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An empirical test of the BS and CSR valuation models for warrants listed in Thailand ☆

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Abstract

The purpose of this paper is to examine the pricing structure of warrants that trade on the Stock Exchange of Thailand. A comparison of model prices with the market prices of the warrants shows that the Black–Scholes (BS) model misprices warrants on average by 4 percent of market price, while the corresponding figure for the Cox Square Root (CSR) model is 3.84 percent. The improvement in pricing performance is statistically significant, with the CSR model providing more "accurate" prices in 70 percent of the cases. On the other hand, the difference in pricing performance is only 0.16 percent, and may not be economically significant.

JEL classification: G13; G15

Keywords: Warrants; Valuation; Pricing errors

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1. Introduction

Warrants are long-term call options issued by firms that give the holder the right to purchase the firm's common stock at a pre-determined price (the exercise price), on or before an expiration date. These securities have been issued with some regularity by firms in the United States, but are a relatively new instrument in Thailand. The purpose of this paper is to examine the pricing structure of warrants that are listed on the Stock Exchange of Thailand (SET), and to determine if pricing models commonly used for warrants traded in U.S. markets are applicable to those traded on the SET.

There have been a number of studies that have provided theoretical models for the pricing of warrants. These include Black and Scholes (1973), Chen (1975), Galai and Schneller (1978) and Schwartz (1977). Black and Scholes, and Galai and Schneller suggest that a model for pricing call options, with some minor modifications, can be used to price warrants. Their model is derived assuming that the underlying stock follows a Gauss–Wiener process, and that the price of the option is obtained by pricing a riskless hedge formed by combining the option and the underlying stock in a market that precludes the existence of arbitrage opportunities. In contrast, Chen uses a dynamic programming approach to price warrants, while Schwartz generalizes the Black–Scholes formulation by employing a finite difference technique to approximate solutions to a partial differential equation that governs the value of a warrant.

Empirical studies of warrant pricing include Schwartz (1977), Noreen and Wolfson (1981), Ferri et al. (1986), Leonard and Solt (1990), and Lauterbach and Schultz (1990). The results in these studies regarding the applicability of the Black–Scholes (BS) model to warrant valuation is mixed. Specifically, Schwartz, Noreen and Wolfson, Ferri, Kremer and Oberhelman, and Leonard and Solt conclude that the Black–Scholes model performs as well as alternative more complicated models for warrant pricing. On the other hand, Lauterbach and Schultz present evidence that suggests that the Black–Scholes model is outperformed by a model that assumes a constant elasticity of variance (CEV) diffusion process for stock price.

The purpose of this study is to examine the validity of the BS model, and compare the pricing performance of this model with some alternatives in the context of pricing warrants that are listed on the SET. One clear advantage of examining the pricing structure of warrants on the SET is that they have two features that are more consistent with the assumptions of the Black–Scholes model as compared to warrants that trade in the United States. First, the SET options are European (as assumed by the BS model) or Pseudo-American, while those that trade in the U.S. are generally American. [1] Second, because of tax laws in the

[1] A European option is one that can be exercised at expiration only, while an American option can be exercised at any time from the date of purchase till the expiration date.

United States, companies tend to extend the expiration of warrants if an "out-of-the-money" expiration is imminent, thus making the time to expiration on the option stochastic. [2] Since such tax laws do not exist in Thailand, the warrants traded there have deterministic expiration dates (as assumed by the BS model). In addition, as suggested by Yau et al. (1990) the analysis of pricing in non-US markets is important since the structures of these markets are different from that in the United States. [3] For example, regulations in Thailand prevent the short selling of stocks, which, in turn, may have an impact on the ability of participants to arbitrage away deviations of market prices of warrants from model values. [4]

The analysis of the pricing structure proceeds in three steps. First, we examine whether the closing transaction prices on the warrants satisfy the arbitrage free lower bound defined by $(S - PV_D - Xe^{-rT})$, where S is the corresponding closing price of the underlying stock, PV_D is the present value of the dividends expected over the life of the warrant, X is the exercise price of the warrant, r is the risk-free rate of interest and T is the time to expiration on the warrant. [5] Our results indicate that 39 percent of the quoted warrant prices are in violation of this arbitrage-free lower bound with the average size of the violation being 28 percent of the warrant price.

In analyzing the causes for this observed deviation one has to recognize some distinctive features of the market for these warrants. Specifically, the observed mispricing can be a result of the fact that these instruments are relatively new to the markets in Thailand, the illiquidity of these instruments, and/or that short

[2] A warrant is considered out-of-the-money if the price of the underlying security is less than its exercise price.

[3] Specifically, they argue that with the existence of short-sale restrictions in the Hong Kong market, arbitrage activity only sets an upper bound for futures prices. There is no lower bound because the arbitrage strategy that establishes the lower bound requires a short sale in the cash market. Thus observed futures prices in Hong Kong below those predicted by the cost-of-carry model would not reflect pricing errors, but would be a manifestation of trading restrictions imposed on market participants.

[4] Since a short sale can be simulated by selling a stock one already owns and buying it back at a future date, the existence of restrictions on short selling can be viewed as an additional transactions cost of trading. In this framework, it can be shown that the model price of the warrant would be lower than that obtained assuming zero transactions costs. On the other hand, restrictions on short selling (writing) warrants would result in a warrant model price that is higher than the zero transaction cost price. See Leland (1985) for more details.

[5] See Merton (1973) for a derivation of this lower bound for call options. To derive the equivalent bound for a warrant, we have to make a dilution adjustment. Specifically, the warrant price $W \geq [N/\{(N/\gamma)+M\}][S+(M/N)W - PV_D - Xe^{-rT}]$, where S is the stock price, X is the strike price, PV_D is the present value of expected dividends over the life of the warrant, r is the risk-free rate of interest, T is the time to expiration, N is the number of shares outstanding, M is the number of warrants outstanding and γ is the number of shares that can be purchased with each warrant. With some simple algebraic manipulation, the above expression simplifies to $W \geq \gamma[S - PV_D - Xe^{-rT}]$. Since $\gamma = 1$ for all warrants listed on the SET, this reduces to the expression used in the main body of the paper.

selling is prohibited. The empirical analysis suggests that the violations are within the bounds of quoted bid–ask spreads on these warrants. In addition, since arbitraging away violations of the lower bound, involves the execution of a trading strategy that requires short selling of the underlying stock, we also cannot rule out the possibility that the deviations exist because of the restrictions on short sales. Specifically, these restrictions effectively increase the transactions costs of short selling, and thereby reduce the lower bound since individuals could simulate a short position by selling the stock they already own and buying it back at a future date. [6] Thus violations of the transactions-free lower bound can be a result of the fact that we ignore the fact that market participants have to use a more costly alternative to short selling.

Second, we determine if the BS model is valid for the warrants traded on the SET. We make this determination by obtaining the volatility implied by the BS model and the closing market prices of the warrants, and regressing this implied volatility on the moneyness of the warrant under consideration, where moneyness is defined as $(S - PV_D - Xe^{-rT})$ divided by Xe^{-rT}. A zero coefficient would lend support to the BS model. Our results indicate that the coefficient in the regression is significantly negative. This suggests that a model that allows for a negative relationship between stock price and volatility may be more appropriate for pricing these warrants. The model we select as the alternative candidate is the Cox Square Root (CSR) model.

In the third step, we compare the pricing accuracy of the BS model with the CSR model. [7] A comparison of model prices with the market prices of the warrants indicates that the BS model misprices warrants on average by 4 percent of market price, while the corresponding figure for the CSR model is 3.84 percent. The improvement in pricing performance is statistically significant, with the CSR model providing more ''accurate'' prices in 70 percent of the cases. On the other hand, the difference in pricing performance is only 0.16 percent, and may not be economically significant.

We examine several possible explanations for the observed deviations. One possible explanation for the observed results is that the dividend adjustment we employ is incorrect. Specifically, we assume that the forecasted dividend is equal to the actual dividends. On the other hand, if the market was anticipating a higher or lower dividend we would systematically misprice the warrants. A second explanation that we consider is that the models we employ assume that the warrants are European even though the securities have some restrictive early

[6] See Nisbet (1992) for a discussion of the impact of transactions costs on arbitrage relationships in the context of put-call parity.

[7] The analysis follows that used in previous studies on the accuracy of option pricing models. For example, see Whaley (1982) for equity options, Bodurtha and Courtadon (1987), and Shastri and Tandon (1987) for foreign currency options, Whaley (1986) and Shastri and Tandon (1986) for options on futures and Lauterbach and Schultz (1990) for equity warrants.

exercise privileges. A third possible explanation is that the mispricing is caused by the fact that we ignore the existence of a 10 percent daily price limit on the SET. A fourth explanation that we explore is that the pricing errors are an artifact of the illiquid market for these securities in terms of both low trading volume and high bid–ask spreads. Finally, we consider the possibility that restrictions on short sales could result in the observed pricing errors. Based on this analysis, we conclude that the pricing errors reflect bid–ask spreads in the market and/or the higher transactions costs associated with short sales.

The remainder of this paper is organized as follows. The next section contains a description of the institutional details of the warrant issues. The description of the data, the empirical methodology that will be employed in the tests, and the hypotheses tested are presented in Section 3. Section 4 provides the empirical results. A discussion of the results and a summary conclude the paper.

2. The institutional background

The SET was founded in 1975 with a total market capitalization for listed securities of Baht 5.39 billion. In 1986, the market capitalization had increased to Baht 75.20 billion. Since 1986, the number of listed securities and trading volume has expanded rapidly, with the market capitalization at the end of 1992 being Baht 1.49 trillion. In December 1992, there were 305 common stocks, 10 preferred stocks, 15 unit trusts, 8 convertible debentures and 3 warrants traded on the SET.

Warrants were first issued in Thailand in April 1989, when the Bangkok Bank of Commerce (BCC) issued 40 million shares of common stock in a package with 80 million warrants to raise capital. The warrants were priced at Baht 3, and provided the holder the option to buy one share of BBC common stock for Baht 10 in three years. Even though there were no specific rules and regulations defined by the SET in place at the time of this issue on the public offering of warrants, BBC was granted a waiver on the regulations and its warrants were listed on the SET in February, 1990. [8] These warrants expired, and were, therefore, delisted from the SET in April 1992. In the expiration period, the price of BBC stock ranged from Baht 11.75 to Baht 12, and a large proportion of the BBC warrants were exercised.

1989 also witnessed the second warrant issue in the Thai capital markets. Specifically, Star Block Company (STAR) issued 21 million new shares with an equal number of warrants attached. The warrants were priced at Baht 5 each, and allowed the holder the option to buy one share of STAR common stock with the expiration month of the warrant being August 1993. STAR warrants were not listed immediately on the SET since they did not comply with the SET regulation

[8] See Appendix A for details on the listing requirements for warrants on the SET.

that the combined number of shares obtained through the exercise of the warrants could not exceed 30 percent of the company's outstanding stock. [9] As a result, STAR had to restructure its warrants, requiring 20 percent of them to be exercised (or expire) in October 1992. The restructuring resulted in STAR satisfying this SET regulation, and these warrants were first listed on the SET in February 1992.

Two new warrants on unit trusts were listed on the SET in November 1992. The warrant on the One Multiple Growth Fund (ONE-G) has an exercise price of Baht 11, and is pseudo-American in contrast to the previous two warrants on BBC and STAR. In particular, this warrant expires in July, 1995, but allows early exercise in the period from June 25 to July 25 in 1993 and 1994. One warrant was issued for every 5 unit trusts held by the stockholder, with the total number of warrants issued being 8.31 million. The second warrant issue for 1992 was on the Sub-Anan Fund (SAN), and has an exercise price of Baht 10. This warrant is also pseudo-American, since it expires in March 1995, but allows early exercise in the period from February 15 to March 15 in 1993 and 1994. As with the ONE-G warrants, one SAN warrant was issued for every 5 unit trusts held, with the total number issued being 17 million.

More recently, Land and Houses Public Company (LH) issued bonds with warrants as a part of its capital funding. The issue consisted of 2 million bonds and 10 million warrants at a price of Baht 1,000 for a package of 1 bond and 5 warrants. Each warrant gave the holder the right to buy LH common stock at an exercise price of Baht 288 with a time to expiration on the option of 4 years and 11 months.

3. The data, models, and empirical methods

3.1. The data

The empirical tests conducted here are based on closing quotations for warrants listed on the SET and the underlying stocks. The stocks underlying these warrants are the Bangkok Bank of Commerce (BCC), the One Multiple Growth Fund (ONE-G), the Sub-Anan Fund (SAN) and the Star Block Company (STAR). The data covers the period from the date of first listing to the last trading date in 1992. Table 1 provides a more detailed description of the warrants in terms of their strike price, expiration date, exercise ratio, the exercise terms, number of units issued, issue (offer) price, and the first day they traded on the SET.

[9] At the time of issuance of the package of 21 million shares and warrants, STAR had 10.5 million shares outstanding. Thus, the number of warrants were equal to 66.67 percent of the number of shares outstanding.

Table 1
The characteristics of warrants traded on the stock exchange of Thailand

Underlying stock	Number issued	Offer price (baht)	Expiration date	Exercise price (*baht*)	Exercise *ratio* [a]	Type of *option*	First day of trade
Bangkok Bank of Commerce (BBC)	80 million	3	April 28, 1992	10 [b]	1	European	February 20, 1990
One Multiple Growth Fund (ONE-G)	41.55 million	5 unit trusts [c]	July 25, 1995	11	1	Pseudo-American [d]	November 10, 1992
Sub-Anan Fund (SAN)	85 million	5 unit trusts [c]	March 15, 1995	10	1	Pseudo-American [e]	November 10, 1992
Star Block Company (STAR)	21 million	5	August, 1993 [f]	30	1	European	February 27, 1992

[a] The exercise ratio is the number of shares received for each warrant exercised.
[b] The exercise price on these warrants was changed to Baht 9.89 after November 10, 1990.
[c] One warrant was issued for every five shares of the unit trust held by a stockholder.
[d] The warrants can be exercised from June 25 to July 25 in the years 1993, 1994 and 1995.
[e] The warrants can be exercised from February 15 to March 15 in the years 1993, 1994 and 1995.
[f] 25 percent of the outstanding warrants expired in October 1992.

Based on the day of the first trade, the maximum number of possible observations on closing price in our sample period are 509, 35, 35 and 208 for the BBC, ONE-G, SAN and STAR warrants, respectively. For the warrants on SAN and STAR, there are closing prices available for each of the above trading days. On the other hand, warrants on BBC and ONE-G do not trade on 61 and 6 days respectively. This non-trading represents 12 and 17 percent of the maximum possible number of observations, and leaves us with 448 and 29 closing prices on these two warrants, respectively.

Table 2 provides some descriptive statistics on these four warrants based on the days they have non-zero trading volume. Specifically, this table provides, means, medians, minimums and maximums over our sample period for the daily trading volume (as a fraction of number of warrants outstanding), the time to expiration, the moneyness, and the market price of the four warrants. Moneyness is defined as $(S - PV_D - Xe^{-rT})/Xe^{-rT}$, where S is the closing price of the underlying stock,

Table 2
Descriptive statistics for the warrants traded on the stock exchange of Thailand

Characteristic	Mean	Minimum	Median	Maximum
Panel A: Bangkok Bank of Commerce (448) [a]				
Trading volume (%) [b]	2.44	0.00	1.35	25.26
Time to expiration (years)	1.03	0.13	1.02	2.15
Moneyness [c]	0.71	0.18	0.57	3.11
Market price (baht)	6.42	1.60	6.00	17.00
Panel B: One Growth Multiple Fund (29)				
Trading volume (%)	1.27	0.02	0.43	15.44
Time to expiration (years)	2.64	2.57	2.64	2.70
Moneyness	0.01	-0.04	-0.02	0.11
Market price (baht)	3.31	2.60	3.30	4.20
Panel C: Sub-Anan Fund (35)				
Trading volume (%)	2.81	0.20	1.07	23.05
Time to expiration (years)	2.28	2.21	2.28	2.34
Moneyness	-0.02	-0.05	-0.04	0.05
Market price (baht)	2.39	1.90	2.40	3.00
Panel D: Star Block Company (208)				
Trading volume (%)	0.16	0.00	0.08	1.54
Time to expiration (years)	1.08	0.67	1.08	1.50
Moneyness	1.31	0.77	1.34	1.84
Market price (baht)	36.13	27.00	37.00	45.20

[a] The figure in parentheses is the sample size.
[b] Trading volume is defined as a fraction of the number of warrants outstanding.
[c] Moneyness is defined as $(S\text{-}PV_D\text{-}Xe^{-rT})/Xe^{-rT}$, where S is the corresponding closing price of the underlying stock, PV_D is the present value of the dividends expected over the life of the warrant, X is the exercise price of the warrant, r is the call money rate, and T is the time to expiration on the warrant.

PV_D is the present value of the dividends expected to be paid over the life of the warrant, X is the exercise price of the warrant, r is the risk-free rate of interest, and T is the time to expiration of the warrant. The present value of the dividends is calculated assuming either that the forecasted dividend and ex-dividend date were the same as the actual occurrence, or that they were such that future dividends were equal to that declared most recently, and that ex-dividend dates followed the same pattern as previous ex-dividends dates. [10] The risk-free rate is based on the call money rate, and is available on a monthly basis.

As can be seen from this table, the trading volume on these warrants averages 1.75 percent of the number of warrants outstanding. The BBC warrants trade most frequently, followed by SAN, ONE-G and STAR. The time to expiration on the warrants ranges from a minimum of 33 to a maximum of 675 trading days, with over 50 percent of the observations being on warrants with more than 250 trading days to expiration. Finally, 94 percent of the observations are on warrants that are in-the-money (moneyness > 0). The 46 out-of-the-money (moneyness < 0) observations are all on ONE-G and SAN warrants.

As a first step in our empirical tests, we need to ensure that the quoted prices satisfy the arbitrage-free lower bound of $(S - PV_D - Xe^{-rT})$ derived in Merton (1973), where all the variables have been defined earlier. [11] The results of this analysis are presented in Panel A of Table 3. The format of the table is as follows: column 1 lists the underlying stock, the percentage of quoted prices that violate the lower bound is contained in column 2, columns 3 and 4 present the mean and median value for the difference between the lower bound and the quoted price for the observations that indicate violations, and the corresponding mean and median values for the size of the violations as a fraction of warrant price are in columns 5 and 6.

As can be seen from the table, all price observations on ONE-G and SAN warrants are greater than their corresponding lower bound. On the other hand, 33 and 45 percent of the observations on BBC and STAR warrants violate this lower bound with the violations averaging Baht 2.93 and 2.96, or 41.82 and 8.00 percent of the warrant price, respectively (see Table 3). If transactions costs and other liquidity related factors are ignored, this represents arbitrage profit opportunities since in 92 percent of the cases the violation persists at the next quoted closing price. [12]

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[10] Based on these assumptions, there were no dividends forecasted for BBC, ONE-G and SAN, while STAR is forecasted to have paid dividends of Baht 1.20 and Baht 0.25 per share with corresponding ex-dividend dates of April 4, 1992 and April 7, 1993.

[11] We need to make sure that the warrant price is above this lower bound because all warrant valuation models preclude the existence of arbitrage opportunities.

[12] The average size of the violations based on the next quoted price is Baht 2.93 and 2.76, or 41.39 and 7.51 percent of warrant price for BBC and STAR, respectively.

Table 3
Statistics on the violation of the arbitrage-free lower bound for European warrant prices for warrants listed on the stock exchange of Thailand [a]

Panel A: Size of the violations

Underlying stock	Percent of observations that violate lower bound	Difference in lower bound and warrant price (baht)		Percentage difference in lower bound and warrant price	
		Mean value	Median value	Mean value	Median value
Bangkok Bank of Commerce (448) [b]	32.54	2.93	2.42	41.82	40.36
Star Block Company (208)	45.19	2.96	2.16	8.00	5.15

Panel B: A comparison of trading volume for the sample of violations and the sample of non-violations [c]

Underlying stock	Difference in mean values of trading volume for the two samples (%)	t-statistic for the two sample test of difference in mean values of trading volume
Bangkok Bank of Commerce	1.21	3.59 *
Star Block Company	0.06	1.95

[a] The lower bound is defined as (S-PV_D-Xe^{-rT}), where S is the corresponding closing price of the underlying stock, PV_D is the present value of the dividends expected over the life of the warrant, X is the exercise price of the warrant, r is the call money rate, and T is the time to expiration on the warrant.
[b] The figure in parentheses is the sample size.
[c] Trading volume is defined as a fraction of the number of warrants outstanding.
* indicates significance at the 1 percent level.

There are a number of possible explanations for this result. First, the violations may be a result of the fact that the market in new, and that there is a ''learning curve'' associated with these securities. This would imply that, for the observations that indicate violations of the lower bound, the time to expiration on the warrants should be greater than that for the remaining observations. This hypothesis is supported by the data since the difference in the time to expiration between the two subsamples is positive and significant at the 1 percent level. [13]

A second possible explanation is that the size of the violations are within the trading costs associated with the arbitrage strategy. Specifically, since bid–ask spreads on the warrants are approximately Baht 2.50, the size of the violations are not economically significant after accounting for the effect of the spread.

[13] The difference in time to expiration averages 0.77 and 0.14 years with associated t-statistics of 20.75 and 4.35, for BBC and STAR, respectively.

A third possible explanation for the existence of the arbitrage profit is that the strategy to take advantage of the violation requires the short selling of the underlying stock, and short selling is not allowed in Thailand. Restrictions on short sales can be circumvented by selling stock you already own and buying it back on a later date. Since this alternative is more costly, the lower bound based on zero transactions costs overstates the "true" lower bound. In this framework, observed warrant prices may not represent arbitrage opportunities even if they are below the zero transaction cost bound since they are above the lower bound that adjusts for the transactions costs implied by short sales restrictions.

A fourth possible explanation is related to the liquidity on the warrants. Specifically, since the trading volume on these securities are low, one could argue that these arbitrage violations are liquidity driven. This would suggest that the observations that violate the lower bound should be associated with lower trading volume than the remaining observations. Panel B of Table 3 provides a comparison of the trading volume (as a fraction of the number of warrants outstanding) for the two groups. Column 1 of the table contains the name of the underlying stock, column 2 presents the difference in the mean values of trading volume, and the t-statistic for the null hypothesis that the difference is zero is in column 3. As can be seen from this table, the data does not support this hypothesis. Specifically, trading volume is significantly larger for the observations that violate the lower bound.

Regardless of the specific reason for the existence of these violations, we eliminate all observations that do not satisfy the lower bound. This leaves us with a total of 490 observations with 312 on BBC, 29 on ONE-G, 35 on SAN and 114 on STAR.

3.2. The models and the empirical methods

The first pricing model considered for analysis is the Black–Scholes (BS) model for call options with appropriate adjustments for dilution. As shown by Galai and Schneller (1978), the BS model for warrant pricing is given by:

$$W = \hat{N}\left[\hat{S}N(d_1) - X\mathrm{e}^{-rT}N(d_2)\right], \quad (1)$$

where

$d_1 = [\ln(\hat{S}/X) + (r + 1/2\sigma^2)T]/\sigma\sqrt{T}$

$d_2 = d_1 - \sigma\sqrt{T}$,

$\hat{N} = N/[(N/\gamma) + M]$,

$\hat{S} = S + (M/N)W - PV_{\mathrm{D}}$

W is the warrant price,

N is the number of shares outstanding,

γ is the number of shares that can be purchased with each warrant,

M is the number of warrants outstanding,

S is the price of the underlying stock,

PV_D is the present value of dividends expected over the life of the warrant,
X is the exercise price of the warrant,
r is the risk-free rate of interest,
σ is the volatility of the underlying stock,
T is the time to expiration on the option,
and
$N(d)$ is the probability that a standard normal variable will take on a value less than equal to d.

To test the Black–Scholes model, the following regression is estimated over the sample period for each warrant:

$$ISD_t = a + b\left[\left(S_{t-1} - PV_D - X e^{-r(T-t+1)}\right)/X e^{-r(T-t+1)}\right] + e_t, \qquad (2)$$

where ISD_t is the volatility implied by setting the warrant market price on day t equal to the BS model price as specified in Eq. (1), and the independent variable is the moneyness of the warrant on day $t-1$. [14] As can been in the above regression, the implied volatility on day t is regressed on the percentage the warrant is in or out of the money on the previous day. This formulation is used to remove any possible biases induced by infrequent trading of the warrants. [15]

If the BS model is appropriate for warrant pricing, then the estimated coefficient b in Eq. (2) should not be statistically different from 0. The results from the estimation of Eq. (2) for each of the four warrants in our sample is presented in Table 4. The first column of the table contains the name of the underlying security, the second column presents the mean value of the time series of implied volatilities, the estimated values of a and b are contained in columns 3 and 4, and finally column 5 presents the adjusted R^2 for the regression.

As can be seen from this table, implied volatilities vary from an average of 54.81 percent to 111.29 percent. In addition, for all four securities, implied volatilities are negatively related to moneyness, since the estimated value of b is significantly negative and varies from -0.75 to -1.33. This suggests that a model that allows for volatility and stock price to be inversely related, or one that allows volatility to be stochastic may be more appropriate than the BS model for the valuation of these warrants.

The existence of stochastic volatility unrelated to equity value implies that the coefficient b would be negative for out-of-the-money options and positive for in-the-money options. [16] Since, 94 percent of our observations are for warrants

[14] Lauterbach and Schultz (1990) use a similar formulation for the analysis of the applicability of the BS model to warrants traded on the New York and American Stock Exchanges. Our formulation differs from theirs in that we do not include the risk-free rate as an independent variable. The elimination of the risk-free rate from our analysis is caused by the fact that we only have monthly observations on this variable.

[15] See Lauterbach and Schultz (1990) for more details.

[16] See Wiggins (1987) for more details.

Table 4
Estimates of the regression of the volatility implied by the market price of the warrant and the Black-Scholes model on the warrant's moneyness for warrants listed on the stock exchange of Thailand [a]

Underlying stock [b]	Mean value of implied volatility (%)	Estimated intercept of regression [c]	Estimated coefficient of moneyness [c]	Adjusted R^2
Bangkok Bank of Commerce (312)	111.29	1.49 (26.58) *	-0.75 (-9.84) *	0.20
One Multiple Growth Fund (29)	62.74	0.64 (48.70) *	-0.95 (-2.69) *	0.27
Sub-Anan Fund (35)	54.81	0.53 (33.81) *	-1.33 (-3.52) *	0.30
Star Block Company (114)	85.61	1.78 (29.74) *	-0.78 (-14.87) *	0.60

[a] Moneyness is defined as $(S\text{-}PV_D\text{-}Xe^{-rT})/Xe^{-rT}$, where S is the corresponding closing price of the underlying stock, PV_D is the present value of the dividends expected over the life of the warrant, X is the exercise price of the warrant, r is the call money rate, and T is the time to expiration on the warrant.
[b] The figure in parentheses is the sample size.
[c] The figure in parentheses is the t-statistic. The statistic is calculated using White's heteroskedasticity adjusted standard errors.
* indicates significance at the 1 percent level.

that are in-the-money, and the estimate of b is significantly negative, our results are not consistent with a stochastic volatility model.

The second model we analyze in the paper allows for volatility to be inversely related to stock price. Specifically, the model used is derived in Cox (1975), and assumes that the elasticity of stock return variance with respect to the stock price is -1. [17] In the Cox Square Root (CSR) model, the price of a warrant is given by: [18]

$$W=\hat{N}\left[\hat{S}N(q(4))-Xe^{-rT}N(q(0))\right], \tag{3}$$

[17] The choice of an elasticity of -1 is predicated by the fact that this parameter value leads to a tractable model.
[18] See Lauterbach and Schultz (1990) regarding the applicability of the CSR model for warrant pricing.

where,

for $v = 0$ or 4,

$q(v) = [1 + h(h-1)p - h(h-1)(2-h)(1-3h)1/2p^2 - (z/(v+y))^h]/[2h^2p(1-(1-h)(1-3h)p)]$,

$h(v) = 1 - [2(v+y)(v+3y)/3(v+2y)^2]$,

$p = (v+2y)/(v+y)^2$,

$y = 4r\hat{S}/\sigma^2(1-e^{-rT})$,

$z = 4rX/\sigma^2(e^{rT}-1)$,

and all other variables have been defined earlier.

To compare the pricing accuracy of the BS and the CSR models on any day t, first separate estimates of implied volatilities are obtained for each warrant using five previous closing prices and the two models. [19] These estimates of implied volatility are then used to price the warrant on day t, and pricing errors are computed for each model by subtracting the model price on day t from the corresponding market price. The values of these pricing errors are then compared using parametric and non-parametric tests. The results of these tests are presented in the next section.

4. Results of the comparison of the Black–Scholes and the Cox Square Root models

Table 5 presents the results of the comparison of the pricing accuracy of the BS and CSR models. Panel A of the table contains the mean and median values for the differences (columns 2 and 3) and the absolute differences (columns 4 and 5) between model and market prices as a fraction of market price, and the fraction of the differences that are positive (column 6). Panel B of the same table contains the mean and median values for the differences in the absolute percentage pricing errors for the BS and the CSR models (columns 2 and 3), the percentage of the differences that are positive (column 4), the t-statistic for the null hypothesis that the difference is zero (column 5), and the p-value from a Wilcoxon Signed Rank test of the same null hypothesis (column 6).

As can be seen from Panel A of this table, on average both models tend to underprice in-the-money warrants and overprice out-of-the-money warrants. [20]

[19] To estimate implied standard deviations, we pursue the following procedure. First, we estimate the model price of the warrant $W_j(\sigma_0)$ based on an initial guess for the implied volatility, σ_0. Second, we estimate the regression, $W_j - W_j(\sigma_0) + \sigma_0(\partial W_j(\sigma_0)/\partial\sigma) = \sigma(\partial W_j(\sigma_0)/\partial\sigma) + e_j$, where W_j is the market price of the option. The coefficient of this regression, σ_1, is a new estimate of implied volatility. Third, we calculate the value of $(\sigma_1 - \sigma_0)/\sigma_0$. If the value is greater than 0.0001, we repeat the three steps using σ_1 as the new guess for implied volatility. If the absolute value is less than 0.0001, then σ_1 is the estimated implied volatility. See Whaley (1982) for details on the technique.

[20] This result is consistent with the volatility skew found in U.S. S&P500 Index option prices. See Choi and Wohar (1994) for more details.

Table 5
A comparison of model and market prices for warrants listed on the stock exchange of Thailand. Model prices are obtained using the Black–Scholes and the Cox Square Root models

Panel A: The percentage difference

Underlying stock [a,b]	Difference between model and market prices as a fraction of market price (%)		Absolute difference between model and market prices as a fraction of market price (%)		Percentage of differences that are positive
	Mean value	Median value	Mean value	Median value	
Panel A1: Difference between the Black–Scholes model and market prices					
BBC (307)	0.38	-0.57	3.90	2.30	43
ONE-G (24)	-1.56	-1.39	9.49	5.65	46
SAN (30)	-0.24	2.09	8.40	4.34	60
STAR (109)	-0.07	-0.15	1.88	1.62	49
Panel A2: Difference between the Cox Square Root model and market prices					
BBC (307)	0.36	-0.50	3.72	2.14	42
ONE-G (24)	-1.22	-1.33	9.23	5.28	46
SAN (30)	-0.17	1.92	8.18	4.15	60
STAR (109)	-0.15	-0.16	1.78	1.52	47

Panel B: The difference between the pricing errors of the Black–Scholes and the Cox Square Root models

Underlying stock [a,b]	Mean difference between percentage absolute pricing errors (%)	Median difference between percentage absolute pricing errors (%)	Percentage of differences in absolute pricing errors that are positive	*t*-statistic for the two sample test of difference in absolute pricing errors	Wilcoxon signed-rank test *p*-value
BBC (307)	0.18	0.14	71	7.45 *	0.00
ONE-G (24)	0.26	0.25	79	1.68	0.00
SAN (30)	0.21	0.03	50	2.18 **	0.11
STAR (109)	0.10	0.12	71	4.73 *	0.00

[a] The underlying stocks are Bangkok Bank of Commerce (BBC), One Multiple Growth Trust (ONE-G), Sub-Anan Fund (SAN), and Star Block Company (STAR).
[b] The figures in parentheses are sample sizes.
* indicates significance at the 1 percent level.
** indicates significance at the 5 percent level.

This follows from (a) the result that the difference between model and market prices tends to be negative for warrants on BBC, ONE-G and STAR, and positive for warrants on SAN, and (b) the fact that 97 percent of the warrants on BBC, ONE-G and STAR are in-the-money, while 80 percent of the warrants on SAN are out-of-the-money. There is no pattern of mispricing with respect to time to expiration except in the case of STAR, where the warrants tend to be more underpriced by the model the longer the time to expiration.

An examination of the absolute pricing errors suggest that the mispricing across the sample of 470 observation averages 4 percent for the BS model and 3.84

percent for the CSR model. This absolute pricing error is negatively related to the moneyness of the warrant in the case of BBC, while the relationship is positive in the case of SAN and STAR. The absolute pricing errors for ONE-G warrants are unrelated to moneyness. There is no systematic relationship between the pricing errors for BBC and ONE-G warrants and time to expiration on the warrants. In contrast, the pricing errors for SAN warrants are positively related to time to expiration, while the opposite relationship is obtained in the case of STAR warrants.

Finally a comparison of the two models suggests that the difference in absolute percentage pricing error averages 0.16 percent for the entire sample. The average error is lower for the CSR model for all four warrants, with the difference in pricing errors between the two models being significantly different from zero in both the t-test and the Wilcoxon Signed rank test. The difference in pricing error is not systematically related to moneyness except in the case of the SAN warrants where the relationship is positive. In addition, there is no systematic relationship between pricing errors and time to expiration except in the cases of the SAN and STAR warrants where the relationship is negative.

5. Some alternative explanations for the results

To summarize, our results indicate that both models tend to underprice in-the-money warrants and overprice out-of-the-money warrants with the CSR model providing a marginal improvement in pricing performance. In this section, we explore some possible explanations for the results related to the mispricing.

One possible reason for the mispricing is that the dividend adjustment we employ is incorrect. Specifically, we assume in the case of BBC and STAR that the forecasted dividends are equal to the actual dividends paid. In addition, since ONE-G and SAN have no history of dividend payments, we assume a zero dividend payment over the life of the warrants in these two cases. Since BBC and STAR cut their dividend payments during the life of the warrants, it is possible that market participants were expecting a higher payout on all four underlying securities, and we are underestimating the magnitude of the dividend adjustment. Since the value of a warrant is negatively related to the magnitude of the dividend payout on the underlying stock, this would imply that we would, on average, overestimate the value of the warrants with our incorrect dividend adjustment. This prediction is not consistent with our results, since on average we tend to underestimate warrant prices using both models. [21] Thus incorrect adjustments for dividends cannot explain our results.

A second possible reason for systematic mispricing is that two of the four warrants under consideration are not European (SAN and ONE-G), while the

[21] In 45 percent of the 470 observations, the two models underprice the warrants under consideration.

pricing models we use are applicable only to European options. [22] Again, since we assume a zero dividend for the stocks underlying these two warrants, the use of an American warrant pricing model would not improve pricing performance. This follows from the fact that early exercise is not optimal for an American call option on a stock with no dividend payments. On the other hand, if a dividend was expected on these stocks, then the fact that we assume a zero dividend will have two counteracting effects. First ignoring the dividend will cause us to overprice warrants, while ignoring the early exercise feature will have the opposite effect. This factor cannot explain our results for two main reasons. First if the dividend expectation was non-zero we would expect it to be small given the fact that these stocks are newly listed. If the dividend is small, the impact on the value both directly and indirectly through its effect on the probability of early exercise would be small. In addition, the argument that the impact of early exercise is small is further strengthened by the fact that these warrants only allow for early exercise at two points in time, thus, further decreasing the value of the early exercise privilege in contrast to American options.

In the case of the STAR warrants, we may have induced systematic errors by not recognizing the fact that a portion of the warrants had to be exercised prior to the expiration date. [23] Ignoring this feature would imply that we would systematically overprice these warrants prior to this early exercise date, and we would expect the pricing error to be an increasing function of time to expiration. This hypothesis is not consistent with our results since both the BS and CSR model pricing errors are decreasing functions of time to expiration for STAR warrants.

A third possible reason for systematic mispricing is that we ignore the fact that there is a 10 percent limit on one-day changes in price for any securities that are listed on the SET. This price limit would effectively reduce the volatility on a security, and would have a greater impact on more volatile underlying securities. This hypothesis would imply that the mispricing we observe would be an increasing function of the volatility of the underlying stock. This is not consistent with our hypothesis since a regression of the pricing errors for both the BS and the CSR on implied volatility indicates no relationship. In addition, since the warrants under consideration here are either European (BBC and STAR) or can be exercised at three different points in time (ONE-G and SAN), the impact of the imposition of a price limit would be small.

A fourth possible explanation for our results is based on the liquidity of these warrants. Specifically, one could argue that the pricing errors are an artifact of the illiquid market for these securities. This would suggest that the pricing errors would be a decreasing function of trading volume. This hypothesis is not consistent with our results since the pricing errors for both models are positively related to trading volume (as a fraction of number of warrants outstanding).

[22] See Table 1 for more details on the pseudo-American features of these warrants.

[23] See Table 1 for more details.

A fifth possible explanation for our results is that the pricing errors reflect the bid–ask spread in the market. Since percentage bid–ask spreads decrease with the price level of the security, this suggests that we should observe a smaller percentage pricing error for the higher priced options. This hypothesis is consistent with our results since the warrants on STAR have the highest price and the lowest absolute percentage pricing errors, while those on ONE-G and SAN have the lowest price and the highest absolute percentage pricing errors.

Finally, a sixth possible explanation for our results is that our model does not properly incorporate the fact that market participants are not allowed to short sell securities in Thailand. Since this restriction effectively increases the transactions costs associated with arbitrage strategies that require short sales, our results are consistent with this hypothesis. This follows from the fact that a short sale of a security can be simulated by the more costly strategy of selling units of the security one already owns and buying it back at a future date. As shown by Leland (1985), this, in turn, implies that the model price of the warrant would lie in between two bounds, one lower and the other higher than the zero-transactions cost value. As more warrants are listed, future research may be better able to distinguish between the relative importance of the aforementioned explanations for the observed pricing error.

Appendix A

Listing requirements for warrants on the stock exchange of Thailand

Certificates representing rights to purchase ordinary shares of preferred shares or debentures should have the following specific qualifications:

- Be certificates for which the approval of the public offering by the Securities and Exchange Commission has been applied for by an applicant which is a public limited company, and such approval has been granted.
- Be certificates issued by a listed company whose ordinary shares have been listed on the Exchange for a period of not less than one year.
- Have no fewer than two hundred and fifty holders on the date of filing of the application.
- The number of ordinary shares or preferred shares to be issued under the exercise of rights under the certificates shall not exceed thirty percent of the total amount of paid-up capital of the applicant at any one time.
- Have definite dates for payment of shares where the rights under the certificates may be exercised more than once, and have a notice period of not less than fifteen days for the final exercise of rights under the certificates.
- Have the ratio of the exercise of rights under the certificates of one unit per ordinary share, preferred shares or debenture as of the date of issuance, or any such other ratio as specified by the Board.

- Have a maturity period of not less than one year but not more than five years from the date of issuance of such certificates provided, however, that the repurchase by the issuer before maturity date under conditions specified in the issuance of such certificates is not prohibited.

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