AN EMPIRICAL TEST OF BS AND CSR VALUATION MODELS FOR THE INDEX OPTIONS LISTED IN TAIWAN

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An Independent Study
Submitted in Partial Fulfillment of the Requirements
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Thammasat University
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By

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has been approved as a partial fulfillment of the requirements
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ABSTRACT

The purpose of this paper is to compare the options pricing performance between Black-Scholes (BS) and Cox Square Root (CSR) models for the index options traded on Taiwan Stock Exchange (TWSE) under different market conditions over the period 2002 to 2004. The result of this study indicates that, based on pricing errors measurement, the CSR model provides more accuracy for index option pricing than the BS model does. For overall period, absolute pricing errors for call and put options are statistically significant by the percentage of 0.07 and 0.04 respectively. The square root constant elasticity of variance model (CSR) that assumes stochastic volatility can improve the pricing performance compared to the model that assumed volatility constant during the life of the options.

I. INTRODUCTION

This paper will examine the TAIEX index options, the financial instruments traded in Taiwan Stock Exchange (TWSE). Generally speaking, options give the options holders the right to buy or sell the underlying asset at the pre-determine price or exercise price only on the maturity date (European type) in spot month, the next two calendar months, and the next two quarterly months. For instance, if the options are issued in January, the expiration months will be January, February, March, June, and September.

In financial markets, it would be better for any derivative writers if they could appropriately choose standard pricing models with accuracy in pricing to price the option-type derivatives such as plain vanilla call-put options, exotic options or warrants. However, in reality, some discrepancies or pricing error between theoretical prices and the market price or fair price could occur. It is beneficial for the institutional investors or individual investors to know which models can provide the most accurate pricing.

Several current researches which examine the pricing errors can be categorized into two groups. The first category concerns about the model specification errors on the underlying stock process, interest rate process and the volatility assumption on its underlying assets. Bakshi, Cao and Chen (1997) examined several alternative models by using S&P500 index options and claimed that stochastic volatility and jumps component part should be incorporated in the pricing model. The second category mainly describes the pricing errors from the aspect of the market microstructure. This paper, however, will focus on the model's biased assumption. The latter group is beyond the scope of this paper.

In theory, Black and Scholes (1973) first derived the traditional option pricing formula by assuming that underlying stock price process follows a geometric Brownian motion, stock price returns distribution is lognormal, and volatility of stock return is constant during the entire life of the options. From many empirical evidences, however, neither constant volatility nor lognormal is difficult to exist. In practice, when the implied volatility is plotted against the exercise price, the shape of volatility exhibited the smile phenomenon so often known as "volatility smile". Given the same expiration month, some options have implied volatility of in-the-money and out-of-the-money options higher than that of at-the-money options, the graphical result approximately exhibited U-pattern which looks like a smile or having either downward with an upward sloping at both ends. Another pattern of the shape is also called "volatility smirk" which appears downward sloping toward in-the-money options. Given the same expiration month, the options might have higher implied volatility for more out-of-the-money options.

Later, Cox and Ross (1976) developed the Constant Elasticity of Variance (CEV) by extending the original Black-Scholes model. This alternative options pricing model can explain the empirical volatility bias in Black-Scholes model by allowing stochastic volatility or volatility can be changed according to an underlying asset change. Moreover, empirical evidences suggest that if there exists an inverse relationship between underlying stock price and volatility, the CEV model could be more accurate for pricing options than Black-Scholes model counterpart.

The purpose of this paper is to examine discrepancies or the pricing errors between the Black-Scholes model and the square root constant elasticity of variance or Cox Square Root model (CSR). Furthermore, since the Black-Scholes model and the square root constant elasticity of variance or Cox Square Root model (CSR) have different assumptions based on stock price process, stock return distribution and especially its volatility, this paper will also examine the option pricing performance between this two models.

II. LITERATURE REVIEW

Numbers of empirical evidences support that time-varying volatilities can provide more accuracy in pricing performance and explain the empirical bias occurred in the Black-Scholes model, which assumes volatility being constant over time. Chu, Yang, and Yuen proposed the new estimation procedure on parameter for constant elasticity of variance model and indicated that their simulation method is more appropriate in practice. They claimed that the constant volatility assumption in stock price process under Black-Scholes framework may not be appropriate in practice. They also analyzed the call option prices of three popular underlying stocks in Hong Kong and showed the varying pattern of volatility on each stock. The result of their study suggests that percentage pricing errors in Black-Scholes model are more distributed away from zero compared to CEV model.

An interesting issue investigated in many papers is the relationship between the underlying stock price and its variance. In reality, volatility of stock return is not constant and underlying stock returns distribution is not lognormal. An inverse relationship between stock price and variance of daily returns is first observed by Black (1976). Later , Beckers (1980) investigated the relationship between stock price and variance of stock return and compared Black-Scholes model with CEV model on both absolute CEV model ($\theta = 0$) and square root CEV model or Cox Square Root model ($\theta = 1$). He claimed that a constant elasticity of variance class could provide a better descriptor on actual price than the traditional Black-Scholes model. Furthermore, there exists an inverse relationship between stock price and its volatility. The price predictions from the CEV model is higher than the Black-Scholes model for in-the-money and at-the-money options and truly reverse on out-of-the-money options. This result is also supported by Macbeth and Merville (1980) who test the Cox call option valuation model against the Black-Scholes model. They found that a stock price is generated by constant elasticity of variance diffusion processes. As a result, Cox valuation model significantly provides more accuracy for market prices than the Black-Scholes model. As Black observed, they also supported that when $\theta < 2$, the model can explain a negative relationship between stock prices and its daily return. Furthermore, their analysis based on the degree of under pricing or over pricing from model prediction is related to the degree of moneyness and time to maturity of the options. Emanuel and Macbeth (1982) also claimed that the constant elasticity of variance model can be a better predictor for pricing an options than the Black-Scholes model when $\theta < 2$, which predict that the volatility and stock price move in the opposite way. However, when $\theta > 2$, meaning that the volatility and stock price move in the same direction, the CEV model predicts worse than Black-Scholes model.

Volatility becomes a major concern based on previous empirical literatures. In theory, any options that have written on the same underlying assets on the same expiry days should have the same volatility although they have different exercise price because volatility connects to the underlying price not the exercise price. However, in reality, this phenomenon is difficult to occur. In practice, there exist volatility smiles on implied volatility, which is the phenomenon that implied volatility is not constant across different exercise prices. Macbeth and Merville (1979) argued that the implied volatility declines as the exercise price increases. Their empirical results showed that implied volatilities depend on degree of moneyness of the options. Lu and Hsu (2005) claimed that the empirical bias of Black-Scholes model can be explained by the CEV model which allows volatility change given the change in underlying price. They also purposed alternative options pricing models with the assumption that underlying asset price process follows CEV model known as binomial CEV model and CEV options formula in terms of non-central chi-square distributions proposed by Nelson & Ramaswamy (1990) and simplified by Schroder (1989) respectively.

From the empirical evidences in Thailand, the studies related to warrant pricing models showed that the dilution adjustments of the Black-Scholes model for pricing warrant does not improve pricing performance. The model uses an implied volatility as an input and yields smaller average pricing error than historical volatility. As a result, it is found that CEV model is superior to the Black-Scholes model. Furthermore, Shastri and Sirodom (1995) claimed that the 70 percent accuracy in pricing all warrant cases came from Cox Square Root model (CSR), compared to the dilution-adjusted Black-Scholes model.

III. THEORETICAL FRAMEWORK

Derivative Pricing Theory is applied in order to calculate the theoretical option price compared to its actual market price. In this study, the first options pricing model is Black-Scholes model which assumes normal distribution of asset returns and constant volatility of asset returns. An alternative options pricing model is the constant elasticity of variance (CEV) model which extends the Black-Scholes model by relaxing constant volatility assumption.

- ***Black-Scholes model***

Black and Scholes (1973) derived the derivative pricing model known as Black-Scholes model which is widely used to determine the option-type derivatives. Under the Black-Scholes model, the stock price process is assumed to follow geometric Brownian motion:

$$dS = \mu S dt + \sigma S dz \tag{1}$$

The call options pricing formula for Black-Scholes model is given by:

$$C = SN(d_1) - Xe^{-rT} N(d_2) \tag{2}$$

where $d_1 = \dfrac{\ln\left(\dfrac{S}{X}\right) + rT}{\sigma\sqrt{T}} + \dfrac{\sigma\sqrt{T}}{2}$

$d_2 = d_1 - \sigma\sqrt{T}$

C is the call option price,

S is the underlying stock price,

X is the option exercise price,

T is the time to maturity of the option,

r is the risk-free interest rate,

σ is the standard deviation of stock return of S,

$N(d)$ is the cumulative distribution function evaluated at d.

- ***Square Root Constant Elasticity of Variance model (CSR)***

Cox (1975) later developed the derivative pricing model known as Constant Elasticity of Variance (CEV). The CEV model follows all of the assumptions of the Black-Scholes valuation model except that the stochastic stock price process follows constant elasticity of variance process. The stock price process of the family of the constant elasticity of variance class can be described by the stochastic differential equation as follows:

$$dS = \mu S dt + \delta S^{\theta/2} dz \tag{3}$$

where dz is a Weiner process and θ is the elasticity of variance with respect to stock price and has the value in the range of $0 \leq \theta < 2$. When we assumes the elasticity of stock returns volatility with respect to the level of stock price (θ) equals to 1, the CEV model is called as "Cox Square Root model". The instantaneous variance of the percentage price change (σ^2) is described by the equation as follows:

$$\sigma^2 = \delta^2 S^{(\theta - 2)} \tag{4}$$

To make a clarification on elasticity of variance factor of the stock price process under CEV model, if $\theta = 2$, the instantaneous variance of the return is constant and equals to δ^2. Therefore, in this case, stock price process under CEV model follows geometric Brownian motion which is consistent with the price process assumed under the Black-Scholes valuation model. Alternatively, If $\theta < 2$, there exist an inverse relationship between the stock prices and volatility as shown in equation (4), meaning that the volatility will be decreased as the stock price increases and vice versa.

By assuming the non-central Chi-square distribution developed by Sankaran (1963), we can obtain an options pricing formula for a square root constant elasticity of variance or the Cox Square Root model.

The call options pricing formula for Cox Square Root model is given by:

$$C = SN(q(4)) - Xe^{-rT} N(q(0)) \tag{5}$$

Let v be a degree of freedom parameter which takes on the value of 0 or 4, where

$$y = \frac{4rS}{\delta^2 (1 - e^{-rT})}$$

$$z = \frac{4rX}{\delta^2 (e^{rT} - 1)}$$

$$p = \frac{v + 2y}{(v + y)^2}$$

$$h(v) = 1 - \frac{2(v + y)(v + 3y)}{3(v + 2y)^2}$$

$$q(v) = \frac{1 - hp\left[1 - h + \frac{1}{2}(2-h)(h-1)(1-3h)p\right] - \left[z/(v+y)\right]^h}{h\sqrt{2p\left[1 + (h-1)(1-3h)p\right]}}$$

$N(q)$ is the cumulative normal distribution function evaluated at q

Based on empirical studies conducted by researchers including Becker (1980), Macbeth and Merville (1980), and Emanuel and Macbeth (1982), the results of their studies consistently suggest that the constant elasticity of variance option pricing model yields the estimated price with the less pricing error relative to traditional option pricing models like the Black-Scholes model. Therefore, this study will hypothesize that the square root constant elasticity of variance or Cox Square Root model, which allows the volatility to change with the underlying price in the negative way, could provide less pricing discrepancies than Black-Scholes model, which assumes constant volatility.

IV. DATA

TXO options traded on Taiwan Futures Exchanges (TAIFEX) have TAIEX as its underlying asset. The types of TXO options launched by TAIFEX are European call and put options. In this study, our observations consist of the daily closing prices of Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) and characteristics of TXO options, including exercise price, time to maturity and daily risk-free interest rate for the non-zero trading volume options.

The TAIEX trend and its returns during the sampling period from January 2, 2002 to May 25, 2004 are shown in Figure I and II respectively. According to the Chan, Chang and Lung (2008) who examined the options trading under different market conditions in Taiwan, they divided full sampling period into three sub periods including downtrend market period running from January, 2002 to April 2003, uptrend market period running from May, 2003 to March, 2004, and the political tension market period starting from March 20, 2004 to the end of the period. On April 30, 2003 the Taiwan Stock index started moving up, so it is selected to be the dividing line between the downtrend and uptrend period. And around March 20, 2004 the political tension in Taiwan started, so it is select to be the line which separates the uptrend and political tension period.

[Figure I and II are here]

Table I shows the descriptive statistics for Taiwan Stock Index in a daily basis. The table contains number of observation, mean, standard deviation, minimum, maximum, skewness and kurtosis of daily Taiwan stock index during the sampling period, which starts from 2 January 2002 to 25 May 2004. The entire sample period is classified into three sub periods, including downtrend market period starting from 2 January 2002 to 30 April 2003, uptrend market period from 1 May 2003 to 19 March 2004, and political tension market period from 20 March 2004 to 25 May 2004.

[Table I is here]

V. METHODOLOGY

Firstly, we examine the validity of the CSR model which allows for an inverse relationship between the stock index and its return volatility by calculating the realized volatility or historical volatility over the most recent 60 trading days and comparing to the index price. In Figure III, we empirically observed the existence of a negative relationship between the TAIEX index and its volatility of index return. According to several empirical researches, if the relationship between stock index and volatility of its return were negatively related, it would be more appropriate to price these options by using the models which allow for that characteristic instead of constant variance models. The alternative model which is selected to compare the pricing performance with the Black-Scholes is the Cox Square Root model.

[Figure III and Table II are here]

According to the range of delta value described in Table II, we classified the options into five different moneyness categories as Bollen and Whaley (2004) suggested. The call (put) options' delta, the sensitivity of the options price to the change in underlying price, is ranged in the value between 0 and 1 (0 and -1) and can be roughly interpreted as a probability of the option will be in-the-money at expiration in a risk-neutral world. The observations will be eliminated when the absolute deltas are below 0.02 or above 0.98 due to their insensitivity to the underlying price change. The European-style call and put options' delta are shown in equation (6) and (7) respectively.

$$\Delta_C = N(d_1) = N\left[\frac{\ln(S/K) + (r + 0.5\sigma^2)T}{\sigma\sqrt{T}}\right] \tag{6}$$

$$\Delta_P = -N(-d_1) = -N\left[-\left(\frac{\ln(S/K) + (r + 0.5\sigma^2)T}{\sigma\sqrt{T}}\right)\right] \tag{7}$$

The options' delta is computed on each series in a daily basis by parameters in the equation above. The realized volatility of the underlying index returns over the most recent 60 trading days is used as a proxy of the volatility parameter in the equations.

To price options using Black-Scholes valuation model, we can directly observe the factors *S, T, X* and *r* from the market. The only unknown parameter in Black-Scholes pricing formula is volatility. We can obtain implied volatility from the observed options prices in the market by using the Newton-Raphson numerical algorithm[1]. Implied volatility, however, should be estimated from actively traded at-the-money options. Because volatility implied from deep-in-the-money and deep-out-of-the-money options prices are likely to be unreliable due to their insensitivity to volatility. Once we find implied volatility at time *t*, the risk-free rate on day *t*, and the underlying index price at the time *t+1* can be used as an input to predict option price at time *t+1*.

Figure IV a) shows the 72.5% positive correlation between the realized return volatility and the average implied volatility of at-the-money options and b) shows the trend of both volatilities. We calculated both volatilities trend by simply comparing them based on monthly basis. The trend of an implied volatility of at-the-money options seems to lead the realized

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[1] The iterative technique begins with expansion C_j into a Taylor series around initial value σ_0.

$$C_j - C_j(\sigma_0) = \frac{\partial C_j(\sigma_0)}{\partial \sigma}(\sigma - \sigma_0) + \ldots h.o.t. \ldots + \varepsilon_j$$

Firstly, we estimate the model price of the option $C_j(\sigma_0)$ based on an initial guess for implied volatility, σ_0. Then, we'll calculate σ_1 and use as a new guess of implied volatility until the $|C_j - C_j(\sigma_0)|$ is less than 0.0001.

$$\sigma_1 = \sigma_0 + \frac{C_j - C_j(\sigma_0)}{\partial C_j(\sigma_0)/\partial \sigma}$$

return volatility. Therefore, the implied volatility of at-the-money options should be closely predicted the realized volatility of the index return.

[Figure IV is here]

To obtain options prices based on CSR model, an estimation of the δ is required according to the value of θ. Apparently from equation (4), if θ equal to 2, δ will be constant and equal to the volatility of the return, σ. When the value of θ is less than 2, the volatility of return will not be constant and changes through time. According to the CSR model which θ value is 1 and the volatility parameter is implied from the actual market price of at-the-money options based on the BS model on the given day t, σ_t , we can rearrange equation (4) in order to solve for the δ, that is $\delta_t = \sigma_t S_t^{(2-\theta)/2}$. Once we find the δ_t for CSR model, we can calculate the CSR model price by replacing d_1 or d_2 in BS model with non-central Chi Square distribution function $q(\cdot)$ according to the degree of freedom 4 and 0, respectively. Thus, based on the same instantaneous volatility we can calculate options price from the BS and CSR valuation model, and find the price discrepancies of the two models by comparing market price with price calculated from the two models.

To compare the pricing performance between the BS and CSR model on the given day t, we computed the percentage pricing errors for each model by dividing the differences between market and model prices by model prices.

$$\%\ pricing\ errors = \frac{P_{\text{market}} - P_{\text{model}}}{P_{\text{model}}} \tag{8}$$

Therefore, we computed the difference between the absolute pricing errors of the BS and CSR models. The parametric and non-parametric tests are applied to compare the pricing errors whether it is significantly different from zero.

VI. EMPIRICAL RESULT

Table III compares mean of the percentage pricing errors of Black-Scholes and Cox Square Root model for the entire period and each sub period according to levels of moneyness and time to maturity of the options. For the overall period, call options prices calculated from both models tend to overprice for the shorter time to expiration and underpriced for the longer time to expiration, while put options prices calculated from both models tend to be more underpriced for the longer time to expiration. During downtrend period and political tension period, put options price obtained from both models tend to be more underpriced for the shorter time to expiration, while during uptrend period call options prices obtained from both model seem to be the more underpriced for the longer time to expiration.

[Table III is here]

The relationship between the absolute pricing error, level of moneyness of the options, and time to expiration of the BS model and CSR model is shown in Table IV. For both models, the absolute pricing error is negatively related to the level of moneyness of the options in all periods. This means that the more level of moneyness of the options, the less pricing error of the models. However, there appears no relationship between absolute pricing errors and the time to expiration of the options, except the case of put options in the downtrend and political tension period. In these two periods, the absolute pricing error is negatively related to time to expiration. For call options, in the political tension period, the absolute pricing error is positively related to time to expiration. Furthermore, the CSR model yields less absolute pricing error in almost all of sub periods for call options except only in the uptrend period, and the CSR model also provides less absolute pricing error in almost all of sub periods for put options except in the downtrend period..

[Table IV is here]

Table V compares pricing errors between Black-Scholes and Cox Square Root models. Panel A1 presents mean and median values of the pricing errors (in columns 2 and 3), mean and median of the absolute pricing errors (in columns 4 and 5), and the percentage of the pricing errors. The percentage of the pricing errors that are positive (in column 6), indicating the underpriced degree of the model in pricing.

Panel B presents the difference between the pricing errors of the Black-Scholes and the Cox Square Root models. Columns 2 and 3 contain mean and median of difference between the pricing errors of the Black-Scholes and that of the Cox Square Root models, respectively.

The percentage of positive differences of absolute pricing errors between the two models in column 4, which is calculated by subtracting absolute pricing error of the CSR from the that of the BS, represents the degree of the CSR's pricing performance. To test null hypothesis of no difference between absolute pricing errors of the two models, the *t*-statistics test and the Wilcoxon signed-rank test are performed. The *t*-statistics and the *p*-value of the Wilcoxon signed-rank test are presented in columns 5 and 6, respectively.

From Panel A1 and A2, both models are likely to overprice call options and underprice put options for overall period. However, an average pricing errors of the CSR model yields the better performance for call and put options than that of BS model for the entire period by the percentage of 0.97 and 1.02 respectively.

By classifying underlying index's trend into 3 different subperiods based on market condition - downtrend, uptrend and political tension period - call (put) options price calculated from both models tend to be overpriced (underpriced) in the economic downtrend and the political tension period and call (put) options calculated from both models tend to be underpriced (overpriced) in the economic uptrend period. Moreover, the absolute pricing errors of the CSR model in subperiods yields the better performance than the BS model except the case of call options in the uptrend period. For the entire period, however; CSR model provides better performance in absolute pricing errors for call and put options relative to BS model by the percentage of 0.07 and 0.40 respectively.

In table V, panel B presents the comparison of pricing error between the two models. The average pricing error of the CSR model is lower for call and put options for the entire period. In column 6, it is shown that mean and median of pricing errors are statistically significantly different from zero based on both t-statistic and Wilcoxon signed rank test. However, only the pricing error from the BS model for call options in the uptrend period is lower than CSR model.

[Table V is here]

In summary, the overall results indicate that both models tend to overprice call options and underprice put options. Furthermore, the CSR model that assumed stochastic volatility can improve the pricing performance compared to the model that assumed volatility constant during the life of the options.

VII. CONCLUSION

The empirical results of this study suggests that call (put) options prices obtained form the BS and CSR models are likely to be overpriced (underpriced) for overall period. When pricing errors of the BS and CSR models are examined by classifying underlying index's trend into 3 subperiods - downtrend, uptrend and political tension , call (put) options prices from both models tend to be overpriced (underpriced) in the economic downtrend and the political tension period and underpriced (overpriced) in the economic uptrend period. According to the results in economic downtrend and political tension period, both models tend to predict call and put options price in the same way. Both BS and CSR model tend to overprice (underprice) the call (put) options in that period. Furthermore, the CSR model yields the better pricing performance for both call and put options. So if we combine these two periods together, the prediction of pricing error and pricing performance between the two models will be the same. As shown in the table VI, the absolute pricing errors of the CSR model in subperiods yield the better performance than that of the BS model except the case of call options in the uptrend period. For the entire period, however; Cox Square Root model provides better performance in absolute pricing errors for call and put options than that of Black-Scholes model.

[Table VI is here]

In addition, the constant elasticity of variance model that assumed stochastic volatility with $\theta < 2$, which predicted the volatility and stock price move in the opposite way, can improve the pricing performance compared to the model that assumed volatility constant during the life of the options. From our findings, in the case of TXO options, which is observed the negatively relationship between its underlying index and return volatility, Cox Square Root model ($\theta = 1$) yields the better pricing performance compared to the Black-Scholes model.

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Table I

Descriptive Statistics

Table I contains descriptive statistics for Taiwan Stock Index in a daily basis during 2 January 2002 through 25 May 2004. The entire sample is classified into three subperiods, beginning with downtrend market during 2 January 2002 to 30 April 2003; uptrend market during 1 May 2003 to 19 March 2004; and political tension market during 20 March 2004 to 25 May 2004.

a) Descriptive Statistics of Daily Taiwan Stock Index

	Obs.	Mean	Std.	Min	Max	Skewness	Kurtosis
Entire period (02/01/2002 - 25/05/2004)	592	5393.86	753.51	3850.04	7034.10	0.077	-1.050
Downtrend (02/01/2002 - 30/04/2003)	324	5067.48	643.93	3850.04	6462.30	0.451	-0.997
Uptrend (01/05/2003 - 19/03/2004)	221	5674.89	680.88	4187.82	7034.10	-0.354	-0.261
Political tension (20/03/2004 - 25/05/2004)	47	6322.40	394.11	5482.96	6880.18	-0.292	-1.226

b) Descriptive Statistics of Daily Taiwan Stock Index Return (%)

	Obs.	Mean	Std.	Min	Max	Skewness	Kurtosis
Entire period (02/01/2002 - 25/05/2004)	592	0.012	1.621	-6.912	5.485	-0.098	1.543
Downtrend (02/01/2002 - 30/04/2003)	324	-0.090	1.765	-5.949	5.485	0.183	0.457
Uptrend (01/05/2003 - 19/03/2004)	221	0.225	1.095	-2.445	3.497	0.201	0.349
Political tension (20/03/2004 - 25/05/2004)	47	-0.286	2.407	-6.912	5.419	-0.405	1.354

Table II

Moneyness Category Definitions

The table II contains categories, labels and the delta ranges column of the options. The call and put options are classified corresponding delta range. The observations are eliminated when the absolute deltas are below 0.02 and above 0.98.

Category	Labels	Range
1	Deep in-the-money (DITM) call	$0.875 < \Delta_C \leq 0.98$
	Deep out-of-the-money (DOTM) put	$-0.125 < \Delta_P \leq -0.02$
2	In-the-money (ITM) call	$0.625 < \Delta_C \leq 0.875$
	Out-of-the-money (OTM) put	$-0.375 < \Delta_P \leq -0.125$
3	At-the-money (ATM) call	$0.375 < \Delta_C \leq 0.625$
	At-the-money (ATM) put	$-0.625 < \Delta_P \leq -0.375$
4	Out-of-the-money (OTM) call	$0.125 < \Delta_C \leq 0.375$
	In-the-money (ITM) put	$-0.875 < \Delta_P \leq -0.625$
5	Deep-out-of-the-money (DOTM) call	$0.02 < \Delta_C \leq 0.125$
	Deep-in-the-money (DITM) put	$-0.98 < \Delta_P \leq -0.875$

Table III

Percentage Pricing Errors Categorized by Moneyness and Time to Maturity

A comparison of the mean difference between market and model prices as a fraction of model price of the Black-Scholes model and Cox Square Root model for the entire period and subperiods according to moneyness and time to maturity of the options.

Moneyness	Call Options				Put Options			
	Mean of the percentage pricing errors of the BS model (%)		Mean of the percentage pricing errors of the CSR model (%)		Mean of the percentage pricing errors of the BS model (%)		Mean of the percentage pricing errors of the CSR model (%)	
	TTM < 90	TTM >= 90	TTM < 90	TTM >= 90	TTM < 90	TTM >= 90	TTM < 90	TTM >= 90
Panel A: Entire period								
OTM	-4.50	0.68	-2.55	4.13	5.64	9.04	3.71	5.67
ATM	-3.63	3.88	-3.53	4.15	2.85	4.54	2.88	4.74
ITM	-1.22	2.85	-1.61	2.10	1.60	2.28	2.06	3.28
Panel B: Downtrend period								
OTM	-7.20	-2.26	-5.02	2.09	15.28	12.25	12.97	8.34
ATM	-6.81	0.71	-6.76	0.92	7.71	3.81	7.75	3.98
ITM	-4.45	0.11	-4.88	-0.79	5.27	0.81	5.75	1.92
Panel C: Uptrend period								
OTM	0.90	7.68	2.56	10.47	-8.61	8.49	-9.84	5.62
ATM	2.13	7.62	2.34	7.98	-6.66	6.12	-6.51	6.44
ITM	2.63	5.42	2.33	4.81	-4.86	4.45	-4.41	5.31
Panel D: Political Tension period								
OTM	-8.47	-18.14	-6.63	-14.87	11.46	1.57	8.84	-2.53
ATM	-5.87	-5.77	-5.86	-5.87	9.59	2.86	9.22	2.76
ITM	-1.41	-3.28	-1.98	-4.27	6.24	2.10	6.64	3.11

Table IV

Absolute Percentage Pricing Errors Categorized by Moneyness and Time to Maturity

A comparison of the mean absolute difference between model and market prices as a fraction of model prices of the Black-Scholes model and Cox Square Root model for the entire period and subperiods according to moneyness and time to maturity of the options.

Moneyness	Call Options				Put Options			
	Mean of the absolute percentage pricing errors of the BS model (%)		Mean of the absolute percentage pricing errors of the CSR model (%)		Mean of the absolute percentage pricing errors of the BS model (%)		Mean of the absolute percentage pricing errors of the CSR model (%)	
	TTM < 90	TTM >= 90	TTM < 90	TTM >= 90	TTM < 90	TTM >= 90	TTM < 90	TTM >= 90
Panel A: Entire period								
OTM	17.66	17.63	17.42	17.90	21.06	18.65	20.33	17.44
ATM	13.38	13.93	13.30	13.89	14.53	14.12	14.51	14.30
ITM	10.22	10.11	10.22	9.98	11.18	11.10	11.25	11.44
Panel B: Downtrend period								
OTM	19.00	19.29	18.64	19.29	24.09	22.37	22.99	20.90
ATM	14.55	13.66	14.44	13.46	15.24	13.68	15.33	13.90
ITM	10.75	8.77	10.82	8.65	11.32	9.24	11.53	9.61
Panel C: Uptrend period								
OTM	15.85	15.20	16.01	16.32	17.01	16.96	17.03	15.76
ATM	11.63	13.78	11.66	13.90	12.45	16.11	12.43	16.35
ITM	9.57	10.86	9.53	10.71	10.75	14.17	10.68	14.51
Panel D: Political Tension period								
OTM	17.26	22.17	16.43	19.90	21.40	15.32	19.81	14.90
ATM	13.13	16.76	12.94	16.37	17.70	9.93	17.18	9.67
ITM	10.25	12.35	10.15	12.23	11.89	9.89	11.81	10.09

Table V

The Comparison of the Pricing Performance between BS and CSR Model

A comparison of model and market prices for index options listed on the Taiwan Stock Exchange (TWSE). Model prices are obtained using the Black-Scholes and the Cox Square Root models.

Panel A : The percentage difference

Options type	Difference between model and market prices as a fraction of model price (%)		Absolute difference between model and market prices as a fraction of model price (%)		Percentage of differences that are positive
	Mean value	Median value	Mean value	Median value	
Panel A1: Difference between the Black-Scholes model and market prices.					
CALL					
Entire period	-1.82	-2.89	14.30	10.49	42.37%
Downtrend	-5.05	-6.51	15.24	11.33	32.28%
Uptrend	3.48	2.08	12.94	9.48	56.50%
Political tension	-7.22	-7.01	15.25	11.21	33.98%
PUT					
Entire period	4.54	2.23	16.38	11.10	55.50%
Downtrend	9.65	6.33	17.72	11.40	65.61%
Uptrend	-2.42	-3.39	14.81	11.29	42.36%
Political tension	7.35	-0.47	16.24	9.26	58.85%
Panel A2: Difference between the Cox-Square Root model and market prices.					
CALL					
Entire period	-0.85	-2.30	14.23	10.42	43.72%
Downtrend	-3.95	-5.79	15.09	11.12	33.79%
Uptrend	4.30	2.68	13.14	9.71	57.74%
Political tension	-6.27	-5.79	14.55	10.90	35.00%
PUT					
Entire period	3.52	1.59	15.98	10.96	53.65%
Downtrend	8.59	5.83	17.26	11.18	63.90%
Uptrend	-3.27	-3.79	14.56	11.16	40.43%
Political tension	5.94	2.15	15.53	9.16	56.75%

Panel B: The difference between the pricing errors of the Black-Scholes and the Cox Square Root models

Options type	Mean difference between percentage absolute pricing errors (%)	Median difference between percentage absolute pricing errors (%)	Percentage of differences in absolute pricing errors that are positive	*t*-statistic for the two sample test of difference in absolute pricing errors	Wilcoxon signed-rank test *p*-value
CALL					
Entire period	0.07	0.10	53.32%	2.89 *	0.000
Downtrend	0.15	0.16	54.83%	3.63 *	0.000
Uptrend	-0.20	-0.05	48.68%	-7.58 *	0.000
Political tension	0.70	0.48	63.68%	11.04 *	0.000
PUT					
Entire period	0.40	0.05	51.38%	15.90 *	0.000
Downtrend	0.46	0.01	50.25%	11.89 *	0.000
Uptrend	0.25	0.05	51.70%	7.93 *	0.049
Political tension	0.70	0.23	54.97%	7.03 *	0.000

* indicates significance at the 1 percent level.

Table VI

The Conclusion of the Pricing Performance between BS and CSR Model

According to the model comparison for index options listed on the Taiwan Stock Exchange (TWSE), the appropriate model which yields the better performance classified by period and time to maturity.

Options type \ Period	Call		Put	
	TTM < 90	TTM >= 90	TTM < 90	TTM >= 90
Overall	CSR	CSR	CSR	CSR
Downtrend	CSR	CSR	CSR	CSR
Uptrend	BS	BS	CSR	CSR
Political Tension	CSR	CSR	CSR	CSR

Figure I

Taiwan Stock Index Spot

Taiwan Capitalized Weighted Stock Index (TAIEX) spot from January 2002 to May 2004.

Figure II

Taiwan Stock Index Return

Taiwan Capitalized Weighted Stock Index (TAIEX) return from January 2002 to May 2004.

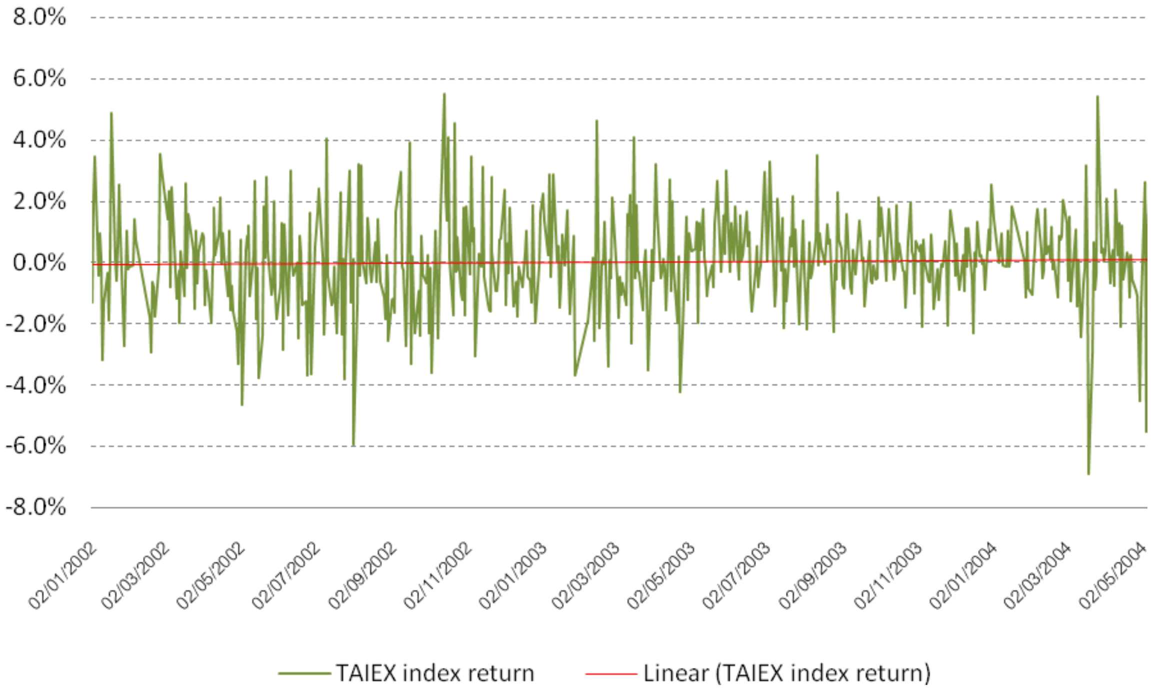


Figure III

Taiwan Stock Index and its realized return volatility

The relationship between Taiwan Capitalized Weighted Stock Index (TAIEX) and its realized return volatility.

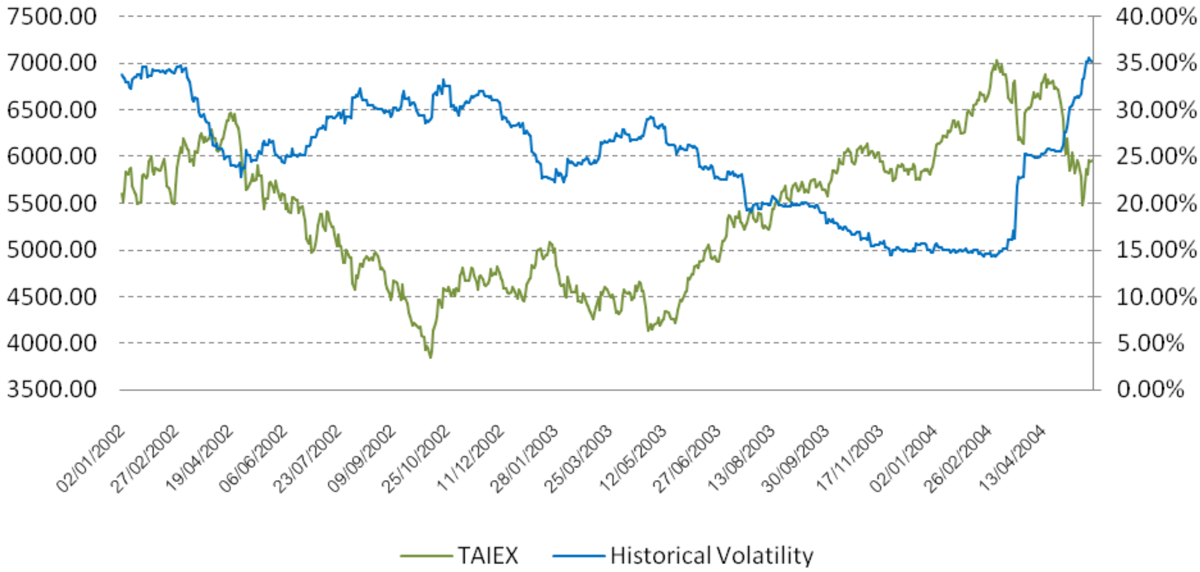
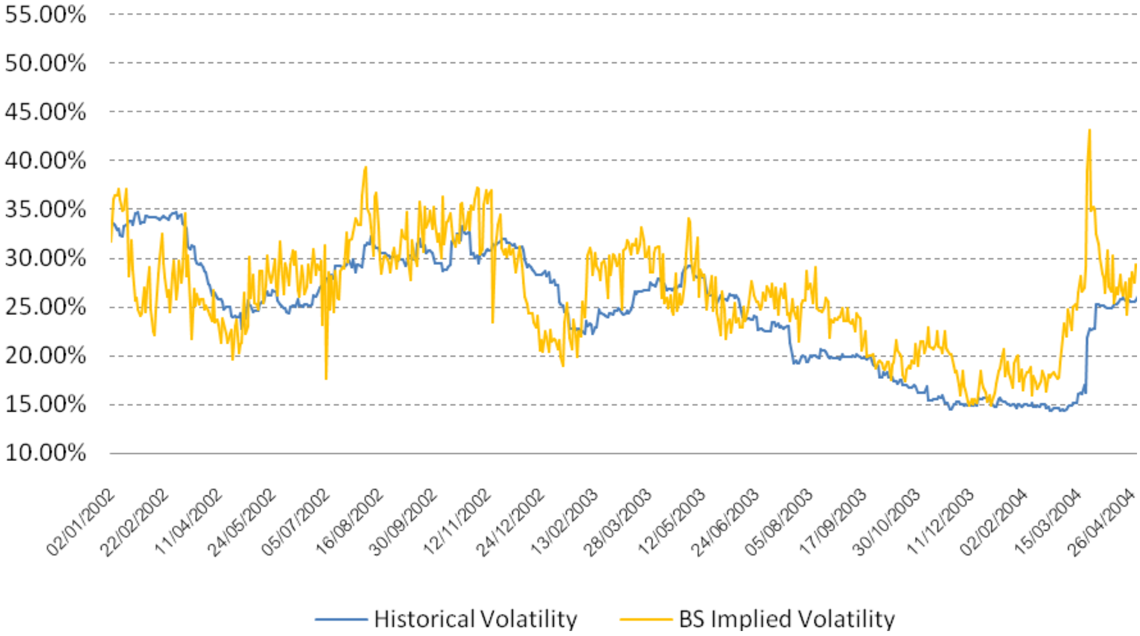


Figure IV

Taiwan Stock Index's realized return volatility and implied volatility

a) The relationship between Taiwan Capitalized Weighted Stock Index (TAIEX) realized return volatility and its average implied volatility of at-the-money options.



b) The trend of the realized return volatility and its average implied volatility of at-the-money options.