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An Empirical Examination of the Black-Scholes Call Option Pricing Model

JAMES D. MACBETH and LARRY J. MERVILLE*

I. Introduction

THIS STUDY IS A descriptive analysis of how market prices of call options compare with prices predicted by the Black and Scholes [2], B-S, option pricing model. Although tests of alternative call option valuation models are not conducted, it is possible to relate some of the deviations which we observe between market and B-S model prices to predictions of other models, and thereby reconcile conflicting statements in the empirical literature regarding the relationship between market prices and B-S model prices.

The B-S model:

$$C = S \cdot N(d_1) - Xe^{-r\tau} \cdot N(d_2)$$

$$d_1 = \frac{\ln\left(\dfrac{S}{X}\right) + (r + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}; \qquad d_2 = d_1 - \sigma\sqrt{\tau} \tag{1}$$

has been discussed extensively in the literature. At any instant of time, C is the market value of the call option; S is the price of the underlying security; X is the exercise price; τ is the time to expiration; r is the short-term interest rate which is continuous and constant through time; σ^2 is the variance rate of return for the underlying security; $N(d_i)$ is the cumulative normal density function evaluated at d_i.

For any time interval of length d, the return on the underlying security has a normal distribution with variance $\sigma^2 d$. It is also assumed in this model that the underlying security pays no cash distributions, that there are no transaction costs in buying or selling the option or underlying security, that there are no taxes, and that there are no restrictions on short sales.

Our investigation focuses on the variance rate, σ^2. Blattberg and Gonedes [3] present evidence that σ^2 changes through time. That is, it appears that observed rates of return on common stock can be characterized as independent drawings from a normal population with presumably constant mean but changing variance. An initial step in testing the B-S model against option pricing models which do not require a constant variance rate (for example, Merton's [7] model) should be a careful documentation of how B-S model predicted prices compare with observed market prices.

The basic method of analysis is similar to that of Latané and Rendelman [6],

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Schmalensee and Trippi [9], and Chiras and Manaster [4]. We substitute the observed market price into equation (1) and numerically solve the equation for its only unobservable quantity, the variance rate, σ^2. We find that on any given day different market prices of options written on the same underlying stock yield different values of σ^2 and that these implied variance rates, for the same option, change through time. However, our results indicate that the differences between the implied variance rates are systematically related to the difference between the stock price, S, the exercise price, X, and the time left to expiration, τ. Then, under certain assumptions, these systematic differences in variance rates are translated into systematic differences between observed market prices of call options and B-S model predicted prices.

II. The Data and Empirical Results

Our sample consists of daily closing prices of all call options traded on the Chicago Board of Trade Options Exchange for American Telephone and Telegraph (ATT), Avon Products (AVON), Eastman Kodak (ETKD), Exxon (EXXN), International Business Machines (IBM), and Xerox (XERX) from December 31, 1975 to December 31, 1976. Option prices and prices of the underlying stocks are taken from the Wall Street Journal. Dividend information comes from Standard and Poors Stock Record. The riskless return is imputed from bid and ask yields reported in the Wall Street Journal for United States Treasury Bills and is updated weekly. That is, when an option is first traded we select a Treasury Bill that expires just beyond the expiration date of the option and then follow the yield on that Bill through time. For each option expiration date we have a different riskless rate. The riskless rates are generally within ½ of 1% of one another and given the lack of sensitivity of the call price to the riskless rate, our results would be virtually identical had we used a single riskless return for a Treasury Bill with, say, one year to maturity for all expiration dates.

For each day, t, a numerical search routine is used to calculate an implied value of σ for each option price. The time to expiration and the riskless return are measured on a daily basis and the current stock price is reduced by the present value, measured by the riskless rate, of any dividends paid between t and the expiration date. At this juncture we have not explicitly addressed the issue resulting from the fact that American options may be exercised early; however, we later demonstrate that our conclusions would not be altered had we considered early exercise in the calculation of each implied value of σ.

The numerical search routine finds implied values of σ in the interval .0001 to .06.[1] For options in the money, $S > Xe^{-r\tau}$, we observe some option prices that are too low to yield a σ as large as .0001. These options are invariably deep in the money with less than ninety days to expiration.[2] We also observe some deep in the money options with less than ninety days to expiration which have implied values of σ greater than .06. Overall, we are able to find values of σ between .0001 and .06 for approximately 97% of the observed option prices. For example, we

[1] The range .0001 to .06 is large enough to include all reasonable values of σ.

[2] Out the money options with positive market prices will always have a positive implied variance rate.

observe 2483 separate prices for options written on IBM and traded on the CBOE in 1976. The implied value of σ exceeds .06 in only 23 cases and is less than .0001 in just 31 cases.

A. *The Relationship Between the Implied Variance Rate, the Exercise Price, and the Time to Expiration*

Table 1 contains a sample of the implied values of σ for IBM. Looking down a column of values of σ in Table 1 we see that the values of σ change from day to day and that on any given day the implied variance rates decline as the exercise price increases. These relationships hold for all of our option data.

To see the importance of these results, consider again the B-S model. In the valuation of a call option price there are two variables (S, τ) and three parameters (r, σ^2, X). Of the parameters, only the variance rate is tied directly to the underlying stock. Thus, consider two call options written on the same stock with the same number of days to expiration, τ. For these two options, S, r, and τ should clearly be the same values in their pricing. However; let one be in the money ($Xe^{-r\tau} < S$) and one out of the money ($X^*e^{-r\tau} > S$) which implies different exercise prices ($X < X^*$). Should different exercise prices imply different variance rates? The answer must be no, as the σ^2 pertains to S not X. Therefore, in the money and out of the money options according to B-S should yield the same implied σ^2 or σ. Our empirical results show that the implied variances are different in general depending on whether or not the option is out, near, or in the money.

There also appears to be a relationship between implied variance rates and time to expiration. In Table 1 one can see a tendency for in the money options with a short time to expiration to have implied values of σ which are larger than those of options with the same exercise price but a longer time to expiration. At the same time, out of the money options with a short time to expiration tend to have implied values of σ smaller than those of options with the same exercise price but a longer time to expiration. We observe the same relationships between the implied variance rates and exercise prices in the data for the other five securities. However, the relationships between time to expiration and implied variance rates in the data for options on the other five securities is not always as systematic as it is for options written on IBM, especially as the time to expiration becomes less than one month.

B. *The Relationship Between B-S Model Prices and Observed Market Prices*

Black [1] states that market prices of call options "tend to differ in certain systematic ways" from the values given by the B-S model for options with less than three months to expiration and for options that are either deep in or deep out of the money. The evidence in Table 1 is consistent with Black's statement. On the basis of Black's assertion and the evidence in Table 1, we assume, for the remainder of our analysis, that the B-S model correctly prices at the money options with at least ninety days to expiration.[3] It follows from this assumption

[3] While this assumption is somewhat arbitrary, it is useful for our purpose of comparing observed market prices and B-S model prices; but it may not be appropriate for all purposes. For example, it may not be useful for a test of the B-S model against Merton's model.

Table 1

Sample Implied Values of Sigma for IBM

Date	Exercise Price	January	April	July	October	Stock Price
1006.1976	200.00	0.0	0.0	0.02103	0.0	255.12
1006.1976	220.00	0.0	0.0	0.01730	0.01655	255.12
1006.1976	240.00	0.01412	0.0	0.01362	0.01423	255.12
1006.1976	260.00	0.01263	0.0	0.01118	0.01254	255.12
1006.1976	280.00	0.01154	0.0	0.00980	0.01164	255.12
1106.1976	200.00	0.0	0.0	0.02900	0.0	257.75
1106.1976	220.00	0.0	0.0	0.02087	0.01674	257.75
1106.1976	240.00	0.01431	0.0	0.01567	0.01496	257.75
1106.1976	260.00	0.01304	0.0	0.01080	0.01272	257.75
1106.1976	280.00	0.01158	0.0	0.00979	0.01153	257.75
1406.1976	200.00	0.0	0.0	0.02841	0.0	260.25
1406.1976	220.00	0.0	0.0	0.01931	0.01869	260.25
1406.1976	240.00	0.01540	0.0	0.01472	0.01542	260.25
1406.1976	260.00	0.01289	0.0	0.01066	0.01282	260.25
1406.1976	280.00	0.01169	0.0	0.01016	0.01164	260.25
1506.1976	200.00	0.0	0.0	0.02865	0.0	259.00
1506.1976	220.00	0.0	0.0	0.02413	0.01727	259.00
1506.1976	240.00	0.01498	0.0	0.01588	0.01502	259.00
1506.1976	260.00	0.01316	0.0	0.01072	0.01317	259.00
1506.1976	280.00	0.01173	0.0	0.01063	0.01174	259.00
1606.1976	200.00	0.0	0.0	0.03368	0.0	262.37
1606.1976	220.00	0.0	0.0	0.02542	0.01947	262.37
1606.1976	240.00	0.01519	0.0	0.01630	0.01584	262.37
1606.1976	260.00	0.01287	0.0	0.01068	0.01303	262.37
1606.1976	280.00	0.01172	0.0	0.01018	0.01132	262.37
1706.1976	200.00	0.0	0.0	0.02794	0.0	267.50
1706.1976	220.00	0.0	0.0	0.01398	0.01490	267.50
1706.1976	240.00	0.01358	0.0	0.01108	0.01335	267.50
1706.1976	260.00	0.01248	0.0	0.00820	0.01220	267.50
1706.1976	280.00	0.01140	0.0	0.00983	0.01097	267.50
1806.1976	200.00	0.0	0.0	0.03929	0.0	266.25
1806.1976	220.00	0.0	0.0	0.02627	0.01835	266.25
1806.1976	240.00	0.01463	0.0	0.01790	0.01568	266.25
1806.1976	260.00	0.01323	0.0	0.01087	0.01299	266.25
1806.1976	280.00	0.01206	0.0	0.01153	0.01182	266.25
2106.1976	200.00	0.0	0.0	0.03496	0.0	270.50
2106.1976	220.00	0.0	0.0	0.02514	0.01650	270.50
2106.1976	240.00	0.01374	0.0	0.01733	0.01632	270.50
2106.1976	260.00	0.01305	0.0	0.01254	0.01324	270.50
2106.1976	280.00	0.01207	0.0	0.01186	0.01196	270.50
2206.1976	200.00	0.0	0.0	0.04245	0.0	268.75
2206.1976	220.00	0.0	0.0	0.02312	0.02017	268.75
2206.1976	240.00	0.01658	0.0	0.01412	0.01346	268.75
2206.1976	260.00	0.01336	0.0	0.01066	0.01228	268.75
2206.1976	280.00	0.01153	0.0	0.01133	0.01137	268.75
2306.1976	200.00	0.0	0.0	0.03953	0.0	271.50
2306.1976	220.00	0.0	0.0	0.02708	0.01796	271.50
2306.1976	240.00	0.01309	0.0	0.01883	0.01555	271.50
2306.1976	260.00	0.01317	0.0	0.01119	0.01374	271.50
2306.1976	280.00	0.01212	0.0	0.01199	0.01203	271.50

that the at the money implied variance rate is the proper or "true" variance rate. Then, given $\frac{\partial C}{\partial \sigma^2} > 0$, the implication of this assumption is that the B-S model must yield call option prices which exceed observed market prices for options out of the money and call option prices which are less than the observed market prices for options in the money because the implied values of σ decline as the exercise price increases.

For example, consider the IBM options traded on June 14. Panel A of Table 2 contains the market prices of the options, Panel B contains the implied values of σ, and Panel C contains the B-S model prices based on an estimated value of σ for an at the money option equal to .012848. The, in the money, $220 October option has a market price $3.60 greater than the B-S model price while the, out of the money, $280 October option has a B-S model price $1.28 greater than the market price.

Table 2 also illustrates that although in the money (out of the money) options with a short time to expiration have implied variance rates that are larger (smaller) than options with the same exercise price but at least ninety days to expiration, the dollar differences between B-S model prices and market prices of these short term options are generally less than the dollar differences between B-S model prices and market prices of options with the same exercise price but at least ninety days to expiration. For example, the implied value of σ for the $220

Table 2

IBM Option Data for June 14, 1976

	Exercise Price	Expiration Dates: January	April	July	October
		Market Prices			
Panel A	$200			$62.00	
	$220			$42.00	$47.50
	$240	$36.50		$23.13	$30.50
	$260	$22.00		$ 7.13	$16.13
	$280	$12.00		$ .94	$ 7.00
		Implied Values of σ			
Panel B	$200			.028412	
	$220			.019312	.018691
	$240	.015404		.014725	.015417
	$260	.012885		.010658	.012824
	$280	.011689		.010158	.011640
		Black-Scholes Model Prices			
Panel C	$200			$61.23	
	$220			$41.38	$43.90
	$240	$33.22		$22.53	$28.13
	$260	$21.95		$ 8.42	$16.15
	$280	$13.73		$ 1.90	$ 8.28
	τ =	220.		33.	124.
	r (Annual) =	5.9%		5.3%	5.6%
	S* =	$255.85		$260.25	258.03

S* is the stock price less the present value of dividends of $2.25 paid September 10, and December 10.

July option is greater than the implied value for the \$220 October option, but the difference between the B-S model price and the market price of these options is \$.62 for the July option and \$3.60 for the October option.

In order to examine the difference between the B-S model prices and observed market prices, we require the implied variance rate for an at the money option. Since we are usually unable to observe an at the money option, we deduce the variance rate for an at the money option on the basis of observed variance rates. That is, to determine an implied value of σ for an at the money option on a particular stock we estimate the following regression model each day:

$$\sigma_{ijt} = \theta_{i0t} + \theta_{i1t} m_{ijt} + \epsilon_{ijt} \qquad j = 1, 2 \cdots J \tag{2}$$

where σ_{ijt} is the implied σ for option j on security i on day t. And

$$m_{ijt} = \frac{S_{it} - X_{ij} e^{-r\tau}}{X_{ij} e^{-r\tau}}$$

where S_{it} is the closing price of a share of security i on day t and $X_{ij} e^{-r\tau}$ is the present value at time t of the exercise price j for an option on a share of security i. The estimated values of θ_{i0t}, $\hat{\theta}_{i0t}$, is our estimate of the σ that would be implied by the B-S model on day t for an at the money option written on security i[4].

Figure 1 is an exemplary scatter diagram with the estimate regression line for the options written on IBM on June 14, 1976. Notice that $\hat{\theta}_{i0t}$ equals .012848 and that the implied values of σ for the two near the money \$260 options with at least ninety days to expiration are .012885 and .012824. Typically, the number of observations, J, in these regressions is in the neighborhood of five. Given the evidence in Table 1 for IBM, it is not surprising that the R^2's from these regressions are large, generally on the order of 0.80.

$\hat{\theta}_{i0t}$ is a weighted sum of the implied values of σ for the options on security i on day t and, therefore, corresponds to the weighted average implied standard deviations of Latané and Rendelman [6], Schmalensee and Trippi [9], and Chiras and Manaster [4].

To facilitate comparisons between options on different securities we relate the difference between C_{ijt}, the market price on day t of option j on security i with exercise price X_{ij}, and the B-S model price of the same option, $C_{BS}(\hat{\theta}_{i0t})$; to the extent to which the option is in or out of the money as measured by m_{ijt}, the independent variable in equation (2). Initially, we relate m_{ijt} to v_{ijt}, the difference between the market price of an option and the B-S model price expressed as a percentage of the B-S model price.[5]

$$v_{ijt} = \frac{C_{ijt} - C_{BS}(\hat{\theta}_{i0t})}{C_{BS}(\hat{\theta}_{i0t})} \tag{3}$$

Figure 2 is a scatter diagram of v against m for options with at least ninety days to expiration written on shares of IBM in the year 1976.[6] It appears that on

[4] Since the B-S model appears to fit the data best for options with at least ninety days to expiration, we include in these regressions only options with that property.

[5] When comparing B-S model prices and market prices using v_{ijt} we do not include options which have B-S model prices less than \$.0625, the lowest quoted market price for options.

[6] Subscripts are omitted when there is little chance of confusion on the part of the reader.

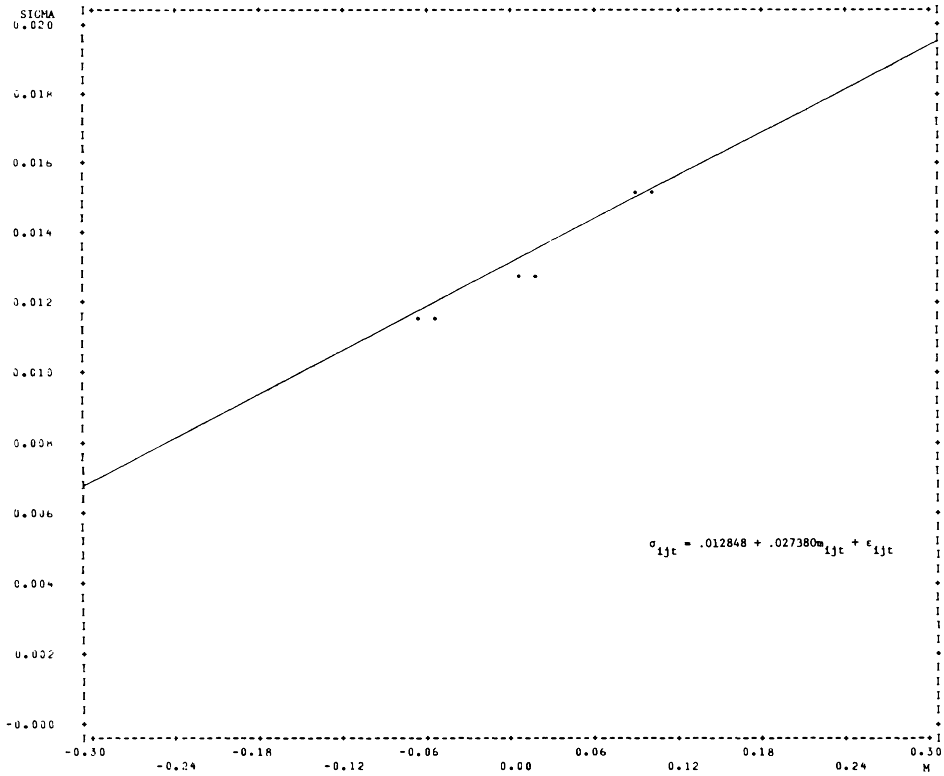

Figure 1. Sigma values regressed on M for IBM

average v is an increasing function of m for values of m less than .05. For options on IBM that are in the money by more than five percent, the percentage difference between observed market prices and B-S model prices appears to be constant, in the neighborhood of five percent of the present value of the exercise price. Scatter diagrams for corresponding options written on the other five stocks appear almost identical to Figure 2 and are thus omitted.

Figure 3 is the same type of scatter diagram as Figure 2 but for options written on IBM with less than ninety days to expiration. The scatter in Figure 3 is basically the same as in Figure 2, but there are some out of the money options with fairly large positive values of v. Scatter diagrams for corresponding options written on the other five stocks are similar to Figure 3 except that there are more out of the money options with positive values of v. For example, out of the money options written on Xerox have nearly as many positive values of v as negative values.

Call options trade on the CBOE can be exercised at any time prior to the stated expiration date and sometimes should be exercised just prior to an ex-dividend day. Assuming that one dividend, known for certain, will be paid prior to the stated expiration date of an option, the option can be valued either by the B-S model with the stock price reduced by the present value of the future dividend (as we have done), or by the B-S model with the observed stock price and the ex-dividend date as the expiration date. The market price of the option will be

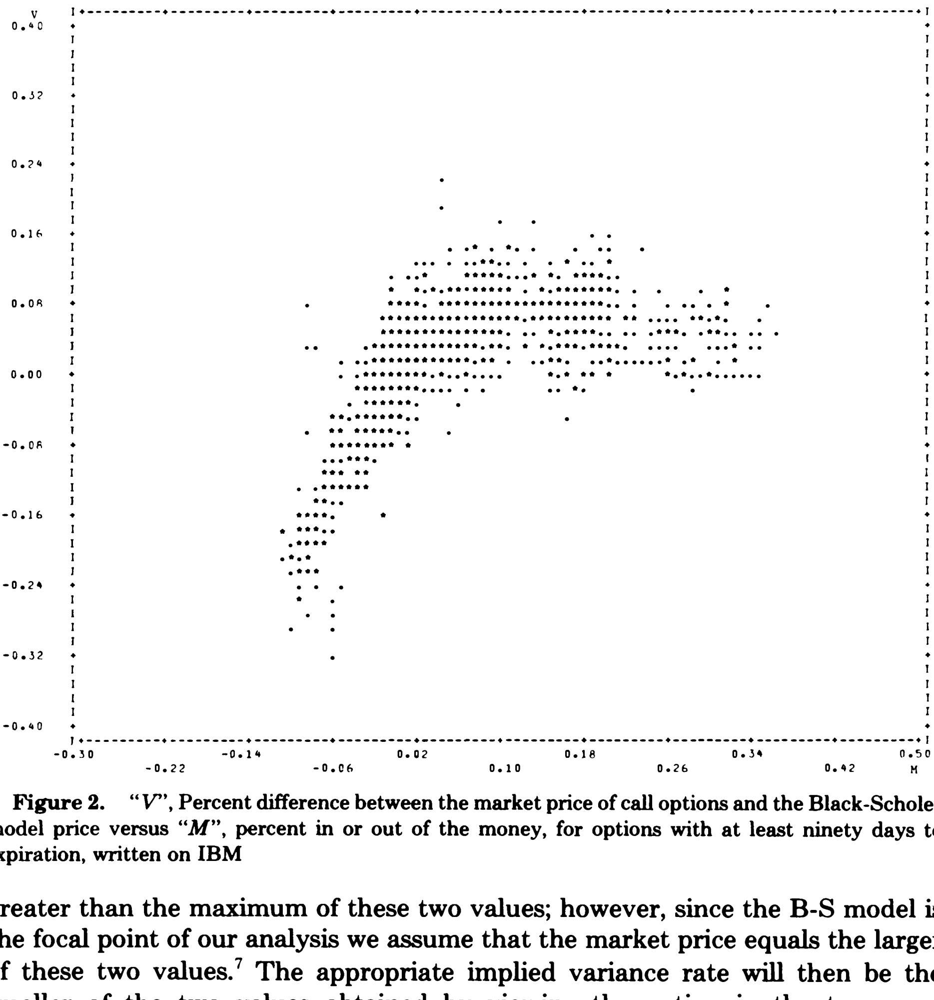

Figure 2. "V", Percent difference between the market price of call options and the Black-Scholes model price versus "M", percent in or out of the money, for options with at least ninety days to expiration, written on IBM

greater than the maximum of these two values; however, since the B-S model is the focal point of our analysis we assume that the market price equals the larger of these two values.[7] The appropriate implied variance rate will then be the smaller of the two values obtained by viewing the option in the two ways mentioned above.

Early exercise is more likely the closer the ex-dividend date is to the expiration date. Since all of our six stocks pay dividends in February, May, August and November there is at least forty-five days between an ex-dividend date and an expiration date. Therefore, apriori there is some reason to believe that our option prices will not be seriously affected by the possibility of early exercise. However, should early exercise be important, our implied value of σ for an option will be too high in the days prior to the ex-dividend date. This could affect our results in two ways. First, $\hat{\theta}_{i0t}$ will be too large and second, we will have some observations in the plots of v against m that should be deleted and replaced with observations based upon larger stock prices and shorter times to expiration.

Since we exclude options with less than ninety days to expiration from the calculation of $\hat{\theta}_{i0t}$, the early exercise effect could only affect $\hat{\theta}_{i0t}$ if it occurs at least

[7] Roll [8] derives an explicit valuation equation for a call option with a single known dividend to be paid before the option's expiration date.

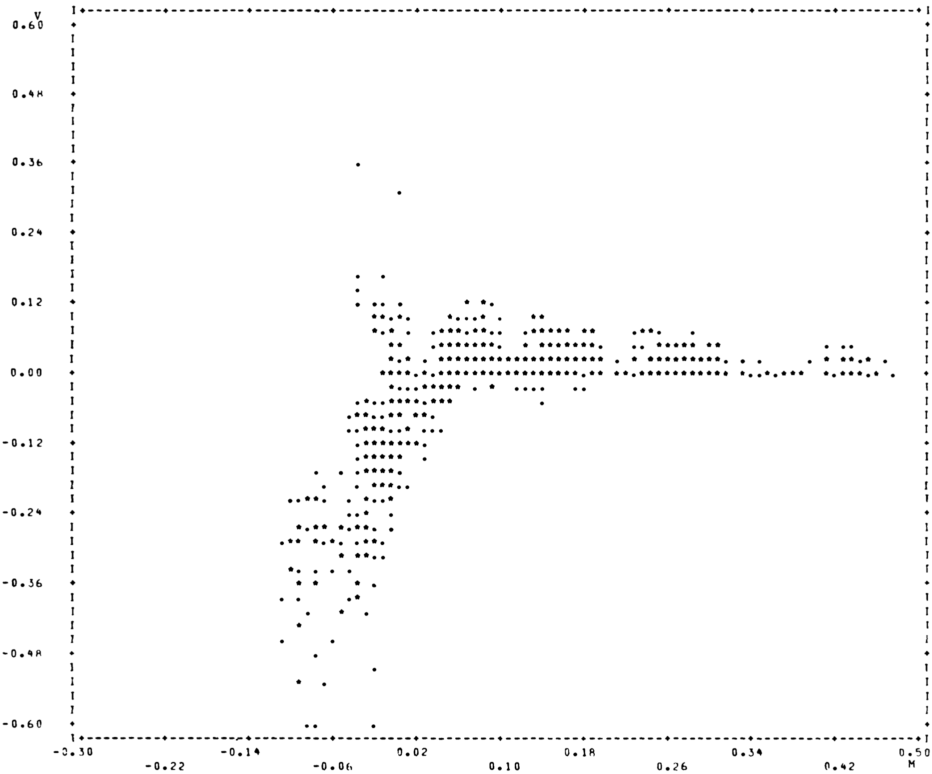

Figure 3. "*V*", Percent difference between market price of call options and the Black-Scholes price versus "*M*", Percent in or out of the money, for options with less than ninety days to expiration, written on IBM

ninety days prior to expiration. As a check, we have examined our data for the period between ninety and one hundred days prior to expiration and find no evidence of an early exercise effect. That is, if we consider that the option will be exercised just prior to the ex-dividend date and compute an implied variance rate, we invariably obtain a value larger than the implied variance we originally calculated.

Since we find no evidence of an early exercise effect in the prices of options with between ninety and one hundred days to expiration we assume that our computed values of v and m for options with at least ninety days to expiration are not contaminated by an early exercise effect. To determine whether the computed values of v and m for options with less than ninety days to expiration are affected, we have examined our data for the two weeks prior to and two weeks following ex-dividend dates. For Eastman Kodak and International Business Machines we find no evidence of an early exercise effect. However, there does appear to be some unusual price behavior for deep in the money options on the other four stocks around ex-dividend days. It is not uncommon for the price of a deep in the money option to drop suddenly to its exercise value in the week prior to the ex-dividend day even though the stock price remains essentially constant. This suggests that the option should have been exercised; however, it is difficult to explain why in the weeks prior to the ex-dividend date the price should reflect the original expiration date and then, three or four days before the ex-dividend day, suddenly reflect the day before the ex-dividend date as the expiration date. On the ex-dividend day and the days following, the option sometimes continues to trade at its exercise value and sometimes trades at a higher value.

Fortunately these days of unusual price behavior have been excluded from our sample since on days when an option trdes at its value if exercised, we are unable to compute a positive value of σ. Eastman Kodak and International Business Machines have significantly fewer instances where we are unable to impute a positive value of σ for deep in the money options. To summarize, we are unable to detect an early exercise effect in prices of options on two of our stocks. The possibility of early exercise appears to affect a small number of prices of options, with less than ninety days to expiration, on the other four stocks, but we have eliminated virtually all of these observations from our sample. Thus, had early exercise been considered for all of our options the plots in Figure 2 would look exactly the same and those in Figure 3 would look essentially the same.

To quantify the visual relationships conveyed in Figures 2 and 3 we report in Table 3 the sample means and standard deviations of v and m for in the money and out of the money options with at least ninety days to expiration (far), and less than ninety days to expiration (near) on all six stocks. To convey an idea of the dollar amounts involved we compute

$$y_{ijt} \equiv C_{ijt} - C_{BS}(\hat{\theta}_{i0t}) \tag{4}$$

and report the sample mean and standard deviation of y where N is the number of options in each category.

The statistics reported in Table 3 reinforce the visual relationships expressed in the Figures. Regardless of whether the time to expiration is at least ninety days or less than ninety days, options that are in the money have, on average, market prices which exceed B-S model prices and options that are out of the money have, on average, market prices which are less than B-S model prices.

Table 3 also contains some information regarding the effect of time to expiration on the overpricing or underpricing of options by the B-S model. Notice that in the money options with τ less than ninety days are on average deeper in the money but that they are on average underpriced by the B-S model to a smaller extent, as measured by $\bar{y}$, than options with at least ninety days to expiration. On the other hand, out of the money options with less than ninety days to expiration are on average about as far out of the money as out of the money options with at least ninety days to expiration but those with less than ninety days to expiration are on average overpriced by the B-S model to a lesser extent than those with at least ninety days to expiration. Thus, it appears that as τ approaches zero so does y even though the implied variance rates in Table 1 for options in or out of the money tend to deviate more from the implied variance rate of an at the money option. But this result is to be expected since B-S model prices and market prices converge to the minimum of zero or $S - X$ as τ approaches zero.

To investigate the relationship between changes in y and changes in m and τ, we estimate the following linear regression model

$$y_{ijt} = \alpha_0 + \alpha_1 m_{ijt} + \alpha_2 \tau_{ijt} + \epsilon_{ijt} \qquad \begin{matrix} j = 1, 2 \cdots J \\ t = 1, 2, \cdots T \end{matrix} \tag{5}$$

Table 3

Difference Between Market Prices and Black-Scholes Model Prices

Option Money	Expiration	N	$\bar{m}$	$s(m)$	$\bar{v}$	$s(v)$	$\bar{y}$	$s(y)$
ATT								
Out	Near	120	−.04	.03	−.16	.16	$−.10	$ .12
Out	Far	420	−.04	.03	−.10	.10	−.13	.13
In	Far	875	.09	.06	.05	.08	.34	.38
In	Near	384	.11	.08	.01	.09	.19	.31
AVON								
Out	Near	283	−.08	.06	−.05	.25	−.07	.18
Out	Far	532	−.07	.06	−.07	.08	−.17	.16
In	Far	928	.13	.10	.04	.04	.28	.29
In	Near	465	.16	.13	.01	.08	.10	.29
ETKD								
Out	Near	406	−.10	.06	−.12	.25	−.10	.26
Out	Far	930	−.10	.07	−.12	.13	−.33	.30
In	Far	632	.09	.07	.03	.04	.39	.63
In	Near	376	.10	.08	.04	.06	.49	.60
EXXN								
Out	Near	230	−.06	.04	−.07	.21	−.09	.14
Out	Far	590	−.07	.05	−.10	.12	−.17	.20
In	Far	684	.08	.06	.02	.05	.16	.35
In	Near	342	.09	.07	−.01	.06	.06	.30
IBM								
Out	Near	206	−.03	.03	−.18	.16	−.83	.73
Out	Far	376	−.04	.03	−.07	.08	−.80	.83
In	Far	1096	.11	.08	.06	.04	2.09	1.55
In	Near	748	.17	.12	.03	.05	1.09	1.16
XERX								
Out	Near	294	−.11	.07	−.02	.27	−.00	.23
Out	Far	567	−.10	.07	−.07	.09	−.17	.19
In	Far	798	.16	.12	.02	.03	.23	.33
In	Near	379	.16	.12	.01	.06	.19	.34

for the various classifications of options reported in Table 3. Summary statistics from the estimated model are reported in Table 4. The estimate value of α_k is denoted $\hat{\alpha}_k$; $t(\hat{\alpha}_k)$ is the Student "t" statistic of the estimated coefficient $\hat{\alpha}_k$; $\rho(\hat{\epsilon})$ is the first order autocorrelation coefficient of the residuals; $s(\hat{\epsilon})$ is the standard error of the residuals; and R^2 is the coefficient of determination.

For in the money options $\hat{\alpha}_1$ is always positive and with one exception larger for options with at least ninety days to expiration. Note also that for in the money options the coefficient of τ, $\hat{\alpha}_2$, is, with one exception, positive. Thus, the extent to which the B-S model underprices in the money options increases with the extent to which the option is in the money and this relationship appears stronger the longer the time to expiration. On the other hand, the extent to which the B-S model underprices in the money options becomes smaller as the expiration date approaches.

Table 4

Summary Statistics from the Regression Model

$$y_{jt} = \alpha_0 + \alpha_1 m_{jt} + \alpha_2 \tau_{jt} + \epsilon_{jt} \quad \begin{array}{l} j = 12\cdots J \\ t = 12\cdots T \end{array}$$

Option Money	Expiration	$\hat{\alpha}_0$	$t(\hat{\alpha}_0)$	$\hat{\alpha}_1$	$t(\hat{\alpha}_1)$	$\hat{\alpha}_2$*	$t(\hat{\alpha}_2)$	$\rho(\hat{\epsilon})$	R^2	$s(\hat{\epsilon})$
ATT										
Out	Near	−.12	−3.03	−.78	−2.20	−.03	−.56	.29	.03	.12
Out	Far	−.05	−1.93	1.78	8.15	−.01	−.47	.19	.13	.12
In	Far	−.40	−9.80	4.01	24.00	.22	11.36	.36	.41	.29
In	Near	−.26	−8.86	2.46	17.99	.31	6.95	.18	.52	.21
AVON										
Out	Near	−.15	−5.97	−1.15	−7.07	−.03	−.66	.61	.15	.17
Out	Far	−.03	−1.04	.98	8.34	−.04	−3.13	−.11	.12	.15
In	Far	−.15	−4.66	1.81	21.93	.11	7.33	.09	.34	.23
In	Near	−.13	−4.25	1.10	12.38	.11	2.34	.49	.26	.25
ETKD										
Out	Near	−.17	−4.90	−.01	−.05	.14	2.36	.63	.01	.26
Out	Far	.28	7.06	1.44	10.29	−.27	−14.56	.14	.21	.27
In	Far	.23	2.44	3.00	9.21	−.07	−1.45	.25	.13	.58
In	Near	−.24	−3.75	3.65	11.24	.72	7.08	.46	.32	.49
EXXN										
Out	Near	−.11	−4.50	−1.41	−6.84	−.11	−2.82	.32	.17	.13
Out	Far	−.03	−1.04	.95	5.39	−.04	−2.74	.38	.06	.20
In	Far	−.36	−7.53	2.74	13.40	.17	7.46	.11	.23	.30
In	Near	−.23	−6.77	2.71	13.97	.06	1.16	−.01	.36	.24
IBM										
Out	Near	−.56	−4.38	4.55	2.51	−.24	−1.05	.83	.04	.72
Out	Far	.11	1.02	21.36	19.82	−.09	−1.71	.12	.51	.58
In	Far	−.25	−1.62	11.69	25.52	.63	8.40	.43	.37	1.23
In	Near	−.24	−2.51	4.47	14.50	1.25	8.41	.45	.27	.99
XERX										
Out	Near	−.13	−3.49	−.15	−.76	.21	3.33	.81	.04	.23
Out	Far	.08	2.61	.59	5.14	−.11	−7.40	.14	.10	.18
In	Far	−.16	−3.91	1.33	14.80	.10	4.91	.09	.22	.29
In	Near	−.10	−2.64	1.27	9.86	.18	2.90	.33	.23	.30

$\hat{\alpha}_2^* = \hat{\alpha}_2 \times 10^2$.

The estimated coefficients of m and τ for out of the money options with at least ninety days to expiration are all positive and negative, respectively. Hence, for this class of options the extent to which the B-S model price exceeds the market price increases with the extent to which the option is out of the money and decreases as the expiration date approaches.

There does not appear to be any consistent relationships between changes in y and changes in m or τ for out of the money options with less than ninety days to expiration.

The Student t statistics of the estimated coefficients in Table 4 cannot be strictly interpreted because they overstate the statistical significance of the estimated coefficients. This results from sizable positive autocorrelation in the error terms, ϵ_{ijt}, which in turn probably results from the effects of one or more explanatory variables left out of the regression model and also from the way the observations for equation (5) are loaded into our regression routine. For any regression reported in Table 3 the observations are loaded each day in order of exercise price (e.g., across the rows of Table 1 for IBM). Thus, it appears that if this unknown explanatory variable causes $y_{i,j,t}$ to be larger (smaller) than the value predicted by the estimated regression equation, then this unknown explanatory variable also tends to make $y_{i,j+1,t}$ larger (smaller) than its predicted value. Since our main intent is to describe general systematic deviations of the B-S model predicted prices from observed market prices rather than to conduct a rigorous statistical test of the B-S model, the statistics reported in Table 4 should be viewed as more descriptive than inferential.

III Summary and Conclusions

Since we have examined daily prices of options on only six underlying securities over a one year time period, inferences drawn from this research must be tentative. However, we have analyzed in excess of twelve thousand option prices and have found striking similarities in the results for options on the six different stocks; therefore, we consider this research to be one of the most extensive empirical examinations of option prices to be reported in the literature to date.

If one assumes that the B-S model correctly prices at the money options with at least ninety days to expiration, then our analysis implies that:

1. The B-S model predicted prices are on average less (greater) than market prices for in the money (out of the money) options.

2. With the lone exception of out of the money options with less than ninety days to expiration, the extent to which the B-S model underprices (overprices) an in the money (out of the money) option increases with the extent to which the option is in the money (out of the money), and decreases as the time to expiration decreases.

3. B-S model prices of out of the money options with less than ninety days to expiration are, on average, greater than market prices; but there does not appear to be any consistent relationship between the extent to which these options are overpriced by the B-S model and the degree to which these options are out of the money or the time to expiration.[8]

We emphasize that our results are exactly opposite to those reported by Black [1], wherein he states that deep in the money (out of the money) options generally have B-S model prices which are greater (less) than market prices; and, our results also conflict with Merton's [7] statement that practitioners observe B-S model prices to be less than market prices for deep in the money as well as deep

[8] In our analysis, we have excluded options which have B-S model prices less than $.0625. The number of excluded observations varies from three for IBM to one hundred twenty-three for Eastman Kodak. Since the lowest option price is $.0625, these options all have market prices greater than their B-S model prices. On the other hand, if market prices less than $.0625 existed these options may not be underpriced by the B-S model. Moreover, the difference between market prices and B-S model prices for these options is on average a few cents; for the one hundred twenty-three Eastman Kodak options the average price difference is $.05. Thus, the inclusion of these options in our analysis would not affect our inferences.

out of the money options. We propose that these conflicting empirical observations may, at least in part, be the result of a non-stationary variance rate in the stochastic process generating stock prices.

One call option valuation model that explicitly incorporates a non-stationary variance rate is Geske's [5] compound option model. Assuming market prices are given by this model, then B-S model prices will also equal market prices on day t provided the variance rate for day t, σ_t^2, is used in the B-S equation. Thus, the compound option model provides theoretical justification for our methodology of computing implied variance rates on a daily basis, but it still does not explain the systematic differences we observe in implied variance rates.

Nevertheless, if one accepts the hypothesis that market prices should correspond to compound option model prices, then it is possible to explain some of the apparently conflicting empirical observations on option pricing which appear in the literature. For example, if B-S model prices are calculated using a constant estimated variance rate, $\hat{\sigma}$, obtained from a time series of past returns to the underlying common stock, then on days when the true variance rate, σ_t, exceeds $\hat{\sigma}$, the B-S model price based upon $\hat{\sigma}$ will be less than the market price; but on other days, when σ_t is less than $\hat{\sigma}$, the reverse will be true. Therefore, it is easy to see how one researcher may find in the money options overpriced by the B-S model while another finds in the money options underpriced if they use different data to estimate the variance rate $\hat{\sigma}$ and/or compute B-S model prices over different time periods.

Finally, our analysis sheds some light on the apparently profitable option trading strategy of Chiras and Manaster [4]. Their strategy involves selling options that have a market price which exceeds the B-S model prices and buying options that have a market price which is less than the B-S model price. Since Chiras and Manaster do not weight the implied σ values exactly as we weight them, their B-S model price for an option is different from ours. Generally, their B-S model price exceeds our B-S model price but the differences are not large. Our analysis suggests that their trading strategy may involve selling deep in the money options and buying deep out of the money options. Whether or not this strategy yields abnormally high returns is another matter.

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