

Tutorial for option pricing using the Black Scholes Model:

About Options:

Option is the right and not the obligation to buy or sell an underlying asset at a particular price within or at a specified period of time. The black scholes model helps in pricing the these options based on the creation of a perfect hedge by simultaneously taking an opposite position in the options market. The return on a completely hedged position will then be equal to the risk-free return on the investment.¹ The value of a call option can be calculated using the following formula:

$$C = SN(d_1) - Xe^{-Rf(T)}N(d_2)$$

When

$$d_1 = (\ln(S / X) + (Rf + \sigma^2 / 2) * T) / (\sigma \sqrt{T})$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

$$N(d_1)^* = NORMSDIST(d_1) \quad (\text{These}$$
$$N(d_2)^* = NORMSDIST(d_2)$$

* These are the normal distribution values of d1 and d2.

Where

C = Value of the Option

S = Value of the underlying asset right now

X = Exercise price or Strike price

Rf= Risk free rate of return

T = Time to expiration

σ = Standard Deviation of the percentage returns on the underlying assets

$N(d_1)$ = Delta, it measures the rate of change of the option price with respect to the price of the underlying asset

$N(d_2)$ = Probability of the option being “in the money”

¹ Pg. 41, Basic Tools of Financial Risk Management: Portfolio and Options

Tutorial:

We begin the spreadsheet by making an input table like the one shown below:

Inputs		
S	underlying price	46.7500
X	strike	45.0000
σ	volatility	0.2800
Rf	interest rate	0.0620
T	time to expiry (years)	0.5452

First, we calculate the values of d1 and d2, using the following formula:

From B&S formula	d1	0.4514	$d_1 = (\ln(S / X) + (Rf + \sigma^2 / 2) * T) / (\sigma \sqrt{T})$
From B&S formula	d2	0.2447	$d_2 = d_1 - \sigma \sqrt{T}$

Then we calculate N(d1) and N(d2) by using the normstdist function in excel.

$$N(d_1) = \text{NORMSDIST}(d_1)$$

$$N(d_2) = \text{NORMSDIST}(d_2)$$

Now we have all the inputs to calculate the call value of the option: \

$$C = SN(d_1) - Xe^{-Rf(T)}N(d_2)$$

The Put value of this option can also be calculated using the formula:

$$P = Xe^{-Rf(T)}(1 - N(d_2)) - S(1 - N(d_1))$$

GREEKS

Now we will calculate the “Greeks”, that is the values for the greek symbols. The greek values would enable us to have a better understanding of the related variables and the working of an options model.

To calculate the Greeks we need to calculate the values of $N'(d_1)$ and $N'(d_2)$ using the formula:

$$N'(d_1) = e^{-(d_1)^2/2} / \sqrt{2\pi}$$

Note: In case of excel the formula would be =EXP(-(d1^2)/2)/(SQRT(2*PI()))

$$N'(d_2) = e^{-(d_2)^2/2} / \sqrt{2\pi}$$

Note: In case of excel the formula would be =EXP(-(d2^2)/2)/(SQRT(2*PI()))

Thereafter, we will calculate the Greek values by making a table that looks like the one shown below and using the corresponding formula:

From B&S formula	d1	$d_1 = (\ln(S/X) + (Rf + \sigma^2/2)*T) / (\sigma\sqrt{T})$
From B&S formula	d2	$d_2 = d_1 - \sigma\sqrt{T}$
N'(d1)		$N'(d_1) = e^{-(d_1)^2/2} / \sqrt{2\pi}$
N(d1)		$N(d_1) = NORMSDIST(d_1)$
N'(d2)		$N'(d_2) = e^{-(d_2)^2/2} / \sqrt{2\pi}$
N(d2)		$N(d_2) = NORMSDIST(d_2)$
theoretical call value		$C = SN(d_1) - Xe^{-Rf(T)}N(d_2)$
Call delta		Same as N(d1)
theoretical put value		$P = Xe^{-Rf(T)}(1 - N(d_2)) - S(1 - N(d_1))$
put delta		$= N(d_1) - 1$
Same for call & put	gamma	$\Gamma = N'(d_1) / S\sigma\sqrt{T}$
Same for call & put	vega	$V = S\sqrt{T}N'(d_1)$
call theta		$\Theta = -(S_0N'(d_1)\sigma / 2\sqrt{T}) - rXe^{-Rf(T)}N(d_2)$
put theta		$\Theta = -(S_0N'(d_1)\sigma / 2\sqrt{T}) + rXe^{-Rf(T)}(1 - N(d_2))$
call rho		$\rho = XTe^{-Rf(T)}N(d_2)$
put rho		$\rho = -XTe^{-Rf(T)}(1 - N(d_2))$

Delta: $N(d_1)$

Delta, measures rate of change of the option's value with respect to the stock price. Suppose, the delta of a call option is 0.6. This means that if the stock price goes up by \$1, the value of the option goes up by 60% of \$1 i.e. 60 cents.

Theta Θ :

Theta measures the rate of change of an option's value with respect to the passage of time, all else remaining the same. The value of an option will change over time even if the stock price remains unchanged. This is mainly because, as the date of maturity comes closer the expected change in the value of the underlying asset reduces. Theta is usually negative because as time passes, the option becomes less valuable. As a result, the value of theta increases and becomes closer to zero as the option moves closer to maturity.

Note: If T represents theta, then when one day passes, the value of the option changes by approximately $T/365$.

Gamma (Γ):

Gamma of an option is the rate of change of the delta with respect to the change in the stock price. Gamma helps us understand delta and how it is going to change as the stock price changes. If the gamma is small, delta changes slowly with respect to price. However, if gamma is large it means that delta is highly sensitive to the changes in the stock price.

Vega (V):

Vega measures the relationship between the stock volatility and option value. If value of Vega is high, the option's value is very sensitive to small changes in volatility whereas if the value of Vega is low volatility changes do not have a significant impact on option value. Moreover, higher the volatility the greater is the probability that the option will end up with a higher price.

Rho (rho):

Rho measures the sensitivity of the option value to the interest rates. It is the rate of change in the option value with respect to the interest rate. For example, if rho is 14.1516, it means that for every percentage point i.e. 0.01 increase in the interest rate, the value of the option increases by 14.1516% or .141516. Moreover, rho is always positive for European calls and always negative for European puts.