
RS1 = 1.3; G1s = 1.3;
RS2 = 1.5; G2s = 1.5;
LS1 = 2.5; L1s = 2.5;
LS2 = 0.5; L2s = 0.5;
LS3 = 0.6; L3s = 0.6;

%% steady state mass fraction
yinss1 = 0.1;
yinss2 = 0.115;
yinss3 = 0.115;
yinss4 = 0.05;
yinss5 = 0.0317;
yinss6 = 0.05;
xinss1 = 0.05;
xinss2 = 0.05;
xinss3 = 0.0900;
xinss4 = 0.035;
xinss5 = 0.010;
xinss6 = 0.010;

youtss1 = 0.05;
youtss2 = 0.04;
youtss3 = 0.08333;
```

```
youtss4 = 0.0317;
youtss5 = 0.025;
youtss6 = 0.025;
xoutss1 = 0.11;
xoutss2 = 0.11;
xoutss3 = 0.109;
xoutss4 = 0.090;
xoutss5 = 0.0433;
xoutss6 = 0.1183;
%% mass accumulation
MG1 = 1000; MG2 = 1000; MG3 = 1000; MG4 = 1000; MG5 = 1000; MG6 = 1000;
ML1 = 1000; ML2 = 1000; ML3 = 1000; ML4 = 1000; ML5 = 1000; ML6 = 1000;
G1 = 1.5; G2 = 1; G3 = 0.3; G4 = 1.5; G5 = 1.5; G6 = 1.3;
L1 = 1.25; L2 = 1.25; L3 = 0.5; L4 = 0.5; L5 = 0.3; L6 = 0.3;
%%Checking for mass load
%%yinbar=(1-fr)*yin + (yout*fr)
    Mp1
    Mp1 = (G1/(1-frss1))*((1-frss1)*yinss1 + (youtss1*frss1) - youtss1)
    Mp1 = (L1/(1-flss1))*(xoutss1 - ((1-flss1)*xinss1) - (xoutss1*flss1))
    Mp2
    Mp2 = (G2/(1-frss2))*((1-frss2)*yinss2 + (youtss2*frss2) - youtss2)
    Mp2 = (L2/(1-flss2))*(xoutss2 - ((1-flss2)*xinss2) - (xoutss2*flss2))
    Mp3
    Mp3 = (G3/(1-frss3))*((1-frss3)*yinss3 + (youtss3*frss3) - youtss3)
    Mp3 = (L3/(1-flss3))*(xoutss3 - ((1-flss3)*xinss3) - (xoutss3*flss3))
    Mp4
    Mp4 = (G4/(1-frss4))*((1-frss4)*yinss4 + (youtss4*frss4) - youtss4)
    Mp4 = (L4/(1-flss4))*(xoutss4 - ((1-flss4)*xinss4) - (xoutss4*flss4))
    Mp5
    Mp5 = (G5/(1-frss5))*((1-frss5)*yinss5 + (youtss5*frss5) - youtss5)
    Mp5 = (L5/(1-flss5))*(xoutss5 - ((1-flss5)*xinss5) - (xoutss5*flss5))
    Mp6
    Mp6 = (G6/(1-frss6))*((1-frss6)*yinss6 + (youtss6*frss6) - youtss6)
    Mp6 = (L6/(1-flss6))*(xoutss6 - ((1-flss6)*xinss6) - (xoutss6*flss6))

%%find KA
a1=0.8; b1=0.002;
a2=0.5; b2=0;
a3=0.2; b3=0;
%xinbar1=0.075
xinbar1=(1-flss1)*xinss1 + xoutss1*flss1
%yinbar2=0.0775
yinbar2=(1-frss2)*yinss2 + youtss2*frss2
%xinbar4=0.0625
xinbar4=(1-flss4)*xinss4 + xoutss4*flss4
%yinbar5=0.0283
yinbar5=(1-frss5)*yinss5 + youtss5*frss5
```

```
KA1 = Mp1/(0.5*((yinss1-b1)/a1-xoutss1+(youtss1-b1)/a1-xinss1))
KA2 = Mp2/(0.5*((yinss2-b1)/a1-xoutss2+(youtss2-b1)/a1-xinss2))
KA3 = Mp3/(0.5*((yinss3-b2)/a2-xoutss3+(youtss3-b2)/a2-xinss3))
KA4 = Mp4/(0.5*((yinss4-b2)/a2-xoutss4+(youtss4-b2)/a2-xinss4))
KA5 = Mp5/(0.5*((yinss5-b3)/a3-xoutss5+(youtss5-b3)/a3-xinss5))
KA6 = Mp6/(0.5*((yinss6-b3)/a3-xoutss6+(youtss6-b3)/a3-xinss6))

%% State space A B C D E
a0101 = (-G1/MG1)-((KA1*(1+frss1))/(2*a1*MG1));
a0102 = (KA1*(1+flss1))/(2*MG1);
a0103=0;a0104=0;a0105=0;a0106=0;a0107=0;a0108=0;a0109=0;a0110=0;a0111=0;a0
112=0;
a0201 = (KA1*(1+frss1))/(2*a1*ML1);
a0202 = (-L1/ML1)-((KA1*(1+flss1))/(2*ML1));
a0203=0;a0204=0;a0205=0;a0206=0;a0207=0;a0208=0;a0209=0;a0210=0;a0211=0;a0
212=0;
a0303 = (-G2/MG2)-((KA2*(1+frss2))/(2*a1*MG2));
a0304 = (KA2*(1+flss2))/(2*MG2);
a0301=0;a0302=0;a0305=0;a0306=0;a0307=0;a0308=0;a0309=0;a0310=0;a0311=0;a0
312=0;
a0403 = (KA2*(1+frss2))/(2*a1*ML2);
a0404 = (-L2/ML2)-((KA2*(1+flss2))/(2*ML2));
a0401=0;a0402=0;a0405=0;a0406=0;a0407=0;a0408=0;a0409=0;a0410=0;a0411=0;a0
412=0;
a0505 = (-G3/MG3)-((KA3*(1+frss3))/(2*a2*MG3));
a0506 = (KA3*(1+flss3))/(2*MG3);
a0508 = (KA3*(1-flss3))/(2*MG3);
a0501=0;a0502=0;a0503=0;a0504=0;a0507=0;a0509=0;a0510=0;a0511=0;a0512=0;
a0605 = (KA3*(1+frss3))/(2*a2*ML3);
a0606 = (-L3/ML3)-((KA3*(1+flss3))/(2*ML3));
a0608 = (L3/ML3)-((KA3*(1-flss3))/(2*ML3));
a0601=0;a0602=0;a0603=0;a0604=0;a0607=0;a0609=0;a0610=0;a0611=0;a0612=0;
a0701=0;a0702=0;a0703=0;a0704=0;a0705=0;a0706=0;a0707=0;a0708=0;a0709=0;a0
710=0;a0711=0;a0712=0;
a0701 = (G4/MG4)-((KA4*(1-frss4))/(2*a2*MG4));
a0707 = (-G4/MG4)-((KA4*(1+frss4))/(2*a2*MG4));
a0708 = (KA4*(1+flss4))/(2*MG4);
a0801=0;a0802=0;a0803=0;a0804=0;a0805=0;a0806=0;a0807=0;a0808=0;a0809=0;a0
810=0;a0811=0;a0812=0;
a0801 = (KA4*(1-frss4))/(2*a2*ML4);
a0807 = (KA4*(1+frss4))/(2*a2*ML4);
a0808 = (-L4/ML4)-(KA4*(1+flss4))/(2*ML4);
a0901=0;a0902=0;a0903=0;a0904=0;a0905=0;a0906=0;a0907=0;a0908=0;a0909=0;a0
910=0;a0911=0;a0912=0;
a0907 = (G5/MG5)-(KA5*(1-frss5))/(2*a3*MG5);
a0909 = (-G5/MG5)-(KA5*(1+frss5))/(2*a3*MG5);
a0910 = (KA5*(1+flss5))/(2*MG5);
```

```
a1001=0;a1002=0;a1003=0;a1004=0;a1005=0;a1006=0;a1007=0;a1008=0;a1009=0;a1
010=0;a1011=0;a1012=0;
a1010 = (-L5/ML5)-(KA5*(1+flss5))/(2*ML5);
a1007 = (KA5*(1-frss5))/(2*a3*ML5);
a1009 = (KA5*(1+frss5))/(2*a3*ML5);
a1101=0;a1102=0;a1103=0;a1104=0;a1105=0;a1106=0;a1107=0;a1108=0;a1109=0;a1
110=0;a1111=0;a1112=0;
a1111 = (-G6/MG6)-(KA6*(1+frss6))/(2*a3*MG6);
a1103 = ((G6/MG6)-((KA6*(1-frss6))/(2*a3*MG6)))*(G2/G1s);
a1105 = ((G6/MG6)-((KA6*(1-frss6))/(2*a3*MG6)))*(G3/G1s);
a1112 = (KA6*(1+flss6))/(2*MG6);
a1201=0;a1202=0;a1203=0;a1204=0;a1205=0;a1206=0;a1207=0;a1208=0;a1209=0;a1
210=0;a1211=0;a1212=0;
a1212 = (-L6/ML6)-(KA6*(1+flss6))/(2*ML6);
a1203 = (KA6*(1-frss6))/(2*a3*ML6)*(G2/G1s);
a1205 = (KA6*(1-frss6))/(2*a3*ML6)*(G3/G1s);
a1211 = (KA6*(1+frss6))/(2*a3*ML6);
A = [a0101 a0102 a0103 a0104 a0105 a0106 a0107 a0108 a0109 a0110 a0111 a0112;
   a0201 a0202 a0203 a0204 a0205 a0206 a0207 a0208 a0209 a0210 a0211 a0212;
   a0301 a0302 a0303 a0304 a0305 a0306 a0307 a0308 a0309 a0310 a0311 a0312;
   a0401 a0402 a0403 a0404 a0405 a0406 a0407 a0408 a0409 a0410 a0411 a0412;
   a0501 a0502 a0503 a0504 a0505 a0506 a0507 a0508 a0509 a0510 a0511 a0512;
   a0601 a0602 a0603 a0604 a0605 a0606 a0607 a0608 a0609 a0610 a0611 a0612;
   a0701 a0702 a0703 a0704 a0705 a0706 a0707 a0708 a0709 a0710 a0711 a0712;
   a0801 a0802 a0803 a0804 a0805 a0806 a0807 a0808 a0809 a0810 a0811 a0812;
   a0901 a0902 a0903 a0904 a0905 a0906 a0907 a0908 a0909 a0910 a0911 a0912;
   a1001 a1002 a1003 a1004 a1005 a1006 a1007 a1008 a1009 a1010 a1011 a1012;
   a1101 a1102 a1103 a1104 a1105 a1106 a1107 a1108 a1109 a1110 a1111 a1112;
   a1201 a1202 a1203 a1204 a1205 a1206 a1207 a1208 a1209 a1210 a1211 a1212;
  ]
b0101=0; b0102=0; b0103=0; b0104=0;
   b0201=0; b0202=0; b0203=0; b0204=0;
   b0301=0; b0302=0; b0303=0; b0304=0;
   b0401=0; b0402=0; b0403=0; b0404=0;
   b0501=0; b0502=0; b0503=0; b0504=0;
   b0601=0; b0602=0; b0603=0; b0604=0;
   b0701=0; b0702=0; b0703=0; b0704=0;
   b0801=0; b0802=0; b0803=0; b0804=0;
   b0901=0; b0902=0; b0903=0; b0904=0;
   b1001=0; b1002=0; b1003=0; b1004=0;
   b1101=0; b1102=0; b1103=0; b1104=0;
   b1201=0; b1202=0; b1203=0; b1204=0;


b0101 = KA1*(xoutss1-xinss1)/(2*MG1);
b0201 = (KA1*(xinss1-xoutss1))/(2*ML1);
b0302 = (KA2*(yinss1-youtss2))/(2*a1*MG2);
b0402 = (KA2*(youtss2-yinss1))/(2*a1*ML2);
```

```
b0703 = (KA4*(xoutss4-xinss4))/(2*MG4);
b0803 = (KA4*(xinss4-xoutss4))/(2*ML4);
b0904 = (KA5*(yinss5-youtss5))/(2*a3*MG5);
b1004 = (KA5*(youtss5-yinss5))/(2*a3*ML5);



B = [b0101 b0102 b0103 b0104;
   b0201 b0202 b0203 b0204;
   b0301 b0302 b0303 b0304;
   b0401 b0402 b0403 b0404;
   b0501 b0502 b0503 b0504;
   b0601 b0602 b0603 b0604;
   b0701 b0702 b0703 b0704;
   b0801 b0802 b0803 b0804;
   b0901 b0902 b0903 b0904;
   b1001 b1002 b1003 b1004;
   b1101 b1102 b1103 b1104;
   b1201 b1202 b1203 b1204;
   ]

c0101=0; c0102=0; c0103=0; c0104=0; c0105=0; c0106=0; c0107=0; c0108=0;
c0109=0; c0110=0; c0111=0; c0112=0;
c0201=0; c0202=0; c0203=0; c0204=0; c0205=0; c0206=0; c0207=0; c0208=0;
c0209=0; c0210=0; c0211=0; c0212=0;
c0301=0; c0302=0; c0303=0; c0304=0; c0305=0; c0306=0; c0307=0; c0308=0;
c0309=0; c0310=0; c0311=0; c0312=0;
c0401=0; c0402=0; c0403=0; c0404=0; c0405=0; c0406=0; c0407=0; c0408=0;
c0409=0; c0410=0; c0411=0; c0412=0;

c0102 = (L1/L1s);
c0104 = (L2/L1s);
c0211 = 1;
c0306 =1;
c0409 =1;

C = [c0101 c0102 c0103 c0104 c0105 c0106 c0107 c0108 c0109 c0110 c0111 c0112;
c0201 c0202 c0203 c0204 c0205 c0206 c0207 c0208 c0209 c0210 c0211 c0212;
c0301 c0302 c0303 c0304 c0305 c0306 c0307 c0308 c0309 c0310 c0311 c0312;
c0401 c0402 c0403 c0404 c0405 c0406 c0407 c0408 c0409 c0410 c0411 c0412;
]

D = [0 0 0 0; 0 0 0 0; 0 0 0 0; 0 0 0 0]

e0101=0; e0102=0; e0103=0; e0104=0; e0105=0;
e0201=0; e0202=0; e0203=0; e0204=0; e0205=0;
e0301=0; e0302=0; e0303=0; e0304=0; e0305=0;
e0401=0; e0402=0; e0403=0; e0404=0; e0405=0;
```

```
e0501=0; e0502=0; e0503=0; e0504=0; e0505=0;
e0601=0; e0602=0; e0603=0; e0604=0; e0605=0;
e0701=0; e0702=0; e0703=0; e0704=0; e0705=0;
e0801=0; e0802=0; e0803=0; e0804=0; e0805=0;
e0901=0; e0902=0; e0903=0; e0904=0; e0905=0;
e1001=0; e1002=0; e1003=0; e1004=0; e1005=0;
e1101=0; e1102=0; e1103=0; e1104=0; e1105=0;
e1201=0; e1202=0; e1203=0; e1204=0; e1205=0;

e0101 = (KA1*(1-flss1))/(2*MG1);
e0104 = (G1/MG1)-(KA1*(1-frss1))/(2*a1*MG1);
e0201 = (L1/ML1) -(KA1*(1-flss1))/(2*ML1);
e0204 = (KA1*(1-frss1))/(2*a1*ML1);
e0302 = (G2/MG2)-(KA2*(1-frss2))/(2*a1*MG2);
e0301 = (KA2*(1-flss2))/(2*MG2);
e0401 = (L2/ML2)-(KA2*(1-flss2))/(2*ML2);
e0402 = (KA2*(1-frss2))/(2*a1*ML2);
e0502 = (G3/MG3) -(KA3*(1-frss3))/(2*a2*MG3);
e0602 = (KA3*(1-frss3))/(2*a2*ML3);
e0703 = (KA4*(1-flss4))/(2*MG4);
e0803 = (L4/ML4)-(KA4*(1-flss4))/(2*ML4);
e0905 = (KA5*(1-flss5))/(2*MG5);
e1005 = (L5/ML5) -(KA5*(1-flss5))/(2*ML5);
e1105 = (KA6*(1-flss6))/(2*MG6);
e1205 = (L6/ML6) -(KA6*(1-flss6))/(2*ML6);



E = [e0101 e0102 e0103 e0104 e0105;
    e0201 e0202 e0203 e0204 e0205;
    e0301 e0302 e0303 e0304 e0305;
    e0401 e0402 e0403 e0404 e0405;
    e0501 e0502 e0503 e0504 e0505;
    e0601 e0602 e0603 e0604 e0605;
    e0701 e0702 e0703 e0704 e0705;
    e0801 e0802 e0803 e0804 e0805;
    e0901 e0902 e0903 e0904 e0905;
    e1001 e1002 e1003 e1004 e1005;
    e1101 e1102 e1103 e1104 e1105;
    e1201 e1202 e1203 e1204 e1205;
    ]

system = ss(A,B,C,D)
g = tf(system)

system2 = ss(A,E,C,0)
g = tf(system2)
```

```
%% Plot Passivity Index
 iw = 200;
w1 = -4;  %Starting Frequency
w2 = 4;   %Ending Frequency


w = logspace(w1,w2,iw);
s = w * sqrt(-1);             %s = jw
I = eye(length (A));          %I = identity matrix

 for z=1:length(s);
 g= C*((s(z)*I-A)\B)+D;      %g = transfer function
 nu1(z)= -min(real(eig(g+g'))/2);  %passivity index
 end

figure(1)
semilogx(w,nu1,'g');
xlabel('Frequency (rad/time)');
ylabel('Passivity Index');
legend('Passivity Index');
grid;

%%Weighting Function %Open this part if you want to find parameter of weighting
function If you want to find it, this file must be opened with myfun.m and mynoncon.m
file

options = optimset('Display','iter','MaxFunEvals',100000);
[x,fval] = fmincon(@myfun,[0.3342 1.0000e-003 1.0000e-003 0.0042],[],[],[],[],[0.001
0.001 0.001 0.000001],[100 100 100 100],@mynoncon,options,nu1,w);

ww01 = x(1)
ww02 = x(2)
ww03 = x(3)
ww04 = x(4)

%%transfer function of weighting function
WtfnG=tf({[x(4) x(1)*x(4) 0]}, {[1 x(2)+x(3) x(2)*x(3)]});
for z=1:length(w)
Gwt = freqresp(WtfnG,w(z));
nu3(z)= -min(real(eig(Gwt+Gwt'))/2);
end
figure(2);
semilogx(w,nu3,'r-');
xlabel('Frequency (rad/hr)');
ylabel('Passivity Index');
legend('passivity Index of Weighting function');
grid;
```

```
I1 =  eye(length (g));
 for z = 1:length(w);
 Gp1 = C*(((s(z)*I)-A)\B) + D ;
 Gplus = Gp1;
 wf = (x(4).*(w(z).*i).*((w(z).*i)+x(1)))/(((w(z).*i)+x(2)).*((w(z).*i)+x(3)));
 Wtfn = wf*I1;                                  %weighting function matrix
 H =Gplus+Wtfn;  %Transfer function after adding the weighting function
 nu4(z)= -min(real(eig(H+H'))/2);  %passivity index of passive system
    ggreal = real(H);
    ggimag = imag(H);
%size of new matrix
[nAAr,nAAc] = size(ggreal);
[nBBr,nBBc] = size(ggimag);
end
% %Plot passivity index of passive system
figure(3);
semilogx(w,nu4,'r-');
xlabel('Frequency (rad/hr)');
ylabel('Passivity Index');
legend('passivity Index of passive system');
grid;

% % Parameter of PI controller
% % Open this part if you want to find the parameter of PI controller
% % If you want to find it, this file must be opened with Pisearch.m file

%%loop1
  for z =1:length(w);
 gPI = C*((((w(z).*i).*I)-A)\B) + D;
 wf = (x(4).*(w(z).*i).*((w(z).*i)+x(1)))/(((w(z).*i)+x(2)).*((w(z).*i)+x(3)));
 Wtfn = wf*I1;
 U = [1 0 0 0;0 1 0 0; 0 0 1 0; 0 0 0 1];
 gPIplus = gPI.*U;
 H =gPIplus+Wtfn ;
 GLoop1=H(1,1);
options = optimset('Display','iter','MaxFunEvals',100000);
 [p,fval] =fmincon(@myfunpi,[0.00001 0.0075 10],[],[],[],[],[0.000001 0.0001 1],[10 5
1000],@mynonconpi,options,nu4,GLoop1,w,z);
gamma1=[p(1)];
kcplus1=[p(2)];
ti1=[p(3)];
  end

gamma1=[p(1)]
kcplus1=[p(2)]
ti1=[p(3)]
PB1=100/kcplus1
```

```
zs3 = kcplus1*ti1
zs2 = kcplus1*ti1*x(3)+kcplus1*ti1*x(2)+kcplus1
zs1 = kcplus1*ti1*x(2)*x(3)+kcplus1*x(2)+kcplus1*x(3)
zs0 = kcplus1*x(2)*x(3)
ps3 = ti1-kcplus1*x(4)*ti1
ps2 = ti1*x(3)+ti1*x(2)-kcplus1*x(4)*ti1*x(1)-kcplus1*x(4)
ps1 = ti1*x(2)*x(3)-kcplus1*x(4)*x(1)
[z,p] = tf2zp([zs3 zs2 zs1 zs0],[ps3 ps2 ps1 0])

%%loop2
for z =1:length(w);
 gPI = C*((((w(z).*i).*I)-A)\B) + D;
 wf = (x(4).*(w(z).*i).*((w(z).*i)+x(1)))/(((w(z).*i)+x(2)).*((w(z).*i)+x(3)));
 Wtfn = wf*I1;
 U = [1 0 0 0;0 1 0 0; 0 0 1 0; 0 0 0 1];
 gPIplus = gPI.*U;
 H =gPIplus+Wtfn ;
 GLoop2=H(2,2);
options = optimset('Display','iter','MaxFunEvals',100000);
 [p,fval] =fmincon(@myfunpi,[0.00001 0.005 10],[],[],[],[],[0.000001 0.00001 1],[1500
5 1000],@mynonconpi,options,nu4,GLoop2,w,z);
gamma2=[p(1)];
kcplus2=[p(2)];
ti2=[p(3)];
  end

gamma2=[p(1)]
kcplus2=[p(2)]
ti2=[p(3)]
PB1=100/kcplus2
zs3 = kcplus2*ti2
zs2 = kcplus2*ti2*x(3)+kcplus2*ti2*x(2)+kcplus2
zs1 = kcplus2*ti2*x(2)*x(3)+kcplus2*x(2)+kcplus2*x(3)
zs0 = kcplus2*x(2)*x(3)

ps3 = ti2-kcplus2*x(4)*ti2
ps2 = ti2*x(3)+ti2*x(2)-kcplus2*x(4)*ti2*x(1)-kcplus2*x(4)
ps1 = ti2*x(2)*x(3)-kcplus2*x(4)*x(1)
 [z,p] = tf2zp([zs3 zs2 zs1 zs0],[ps3 ps2 ps1 0])

%%loop3
for z =1:length(w);
 gPI = C*((((w(z).*i).*I)-A)\B) + D;
 wf = (x(4).*(w(z).*i).*((w(z).*i)+x(1)))/(((w(z).*i)+x(2)).*((w(z).*i)+x(3)));
 Wtfn = wf*I1;
 U = [1 0 0 0;0 1 0 0; 0 0 1 0; 0 0 0 1];
 gPIplus = gPI.*U;
```

```
 H =gPIplus+Wtfn ;
 GLoop3=H(3,3);
options = optimset('Display','iter','MaxFunEvals',100000);
 [p,fval] =fmincon(@myfunpi,[0.00001 0.00075 10],[],[],[],[],[0.000001 0.00001
1],[1500 5 1000],@mynonconpi,options,nu4,GLoop3,w,z);
gamma3=[p(1)];
kcplus3=[p(2)];
ti3=[p(3)];
  end

gamma3=[p(1)]
kcplus3=[p(2)]
ti3=[p(3)]
PB1=100/kcplus3
zs3 = kcplus3*ti3
zs2 = kcplus3*ti3*x(3)+kcplus3*ti3*x(2)+kcplus3
zs1 = kcplus3*ti3*x(2)*x(3)+kcplus3*x(2)+kcplus3*x(3)
zs0 = kcplus3*x(2)*x(3)

ps3 = ti3-kcplus3*x(4)*ti3
ps2 = ti3*x(3)+ti3*x(2)-kcplus3*x(4)*ti3*x(1)-kcplus3*x(4)
ps1 = ti3*x(2)*x(3)-kcplus3*x(4)*x(1)
 [z,p] = tf2zp([zs3 zs2 zs1 zs0],[ps3 ps2 ps1 0])

%%loop4
  for z =1:length(w);
 gPI = C*((((w(z).*i).*I)-A)\B) + D;
 wf = (x(4).*(w(z).*i).*((w(z).*i)+x(1)))/(((w(z).*i)+x(2)).*((w(z).*i)+x(3)));
 Wtfn = wf*I1;
 U = [1 0 0 0;0 1 0 0; 0 0 1 0; 0 0 0 1];
 gPIplus = gPI.*U;
 H =gPIplus+Wtfn ;
 GLoop4=H(4,4);
options = optimset('Display','iter','MaxFunEvals',100000);
 [p,fval] =fmincon(@myfunpi,[0.00001 0.0075 10],[],[],[],[],[0.000001 0.00001
1],[1500 5 1000],@mynonconpi,options,nu4,GLoop4,w,z);
gamma4=[p(1)];
kcplus4=[p(2)];
ti4=[p(3)];
  end

gamma4=[p(1)]
kcplus4=[p(2)]
ti4=[p(3)]
PB4=100/kcplus4


zs3 = kcplus4*ti4
```

```
zs2 = kcplus4*ti4*x(3)+kcplus4*ti4*x(2)+kcplus4
zs1 = kcplus4*ti4*x(2)*x(3)+kcplus4*x(2)+kcplus4*x(3)
zs0 = kcplus4*x(2)*x(3)

ps3 = ti4-kcplus4*x(4)*ti4
ps2 = ti4*x(3)+ti4*x(2)-kcplus4*x(4)*ti4*x(1)-kcplus4*x(4)
ps1 = ti4*x(2)*x(3)-kcplus4*x(4)*x(1)

 [z,p] = tf2zp([zs3 zs2 zs1 zs0],[ps3 ps2 ps1 0])
```

CURRICULUM VITAE

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