

# Implied Volatility and Smile Phenomenon

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**Abstract:** The paper examines the implied volatility of the options and explains the smile phenomenon without rejecting the option pricing model. Furthermore, it shows that the market uses more factors with respect to the historical volatility (ex-ante) to determine the instantaneous volatility of the underlying of an option and that when we use the effective volatility (ex-post) we get the effective market price of the options. Moreover, it determines the expectation of the market such that to get an investment strategy.

The first empiric test on the option pricing model was made by Black-Scholes(1972), they showed that if you consider the effective volatility (ex-post) to make the hedged portfolios gives profits but once you consider the transaction cost the profits disappear. Instead, when they used the historical volatility (ex-ante) to make the hedged portfolios gave loss.

This result shows that the market use more factors to value the volatility and that if you use the effective volatility you get the effective market price of the options.

All this suggests us that we can get the implied volatility by calibration to the market price, formally:

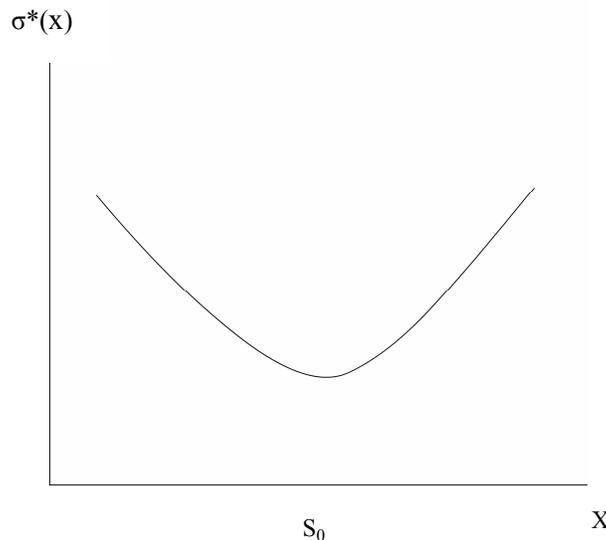
$$W_c(S,T,X, r, \sigma^*) = W_c^*(S,T,X, r, \sigma)$$

Where:

$W_c^*(S,T,X, r, \sigma)$  denotes the effective market price of the Call option on the underlying  $S$ , maturing at time  $T$  and with exercise price  $X$ , given the interest-rate  $r$ .

Solving the equation for  $\sigma^*$ , given the others factors  $S$ ,  $T$ ,  $X$ ,  $r$ , we get the implied volatility of the market.

Now, if the option pricing model is right the implied volatility must be constant with respect to the variation of the exercise price, but it is not the case because we observe the following relation usually called *Smile phenomenon*:



This result can bring us to reject the option pricing model, but we can note that the phenomenon is first a decreasing function and after an increasing function with minimum around  $X = S_0$ , or alternatively, it is an increasing function of the following pay-off :

$$\Pi(x) = \text{Max} [ S_0 - X, X - S_0 ]$$

It is easy to see that the first side is the initial pay-off of a Call option, and that the second side is the initial pay-off of a Put option. In fact, the initial pay-off of an option (Call and Put) affects the probability that the option will be in-the-money at expiration time. Moreover, it is obvious to expect that the more sensitive to the initial pay-off are the sellers of the options because they are exposed to an unlimited loss and to limited earnings. Thus, we can try to explain the distortions of the model changing some assumptions.

We know that in the option pricing model none assumption is made about the market preference because it is based on the hedging relation that permits us to get a risk-free portfolio in a dynamic way. We can note that the hedging relation is determined in a probabilistic context and that it is based on the assumption that the underlying follows a continuous process with possibility to trade in every instant. Thus, if we assume that the underlying can have a jump process, as it is in reality, we can note that the hedging relation is just approximate.

A way to avoid this problem is to make a large number of such portfolios, but in reality it is improbable that investors do that because there is not profit once you consider the transaction cost. Thus, we can accept that the price moves in a tunnel without removing the assumption that the market is competitive. The width of the tunnel is given by the risk-aversion of the market. Instead, its limits are given by the amount of the transaction cost. Now it is obvious to assume that the market is adverse to the risk with a local concavity around:

$$X = S_0$$

This means that the market wants an increasing premium to the risk with respect to the grade that the initial pay-off is in-the-money. This is consistent with the empiric test of MacBeth-Merville('79). In fact, they showed that the price of the model are in mean less than the market price of the options in-the-money and that the grade of the underestimate increases with the grade that the options are in-the-money and with the time of maturity. Now we can note that the parity-relation Put-Call to be valid doesn't need assumption about the distribution of the underlying and of the interest-rate. Hence, we can consider it a condition of pure arbitrage much stronger than the hedging relation. In fact, the consistent of the parity-relation was showed by the empiric test of Galai('77) and Klemkosky-Resnick('79).

Thus, for  $X < S_0$  the market of the Call options leads the market of the Put options, while, for  $X > S_0$  the market of the Put options leads the market of the Call options.

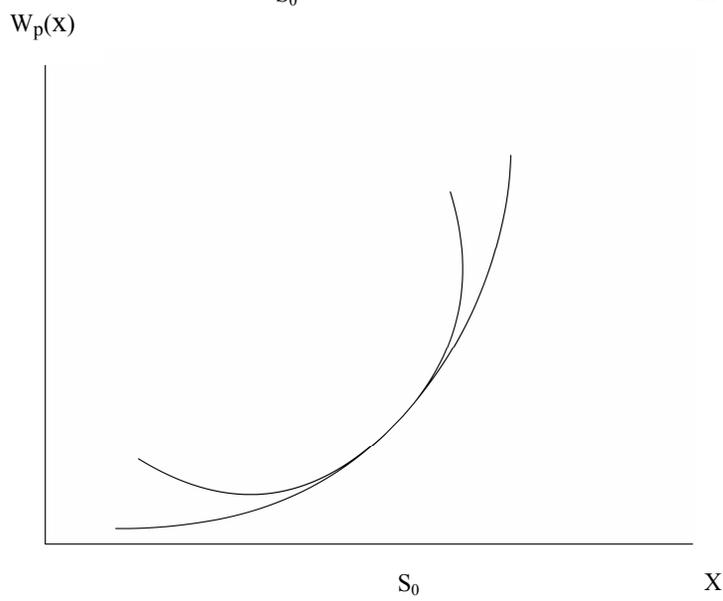
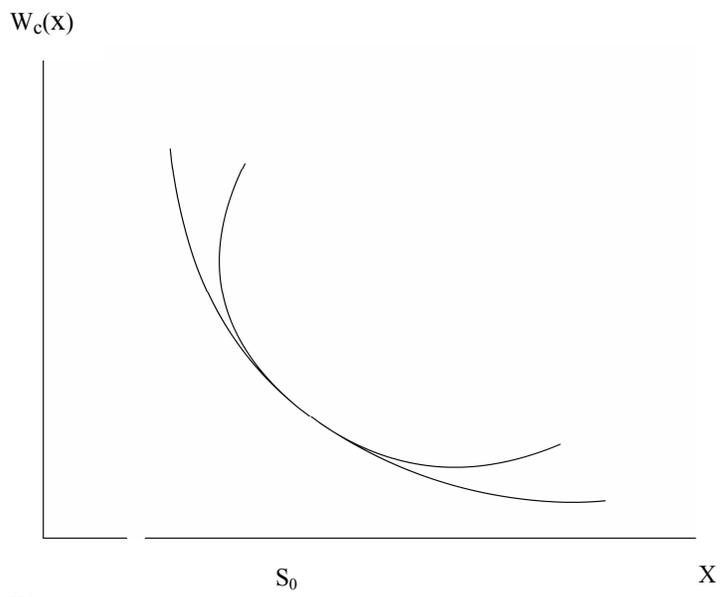
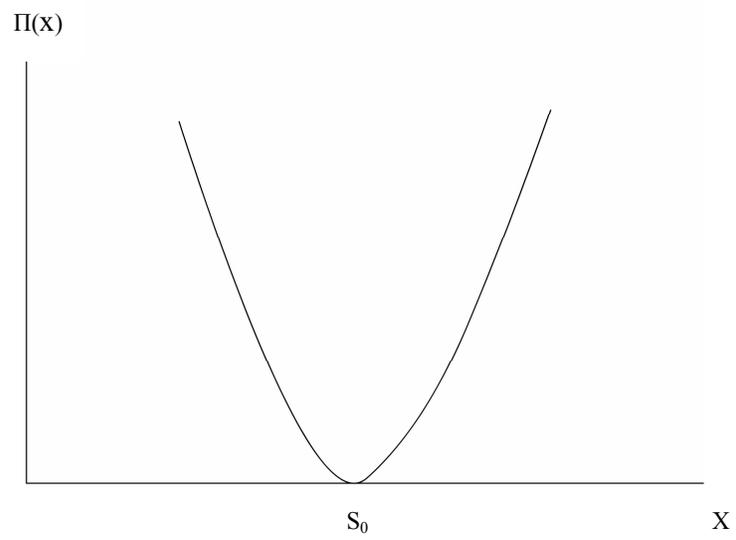
We have assumed that in the market prevail the requests of the sellers and that the buyers accept them because increases the probability to exercise the options.

We can expect that the point where the market is more neutral to the risk is when the initial pay-off is at-the-money:

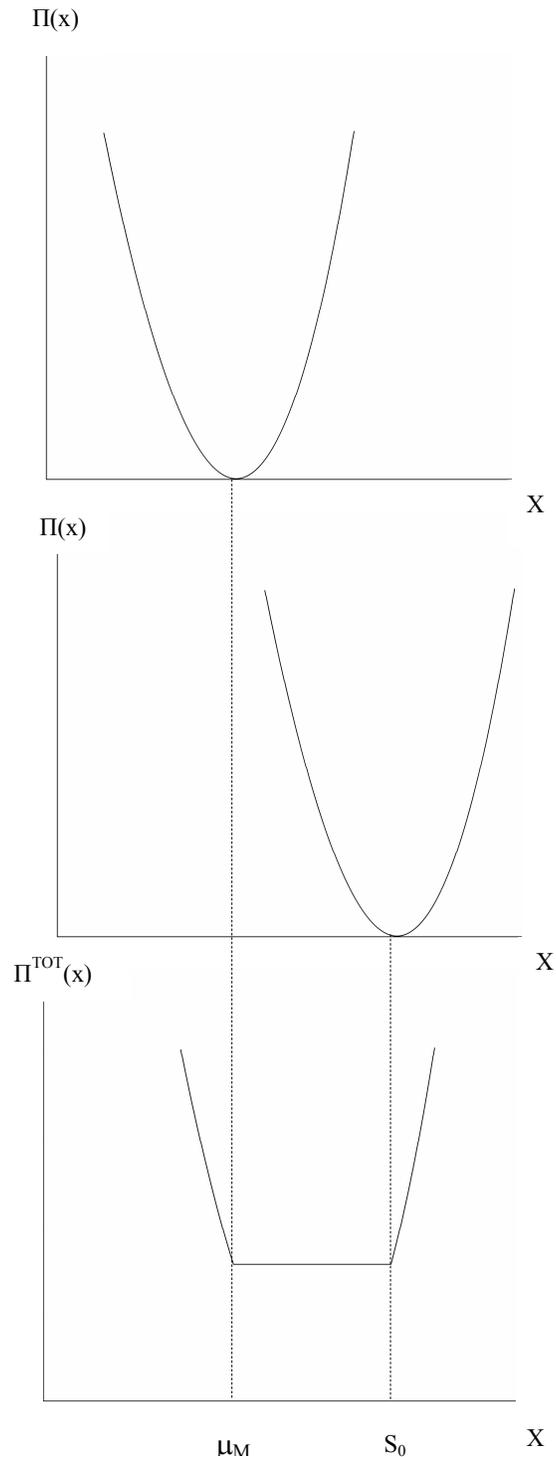
$$X = S_0$$

Thus, for  $X = S_0$  we can get the implied volatility, or its best approximation, but we have to observe that the utility function of the market can be determined only by the market preference for each maturity. In fact, Bhattacharya('80) showed that the option pricing model tends to overestimate the option at-the-money near to expire. Whereas, MacBeth-Merville('79) showed the overestimation of the options out-of-money with time of maturity less than ninety days.

Anyway, if we assume that the market is neutral to the risk when the option is at-the-money, we get the following representation:



Another assumption in the option pricing model is that the distribution of the underlying is independent from the level of the current price. Thus, it doesn't consider the inefficiency of the market in the period of Bull&Bear. Anyway, we can include this case in the model by using the mean-reverting-process because as we know the hedging relation permits us to consider the spot-rate like *drift*, but if the hedging relation is just approximate we can note as it is important for an investor the level of the current price with respect to the moving mean. Thus, we can include in the model this case by assuming that there is another source of risk that it is an increasing function of the following pay-off:  $\Pi(x) = \text{Max}[ \mu_M - X , X - \mu_M ]$ , where  $\mu_M$  is the moving mean. Thus, we have the following representation:



Putting together the two sources of risk we got a model that considers the spread between the current price and the moving mean too. We have to note that we have assumed the symmetry of the market preference, so the risk premium requested for a spread from the sellers of the Call options is the same of that requested from the sellers of the Put options. If it is not the case we have a point of minimum in  $S_o$ , or in  $\mu_M$ , with a flexion in  $\mu_M$ , or in  $S_o$ . We can note that the risk premium is an increasing function of the spread between the current price and the moving mean. Once again we can get the implied volatility, or its best approximation, when:

$$X = S_o = \mu_M .$$

We remind that the effective position and shape of the utility function is given by the market preference. In fact, the possibility to get the implied volatility is subordinated to the assumption that the market is neutral to the risk when the option is at-the-money, a fact that can be just proved ex-post. Anyway, we got an intuition that can help us to better understand what there is beyond the smile phenomenon without rejecting the option pricing model.

We can note that if the market expects that the price of underlying will revert to the moving mean it will revert because it is the market that makes the price. At same time, we can expect that the sellers of the Put options believe in an increasing of the price. Thus, for greater value of the exercise price the risk premium increases more and the buyers agree so that they pay more such that we get a resistance price in the flexion because the market doesn't believe that the price of underlying will be greater. Hence, the same result can be getting for the sellers of the Call options that believe in a decrease of the price of underlying such that we get in the flexion a support price because the market doesn't believe that the price can be less. We can note that the two approaches bring to the same investment strategy. In fact, we must buy when the underlying price is near to the flexion on the left and to sell when it is near to the flexion on the right. We definitely conclude by noting that the paper permits to determine the effective future volatility of the underlying of an option and the expectation of the market such that we can get an investment strategy.

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