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Author(s): David C. Emanuel and James D. MacBeth

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Further Results on the Constant Elasticity of Variance Call Option Pricing Model

David C. Emanuel and James D. MacBeth

I. Introduction

The Black-Scholes [4] call option model is a member of the class of constant elasticity of variance call option models proposed by Cox [6]. While the Black-Scholes model assumes that the volatility or instantaneous variance of return is constant through time, the other members of the class allow the volatility to change with the stock price. This property is of interest because empirical evidence suggests that returns to common stock are heteroscedastic and also that volatilities, implied from the Black-Scholes model and market prices of call options, are not constant.¹

From an analysis of 1976 data on call options written on six stocks, MacBeth and Merville [10] find that other members of the class of constant elasticity of variance models yield prices which correspond more closely to market prices than do prices from the Black-Scholes model. The purpose of this research is to generalize the class of constant elasticity of variance call option models by relaxing a constraint on one of the parameters of the model and to subject this class of call option models to additional empirical analysis.

II. The Constant Elasticity of Variance Model

The class of constant elasticity of variance diffusion processes can be described by the stochastic differential equation:

Both authors, The University of Texas at Dallas. The authors would like to thank John Cox for helpful discussions concerning the theoretical derivation. Cox has independently derived equation (10) in unpublished work.

¹See Blattberg and Gonedes [5], Rosenberg [12], Schmallensee and Trippi [15], and Black [1], [2], [3].

$$(1) \quad dS = \mu S dt + \delta S^{\theta/2} dz$$

where:

dS is the change in the stock price S over the short increment of time dt ;
 μ , δ , and θ are constants; and
 dz is a Wiener process.

The standard deviation of return, or volatility, at any instant of time t is

$$(2) \quad \sigma_t = \delta \cdot S_t^{(\theta-2)/2}.$$

The elasticity of the instantaneous variance of the stock price with respect to the stock price equals θ . When θ equals two, the volatility, σ_t , is a constant δ , and market prices of call options are given by the Black-Scholes model. Cox [6] finds the equilibrium value of a call option for values of θ less than two. For values of θ greater than two, the process can also be analyzed and equilibrium option values determined. The analysis closely follows that developed in Cox [6], Cox and Ross [7], and based on theory due to Feller [8]. For mathematical reasons and because of different boundary behavior, the analysis is not identical and the final formula with $\theta > 2$ differs from the formula for $\theta < 2$. The key steps and logic are explained briefly here.

Let r be the riskless rate of interest and α be the continuous dividend yield (later set to zero). The option price C will be a continuous, twice-differentiable (at interior points) function of stock price S . Riskless hedging will thus be available in continuous time. A differential equation for C can be obtained which does not depend on μ (in equation (1)):

$$(3) \quad \frac{1}{2} \delta^2 S^{2\theta} C_{SS} + (rS - \alpha S) C_S - rC = -C_t.$$

Since equation (3) does not depend on μ , we may assume (if it happens to be convenient) that μ has the value $r - \alpha$, which would imply that the expected return on S is the riskless rate. Then, instead of solving equation (3) directly, we can value the option by discounting its expected terminal value at the riskless rate, since options on securities that do not earn risk premiums likewise earn no reward for risk-bearing. This can be done by finding $f(S_T, T; S_t, t)$, the transition density function for the stock price being S_T at time T , conditional on having previously been S_t at time t .

As in Cox [6], substitute $y = S^{2-\theta}$. This is a well-defined transformation when $\theta > 2$, but care must be taken to remember that it is an inverse

transformation ($dy/dS < 0$) and that the inaccessible boundary at $S=0$ corresponds to $y = \infty$. y has an infinitesimal mean function of $(r-\alpha)(2-\theta)y + 1/2\delta^2(\theta^2 - 3\theta + 2)$ and variance function of $\delta^2(2-\theta)^2y$. Switching to the notation of Feller [8] we define:

$$\begin{aligned} a &= 1/2 \delta^2 (2-\theta)^2; \quad (\delta > 0, \text{ of course}) \\ b &= (r-\alpha)(2-\theta); \text{ and} \\ c &= 1/2 \delta^2 (\theta-1)(\theta-2) \quad (\theta > a). \end{aligned}$$

Because $c > a$, the process is qualitatively different from the situation when $\theta < 2$, which implies $c < a$. Feller gives the Laplace transform of the density function for the case where $c > a$:

$$\begin{aligned} (4) \quad \omega(\tau, q) &= \left[\frac{b}{qa(e^{b\tau}-1)+b} \right] c/a \quad \Pi \left[\frac{qbe^{b\tau}}{qa(e^{b\tau}-1)+b} \right] \\ (5) \quad &= \left[\frac{b}{qa(e^{b\tau}-1)+b} \right] c/a \quad e^{-\left[\frac{\xi qbe^{b\tau}}{qa(e^{b\tau}-1)+b} \right]} \end{aligned}$$

where:

q is the transforming variable;

τ is used instead of $T-t$;

$\Pi(\cdot)$ is a functional form; and

ξ is the value of y corresponding to S_t at time t ($=S_t^{2-\theta}$). S_t is known at time t and its density function is thus concentrated at a point. (This implies a density function transform of $e^{-\xi q}$ when $\tau=0$.)

$\omega(\tau, q)$ is the Laplace transform of:

$$(6) \quad \left[\frac{b}{a(e^{b\tau}-1)} \right] c/a \quad e^{-\left[\frac{\xi be^{b\tau} + by}{a(e^{b\tau}-1)} \right]} \left[\frac{ya^2(e^{b\tau}-1)^2}{\xi b^2 e^{b\tau}} \right] I_{\frac{c-a}{a}} \left[2 \sqrt{\frac{\xi b^2 y e^{b\tau}}{a^2 (e^{b\tau}-1)^2}} \right]$$

where $I_{\frac{c-a}{a}}(\cdot)$ is the modified Bessel function of the first kind of order $\frac{c-a}{a}$.

The density function (6) can be transformed into the density function in terms of the original variables. This is done by substituting for ξ , y , a , b , and c in (6) and multiplying by $\frac{-dy}{dS}$. (This is a standard operation in changing variables for density functions. $\frac{-dy}{dS}$ is used instead of $\frac{dy}{dS}$ because y is an inverse function of S .) We thus obtain:

$$(7) \quad f(S_T, T; S_t, t) = (\theta-2) K^{\frac{1}{2-\theta}} e^{-Z-X} I_{\frac{1}{\theta-2}}(2\sqrt{XZ}) (XZ)^{1-2\theta} \frac{1}{2(2-\theta)}$$

where:

$$K = \frac{2(r-\alpha)}{\delta^2(2-\theta)(e^{(r-\alpha)(2-\theta)\tau}-1)} ;$$

$$X = K S_t^{2-\theta} e^{(r-\alpha)(2-\theta)\tau} ; \text{ and}$$

$$Z = K S_T^{2-\theta} .$$

Integration of the density function in equation (7) reveals that it has a mean that is not equal to $S_t e^{(r-\alpha)\tau}$. This is the mean that it should have, given the dynamics of equation (1) in a risk-neutral world. The problem is caused by the explosive nature of the stochastic process when $\theta > 2$. Even though infinite values of S are inaccessible in finite time, according to Feller's classification of boundaries, the density function has a fat upper tail. Because of this, the space inversion $y = S^{2-\theta}$ can create problems due to its singular nature at $y=0$. This is not serious, as we could, in principle, obtain a density function like equation (6) with a boundary imposed at $y=\epsilon$. Since a boundary at $y=0$ is inaccessible, imposing a boundary at ϵ will change equation (6) immeasurably (in the limit as ϵ tends to zero). This means that the density function (equation 7) expressed in terms of S would also be immeasurably changed by the use of a boundary at $y=\epsilon$. However, since y values close to zero correspond to S values that are very large, immeasurable differences in the tail behavior of the density function equation (7) can have a significant effect on the mean. This problem can be avoided by careful use of the density function equation (7). Integrals should always be taken over domains that do not include $+\infty$. This is done below in deriving equation (8).

The foregoing discussion is necessarily a bit mysterious. The accessibility of infinite boundaries is not adequately treated in the standard mathematics texts. As further verification of the analysis, the pricing differential equation (3) was solved numerically. The numerical solutions obtained are essentially identical to the analytical solutions derived from the density function equation (7).

The value of a binding commitment to pay E for one share at time T is simply:

$$S_t e^{-\alpha\tau} - E e^{-r\tau} .$$

The first term represents the current stock price S_t , adjusted for the attenuation caused by dividends, to rate α for time τ . The second term is simply

the present value of the exercise price.

An option to pay E for one share at time T is worth more than a binding commitment. The difference is the present value of the money saved when the option is not exercised. This difference is worth:

$$\int_0^E f(S_T, T; S_t, t) e^{-r\tau} (E - S_T) dS_T$$

and, thus, the total value of the option is given by:

$$(8) \quad C = S_t e^{-\alpha\tau} - E e^{-r\tau} + \int_0^E f(S_T, T; S_t, t) e^{-r\tau} (E - S_T) dS_T.$$

The integration in equation (8) is facilitated by substituting Z for S_T . Then we obtain:

$$(9) \quad C = S_t e^{-\alpha\tau} - E e^{-r\tau} + \int_{KE}^{\infty} \frac{1}{Z} \frac{1}{2(2-\theta)} e^{-X-Zr\tau} \frac{1}{(2\sqrt{XZ})} \left(\frac{Z}{X} \right)^{\frac{1}{2-\theta}} - E dZ$$

where the change in limits again comes from the fact that Z varies inversely with S_T . Equation (9) can be broken up and reduced to:

$$(10) \quad C = S e^{-\alpha\tau} \left[1 - \sum_{n=0}^{\infty} \frac{e^{-X} X^{n+\frac{1}{\theta-2}} G(KE^{2-\theta} | n+1)}{\Gamma(n+1+\frac{1}{\theta-2})} \right] - E e^{-r\tau} \left[1 - \sum_{n=0}^{\infty} \frac{e^{-X} X^n G(KE^{2-\theta} | n+1+\frac{1}{\theta-2})}{\Gamma(n+1)} \right].$$

Inspection of equation (10) reveals that the option price does not tend to zero as the exercise price tends to infinity. This is again a result of the fat tail behavior discussed earlier. No matter how large the exercise price, there is some small probability of S exceeding it and that eventuality contributes significantly to the mean of S. Thus, an option with virtually zero probability of being exercised has a nonzero price.² Equation (10) is restated below for the no-dividend case when $\alpha=0$. Included is the statement

²This is not as paradoxical as it sounds. For example, consider the standard Black-Scholes call option scenario. Let $\sigma^2 = 0.2$ and $r=0.05$. In a risk-neutral setting, it is easy to show that the stock price S will almost surely eventually fall below any exercise price. Thus, a very long-lived option will almost surely expire worthless. Yet, it is well known that a perpetual call option on a stock that pays no dividends is worth the same as the stock.

of Cox's formula for $\theta < 2$.

$$(11) \quad C = S \cdot N_1 - Ee^{-r\tau} N_2$$

$$(12-a) \quad N_1 = \sum_{n=0}^{\infty} g(S'|n+1) \cdot G(E'|n+p) \quad ; \theta < 2$$

$$(12-b) \quad = 1 - \sum_{n=0}^{\infty} g(S'|n+p) \cdot G(E'|n+1) \quad ; \theta > 2$$

$$(13-a) \quad N_2 = \sum_{n=0}^{\infty} g(S'|n+p) \cdot G(E'|n+1) \quad ; \theta < 2$$

$$(13-b) \quad = 1 - \sum_{n=0}^{\infty} g(S'|n+1) \cdot G(E'|n+p) \quad ; \theta > 2$$

where:

$$(14) \quad S' = \left[\frac{2re^{r\tau(2-\theta)}}{\delta^2(2-\theta)(e^{r\tau(2-\theta)} - 1)} \right] \cdot S^{2-\theta}$$

$$(15) \quad E' = \left[\frac{2r}{\delta^2(2-\theta)(e^{r\tau(2-\theta)} - 1)} \right] \cdot E^{2-\theta}$$

$$g(x|m) = \frac{e^{-x} x^{m-1}}{\Gamma(m)} \quad ; \text{ the gamma density function}$$

$$G(x|m) = \int_x^{\infty} g(y|m) dy$$

$$p = 1 + \frac{1}{|2-\theta|} .$$

For values of θ less (greater) than two, the volatility, σ_t , increases (decreases) as the stock price decreases. When θ is less (greater) than two, the Black-Scholes model price, with the correct volatility, σ_t , given by equation (2), will be greater (less) than the market price for out-of-the money options and less (greater) than the market price for in-the-money options. As long as the Black-Scholes model has the correct volatility, σ_t , it will yield the market price for an (approximately) at-the-money option. Thus, in a constant elasticity of variance world, the constant elasticity of variance call option model yields prices that are better than Black-Scholes prices for two reasons. First, the model corrects the mispricing of out-of-the money and in-the-money options; and, second, it predicts changes in volatility.

There are economic scenarios that are consistent with all values of θ .³

³See Geske [9], Rubinstein [13], and Thorpe [15].

III. Previous Research

MacBeth and Merville [10] analyzed daily closing prices of all call options written in 1976 on American Telephone and Telegraph, Avon Products, Eastman Kodak, Exxon, International Business Machines, and Xerox. Each day they computed the Black-Scholes implied volatility of an at-the-money option as the "true" volatility, since in any constant elasticity of variance world the Black-Scholes model (approximately) correctly prices at-the-money call options. They found that for their sample of long-maturity option prices the Black-Scholes model, with this "true" volatility, systematically underpriced in-the-money options and overpriced out-of-the money options (and by construction, correctly priced at-the-money options). For their data, constant elasticity of variance models with parameter θ values ranging from -4 . for IBM to $+1$. for Xerox yielded prices that fit market prices better than Black-Scholes model prices. Since MacBeth and Merville used their complete sample of data to estimate a value for θ , their tests could be interpreted as favoring the constant elasticity of variance model. In our comparison of the two models, we will not "look ahead." Estimates of model parameters will always be based upon currently available data.

IV. Predictive Comparison of the Black-Scholes and Constant Elasticity of Variance Models

A. The Data

We expand the MacBeth-Merville data to include daily closing prices of call options written on the same six stocks for each day in 1978. Option prices and prices of the underlying stocks are taken from the *Wall Street Journal*. Dividend information comes from Standard and Poor's Stock Record. We use call money rates as our riskless return.⁴ These rates are changed weekly.

B. Estimation of the Black-Scholes Model Parameter: The Implied Volatility of an At-The-Money Option

To compare the predictive power of the Black-Scholes and constant elasticity of variance models, we use parameters estimated as of day t along with the stock price of day $t+k$ to predict option prices on day $t+k$. For Black-Scholes model predictions, the implied volatility of an at-the-money option on day t is used as the estimate of σ for day $t+k$.

For each stock and each trading day in 1976 and 1978, we calculate the

⁴We have also used treasury bill rates for the riskless return and obtained results virtually identical to those we report.

Black-Scholes implied volatility of all call options with at least 90 days until expiration and from these implied volatilities we impute the volatility of an at-the-money option. We exclude options with less than 90 days to expiration, because as expiration approaches, option prices become rather insensitive to volatility and this causes implied volatilities to fluctuate a good deal with small changes in stock and/or option prices. Moreover, prices of options near expiration appear to exhibit effects of taxes and transaction costs and, for now, we wish to avoid these complications.

One complication which we handle explicitly is the possibility of early exercise caused by dividends. In calculating implied volatilities, and later in computing Black-Scholes option prices, we use Roll's [12] exact model for all options with one dividend remaining. For options written on Avon, IBM, Eastman Kodak, and Xerox, we find no evidence that option prices reflect a positive probability of early exercise. That is, the implied volatilities from the original Black-Scholes model and Roll's model are equal. Thus, for options written on these stocks, we can safely account for dividends by subtracting their present value off the stock price.

The prices of a good number of in-the-money Exxon options in 1978 and ATT options in 1976 and 1978 do reflect the possibility of early exercise and it appears that a number of these options should have been exercised prior to an ex-dividend date. We were surprised to find that prices of a number of ATT and Exxon options with two dividends remaining also reflect the possibility of early exercise. Since the calculation of implied volatilities is an iterative process and Roll's exact price for options with two dividends remaining requires computation of trivariate normal probabilities, we have not computed exact implied volatilities for these options in interest of economy. Consequently, the implied volatilities from about 500 of the (approximately) 4,000 ATT option prices and 200 of the (approximately) 4,000 Exxon option prices are too big and we have done only limited analysis of these data.

Since we seldom observe an option that is exactly at-the-money, we generally have to infer a value for the implied volatility of an at-the-money option. Our estimate is the estimated intercept from a regression of implied volatilities on the difference between the stock price and the present value of the exercise price expressed as a percent of the present value of the exercise price. We have used other measures such as linear interpolation between the implied volatilities of the nearest in-the-money and out-of-the-money options and we find virtually no difference in our results.

C. Estimation of the Parameters of the Constant Elasticity of Variance Model

The implied volatility of an at-the-money option can also be used to estimate the parameters of the constant elasticity of variance model because it is a reasonable approximation of the true volatility on any day regardless of the value of θ . Thus, for any value of θ , equation (2) can be used with the current stock price and the implied volatility of an at-the-money option to find the corresponding value of δ . We select the "best" value of θ on each day by searching integer values to find the value that minimizes the squared deviation between model prices and market prices of options with at least 90 days to expiration. Admittedly, this approach is not optimal. Moreover, since at the present time the constant elasticity of variance model does not have a counterpart to Roll's exact dividend model, it will not yield accurate prices of ATT and 1978 Exxon options. This source of error coupled with the low volatility of ATT caused the estimated values of θ for ATT to behave erratically. Thus, our Black-Scholes model prices and constant elasticity of variance model prices for ATT and for 1978 Exxon options are not reliable.

D. Predicted Prices

To compare the predictive power of each model, we use parameter (σ , θ , δ) estimates calculated as of day t , the riskless rate as of day t , and the stock price as of day $t+k$ to predict option prices on day $t+k$. We report results for k equal to 0 days, 1 day, 5 days (one week), and 17 days (one month). For each value of k greater than zero, the prediction intervals do not overlap. In order to obtain a reasonable number of one-month predictions, we use parameter estimates for day zero to predict prices for days 18 through 22; the next day 0 is the day following day 22.

The root-mean-squared forecast errors for various option categories and values of k are reported in Tables 1 through 4. For short-term predictions of 0 or 1 day, the constant elasticity of variance model yields more accurate predictions in nearly all cases. The improvement over Black-Scholes predictions is greater in 1976 than it is in 1978 and is, in general, more pronounced for long maturity options.

The superiority of the constant elasticity of variance model wanes as the prediction interval increases. In general, for prediction intervals of one month or longer, the Black-Scholes model does as well as the constant elasticity of variance model. The explanation of these results can be found in the time series behavior of the estimated parameters of our models.

V. Analysis of the Time Series Behavior of the Parameters of the Black-Scholes and Constant Elasticity of Variance Models

At this juncture, our objective is to analyze the time series of parameter values in hope of providing clues as to why the Black-Scholes model, in particular, and the class of constant elasticity of variance models, in general, is not a completely satisfactory model of call option pricing.

Since we know the parameter estimates for ATT and Exxon data contain errors induced by imprecise adjustments for dividends, we exclude these data from further analysis.

A. Implied Volatility

The daily implied volatility for the four remaining stocks as of each Wednesday is plotted in Figures 1 and 2 for 1976 and 1978, respectively. The plots show not only that implied volatilities change, but also that there is a good deal of covariability between implied volatilities of different stocks. The constant elasticity of variance model attempts to explain these changes in volatility via changes in the stock price.

To obtain a measure of the ability of the constant elasticity of variance model to explain these changes, we regress the actual daily percent change in implied volatility, $\hat{\sigma}_t$, on the daily percent change predicted by the constant elasticity of variance model from equation (2). The estimated correlation coefficients from these regressions are reported in Table 5. The constant elasticity of variance model does have some ability to predict change in volatility in 1976, explaining between 9 percent and 24 percent of the variability in percent changes in volatility.⁵ In 1978, the power of the constant elasticity of variance model to predict percent changes in volatility is essentially zero. Two of the four correlation coefficients are negative and all four are small. This is disturbing because prediction of future volatility is one reason why the constant elasticity of variance model yields better predictions (conditional upon the future stock price) of future option prices. This change in the ability of the constant elasticity of variance model to explain changes in volatility coincides with a change or shift in the values of θ .

⁵These numbers actually overstate the power of the constant elasticity of variance model because the parameters θ and δ are allowed to change daily in computing predicted percent changes in volatility.

B. Estimated Values of θ

The time series of estimated values of θ are reported in Figures 3 and 4.⁶ These figures show clearly that θ is not constant and also that there is positive covariability of the values of θ across stocks. For almost all of 1976, the estimated values of θ are less than two. Equation (2) predicts that during this period volatility will increase as the stock price declines and the evidence in Table 5 suggests that these predictions are to some extent valid. In 1978, estimated values of θ rise in April from the neighborhood of two and then decline to the neighborhood of two in November. When θ equals two, equation (2) predicts that volatility will not change; however, when θ exceeds two, equation (2) predicts that volatility will increase as the stock price increases. The bottom row of Table 5, labeled 1978 ($\theta > 2$), contains correlation coefficients between predicted percent changes in volatility and actual percent changes in volatility estimated over those days when the estimated value of θ exceeds two. These correlations are all more negative than those estimated over all days in 1978.

The explanation of these results can be seen in Table 6. Percent changes in volatility are negatively correlated with percent changes in stock prices in both years. In 1976, the constant elasticity of variance model predicts negative correlation, but in 1978 it predicts positive correlation. For much of 1978, the constant elasticity of variance model's predictions of future volatility are worse than the Black-Scholes predictions of constant future volatility. This explains why the superiority of the constant elasticity of variance model predicted prices, relative to Black-Scholes predicted prices, is less pronounced in 1978 than it is in 1976.

VI. Summary and Conclusions

Although our relatively small sample of option prices leaves the generality of our empirical results in question, the consistency of our results across options on different stocks is encouraging.

The constant elasticity of variance model implies that the Black-Scholes call option model will (approximately) correctly price at-the-money call options and, depending upon parameter values, it will either consistently overprice in-the-money options and underprice out-of-the-money options or vice versa. Assuming that the Black-Scholes model correctly prices long maturity in-at-the-money call options, we find periods in which it underprices in-the-money

⁶The plots are as of each Wednesday and are based upon an average of the values of θ over a two-week period centered at Wednesday.

options and overprices out-of-the-money options as well as periods when it overprices in-the-money options and underprices out-of-the-money options. This changing nature of the mispricing is consistent across the stocks in our sample. Thus, a constant elasticity of variance model with stationary parameters does not appear to be able to explain the mispricing of call options by the Black-Scholes model.⁷

At any point in time, one can fit a constant elasticity of variance model to option prices and, in general, predict future option prices, conditional upon future stock prices, better than one can with the Black-Scholes model provided that the prediction period is less than one month. In a constant elasticity of variance world, these predictions are better than Black-Scholes predictions for two reasons. First, the parameter θ is essentially an extra parameter which allows the model to capture systematic mispricing by the Black-Scholes model for a given level of volatility. The other reason stems from the ability of the model to predict future volatility.

Empirically, we observe that implied volatility is inversely related to stock price. This relationship is what the constant elasticity of variance model predicts when θ is less than two. However, when θ is greater than two, the constant elasticity of variance model predicts that volatility and stock price move in the same direction and the model's predictions of future volatility are actually worse than the Black-Scholes prediction of constant volatility. Thus, the constant elasticity of variance model works best when θ is less than two. However, even when θ is less than two, the constant elasticity of variance model can account for approximately only 20 percent of the variation in percent changes in volatility. Essentially, volatility changes in ways that are at best, only partially predicted by the constant elasticity of variance model.

Our findings suggest that careful empirical analysis of call option prices should include exact adjustments for dividends on prices of in-the-money options. For some low volatility, high dividend stocks, we find evidence that prices of call options with as many as two quarterly dividends remaining reflect the possibility of early exercise.

⁷Although our closing prices surely contain some stock and option prices which are nonsynchronous, we do not believe that inclusion of these observations could induce the systematic mispricing which we observe. Moreover, the number of nonsynchronous observations is likely to be small for two reasons. First, the options in our sample are among the most heavily traded. Second, in estimating implied volatility, we identified and excluded many of these observations because they violated boundary conditions or else yielded extremely large values of implied volatility.

TABLE 1
 ROOT MEAN SQUARED PREDICTION ERRORS
 Prediction Interval: 0 Day

Stock Model	1976 Option Prices				1978 Option Prices			
	$\tau \geq 90$ Days		$\tau < 90$ Days		$\tau \geq 90$ Days		$\tau < 90$ Days	
	In-the-Money	Out-of-the-Money	In-the-Money	Out-of-the-Money	In-the-Money	Out-of-the-Money	In-the-Money	Out-of-the-Money
ATT	.33	.13	.20	.13	.22	.15	.19	.11
B.S.								
CEV	.31	.11	.30	.11	.22	.09	.24	.09
AVON								
B.S.	.37	.21	.30	.18	.26	.18	.29	.19
CEV	.19	.17	.26	.24	.20	.12	.29	.18
ETKD								
B.S.	.68	.40	.73	.27	.29	.16	.33	.21
CEV	.37	.24	.62	.31	.25	.16	.33	.20
EXXN								
B.S.	.31	.21	.27	.15	.61	.26	.32	.28
CEV	.24	.14	.26	.16	.53	.18	.35	.17
IBM								
B.S.	2.35	1.06	1.51	.99	.86	.39	.74	.63
CEV	.98	.75	1.19	.73	.67	.38	.74	.60
XERX								
B.S.	.35	.22	.36	.23	.31	.17	.29	.20
CEV	.24	.19	.34	.26	.23	.12	.29	.17

TABLE 2
 ROOT MEAN SQUARED PREDICTION ERRORS
 Prediction Interval: 1 Day

Stock Model	1976 Option Prices		1978 Option Prices	
	$\tau \geq 90$ Days In-the-Money	Out-of-the-Money	$\tau < 90$ Days In-the-Money	Out-of-the-Money
ATT	.36	.18	.21	.14
	\$	\$	\$	\$
B.S.				
CEV	.35	.16	.30	.12
AVON	.42	.28	.31	.22
B.S.				
CEV	.29	.22	.28	.26
ETKD	.80	.45	.75	.32
B.S.				
CEV	.59	.31	.66	.35
EXXN	.38	.28	.29	.18
B.S.				
CEV	.38	.23	.30	.21
IBM	2.41	1.28	1.54	1.08
B.S.				
CEV	1.14	.99	1.23	.81
XERX	.40	.29	.38	.25
B.S.				
CEV	.38	.25	.38	.29
			$\tau > 90$ Days In-the-Money	Out-of-the-Money
			$\tau < 90$ Days In-the-Money	Out-of-the-Money
			\$	\$
			.19	.29
			.14	.29
			.23	.32
			.19	.33
			.23	.36
			.23	.39
			.30	.64
			.22	.60
			.61	.99
			.66	.96
			.28	.37
			.24	.35
			.20	.12
			.25	.10
			.30	.22
			.30	.19
			.35	.24
			.35	.23
			.34	.29
			.35	.19
			.82	.75
			.81	.75
			.32	.23
			.32	.20

TABLE 3
 ROOT MEAN SQUARED PREDICTION ERRORS
 Prediction Interval: 1 Week

STOCK	MODEL	1976 Option Prices		1978 Option Prices	
		$\tau \geq 90$ Days In-the-Money	$\tau < 90$ Days Out-of-the-Money	$\tau \geq 90$ Days In-the-Money	$\tau < 90$ Days Out-of-the-Money
ATT	B.S.	\$.32	\$.32	\$.19	\$.23
	CEV	.31	.32	.29	.25
AVON	B.S.	.45	.38	.32	.26
	CEV	.40	.32	.30	.31
ETKD	B.S.	.94	.56	.76	.41
	CEV	.81	.43	.74	.42
EXXN	B.S.	.44	.38	.30	.24
	CEV	.44	.34	.30	.27
IBM	B.S.	2.46	1.63	1.47	1.25
	CEV	1.42	1.17	1.19	.90
XERX	B.S.	.53	.41	.43	.29
	CEV	.48	.41	.42	.33
				$\tau \geq 90$ Days In-the-Money	$\tau < 90$ Days Out-of-the-Money
				\$.31	\$.23
			.31	.20	.11
			.47	.32	.26
			.46	.30	.24
			.38	.28	.25
			.41	.24	.22
			.71	.36	.26
			.68	.28	.19
			1.26	.94	.81
			1.17	.93	.72
			.38	.31	.22
			.40	.30	.22

TABLE 4
 ROOT MEAN SQUARED PREDICTION ERRORS
 Prediction Interval: 1 Month

Stock Model	1976 Option Prices				1978 Option Prices			
	$\tau \geq 90$ Days		$\tau < 90$ Days		$\tau \geq 90$ Days		$\tau < 90$ Days	
	In-the-Money	Out-of-the-Money	In-the-Money	Out-of-the-Money	In-the-Money	Out-of-the-Money	In-the-Money	Out-of-the-Money
ATT	.47	.43	.26	.25	.50	.39	.29	.15
CEV	.44	.49	.30	.35	.52	.38	.35	.13
AVON	.42	.57	.36	.32	.44	.47	.32	.29
CEV	.52	.48	.38	.30	.41	.31	.32	.21
ETKD	1.24	.78	.98	.60	.69	.61	.47	.48
CEV	1.37	.95	1.08	.62	.75	.56	.48	.47
B.S.	.52	.35	.35	.20	.86	.40	.41	.26
CEV	1.08	.90	.53	.65	.90	.40	.58	.30
IBM	2.50	2.49	1.54	1.35	1.25	1.32	.99	1.21
CEV	2.64	2.20	1.46	1.88	1.15	1.11	.87	.96
XERX	.68	.57	.38	.24	.47	.49	.30	.30
CEV	.57	.57	.18	.42	.48	.53	.28	.32

TABLE 5
CORRELATION BETWEEN ACTUAL PERCENT CHANGES
IN $\hat{\sigma}_t$ AND PREDICTED PERCENT CHANGES: DAILY

<u>Period</u>	O P T I O N			
	AVON	ETKD	IBM	XERX
1976	.49	.28	.48	.30
1978	-.12	.06	-.18	.10
1978($\hat{\theta} > 2$)	-.27	.00	-.39	.00

TABLE 6
CORRELATION BETWEEN PERCENT CHANGES
IN $\hat{\sigma}_t$ AND PERCENT CHANGES IN S_t : DAILY

<u>Period</u>	O P T I O N			
	AVON	ETKD	IBM	XERX
1976	-.51	-.34	-.42	-.39
1978	-.42	-.28	-.21	-.12
1978($\hat{\theta} > 2$)	-.32	-.10	-.40	.00

FIGURE 1
DAILY IMPLIED VALUE OF VOLATILITY 1976

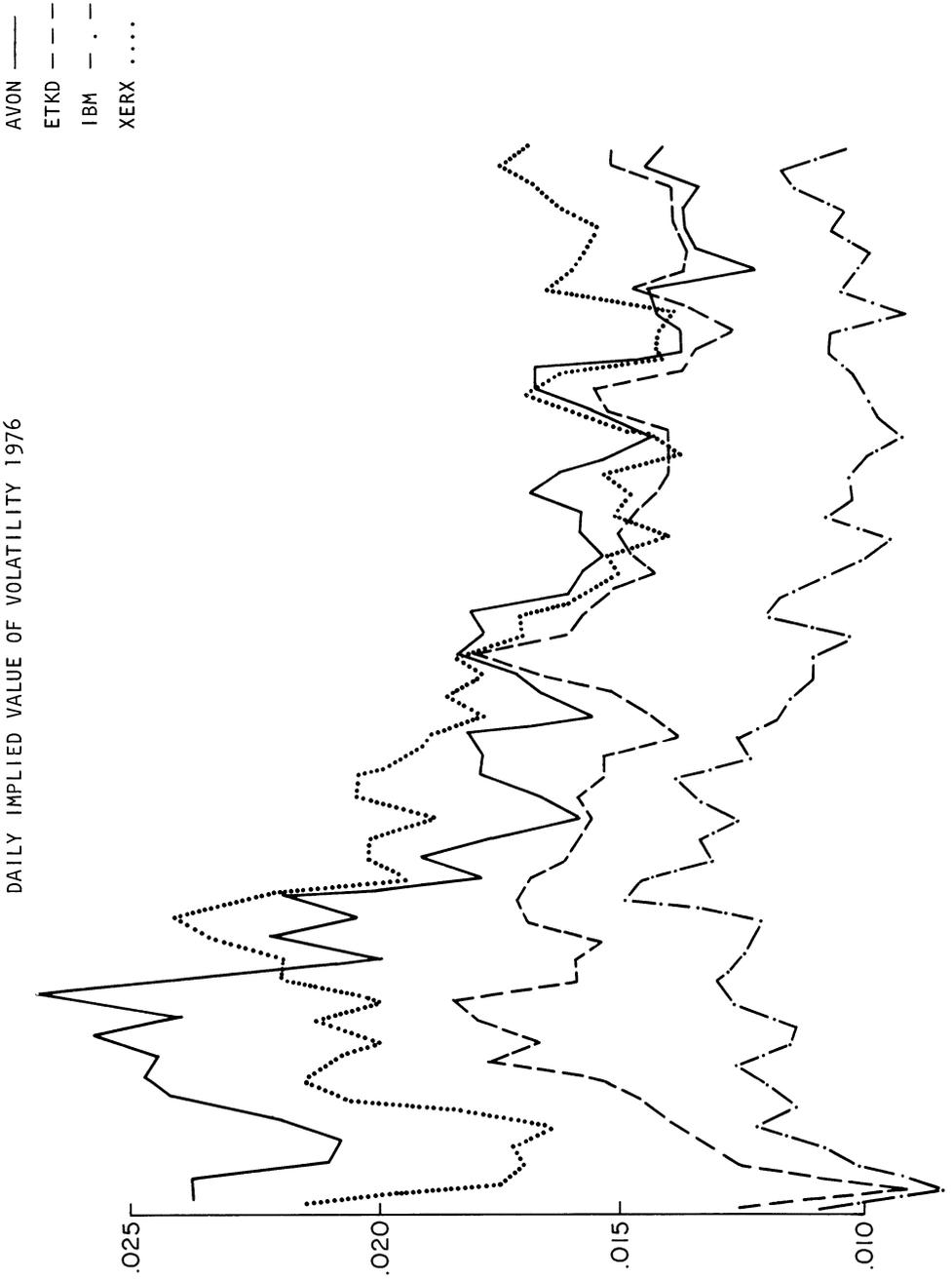


FIGURE 2
DAILY IMPLIED VALUE OF VOLATILITY 1978

AVON —
ETKD - - -
IBM - . -
XERX

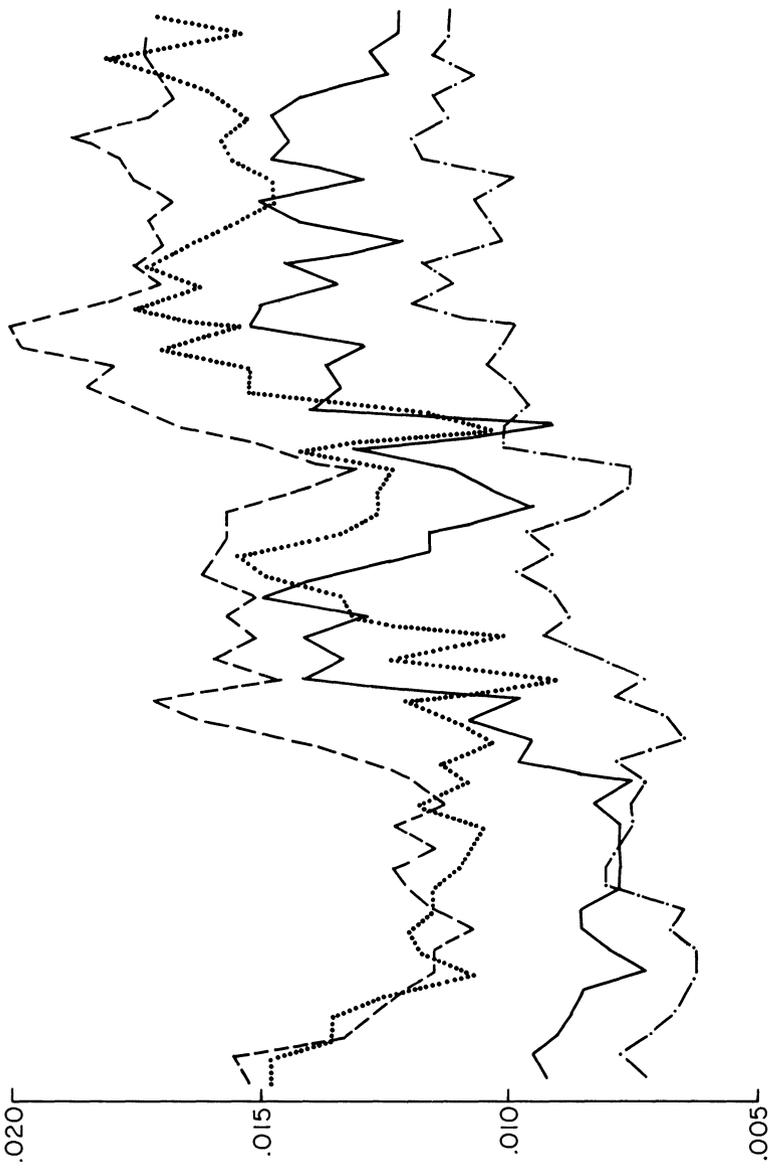


FIGURE 3
ESTIMATED VALUE OF THETA 1976

AVON ———
ETKD - - -
IBM - . -
XERX

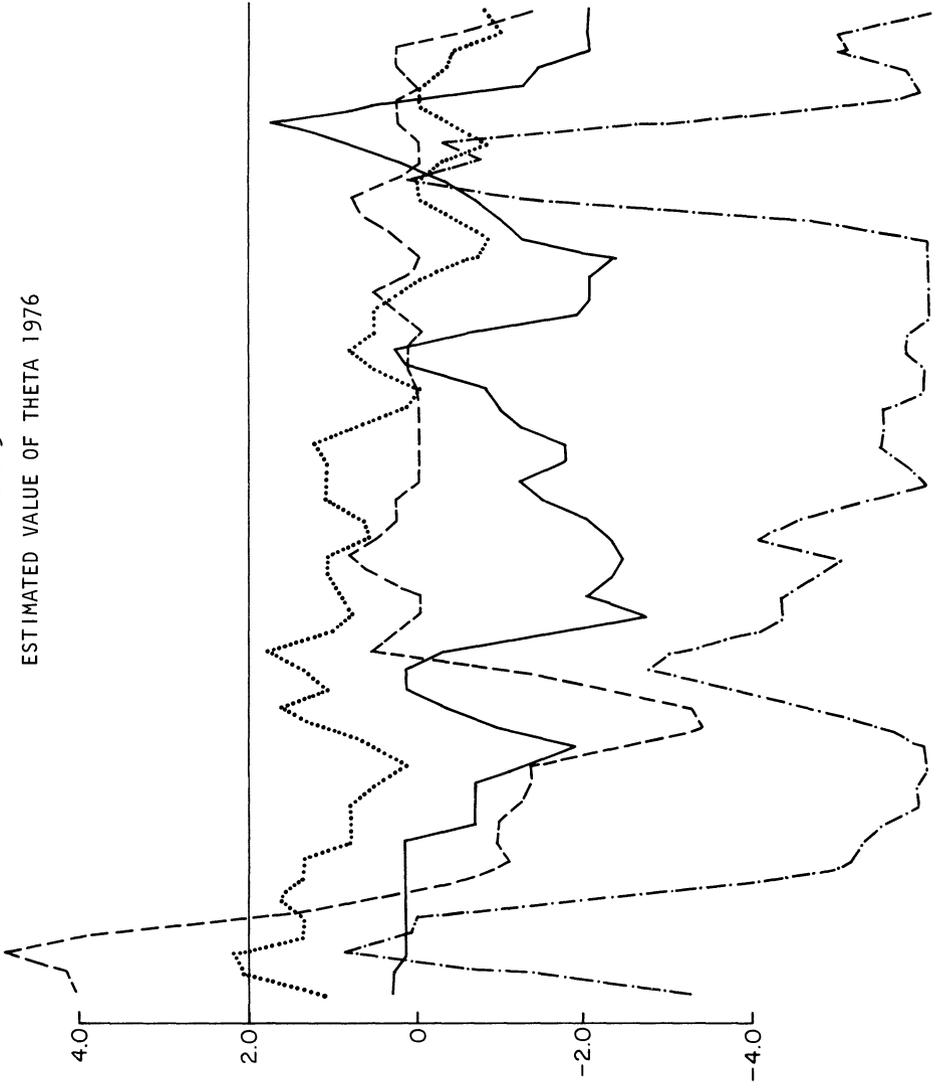
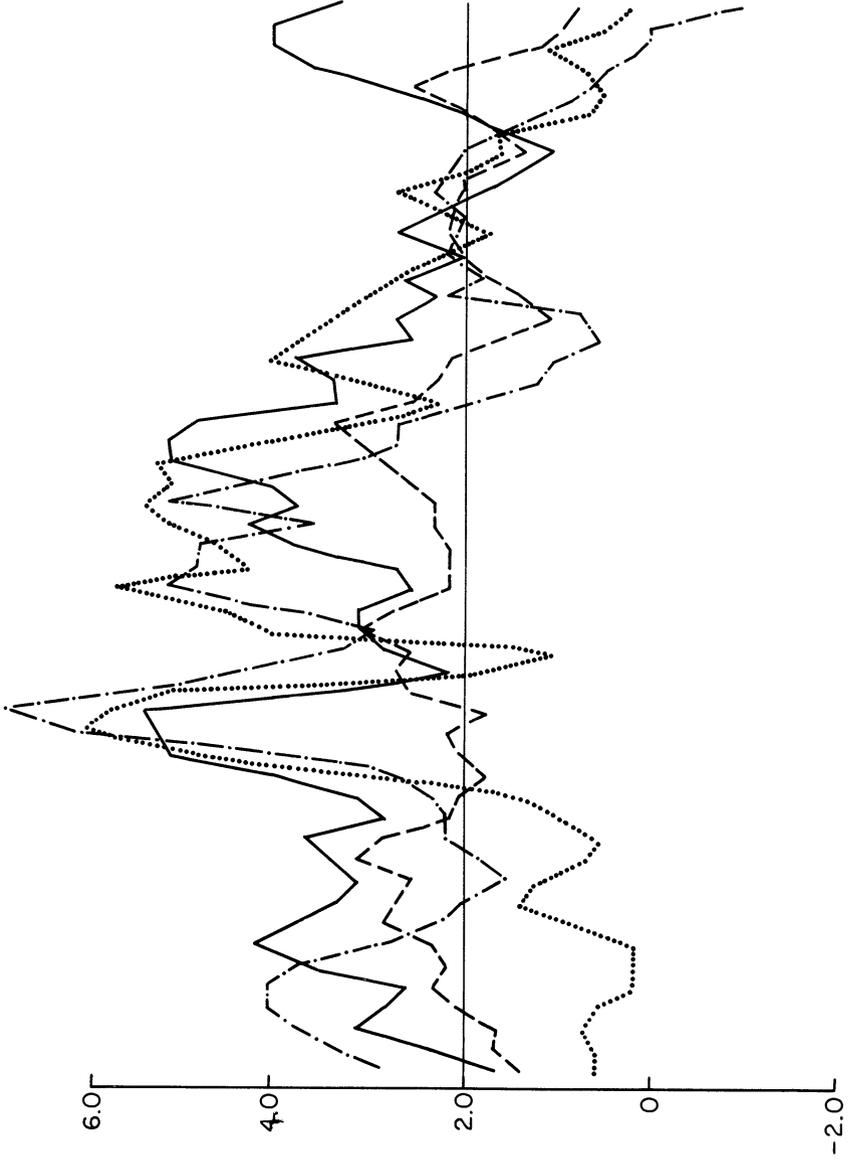


FIGURE 4
ESTIMATED VALUE OF THETA 1978

AVON ———
ETKD - - -
IBM - . -
XERX



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