APPENDIX A

Design and Simulation Example

A.1 Design example of PR connection

Example design of beam-to-column connection for the KBMF

The connection at the third floor will be designed as an example. The design check is carried out according to AISC-LRFD as follows:

AISC Nominal Strength

Sectional properties

Beam

Grade of steel	=	A36 carbon steel
F_y	=	36 ksi
F_u	=	60 ksi
Sectional properties of beam	=	W24×55

Column

Grade of steel	=	A572 Gr50
F_y	=	50 ksi
F_u	=	65 ksi
Sectional properties of column	=	W14×132

Trial

Bolts

Use bolts type of A490N

Tension strength of bolt	=	84.8	ksi
Shear strength of bolt	=	56.3	ksi
Bolt diameter (top and seat angle)	=	$1\frac{1}{8}$	in.
Bolt diameter (web angle)	=	1	in.

Angles

Grade of steel	=	A36 carbon steel
F_y	=	36 ksi
F_u	=	60 ksi

Sectional properties of angle

Top angle	=	$L8\times8\times\frac{5}{8}$	in
Seat angle	=	$L8\times8\times\frac{5}{8}$	in
Web angle	=	$L6\times6\times\frac{9}{16}$	in

Top and Seat angle design

The bolts at the top and seat angle and the beam flange

M_p	=	546.065	kN-m.
d_b	=	600	mm.
T	=	$\frac{546.065 \times 1000}{600}$	
	=	910	kN
Bolt diameter	=	$1\frac{1}{8}$	in
	=	28.575	mm.
Cross section area	=	641.56	mm^2
Shear strength of a bolt	=	$0.75 \times 0.517 \times 641.56$	
	=	250	kN / bolt
n	=	$\frac{910}{250}$ = 3.66	

$\therefore$ **Use 4** $\phi\, 1\frac{1}{8}$ **in.**

The bolts at the top and seat angle and the Column flange

tension stress of a bolt	=	$0.75 \times 0.78 \times 641.56$	
	=	375	kN/bolt
n	=	$\frac{910}{375}$ = 2.43	

$\therefore$ **Use 4** $\phi\, 1\frac{1}{8}$ **in.**

The top and seat angle

Plate Yielding	=	$0.9 \times 0.248 \times 15.875 \times 373$	
	=	1321.65 > 912.117 kN	**OK**
Plate fracture	=	$0.75 \times 0.4 \times 5876.76$	
	=	1750 > 912.177 kN	**OK**

Double web angle design

Length (beam web)	=	600 – 2(13)	
	=	574	mm.
$0.6F_y$	=	$0.6 \times 0.248 \times 574 \times 10$	
	=	856	kN
Bolt diameter	=	25.4	mm.
Cross section area	=	507	mm^2

The bolts at the double web angles and the beam web

Double shear strength	=	$2\times0.75\times0.517\times507$	
	=	393.54	kN / bolt
Bearing strength of bolt	=	$0.75\times0.96\times25.4\times10$	
	=	183.44	kN / bolt
n	=	$\dfrac{856}{183.44}$ = 4.67	

$\therefore$ **Use 5 ϕ 1 in.**

The bolts at the double web angles and the column flange

Shear strength of bolt	=	$0.75\times0.517\times507$	
	=	196.77	kN / bolt
Bearing strength of bolt	=	$0.75\times0.96\times25.4\times15.875$	
	=	290.25	kN / bolt
n	=	$\dfrac{856}{196.77}$ = 4.35	

$\therefore$ **Use 5 ϕ 1 in.**

Three Parameter Power Model

The connection in chapter 4 is designed using the three-parameter power model. The connection behavior is represented by its moment-rotation relationship. The general form of this model is

$$M = \frac{R_{ki}\theta_r}{\left[1+\left(\frac{\theta_r}{\theta_0}\right)^n\right]^{1/n}} \quad (A.1)$$

where M_u is the ultimate moment capacity, R_{ki} is the initial parameter, θ_0 is the reference plastic rotation, and n is the shape parameter. The initial connection stiffness (R_{ki}) and the ultimate connection moment (M_u) can be determined from the mechanics.

1. Determination of the Initial Connection Stiffness, R_{ki}

The initial stiffness can be formulated from simple elastic beam theory

$$K_{it} = \frac{3EI_t\left(d_1\right)^2}{g_1\left(g_1^2+0.78t_t^2\right)} \quad (A.2)$$

$$K_{is} = \frac{4EI_s}{I_{so}} \quad (A.3)$$

$$K_{ia} = \frac{6EI_a\left(d_3\right)^2}{g_3\left(g_3^2+0.78t_a^2\right)} \quad (A.4)$$

Where

K_{it} = initial stiffness contributed by the top angle

K_{is} = initial stiffness contributed by the seat angle

K_{ia} = initial stiffness contributed by the web angle

EI_t = bending rigidity of the top angle

$= \frac{E\times l_t\left(t_t\right)^3}{12}$

EI_s = bending rigidity of the seat angle

$= \frac{E\times l_s\left(t_s\right)^3}{12}$

EI_a = bending rigidity of the web angle

$= \frac{E\times l_a\left(t_a\right)^3}{12}$

$g_1 = g_t - \frac{w}{2} - \frac{t_t}{2}$

$g_3 = g_c - \frac{w}{2} - \frac{t_t}{2}$

$d_1 = d + \frac{t_s}{2} + \frac{t_t}{2}$

l_s = distance from the critical section to the toe of the outstanding leg of the seat angle

d_3 = $\frac{d}{2}+\frac{t_s}{2}$

The initial stiffness for the bolted top and seat angle connection with double web angles is

$$R_{ki} \quad = \quad K_{it}+K_{is}+K_{ia} \tag{A.5}$$

2. Determination of the Ultimate Moment Capacity, M_u

The mechanism moment capacity of a connection is reach when an idealized elastic-plastic collapse mechanism is developed in the assembly angles. On the basis of experimental studies, the collapse mechanism of a connection may be modeled from the individual angles. The mechanism moment of a connection may be obtained by summation of the plastic moment capacities contributed by assembly angles. Herein, plastic theory considering the bending moment-shear interaction is used to drive the expression for the mechanism moment. The ultimate moment of the bolted top and seat angle connection with double web angles is

$$M_u \quad = \quad M_{os}+M_{pt}+V_{pt}d_2+2V_{pa}d_4 \tag{A.6}$$

where

M_{pt} = plastic moment in the top angle

= $\frac{V_{pt}\times g_2}{2}$

M_{os} = plastic moment in the seat angle ($\sigma_y = F_y$)

= $\frac{\sigma_y l_s (t_s)^2}{4}$

V_{pt} = plastic shear force in vertical leg of the top angle

V_{pt} is determined by solving the following equation

$$\left(\frac{V_{pt}}{V_{ot}}\right)^4+\frac{g_2}{t_t}\left(\frac{V_{pt}}{V_{ot}}\right)-1 \quad = \quad 0 \tag{A.7}$$

Where

V_{ot} = $\frac{\sigma_y l_s t_s}{2}$

g_2 = $g_t-k_t-\frac{w}{2}-\frac{t_t}{2}$

V_{pa} = the resultant of plastic shear force in a single web angle

= $\frac{\left(V_{pu}+V_{oa}\right)l_p}{2}$

V_{pu} is determined by solving the following equation

$$\left(\frac{V_{pu}}{V_{oa}}\right)^4 + \frac{g_y}{t_a}\left(\frac{V_{pu}}{V_{oa}}\right) - 1 = 0 \qquad \text{(A.8)}$$

Where

V_{oa} = $\frac{\sigma_y t_a}{2}$

g_y = $g_c - k_a$

3. Determination of the Shape Parameter

Shape parameter (*n*) is determined by using the method of least squares for the differences between the experimental and the predicted moment data. From this, the shape of curve can be defined when the shape parameter changed. Numerical values of *n* are then plotted against $\log_{10}\theta_0$. The shape parameter is assumed to be linear function of $\log_{10}\theta_0$.

$$n = 1.398\log_{10}\theta_0 + 4.631, \log_{10}\theta_0 > -2.721 \quad \text{otherwise} \quad n = 0.827 \qquad \text{(A.9)}$$

Sectional properties

Column

Grade of steel	=	A572 Gr50
F_y	=	50 ksi
F_u	=	65 ksi
Sectional properties of column	=	W14×132

Beam

Grade of steel	=	A36 carbon steel
F_y	=	36 ksi
F_u	=	60 ksi
Sectional properties of beam	=	W24×55

A section of third floor beam is W24x55.

M_p = $Z_x F_y$
= (2,200,000)(0.2482)
= 546,064.754 kN-mm.
= 546 kN-m.

The ultimate moment capacity of the connections was defined similar to the plastic moment of beam.

Member section of connection component

Top and seat angle	=	$L\,8\times8\times\frac{5}{8}$	in
Double web angle	=	$L\,6\times6\times\frac{9}{16}$	in
Bolt diameter for top and seat angle	=	$1\frac{1}{8}$	in
Bolt diameter for double web angle	=	1	in

Determination of the Initial Connection Stiffness, R_{ki}

$$EI_t = \frac{197.18\times109.34(15.875)^3}{12}$$
$$= 24{,}323{,}538 \quad \text{kN-mm}^2$$

$$EI_s = \frac{197.18\times109.34(15.875)^3}{12}$$
$$= 24{,}323{,}538 \quad \text{kN-mm}^2$$

$$EI_a = \frac{197.18\times380(14.287)^3}{12}$$
$$= 18{,}211{,}098 \quad \text{kN-mm}^2$$

$$g_1 = 65-\frac{46}{2}-\frac{15.875}{2}$$
$$= 34.0625 \quad \text{mm.}$$

$$g_3 = 100-\frac{41}{2}-\frac{14.287}{2}$$
$$= 72.356 \quad \text{mm.}$$

$$d_1 = 598.678+\frac{15.875}{2}+\frac{15.875}{2}$$
$$= 614.553 \quad \text{mm.}$$

$$l_{so} = 203.2 - 31.75$$
$$= 171.45 \quad \text{mm.}$$

$$d_3 = \frac{598.678}{2}+\frac{15.875}{2}$$
$$= 307.276 \quad \text{mm.}$$

$$K_{it} = \frac{3EI_t\left(d_1\right)^2}{g_1\left(g_1^2+0.78t_t^2\right)} = 596{,}301.61 \quad \text{kN-m/rad.}$$

$$K_{is} = \frac{4EI_s}{I_{so}} = 567.48 \quad \text{kN-m/rad.}$$

$$K_{ia} = \frac{6EI_a\left(d_3\right)^2}{g_3\left(g_3^2+0.78t_a^2\right)} = 26{,}430.58 \quad \text{kN-m/rad.}$$

$\therefore \quad R_{ki} = 596{,}301.61 + 567.48 + 26{,}430.58$

$= 623{,}300$ kN-m/rad.

Determination of the Ultimate Moment Capacity, M_u

$$V_{ot} = \frac{0.248 \times 370 \times 15.875}{2}$$

$= 728.932$ kN

$$g_2 = 65 - 31.75 - \frac{28.575}{2} - \frac{15.875}{2}$$

$= 11.025$ mm.

V_{pt} is determined by solving the following equation

$$\left(\frac{V_{pt}}{V_{ot}}\right)^4 + \frac{g_2}{t_t}\left(\frac{V_{pt}}{V_{ot}}\right) - 1 = 0$$

$V_{pt} = 592.237$ kN

$$M_{pt} = \frac{592.237 \times 11.025}{2}$$

$= 3{,}264.706$ kN-mm.

$$M_{os} = \frac{0.248 \times 370 \times (15.875)^2}{4}$$

$= 5{,}785.901$ kN-mm.

$$V_{oa} = \frac{0.248 \times 14.29}{2}$$

$= 1.77$ kN

$g_y = 100 - 26.98$

$= 73.01$ mm.

V_{pu} is determined by solving the following equation

$$\left(\frac{V_{pu}}{V_{oa}}\right)^4 + \frac{g_y}{t_a}\left(\frac{V_{pu}}{V_{oa}}\right) - 1 = 0$$

$V_{pu} = 0.346$ kN

$$V_{pa} = \frac{(0.346 + 1.77)380}{2}$$

$= 402.712$ kN

$\therefore \quad M_u = 5,785.9 + 3,294.71 + (592.24 \times 638.36) + 2(402.71 \times 264.65)$

$= 600,268.4$ kN-mm.

$= 600$ kN-m.

Check

M_u = 600 kN-m. > M_{pb} = 546 kN/m. **OK**

Determination of the Shape Parameter

$$\theta_0 = \frac{M_u}{R_{ki}}$$

$$= \frac{600268}{623300} \times 10^{-3}$$

$= 0.001$ rad.

$\log\theta_0 = -3$

$\therefore n = 0.827$

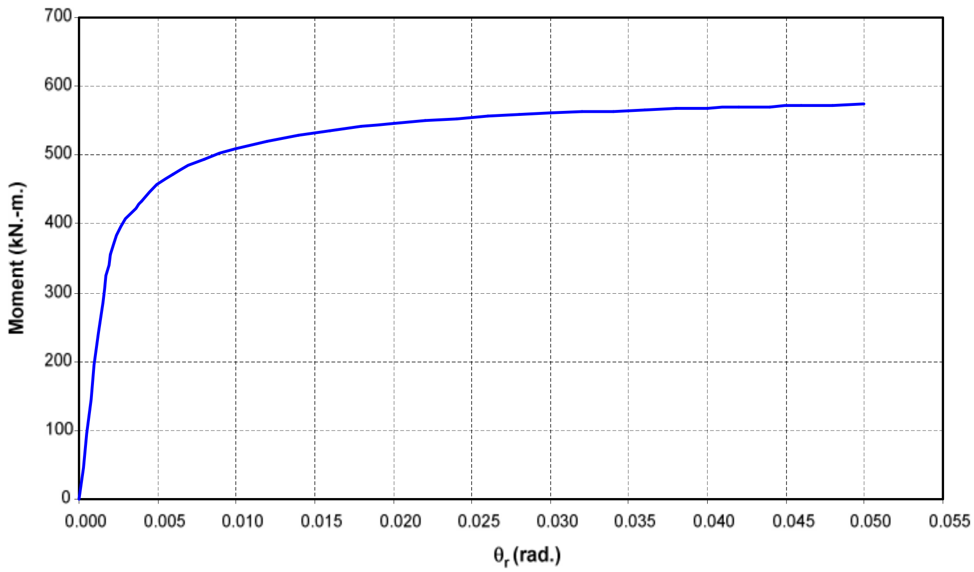

Figure A.1 Moment-Rotation Curve.

A.2 Simulation example of KBMF with PR Connection

1. Partially-restrained connection model

Trilinear Inelastic Element of angle

Key parameters including the initial stiffness (K_0), the first yielding load (P_y), the transition stiffness (K_t), the second yielding load (P_s) can be computed using;

$$K_o = \frac{12EI}{g_1^3}\left[1-\frac{3g_1}{4(g_1+g_2)}\right] \tag{A.10}$$

$$P_y = \frac{4g_1+g_2}{g_1(2g_1+g_2)}M_y \tag{A.11}$$

$$K_t = \frac{12EI}{(g_1-t)^2}\left[\frac{1}{4(g_t-t)+3g_2}\right] \tag{A.12}$$

$$P_s = \frac{2M_p}{g_1-t-\frac{d_h}{2}} \tag{A.13}$$

$$K_u = Steel-hardening\,coefficient\times K_0 \tag{A.14}$$

where, g_1 and g_2 are the distance from the back of the angle to the center line of the bolts on the column and on the beam respectively, t is the thickness of the angle, M_y is the yield moment capacity of the angle section, M_p is the plastic moment capacity of the angle section, d_h is the diameter of the bolt hole, w is the angle width per bolt, EI is Bending rigidly of angle. The post-yielding stiffness (K_u) depends on the steel-hardening of each structure.

Contact and detachment

$$K_{cwc} = E\frac{\lfloor 2t_{sa}+0.6r_{sa}+2(t_{cf}+s)\rfloor t_{wc}}{(h_c-2t_{cf}-2r_c)} \tag{A.15}$$

where, t_{sa}, r_{sa}, t_{cf}, and d_{wc} are the thickness of the seat angle, the fillet radius of the seat angle, the thickness of the column flange, and the depth of the column respectively. The value of s is equal to r_c for a rolled section or $2a_c$ for a built-up section where r_c and a_c are the web-to-flange radius of the column and the throat thickness of the welds, respectively.

Example simulation of beam-to-column connection for the KBMF

The top angle in the connection at the third floor will be calculated the values as an example as follows:

Trilinear Inelastic Element of angle

g_1	=	80	mm.
g_2	=	65	mm.
d_p	=	28.58	mm.
d_h	=	30.58	mm.
t	=	15.88	mm.
E	=	197.181	mm.
I	=	61,678.2	mm.
F_y	=	0.345	mm.
F_u	=	0.49	mm.

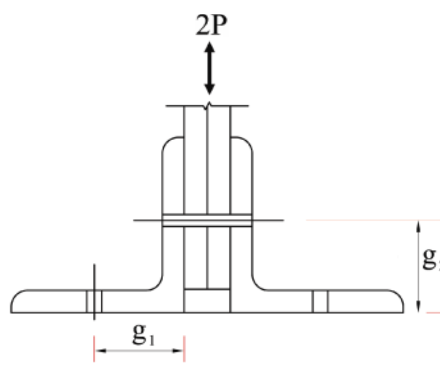

Figure A.2 Double Web Angle

K_0	=	190	kN / mm
K_t	=	78.6	kN / mm
K_u	=	15.14	kN / mm
M_y	=	2,678.48	kN-mm
M_p	=	4,017.73	kN-mm
P_y	=	57.3	kN
P_s	=	125.3	kN

Contact and detachment

$_{cf}$	=	3.35	mm.
h_c	=	373	mm.
t_{wc}	=	20.83	mm.
s	=	17.45	mm.
t_{sa}	=	15.88	mm.
r_{sa}	=	15.88	mm.
r_c	=	17.45	mm.
$\therefore K_{cwc}$	=	2,053.44	kN / mm.

2. Knee-braces model

Regular buckling braces

Elastic modulus	=	199.98	kN / mm^2
Cross section area of knee brace	=	885.83	mm^2
Length of knee braces	=	1.13	m
Yield Strength	=	0.345	kN / mm^2
P_y	=	305	kN
Δ_y	=	$\frac{0.345}{199.98}\times 1310$	
	=	2.25	mm.
Compressive strength	=	0.327	kN / mm^2
P_c	=	290	kN
Δ_c	=	$\frac{0.327}{199.98}\times 1310$	
	=	2.14	mm.
αP_c	=	0.8×290	
	=	230	kN

Buckling-Restrained Braces (BRBs)

Elastic modulus	=	199.98	kN / mm^2
Cross section area of BRB (core)	=	885.83	mm^2
Length of BRB (BRB length)	=	0.92	m
Yield Strength	=	0.345	kN / mm^2
Ultimate Strength	=	0.448	
Initial stiffness	=	$\frac{199.98\times 885.83}{920}$	
	=	192.6	kN / mm.
Post-yield stiffness	=	0.01×192.6	
	=	1.93	kN / mm.
P_{u0}	=	310	kN
Δ_y	=	$\frac{0.345}{199.98}\times 920$	
	=	1.59	mm.

3. Beam and column model

Beam

Plastic moment of beam (M_{pb})	=	121,390	kN-mm.
Initial stiffness (K_i)	=	$\frac{6EI}{L}$	
	=	1.94×10^6	kN/mm.
Yielding rotation (θ_y)	=	0.0062	radian
Post-yielding stiffness (K_u)	=	$0.08\times K_i$	

Column

Plastic moment of column (M_{pc})	=	323,227	kN-mm.
Initial stiffness (K_i)	=	$\frac{6EI}{L}$	
	=	6.5×10^{7}	kN/mm.
Yielding rotation (θ_y)	=	0.005	radian
Post-yielding stiffness (K_u)	=	$0.08\times K_i$	
Axial yielding	=	3,104 kN	
Initial stiffness	=	$\frac{EA}{L}$	
	=	924 kN/mm.	
Post-yielding stiffness (K_u)	=	$0.08\times K_i$	