CHAPTER 7

DESIGN OF AUDITORY WARNING SYSTEM

This chapter presents an analytic procedure to evaluate the audibility of an existing auditory warning system in the workplace. Two optimization models to determine a minimum number of alarm devices and their location based on the workplace noise conditions are formulated and discussed. Then, we propose a heuristic algorithm to determine the number of alarm devices (with known signal sound level) and their locations for generating audible auditory warnings. The proposed algorithm is also intended to minimize a maximum combined signal sound level among the given worker locations. Numerical examples are presented and solved by the heuristic and optimization approaches. Then, their solutions are compared and discussed. A computational experiment is also conducted to investigate the efficiency of the heuristic algorithm.

7.1 Problem Description

According to the safety regulations and standards, employers are required to install alarm devices in their facilities to alert workers of hazardous and/or dangerous situations. Alarm devices may generate auditory signals, visual signals, or both types of signals. The use of auditory signals seems to be a better choice for industrial facilities than the use of other types of signals because workers can perceive (hear) the signals even if they are not watching or are working in areas where they cannot see the alarm devices.

According to the safety regulations and standards relevant to the auditory warning system, it is found that some parts are stated as "specifications" while some are stated as "performance." For instance, the Occupational Safety and Health standards discuss the "employee alarm system" in Part 1901.95 (Title 29 – Code of Federal Regulations) as a reference for the design of an alarm system (OSHA, 1983). However, OSHA only enforces the installation of the alarm system without giving the details such as the number of alarm devices and their locations. Thus, safety practitioners may face the difficulty in designing the auditory warning system for adequate audibility. This is the reason that why the installation of auditory warning devices found in many workplaces uses a convenience basis rather than an objective basis. In many facilities, it is found that alarm locations are located at the corner of the facility or on the wall on top of the entrance/exit regardless of the proper evaluation of audibility at any worker location. As a result, some worker locations may not receive sufficient warning signal levels.

When given the number of alarm devices and the locations where they are presently installed, the adequacy of the audibility of the auditory warning system for alerting workers of dangerous situations must be evaluated. If such warning systems fail to alert workers at some worker locations, redesign of the auditory warning system is also required so that all worker locations will receive adequate warning signal. By using the proposed design procedure, the number of alarm devices and their locations will be specified in order to allow all workers to perceive the warning signal.

7.2 Audibility Evaluation of the Auditory Warning System

This research considers an auditory warning system to be adequately audible if it meets the signal intensity requirement of the International Standard, ISO 7731, which states that "the auditory signal is clearly audible if the signal sound level exceeds the level of ambient noise by at least 15 dB*A*." For workers with normal hearing or mild hearing loss, the signal sound level (measured at the worker's ear) shall be not less than 65 dB*A* to ensure its audibility (ISO 7731:1986). For convenience, the term "sound level" is used in this chapter to represent the "sound pressure level."

7.2.1 Evaluation Procedure

For practicality, it is assumed that all noise sources are pointed sources and their heights are at the same level as the worker's ear. The facility is assumed to be an open area so that the effect of sound absorption/reflection can be neglected. In practice, if the facility size is large with high ceiling or noise sources are located not too close to a wall or corner, this assumption satisfactorily holds.

The procedure to evaluate the audibility of an auditory warning system is as follows.

1. From the layout map of a workplace, determine the (x, y) coordinates (in meter, m) of all machine and worker locations (on the factory floor), and of the existing alarm devices (at the ceiling or on the walls). Also, determine the ceiling height of the workplace.
2. Using the Euclidean distance system, determine all paired distances between the machine and worker locations, and between the alarm device and worker locations.
3. For each machine and alarm device, determine the machine noise and alarm signal sound level (at 1-m distance from the source), in dB*A*.
4. Determine the ambient noise level (in dB*A*) without the presence of machine noise and alarm signal sound levels.
5. At each worker location, determine the combined machine noise level (from all machines) and the combined signal sound level (from all alarm devices) separately using the following formula:

$$\overline{L}_i = 10\log_{10}\left[10^{\frac{L_{ab}-120}{10}} + \sum_{j=1}^{q}\left(\frac{10^{\frac{L_j-120}{10}}}{(x_i-a_j)^2+(y_i-b_j)^2}\right)\right]+120 \qquad (7.1)$$

where:

L_{ab}	ambient noise level, dB*A*;
L_j	sound level generated by machine or alarm device j (measured at 1 m), dB*A*;
$\overline{L}_i$	combined sound level (from all machines or all alarm devices) at worker location i, dB*A*;
(x_i, y_i)	coordinate of worker location i, m;
(a_j, b_j)	coordinate of machine or alarm device j, m;

Note that when using Eq. (7.1) to determine the combined signal sound level from all alarm devices, the first term in the square brackets, $10^{\frac{L_{ab}-120}{10}}$, is neglected and set $q = r$.

6. If a difference between the combined signal sound level and the combined noise level (from the ambient and all machine noises) is less than 15 dB*A* at any worker location, the auditory warning system is not adequately audible.

The notation used in this chapter is as shown below:

da_{ik} Euclidean distance between worker location i and alarm device k

$$= \left[\left(xw_i - xa_k\right)^2 + \left(yw_i - ya_k\right)^2 + h^2\right]^{1/2}; \quad (7.2)$$

dm_{ij} Euclidean distance between worker location i and machine j

$$= \left[\left(xw_i - xm_j\right)^2 + \left(yw_i - ym_j\right)^2\right]^{1/2}; \quad (7.3)$$

h ceiling height (m);
I_{ab} ambient sound intensity (W/m^2);
I_j sound intensity (W/m^2) of machine j, at a distance of 1 m;
L_j sound level (dBA) generated by machine j, at a distance of 1 m;
P_{alarm} sound power (W) of the alarm device;
$P_{\text{alarm}}^{\text{max}}$ maximum allowable sound power (W) of the alarm device;
r number of alarm devices needed for the considered facility;
(xa_k, ya_k) (x, y) coordinate of alarm device k;
(xm_j, ym_j) (x, y) coordinate machine j;
(xw_i, yw_i) (x, y) coordinate of t worker location i.

7.2.2 Numerical Example of the Evaluation Procedure

Suppose that a workplace is a rectangular-shaped machine shop, with its width and length of 20 and 12 m ($x \times y$). The ceiling height is 6 m. In this machine shop, there are seven machines and six locations where workers are present. The machine location coordinates and noise levels generated by these machines (at 1-m distance) are displayed in Table 7.1. The ambient noise level is 65 dBA. The (x, y) coordinates of the six worker locations are also shown in Table 7.2. Currently, an alarm device with its signal sound level of 120 dBA (at 1-m distance) is installed at the ceiling at the (10, 6) coordinate.

Table 7.1 Location coordinates and sound levels generated by the machines

Machine (M)	Location Coordinate (m)		Sound Level (dBA)
	x-coordinate	y-coordinate	
M1	3	2	87
M2	8	2	95
M3	12	2	94
M4	17	4	90
M5	17	7	95
M6	10	10	100
M7	3	7	95

Firstly, we determine the paired distances between the seven machines (xm, ym) and the six worker locations (xw, yw). Letting dm_{ij} be a distance between worker location i (where i = 1, 2, …, 6) and machine j (where j = 1, 2, …, 7). The worker location – machine paired distances are shown in Table 7.3.

Table 7.2 Location coordinates of worker locations

Worker Location (WL)	Location Coordinate (m)	
	x-coordinate	y-coordinate
WL1	3	4
WL2	8	4
WL3	12	4
WL4	17	4
WL5	15	7
WL6	5	7

Table 7.3 Distances between worker location (WL) and machine location (M), dm_{ij} (in m)

Worker Location	Machine						
	M1	M2	M3	M4	M5	M6	M7
WL1	2.0	5.4	9.2	14.0	14.3	9.2	3.0
WL2	5.4	2.0	4.5	9.0	9.5	6.3	5.8
WL3	9.2	4.5	2.0	5.0	5.8	6.3	9.5
WL4	14.1	9.2	5.4	0.0	3.0	9.2	14.3
WL5	13.0	8.6	5.8	3.6	2.0	5.8	12.0
WL6	5.4	5.8	8.6	12.4	12.0	5.8	2.0

Next, we determine the paired distances between all worker locations (xw, yw) and the single alarm device (xa, ya), or da_{ik}, where i = 1, 2, …, 6, and k = 1. Note that h represents the ceiling height (in m).

From Eq. (7.2), the worker location – alarm device paired distances are:

da_{11} = 7.3 m
da_{21} = 2.8 m
da_{31} = 2.8 m
da_{41} = 7.3 m
da_{51} = 5.1 m
da_{61} = 5.1 m

Using Eq. (7.1), the combined noise level from the ambient and all machines and the signal sound level from the alarm device at any worker location can be calculated. Table 7.4 shows the combined noise levels, signal sound levels, and their differences at all six worker locations.

Table 7.4 Combined noise and signal sound levels and their differences

Worker Location	Combined Sound Level (dB*A*)		Difference (dB*A*)
	From Seven Machines	From Alarm Device	
WL1	88.79	100.51	11.72
WL2	91.23	103.57	12.34
WL3	90.84	103.57	12.73
WL4	92.08	100.51	8.43
WL5	91.16	102.08	10.92
WL6	91.01	102.08	11.07

Note: Difference = Alarm signal sound level – combined noise level

It can be concluded that the alarm signal sound levels reaching all worker locations are not adequately audible since all differences are less than 15 dB*A*. This is perhaps due to the following reasons: (1) only one alarm device is not sufficient, (2) its location is not appropriate, and (3) the signal sound level generated by the alarm device is not high enough.

7.3 Alarm Location Models

The alarm location problem is intended to determine a minimum number of alarm devices and the locations where they should be installed so that a maximum combined signal sound level at any worker location in the workplace is minimized. One important requirement of the alarm location problem is that the combined signal sound level at any worker location must exceed the combined noise level at that location by at least 15 dB*A*. While an increasing in the alarm signal sound level will result in a fewer number of alarm devices that are required, the differences between the combined signal sound level and the combined noise level (called the "signal – noise" differences) at some worker locations might, however, be increased. On the other hand, when decreasing the alarm signal sound level, the "signal – noise" differences at some worker locations might decrease, but the number of alarm devices that are required for audible auditory warnings will increase. The former argument is in favor of cost reduction, not the workplace noise control. The latter argument gives more emphasis on the noise situation than the cost of the auditory warning system. It is essential that an appropriate alarm location model must consider both arguments and attempt to minimize not only the number of alarm devices but also the "signal – noise" difference at any worker location.

The assumptions for the alarm location problem are as follows:

1. The workplace is assumed to be a large facility. When considering it as an open area, the effect of sound absorption/reflection from walls, ceiling, and corners can be neglected
2. A noise source (machine) is viewed as a pointed source. The height of the noise source is at the same level as that of the worker's ear
3. All alarm devices are identical. That is, they generate equal signal sound level
4. All alarm devices will be installed at the ceiling of the facility
5. The signal sound level and machine noise level are not time-dependent.

7.3.1 Alarm location model (with unknown signal sound level)

Nanthavanij and Yenradee (1999) developed an alarm location model that not only provides an optimal number of alarm devices and their locations, but also the recommended signal sound level of the alarm device. One of the requirements in their model is that, at any worker location, the combined alarm signal sound level must exceed the combined noise level by at least 10 dB*A*. The alarm location model is a nonlinear programming (NLP) model and does not guarantee a global optimal solution.

To enhance its usefulness, the original alarm location model is slightly modified to cope with the situation in which workers might be located at other locations than the machine locations and the difference of 15 dB*A* is now required. The modified alarm location model with unknown signal sound level can be written as shown below:

$$\text{Minimize} \quad P_{\text{alarm}} \tag{7.4}$$

subject to

$$B_i \left[\sum_{k=1}^{r} \frac{1}{\left(xw_i - xa_k\right)^2 + \left(yw_i - ya_k\right)^2 + h^2} \right]^{-1} \leq P_{\text{alarm}} \quad \forall\, i \tag{7.5}$$

$$P_{\text{alarm}} \leq P_{\text{alarm}}^{\max} \tag{7.6}$$

$$xa_k,\, ya_k \geq 0 \quad \forall\, k \tag{7.7}$$

$$P_{\text{alarm}} \geq 0 \tag{7.8}$$

where

$$B_i = 60\pi \left[I_{ab} + \sum_{j=1}^{q} \frac{I_j}{dm_{ij}^2} \right] \tag{7.9}$$

To solve the above alarm location model, it is necessary to know the number of alarm devices r. Therefore, a trial-and-error procedure is used. Firstly, assume that $r = 1$ and substitute it in the model. If an optimal solution can be found, then only one alarm device is needed. If it is infeasible to find the solution, the number of alarm devices is then increased by one ($r = r + 1$) and the trial-and-error procedure continues. The solution will provide the locations of individual alarm devices and the signal sound level of the alarm device. For more details on the model formulation, see Nanthavanij and Yenradee (1999).

One drawback of the above alarm location model is that it might yield a solution that may not be usable. Specifically, the alarm device that will generate the signal sound level equal to the recommended level may not be commercially available. Often, alarm devices are manufactured with preset signal sound levels which cannot be adjusted. It is more reasonable to assume that the alarm signal sound level is known in advance and is a constant in the alarm location model. As a result, the problem objective is only to find a minimum number of alarm devices and their locations.

7.3.2 Alarm location model (with known signal sound level)

Here, we propose a *revised* alarm location model by assuming that the signal sound level of the alarm device is known. The objective function and the constraints are revised since the constraint on the alarm signal sound level is no longer needed.

Additional model variables are required for the *revised* alarm location model.

- $\overline{Ia}_i$ total alarm signal sound intensity (W/m^2) at worker location i
- $\overline{Im}_i$ total (ambient and machine) noise intensity (W/m^2) at worker location i
- L_{alarm} signal sound level (dB*A*) of the alarm device, measured at 1 m
- $\overline{La}_i$ combined alarm signal sound level (dB*A*) at worker location i
- $\overline{Lm}_i$ combined noise level (dB*A*) at worker location i

For convenience, we once again describe the two basic formulas for converting between sound power (P) and sound intensity (I), and between sound intensity (I) and sound pressure level (L) since they play a significant part in developing the alarm location model. Letting d be the distance between a noise source and a location where sound measurement takes place, and I_0 be a reference sound intensity (= 10^{-12} W/m^2), we obtain

$$I = \frac{P}{4\pi d^2} \qquad \text{(in W/m}^2\text{)} \tag{7.10}$$

$$L = 10\log_{10}\left[\frac{I}{10^{-12}}\right] \qquad \text{(in decibel)} \tag{7.11}$$

For more details, consult Harris (1979), Piercy and Embleton (1979), Ostergaard (1986), Beraneck (1992), and Bies and Hansen (1996).

As stated earlier, for the alarm signal to be clearly audible at any worker location, the combined signal sound level must exceed the combined noise level (including the ambient noise) by at least 15 dB*A*. Thus, at worker location *i*,

$$\bar{L}a_i - \bar{L}m_i \geq 15 \qquad \forall\, i \tag{7.12}$$

From Eq. (7.11) and Inequality (7.12), it can be shown that

$$\bar{I}a_i \geq 10^{1.5}\,\bar{I}m_i \qquad \forall\, i \tag{7.13}$$

From Eqs. (7.10) and (7.11), we can derive the formulas for $\bar{I}a_i$ and $\bar{I}m_i$.

$$\bar{I}a_i = \sum_{k=1}^{r} \frac{10^{\left(\frac{L_{\text{alarm}}-120}{10}\right)}}{da_{ik}^2} \qquad \forall\, i \tag{7.14}$$

$$\bar{I}m_i = I_{ab} + \sum_{j=1}^{q} \frac{10^{\left(\frac{L_j-120}{10}\right)}}{dm_{ij}^2} \qquad \forall\, i \tag{7.15}$$

From Inequality (7.13) and Eqs. (7.14) and (7.15), the following inequality can be obtained.

$$\sum_{k=1}^{r} \frac{10^{\left(\frac{L_{\text{alarm}}-120}{10}\right)}}{da_{ik}^2} \geq 10^{1.5}\left[I_{ab} + \sum_{j=1}^{q} \frac{10^{\left(\frac{L_j-120}{10}\right)}}{dm_{ij}^2}\right] \tag{7.16}$$

$$\sum_{k=1}^{r} \frac{1}{da_{ik}^2} \geq \frac{10^{1.5}}{10^{\left(\frac{L_{alarm}-120}{10}\right)}}\left[I_{ab} + \sum_{j=1}^{q} \frac{10^{\left(\frac{L_j-120}{10}\right)}}{dm_{ij}^2}\right] \forall\, i \tag{7.17}$$

Setting the right-hand-side of Inequality (7.17) to A_i, the expression is reduced to

$$\sum_{k=1}^{r} \frac{1}{da_{ik}^2} \geq A_i \qquad \forall\, i \tag{7.18}$$

Let us denote y_k as a binary integer variable such that $y_k = 1$ if alarm device k is chosen to be installed in the facility, and $y_k = 0$ otherwise. Thus, the *revised* alarm location model can be written as follows.

$$\text{Minimize} \qquad \sum_{i=1}^{n}\left[\left(\sum_{k=1}^{r}\frac{1}{(xw_i - xa_k)^2 + (yw_i - ya_k)^2 + h^2}\cdot y_k\right) - A_i\right] \tag{7.19}$$

subject to

$$\sum_{k=1}^{r}\frac{1}{\left(xw_i - xa_k\right)^2 + \left(yw_i - ya_k\right)^2 + h^2}\cdot y_k \quad \geq \quad A_i \qquad \forall\, i \tag{7.20}$$

$$xa_k, ya_k \quad \geq \quad 0 \qquad \forall\, k \tag{7.21}$$

$$y_k \quad = \quad \{0, 1\} \qquad \forall\, k \tag{7.22}$$

where

$$A_i \quad = \quad \frac{10^{1.5}}{10^{\left(\frac{L_{alarm}-120}{10}\right)}}\left[I_{ab} + \sum_{j=1}^{q}\frac{10^{\left(\frac{L_j-120}{10}\right)}}{dm_{ij}^2}\right] \qquad \forall\, i \tag{7.23}$$

Firstly, the number of alarm devices r must be specified. If r is too small, a feasible solution would not be found. If r is too large, some alarm devices will not be installed (some y_k's = 0). Additionally, if r is set too large, the size of the alarm location problem becomes large and the problem may not be solvable.

7.4 Heuristic Approach

In this section, we introduce a heuristic algorithm to determine a *near-optimal* number and location of alarm devices when the alarm signal sound level is known. The algorithm systematically installs one alarm device at a time, at a location considered to be the most appropriate under the given situation. A required condition (adequate signal perception) must be checked every time an alarm device is installed. If the required condition is not satisfied, another alarm device will then be installed.

When the first alarm device is being considered, its location will be at the ceiling between the worker location having the largest "signal – noise" difference and another worker location having the next largest "signal – noise" difference. From these two worker locations, the algorithm finds the radius of a circle at the ceiling (representing the coverage of the alarm signals) in which the worker location having the largest "signal – noise" difference is located on its circumference. The location of the alarm device will be on a straight line that connects between the two worker locations, and is far from the worker location having the largest "signal – noise" difference by a distance equal to the circle radius. Then, Inequality (7.18) will be checked. If the "signal – noise" difference at any worker location is smaller than 15 dB*A*, an additional alarm device will be considered. The location of the next alarm device will be determined using the same logic as that for the first alarm device. The procedure will stop when Inequality (7.18) is satisfied at all worker locations.

A heuristic algorithm to determine the number and location of alarm devices consists of the following steps.

Step 1: Determine the (x, y) coordinates of all machines (xm_j, ym_j) and worker locations (xw_i, yw_i). Also, determine the ceiling height (h).

Step 2: Determine the ambient noise level (L_{ab}) and convert it to the ambient noise intensity (I_{ab})

Step 3: Determine the machine noise level (at 1-m distance) generated by machine j (L_j) for all j's. Then, determine the total noise intensity at worker location i $(\overline{Im}_i)$ for all i's.

Step 4: At each worker location i, determine A_i from

$$A_i = \left[\frac{10^{1.5}}{10^{\left(\frac{L_{alarm}-120}{10}\right)}}\right]\overline{Im}_i \qquad \forall\ i \tag{7.24}$$

Step 5: Set the number of alarm devices $r = 1$.

Step 6: Calculate C_i for all i's from the following formula. For $r = 1$, set $C_i = A_i$. For $r \geq 2$,

$$C_i = A_i - \sum_{k=1}^{r-1}\frac{1}{da_{ik}^2} \tag{7.25}$$

Let D_i be the Euclidean distance between worker location i and alarm device k. Calculate D_i for all i's from the following formula. For $r = 1$, set $D_i = \dfrac{1}{\sqrt{C_i}}$. For $r \geq 2$,

$$D_i = \left[A_i - \sum_{k=1}^{r-1}\frac{1}{da_{ik}^2}\right]^{-1/2} \tag{7.26}$$

If $C_i \leq 0$, set $D_i = M$ where M is a very large number.

Step 7: Let R_i be the radius of a circle with its center at worker location i. Determine R_i for all i's from the following formulas:

For $D_i \geq h^2$, $\quad R_i = \sqrt{D_i^2 - h^2}$

For $D_i < h^2$, $\quad R_i = 0$

For $D_i = M$, $\quad R_i = M$

Step 8: Among all worker locations, select the worker location i with the largest C_i. Let the selected worker location be worker location i^*, D_i be D_{i^*}, and R_i be R_{i^*}.

Step 9: Find worker location i^{**} ($i^{**} \neq i^*$) where $C_{i^{**}}$ is the largest among the remaining C_i's, not including C_{i^*}.

Step 10: If $R_{i^*} = 0$, install an alarm device above worker location i^*. Its location will then be at the ($xa_r = xw_{i^*}$, $ya_r = yw_{i^*}$) coordinate. Then, proceed to Step 12.

Step 11: If $R_{i^*} > 0$, find the location coordinate of the alarm device from the following formulas. Firstly, let $\theta = \tan^{-1}\left[\left|\dfrac{yw_{i^{**}} - yw_{i^*}}{xw_{i^{**}} - xw_{i^*}}\right|\right]$. If $xw_{i^*} = xw_{i^{**}}$, then set $\theta = 90°$.

For xa_r:
- if $xw_{i^*} = xw_{i^{**}}$, then $xa_r = xw_{i^*}$
- if $xw_{i^*} < xw_{i^{**}}$, then $xa_r = xw_{i^*} + R_i\cos\theta$
- if $xw_{i^*} > xw_{i^{**}}$, then $xa_r = xw_{i^*} - R_i\cos\theta$

For ya_r:
- if $yw_{i^*} = yw_{i^{**}}$, then $ya_r = yw_{i^*}$
- if $yw_{i^*} < yw_{i^{**}}$, then $ya_r = yw_{i^*} + R_i\sin\theta$
- if $yw_{i^*} > yw_{i^{**}}$, then $ya_r = yw_{i^*} - R_i\sin\theta$

Note that both xa_r and ya_r must be within the facility area. That is, the (x, y) coordinate of alarm device r must be such that $0 \leq xa_r \leq x_f$ and $0 \leq ya_r \leq y_f$, where x_f and y_f are the limits on the x-coordinate and y-coordinate of the facility, respectively. If xa_r or ya_r is beyond x_f or y_f, respectively, set the coordinate equal to the corresponding limit.

Step 12: Check if the following condition is satisfied:

$$\sum_{k=1}^{r} \frac{1}{da_{ik}^2} \qquad \geq \qquad A_i \qquad\qquad \forall\ i$$

If yes, proceed to Step 13 (and r becomes r^*). Otherwise, set $r = r + 1$ and return to Step 6.

Step 13: The number of alarm devices that are needed for the given facility is r^*. Each alarm device is to be installed at the (xa_k, ya_k) coordinate, where $k = 1$ to r^*.

Two alarm location problems are presented in this section. In each problem, the number and location of alarm devices are determined using the heuristic and optimization approaches. The solutions from both approaches are then compared. Additionally, the "signal – noise" differences are compared at all worker locations.

7.4.1 Facility with 7 Machines – 4 Worker Locations

Consider a production facility with its dimensions of 30 m × 25 m ($x \times y$). The ceiling height is six meters. There are seven machines and four worker locations in this facility. The machine location coordinates and noise levels (at 1-m distance) generated by the machines are presented in Table 7.5. Table 7.6 shows the (x, y) coordinates of four worker locations. The ambient noise level when none of the machines is operating is 60 dBA (yielding the ambient noise intensity of 1.00×10^{-6} W/m^2). An auditory warning system is being designed for this facility. The signal sound level of an alarm device is 125 dBA (at 1-m distance).

Table 7.5 Location coordinates and sound levels generated by the machines (Facility with 7 machines – 4 worker locations)

Machine (M)	Location Coordinate (m)		Sound Level (dB*A*)
	x-coordinate	*y*-coordinate	
M1	5.00	5.00	85.00
M2	5.00	20.00	100.00
M3	15.00	5.00	90.00
M4	15.00	20.00	90.00
M5	25.00	5.00	95.00
M6	25.00	12.50	90.00
M7	25.00	20.00	85.00

Table 7.6 Location coordinates of worker locations (Facility with 7 machines – 4 worker locations)

Worker Location (WL)	Location Coordinate (m)	
	x-coordinate	*y*-coordinate
WL1	5.00	18.00
WL2	10.00	6.00
WL3	15.00	18.00
WL4	24.00	12.50

From the data in Tables 7.5 and 7.6, A_1, A_2, A_3, and A_4 can be calculated.

Location WL1: A_1 = 0.02525
Location WL2: A_2 = 0.00120
Location WL3: A_3 = 0.00377
Location WL4: A_4 = 0.01101

Locating alarm device No. 1

Initially, set $r = 1$. From Steps 6 and 7 of the heuristic algorithm, calculate C_i, D_i, and R_i (i = 1 to 4).

Location WL1: C_1 = 0.02525 D_1 = 6.29 R_1 = 1.90
Location WL2: C_2 = 0.00120 D_2 = 28.88 R_2 = 28.25
Location WL3: C_3 = 0.00377 D_3 = 16.29 R_3 = 15.15
Location WL4: C_4 = 0.01101 D_4 = 9.53 R_4 = 7.40

It is seen that worker location WL1 has the largest C_i (C_1 = 0.02525). Therefore, set D_{1*} = 6.29 and R_{1*} = 1.90. Next, it is seen that worker location WL4 has the next largest C_i (C_4 = 0.01101). Since $R_{1*} > 0$, the location of the first alarm device is determined using the formulas in Step 11.

$$\theta \quad = \quad \tan^{-1}\left[\left|\frac{12.50-18.00}{24.00-5.00}\right|\right] \quad = \quad 16.14°$$

Therefore,

$$xa_1 \quad = \quad 5.00 + (1.90)\cos(16.14°) \quad = \quad 6.83 \quad \text{m}$$

$$ya_1 = 18.00 - (1.90)\sin(16.14°) = 17.47 \text{ m}$$

Next, Inequality (11) is checked if the required condition is satisfied at all worker locations.

Location WL1: $\left[da_{11}^2\right]^{-1} = 0.02525, \quad A_1 = 0.02525,$ (Satisfied)

Location WL2: $\left[da_{21}^2\right]^{-1} = 0.00563, \quad A_2 = 0.00120,$ (Satisfied)

Location WL3: $\left[da_{31}^2\right]^{-1} = 0.00970, \quad A_3 = 0.00377,$ (Satisfied)

Location WL4: $\left[da_{41}^2\right]^{-1} = 0.00281, \quad A_4 = 0.01101,$ (Unsatisfied)

At worker location WL4, the required condition is not satisfied. Therefore, another alarm device is added.

Locating alarm device No. 2

Next, set $r = 2$. The above computations are repeated.

Location WL1:	$C_1 = 0.00000$	$D_1 = M$	$R_1 = M$
Location WL2:	$C_2 = -0.00443$	$D_2 = M$	$R_2 = M$
Location WL3:	$C_3 = -0.00593$	$D_3 = M$	$R_3 = M$
Location WL4:	$C_4 = 0.00820$	$D_4 = 11.04$	$R_4 = 9.27$

Since worker location WL4 has the largest C_i ($C_4 = 0.00820$), set $D_{4*} = 11.04$ and $R_{4*} = 9.27$. It is also seen that worker location WL1 has the next largest C_i ($C_1 = 0.00000$). The location of the second alarm device can be determined from the following formulas.

$$\theta = \tan^{-1}\left[\left|\frac{18.00 - 12.50}{5.00 - 24.00}\right|\right] = 16.14°$$

Therefore,

$$xa_1 = 24.00 - (9.27)\cos(16.14°) = 15.10 \text{ m}$$
$$ya_1 = 12.50 + (9.27)\sin(16.14°) = 15.08 \text{ m}$$

Once again, Inequality (11) is checked if the required condition is satisfied at all worker locations.

Location WL1: $\left[da_{11}^2\right]^{-1} + \left[da_{12}^2\right]^{-1} = 0.03208, \quad A_1 = 0.02525,$ (Satisfied)

Location WL2: $\left[da_{21}^2\right]^{-1} + \left[da_{22}^2\right]^{-1} = 0.01255, \quad A_2 = 0.00120,$ (Satisfied)

Location WL3: $\left[da_{31}^2\right]^{-1} + \left[da_{32}^2\right]^{-1} = 0.03215, \quad A_3 = 0.00377,$ (Satisfied)

Location WL4: $\left[da_{41}^2\right]^{-1} + \left[da_{42}^2\right]^{-1} = 0.01101, \quad A_4 = 0.01101,$ (Satisfied)

Since Inequality (7.18) is satisfied at all four worker locations, the solution is found. This facility needs two alarm devices (with each device generating a 125-dB*A* auditory signal sound level). Both alarm devices should be installed on the ceiling at (6.83, 17.47) and (15.10, 15.08) coordinates, respectively.

We also solve this problem using an optimization approach. By formulating the problem using the *revised* alarm location model (see Section 3.2) and solving it, it is found that the minimum number of alarm devices r^* needed for this facility is also two devices. They are to be installed at (2.85, 19.02) and (30.00, 6.35) coordinates, respectively. Table 7.7 shows the combined alarm signal sound levels and the combined noise levels based on both solution approaches at the four worker locations. It is seen that both approaches yield the results that satisfy the "15-dB*A* difference" constraint. The optimization approach yields a better solution since the differences are closer to 15 dB*A* than those from the heuristic approach. From Table 7.7, the average "signal – noise" difference from the optimization approach is 17.59 dB*A*, while the one from the heuristic approach is 20.14 dB*A*.

An ideal lower bound of the "signal – noise" difference is used as a benchmark for an evaluation of the solution. Based on the "15-dB*A* difference" constraint, the ideal solution is the one in which all "signal – noise" differences are 15 dB*A* (at all worker locations). The ratio of the average difference to the ideal lower bound is then defined as a quantitative efficiency index. Note that the best efficiency index is 1.00. Thus, the closer to 1.00 the efficiency index is, the higher the efficiency of the solution approach. From Table 7.7, the efficiency index of the solution from the optimization approach is 1.17, while the one from the heuristic approach is 1.34.

Table 7.7 Comparison of the combined signal sound level and the combined noise level based on the heuristic ($r = 2$) and optimization ($r^* = 2$) approaches (Facility with 7 machines – 4 worker locations)

Worker Location	Combined Noise Level (dB*A*)	Combined Signal Level (dB*A*)		Signal – Noise (dB*A*)*	
		Heuristic	Optimization	Heuristic	Optimization
WL1	94.02	110.06	109.02	16.04	15.00
WL2	80.79	105.99	102.92	25.20	22.13
WL3	85.76	110.07	104.00	24.31	18.24
WL4	90.42	105.42	105.42	15.00	15.00
Average "Signal – Noise" Difference				20.14	17.59
Standard Deviation				5.36	3.39
Maximum Difference				25.20	22.13
Minimum Difference				15.00	15.00
Efficiency Index				1.34	1.17

*The required "signal – noise" difference is at least 15 dB*A*.

7.4.2 Facility with 13 Machines – 7 Worker Locations

Next, we test the heuristic algorithm on a larger alarm location problem. Let us now consider a rectangular facility with its dimensions of 45 m × 35 m ($x \times y$), respectively. Its ceiling height is six meters. In this facility, there are 13 machines and seven locations where workers might be present. The location coordinates and noise levels (at 1-m distance) generated by these machines are shown in Table 7.8. The ambient noise level is 65 dB*A*. The alarm signal sound level is 120 dB*A* (at 1-m distance). The location coordinates of the seven worker locations are shown in Table 7.9.

Table 7.8 Location coordinates and sound levels generated by the machines
(Facility with 13 machines – 7 worker locations)

Machine (M)	Location Coordinate (m)		Sound Level (dB*A*)
	x-coordinate	*y*-coordinate	
M1	5.00	5.00	95.00
M2	5.00	15.00	90.00
M3	5.00	25.00	94.00
M4	15.00	5.00	90.00
M5	15.00	15.00	95.00
M6	15.00	25.00	90.00
M7	25.00	5.00	95.00
M8	25.00	15.00	96.00
M9	25.00	25.00	90.00
M10	33.00	5.00	87.00
M11	33.00	15.00	86.00
M12	33.00	25.00	88.00
M13	38.00	15.00	99.00

Table 7.9 Location coordinates of worker locations
(Facility with 13 machines – 7 worker locations)

Worker Location (WL)	Location Coordinate (m)	
	x-coordinate	*y*-coordinate
WL1	5.00	3.50
WL2	5.00	13.50
WL3	5.00	23.50
WL4	25.00	3.50
WL5	25.00	13.50
WL6	25.00	23.50
WL7	40.00	15.00

To facilitate the computation procedure, the heuristic algorithm is coded using the Visual Basic application in Microsoft Excel. From the data given in Tables 7.8 and 7.9, it is found that the recommended number of alarm devices r is 8 devices. The optimization approach also yields the minimum number of alarm devices r^* of 8 devices. The location coordinates of the eight alarm devices determined from both approaches are shown in Table 7.10.

Table 7.11 shows the comparison of the combined noise level and the combined alarm signal sound level between both solution approaches for all worker locations. It is seen that the "signal – noise" differences are quite close to 15 dB*A* for both solution approaches. The average "signal – noise" differences from the heuristic and optimization approaches are 16.04 dB*A* and 15.29 dB*A*, respectively. Although the optimization approach still yields a better solution than the heuristic approach, it is surprising to see that the heuristic approach becomes more efficient when solving this problem than the previous one (in Facility with 7 machines – 4 worker locations). Its efficiency index in this problem is 1.07 while the one in previous problem is 1.34. Thus, it is necessary to investigate more alarm location problems with different problem sizes.

Table 7.10 Location coordinates (in m) of the eight alarm devices
(Facility with 13 machines – 7 worker locations)

Alarm Device	Heuristic Approach		Optimization Approach	
	x-coordinate	*y*-coordinate	*x*-coordinate	*y*-coordinate
A1	40.00	15.00	40.85	14.50
A2	25.00	13.50	40.81	14.48
A3	5.00	3.50	23.71	9.63
A4	25.00	3.50	3.45	0.00
A5	5.00	23.50	25.47	9.52
A6	40.00	15.00	4.95	23.78
A7	19.98	10.99	19.83	8.62
A8	5.00	13.18	0.83	0.00

Table 7.11 Comparison of the combined signal sound level and the combined noise level based on the heuristic (r = 8) and optimization (r^* = 8) approaches
(Facility with 13 machines – 7 worker locations)

Worker Location	Combined Noise Level (dB*A*)	Combined Signal Level (dB*A*)		Signal – Noise (dB*A*)*	
		Heuristic	Optimization	Heuristic	Optimization
WL1	91.68	106.68	106.68	15.00	15.00
WL2	87.63	107.15	104.07	19.53	16.45
WL3	90.78	106.44	105.78	15.66	15.00
WL4	91.81	107.33	106.81	15.53	15.00
WL5	92.87	108.02	107.87	15.16	15.00
WL6	87.79	104.08	103.37	16.29	15.57
WL7	93.11	108.22	108.11	15.11	15.00
Average “Signal – Noise” Difference				16.04	15.29
Standard Deviation				1.60	0.55
Maximum Difference				19.53	16.45
Minimum Difference				15.00	15.00
Efficiency Index				1.07	1.02

*The required (signal – noise) difference is at least 15 dB*A*.

7.5 Computational Experiment

To further investigate the efficiency of the heuristic algorithm, sixteen alarm location problems were created. The number of machines in the facility ranged from 6 to 20 machines. The number of worker locations ranged from 10 to 20 locations. The ambient noise level for each problem was randomly set to 60, 65, or 70 dB*A*. The ceiling height was fixed at 6 meters. The machine noise levels randomly varied between 80 dB*A* to 105 dB*A*.

Both solution approaches (heuristic and optimization) were used to find the alarm location solution. The performance indices used in the comparison of solutions are:

- number of alarm devices required for the workplace,
- average “signal – noise” difference, and
- efficiency index.

Table 7.12 shows the comparison of the solutions from the heuristic and optimization approaches.

From the 16 test problems, the heuristic approach is able to yield the same numbers of alarm devices as those from the optimization approach in 13 problems (or 81.25%). For the remaining three problems, the difference in the number of alarm devices is only one device. When comparing the average "signal – noise" differences, it is seen that the average difference from the heuristic approach is greater than that from the optimization approach by not more than 2 dB*A*, irrespective of the problem size. However, when comparing the efficiency index values, it is found that the efficiency index tends to decrease with the problem size. This seems to indicate that as the problem size grows larger, the heuristic approach becomes more efficient and would yield a solution that is nearer to an optimal solution.

Table 7.12 Comparison of the solutions from the heuristic and optimization approaches

		Number of Alarm Devices		Average "Signal – Noise" Difference		Efficiency Index	
n	*m*	Heuristic	Optimization	Heuristic	Optimization	Heuristic	Optimization
6	18	4	4	24.29	23.11	1.62	1.54
6	10	3	3	18.27	17.43	1.22	1.16
8	15	6	6	20.52	20.17	1.37	1.34
8	17	6	6	20.80	20.37	1.39	1.36
10	11	5	5	19.86	18.31	1.32	1.22
10	14	7	7	19.21	18.65	1.28	1.24
12	16	7	6	19.63	19.05	1.31	1.27
12	20	6	6	21.22	19.91	1.41	1.33
14	17	5	5	18.56	17.88	1.24	1.19
14	13	6	6	18.68	17.80	1.25	1.19
16	13	8	7	18.86	17.82	1.26	1.19
16	10	6	6	18.55	18.33	1.24	1.22
18	12	7	7	18.47	18.33	1.23	1.22
18	10	7	7	17.24	17.00	1.15	1.13
20	19	8	7	17.63	16.99	1.18	1.13
20	15	7	7	17.76	16.94	1.18	1.13

Note: *n* = number of machines
m = number of worker locations

Additionally, readers should note that one important issue in the use of the optimization approach is an upper bound of the number of alarm devices in the alarm location model. If this upper bound is set to be much higher than the optimal number, the problem may not be solvable. In our computational experiment, we used the solution (the number of alarm devices) from the heuristic approach, which is either equal to or greater than the optimal number by one, as the upper bound. With this technique, it is possible to obtain an optimal solution for large alarm location problems. When we tried to set the upper bound to be three or four devices more than the optimal number, the optimal solution could not be found. Regarding the computation time, the heuristic approach is able to yield the *near-optimal* solution within a few seconds whereas the optimization approach needs several minutes or several hours of computation time.

Although this research emphasizes the audibility of alarm systems, readers should be aware that it is not the only factor that warrants the effectiveness of auditory warnings. There are other cognitive and behavioral issues that also need to be considered. Additionally, the heuristic algorithm proposed in this work does not consider the effect of age on auditory signal detection.