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An LMI approach to output feedback H_∞ control design with circular pole constraints for vehicle suspension systems

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Abstract

This investigative paper is aimed to design a dynamic output feedback H_∞ controller with circular pole constraints for vehicle suspension systems. The closed-loop system satisfies both the H_∞ norm on the closed-loop transfer function from the disturbance input to the system output and D-stability constraint on the close-loop system matrix. A condition for finding the desired controller is illustrated via a linear matrix inequality (LMI). Additionally, it is shown that this existing condition is equivalent to the feasibility of a certain matrix inequality which is jointly convex in all variables. The proposed controller performance is carried out through simulation. Also, it is compared with passive system and the dynamic output feedback H_∞ controller.

Keywords: *Output feedback H_∞ control, circular pole constraints, vehicle suspension systems, linear matrix inequality*

1. Introduction

Recently, a lot of work reported in the literature relates to design different kinds of vehicle suspensions via various control strategies. Over two decades, numerous control design techniques have been developed for the vehicle suspension systems. The particular interest is to use advanced control technique to attain performance requirement for vehicle suspensions that include: i) isolating passengers from vibration and shock occurring from road roughness; ii) suppressing the hop of the wheels to maintain firm and uninterrupted contact of wheels to road; and iii) keeping suspension strokes within an allowable maximum (Hrovat, 1997). To the best knowledge of the authors, there are some relevant instance of control schemes employed in this field such as fuzzy control (Du & Zhang, 2009), optimal control (Prabakar, Sujatha, & Narayanan, 2013), H_∞ control (Rubio-Massegu, Palacios-Quinonero, Rossell, & Karimi, 2013; Chen & Guo, 2005), gain scheduling (Fialho & Balas, 2002), model predictive control (Chen & Scherer, 2004), passivity control (Xiao & Zhu, 2014), adaptive control (Koch & Kloiber, 2014) and so on.

In general, a desirable controller is designed to achieve various control objectives i.e. disturbance attenuation, robust stabilization of uncertain systems, or shaping of the open-loop responses capable of expressing in terms of H_∞ performance and tackled by H_∞ -synthesis techniques. In the study of H_∞ control problems, it is particularly aimed to design a feedback linear controller, leading to the fact that the closed-loop system is stable and the minimization of H_∞ norm of a closed-loop transfer function is achieved.

It is known well that one of the practical concerns of control design is its time-domain performance. Certainly, many time-domain performance specifications are determined by zeros and poles of the closed-loop system. The systems are, therefore, constructed so that better dynamic performance becomes achieved. To be more practical, the closed-loop poles are placed in a suitable region of the complex plane, especially in circular region. As such placing in a suitable disk, an upper bound on the damping ratio, the natural frequency, and the damped natural frequency can be guaranteed. In addition, it is possible to conclude that the closed-loop poles in a specified region guarantees both stability; all closed-loop poles forced in the circular region of Left Half Plane (LHP), and the transient performance i.e. settling time, maximum overshoot, and rise time. For the closed-loop pole placement in a specified region, the design of a controller in both nominal and uncertain systems is of great interest. Many researchers have investigated this problem (Chu, 1991; Haddad & Bernstein, 1992) for linear systems without uncertainties. Garcia and Bernussou (1995) extended to the system with uncertainties for state feedback control. Moreover, various researchers include the problem with the guaranteed cost control (Garcia, 1997; Yu, Chen, & Nan, 2002), and with the multi-objective control (Chilali & Gahinet, 1996; Scherer, Gahinet, & Chilali, 1997). For further improving the system transient dynamic performances, the H∞ strategy combining with the transient behavior of the closed-loop system becomes a promising and effective approach. Unfortunately, there are less attentive studies to the combination of the H∞ control and transient behavior improvement simultaneously (Yedavalli & Liu, 1995; Wang, 1998; Wang, Zeng, Ho, & Unbehauen, 2002; Kanchanaharuthai & Ngamsom, 2005). For the particular interest of vehicle suspension systems, no recent report combines H_∞ control design with circular pole constraints. Such combination is not only to assure in the disturbance attenuation problem but also to improve better transient performance.

This paper is organized as follows. Section 2 is a problem statement and Section 3 stated performance and stability analysis. In Section 4, for an output feedback controller, the existing conditions of H∞ controller with pole constraints are illustrated via a linear matrix inequality (LMI) to construct a desired controller and the proposed scheme is applied to design the dynamic output feedback controller for a quarter-car suspension system in Section 5. Section 6 is dedicated to comparative analysis of simulation results with the existing uncontrolled system and the study is then concluded in Section 7.

Throughout the study, R^n and $R^{n\times m}$ denote the set of n-dimensional real vectors and the set of $n\times m$ –dimensional real matrices, respectively. M^T is the transpose of matrix M and the notion $X>Y$ where X and Y are symmetric matrices, meaning that $X-Y$ yields positive definite result. I_n denotes the identity matrix.

2. Problem statement

Since a linear system is of particular interest, the following equations have been described:

$$\begin{aligned}\dot{x} &= Ax + B_w w + B_u u \\ z &= C_z x + D_{zw} w + D_{zu} u \ , \\ y &= C_y x + D_{yw} w + D_{yu} u\end{aligned} \qquad (1)$$

where $x \in R^{n\times 1}$ is the state-vector, $u \in R^{m\times 1}$ is the control-vector, $z \in R^{r\times 1}$ denotes the regulated output-vector, $y \in R^{p\times 1}$ is the output-vector, and $w \in R^{m\times 1}$ denotes the exogenous vector. $A, B_w, B_u, C_z, C_y, D_{zw}, D_{zu}, D_{yw},$ and D_{yu} have appropriate dimensions. In addition, assume that the system considered is completely controllable and observable.

The interested problem is formulated to determine a linear output feedback control such that the following performance requirements are simultaneously achieved;

(a) All closed-loop poles are confined in a stable circular region $D(\alpha, r)$ in the complex plane together with the center at $-\alpha + j0, \alpha > 0$ the radius $r \ (r < \alpha)$ as illustrated in Figure 1.

(b) The H_∞ norm of the transfer function $T_{zw}(s)$ from $w(t)$ to $z(t)$ meets the constraint $\|T_{zw}(s)\|_\infty = \sup_{\omega\in R} \sigma_{\max}[T_{zw}(j\omega)] \le \gamma$ where $\sigma_{\max}[\cdot]$ is the maximum singular value, γ is a pre-specified constant and $T_{zw}(s) = C_{cl}\left(sI - A_{cl}\right)^{-1} B_{cl} + D_{cl}$ where A_{cl}, B_{cl}, C_{cl}, and D_{cl} are matrices of the overall closed-loop system that will be given in Section 4.

In the next section, we will provide a procedure to determine a dynamic output feedback controller which generates an actuating signal to regulate the regulated output of the vehicle suspension system and satisfies with the requirement of (a)-(b).

3. Performance and stability analysis

In this section, some useful important lemmas in the derivation of our main results are provided.

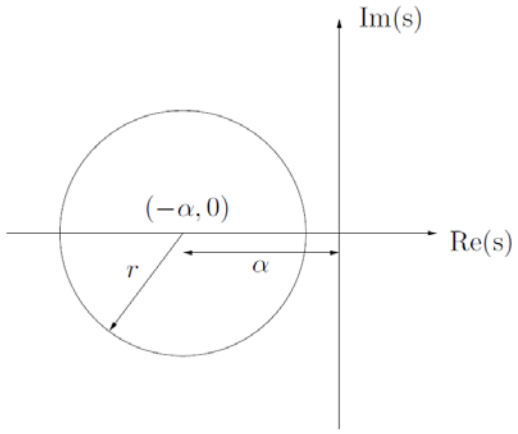

Figure 1 $D(\alpha, r)$ region

Lemma 1: (Garcia & Bernussou, 1995) Let $A \in R^{n\times n}$ be a given matrix. Then all the poles of the closed-loop system are located with a given circular region $D(\alpha, r)$, i.e., $\lambda(A) \subset D(\alpha, r)$, if and only if there exists $Q > 0$ such that

$$A_r^T Q A_r - Q < 0, \tag{2}$$

where $A_r = \dfrac{A + \alpha I}{r}$.

Lemma 2: (Kanchanaharuthai & Ngamsom, 2005) Given a constant $\gamma > 0$ and a disk $D(\alpha, r)$. Then both requirements (1) and (2) are satisfied if the following matrix inequality has a positive $Q > 0$ such that

$$\begin{bmatrix} \tilde{Q} & A_\alpha^T \\ A_\alpha & -Q^{-1} \end{bmatrix} < 0 \tag{3}$$

with

$$\tilde{Q} = -r^2 Q + \alpha(A^T Q + QA + C_{zw}^T C_{zw} + \gamma^{-2} Q B_w B_w^T Q)$$

where $A_\alpha = A + \alpha I$. In addition, from a Schur complement, equation (4) can be rewritten as:

$$\begin{bmatrix} -r^2 Q & A_\alpha^T & \sqrt{\alpha} Q B_w & \sqrt{\alpha} C_z^T \\ A_\alpha & -Q^{-1} & 0 & 0 \\ \sqrt{\alpha} B_w^T Q & 0 & -\gamma^2 & D_{zw}^T \\ \sqrt{\alpha} C_z & 0 & D_{zw} & -I \end{bmatrix} < 0 \tag{4}$$

If we consider the system represented in (1), then a necessary and sufficient LMI condition can be expressed as follows:

Theorem 1: As for the linear system, let the desired circular pole region $D(\alpha, r)$ and the H∞ norm bound constraint $\gamma > 0$ be given. The system (1) is satisfied with requirements (a)-(b) if and only if there exists a symmetrical positive definite matrix $P \in R^{n\times n}$ such that

$$\begin{bmatrix} -r^2 P & P A_\alpha^T & \sqrt{\alpha} B_w & \sqrt{\alpha} P C_z^T \\ A_\alpha P & -P & 0 & 0 \\ \sqrt{\alpha} B_w^T & 0 & -\gamma^2 & D_{zw}^T \\ \sqrt{\alpha} C_z P & 0 & D_{zw} & -I \end{bmatrix} < 0 \tag{5}$$

Proof: We start with pre- and post-multiplying (4) by the matrix $U = \text{diag}\{P, I\}$ to yield (5).

Remark 1 According to Theorem 1, it is straightforward to find a full state control law $(u = Kx)$ with pole constraints that can minimize a pre-specified H∞ norm constraint simultaneously. Hence, the design problem is reformulated as the following optimization problem:

Minimize $\gamma > 0$ subject to LMI (6) and $P > 0$

where

$$\begin{bmatrix} -r^2 P & (A_\alpha P + B_u Y)^T & \sqrt{\alpha} B_w & \sqrt{\alpha} P C_z^T \\ A_\alpha P + B_u Y & -P^{-1} & 0 & 0 \\ \sqrt{\alpha} B_w^T & 0 & -\gamma^2 & D_{zw}^T \\ \sqrt{\alpha} C_z P & 0 & D_{zw} & -I \end{bmatrix} < 0 \tag{6}$$

As a result, a full state control gain can be selected as $K = YP^{-1}$ which can be efficiently found through convex optimization algorithm.

4. Dynamic output feedback controller design

We consider the system in (1), place all closed-loop poles of linear systems in $D(\alpha, r)$ region and satisfy the pre-specified H∞ norm constraint simultaneously via a full-order output feedback controller. Thus the state space representation of the desired controller can be shown as follows:

$$\begin{bmatrix} \dot{x}_K(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} A_K & B_K \\ C_K & 0 \end{bmatrix} \begin{bmatrix} x_K(t) \\ y(t) \end{bmatrix} \quad (7)$$

where $x_K(t) \in R^{n_K \times n_K}$ is the state of the controller, and A_K, B_K,and C_K are matrices with the appropriate dimensions that can be founded. Hence, the overall closed-loop system is given by:

$$\begin{bmatrix} \dot{x}_{cl}(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} A_{cl} & B_{cl} \\ C_{cl} & D_{cl} \end{bmatrix} \begin{bmatrix} x_{cl}(t) \\ w(t) \end{bmatrix} \quad (8)$$

where

$$x_{cl}(t) = \begin{bmatrix} x(t) \\ x_K(t) \end{bmatrix}, A_{cl} = \begin{bmatrix} A & BC_K \\ B_K C & A_K \end{bmatrix},$$
$$B_{cl} = \begin{bmatrix} B_w \\ 0 \end{bmatrix}, C_{cl} = \begin{bmatrix} C_z & 0 \end{bmatrix}. \quad (9)$$

Theorem 2: As for the linear systems (1). Given the desired circular pole region $D(\alpha, r)$ and the H_∞ norm bound constraint $\gamma > 0$, the closed-loop system can achieve the expected performance requirements (a)-(b) if and only if there exist $X, Y, \bar{A}, \bar{B},$ and $\bar{C}$ such that

$$\begin{bmatrix} \Omega_{11} & \Omega_{21}^T & \Omega_{31}^T & \Omega_{41}^T \\ \Omega_{21} & \Omega_{22} & 0 & 0 \\ \Omega_{31} & 0 & -\gamma^2 & D_{zw}^T \\ \Omega_{41} & 0 & D_{zw} & -I \end{bmatrix} < 0 \quad (10)$$

where

$$\Omega_{11} = -\begin{bmatrix} r^2 X & r^2 I \\ r^2 I & r^2 Y \end{bmatrix},$$
$$\Omega_{21} = \begin{bmatrix} AX + \bar{B}C + \alpha X & A + \alpha I \\ \bar{A} + \alpha I & YA + B_u \bar{C} + \alpha Y \end{bmatrix},$$
$$\Omega_{22} = -\begin{bmatrix} X & I \\ I & Y \end{bmatrix}, \Omega_{31} = \sqrt{\alpha}\begin{bmatrix} C_z X & C_z \end{bmatrix},$$
$$\Omega_{41} = \sqrt{\alpha}\begin{bmatrix} B_w^T & B_w^T Y \end{bmatrix} \quad (11)$$

Therefore, a desired dynamic control law can be constructed as:

$$A_K = (N^T)^{-1}(\bar{A} - YAX - N^T B_K C_y X - YB_u C_K M)M^{-1},$$
$$B_K = (N^T)^{-1}\bar{B},$$
$$C_K = \bar{C}M^{-1} \quad (12)$$

where X and Y are arbitrary non-singular matrices satisfying $M^T N = I - XY.$

Proof: We apply a changing variables method and the define the matrix P and P^{-1} as follows

$$P := \begin{bmatrix} X & M^T \\ M & U \end{bmatrix}, P^{-1} := \begin{bmatrix} Y & N^T \\ N & Q \end{bmatrix}, \quad (13)$$

where the order of controller n_K is equal to the order of plant n. After pre-multiplying and post-multiplying (5) by $\mathrm{diag}\{\Theta_2, \Theta_2, I, I\}$ and its transpose, respectively, we obtain:

$$\begin{bmatrix} \Theta_2^T(-r^2 P)\Theta_2 & \Theta_2^T P A_{cl}^T \Theta_2 & \Theta_2^T \sqrt{\alpha} B_{cl} & \Theta_2^T \sqrt{\alpha} P C_{cl}^T \\ \Theta_2^T A_{cl} P \Theta_2 & \Theta_2^T(-P)\Theta_2 & 0 & 0 \\ \sqrt{\alpha} B_{cl}^T \Theta_2 & 0 & -\gamma^2 & D_{zw}^T \\ \sqrt{\alpha} C_{cl} P \Theta_2 & 0 & D_{zw} & -I \end{bmatrix} < 0 \quad (14)$$

where

$$\Theta_1 := \begin{bmatrix} X & I \\ M & 0 \end{bmatrix}, \Theta_2 := \begin{bmatrix} I & Y \\ 0 & N \end{bmatrix} \quad (15)$$

Subsequently, it is apparent that

$$P\Theta_1 = \Theta_2,$$
$$\Theta_1^T P \Theta_1 = \Theta_2^T \Theta_1 = \begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0. \quad (16)$$

We substitute A_{cl} and P in (5) and the controller variables are then renamed as:

$$\bar{A} := YAX + N^{-1} B_K C_y X + YB_u C_K M + N^T A_K M$$
$$\bar{B} := N^T B_K$$
$$\bar{C} := C_K M.$$

Additionally, we can easily check each term in (5) as follows:

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$$\Theta_2^T(-r^2P)\Theta_2 = -\begin{bmatrix} r^2X & r^2I \\ r^2I & r^2Y \end{bmatrix},$$
$$\Theta_2^T A_{cl} P\Theta_2 = \begin{bmatrix} AX+\bar{B}C+\alpha X & A+\alpha I \\ \bar{A}+\alpha I & YA+B_u\bar{C}+\alpha Y \end{bmatrix}$$
$$\Theta_2^T P\Theta_2 = \begin{bmatrix} X & I \\ I & Y \end{bmatrix},$$
$$\sqrt{\alpha}C_{cl}P\Theta_2 = \sqrt{\alpha}\begin{bmatrix} C_zX & C_z \end{bmatrix}, \qquad (17)$$
$$\sqrt{\alpha}B_{cl}^T\Theta_2 = \sqrt{\alpha}\begin{bmatrix} B_w^T & B_w^TY \end{bmatrix}$$

which imply that (11) holds.

5. Vehicle suspension systems

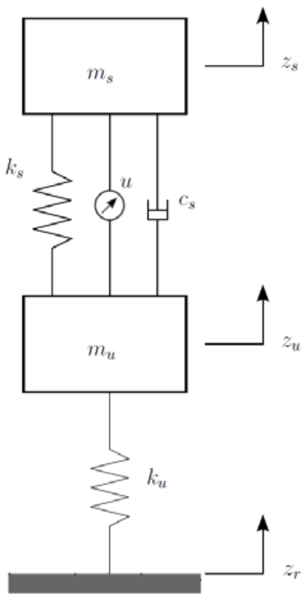

Figure 2 Quarter-car suspension model with active suspension

As mentioned earlier, the developed design method is applied for designing a dynamic output feedback H∞ controller for a quarter-car suspension system. Applying a first-order state space model, the following expression is for the quarter-car suspension system with a suitable vector of controlled output.

Considering the quarter-car system model depicted in Figure 1, its dynamic model can be written as

$$\begin{aligned} m_s\ddot{z}_s(t) &= -c_s[\dot{z}_s(t)-\dot{z}_u(t)] \\ &\quad -k_s[z_s(t)-z_u(t)]+u(t) \\ m_u\ddot{z}_u(t) &= c_s[\dot{z}_s(t)-\dot{z}_u(t)]+k_s[z_s(t)-z_u(t)] \\ &\quad -k_u[z_u(t)-z_r(t)]-u(t) \end{aligned} \qquad (18)$$

where m_s and m_u denotes the sprung and the unsprung masses representing the chassis mass and wheel mass, respectively; k_s and c_s stem from the stiffness and damping of the suspension system; k_u represents the tire stiffness; z_r are the vertical displacement; $z_s(t)$ and $z_u(t)$ denotes the vertical displacement of the sprung and the unsprung masses, respectively; and $u(t)$ denotes the control input of the system considered. Let us define the following state variables:

$x_1(t)=z_s(t), x_2(t)=z_u(t), x_3(t)=\dot{z}_s(t),$

$x_4(t)=\dot{z}_u(t)$. As a result, we have a first-order state-space equation in the following form:

$$\dot{x}(t) = Ax(t)+B_ww(t)+B_uu(t) \qquad (19)$$

where $x(t)$ denotes the state vector, $u(t)$ is the active control to be designed, $w(t)=z_r(t)$ stands for the road disturbance input and the matrices A, B_w and B_u are given by

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_s}{m_s} & \frac{k_s}{m_s} & -\frac{c_s}{m_s} & \frac{c_s}{m_s} \\ \frac{k_s}{m_u} & -\frac{k_s+k_u}{m_u} & \frac{c_s}{m_u} & -\frac{c_s}{m_u} \end{bmatrix},$$
$$B_w = \begin{bmatrix} 0 & 0 & \frac{1}{m_s} & -\frac{1}{m_u} \end{bmatrix}^T,$$
$$B_u = \begin{bmatrix} 0 & 0 & 0 & \frac{k_u}{m_u} \end{bmatrix}^T \qquad (20)$$

Furthermore, we define the vector of controlled output to be main performance criteria; especially, ride comfort, suspension stroke, road holding ability, and the required control effort. These criteria are able to be quantified through the sprung mass acceleration $\ddot{z}_s(t)$, the suspension deflection $z_s(t)-z_u(t)$, the tire deflection $z_u(t)-z_r(t)$ and the control input force $u(t)$, respectively. To attain good vehicle suspension characteristics, such criteria are necessarily made as small as possible, thereby considering the following vector of controlled outputs:

$$z=\begin{bmatrix}\ddot{z}_s(t)\\ \lambda_1\left(z_s(t)-z_u(t)\right)\\ \lambda_2\left(z_u(t)-z_r(t)\right)\\ \lambda_3 u(t)\end{bmatrix} \qquad (21)$$

$$\Rightarrow C_z x(t)+D_{zw}w(t)+D_{zu}u(t)$$

with

$$C_z=\begin{bmatrix}-\dfrac{k_s}{m_s} & \dfrac{k_s}{m_s} & -\dfrac{c_s}{m_s} & \dfrac{c_s}{m_s}\\ \lambda_1 & -\lambda_1 & 0 & 0\\ 0 & \lambda_2 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix},$$

$$D_{zw}=\begin{bmatrix}0 & 0 & -\lambda_2 & 0\end{bmatrix}^T, \qquad (22)$$

$$D_{zu}=\begin{bmatrix}\dfrac{1}{m_s} & 0 & 0 & \lambda_3\end{bmatrix}^T$$

where $\lambda_i,\ (i=1,2,3)$ denote adjustable weights that can manage the tradeoff between the above performance requirements. Apart from this, assuming that the suspension deflection and the sprung mass velocity was only the available feedback information. Thus, we obtain the observed output vector as follows:

$$y=\begin{bmatrix}\left(z_s(t)-z_u(t)\right)\\ \dot{z}_s(t)\end{bmatrix} = \begin{bmatrix}1 & -1 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}x(t)=C_y x(t) \qquad (23)$$

where $x(t)$ is the state vector defined previously, and C_y is the observed output matrix.

In summary, the vehicle suspension control problem is formulated to find a dynamic output feedback controller in (7) so that the desired performance requirements stated in Section 2 are simultaneously achieved.

6. Simulation results

In this section, the developed control methodology is implemented on a vehicle suspension system and the closed-loop performance is evaluated by the computer simulation under a transient condition. That is, MATLAB LMI Control Toolbox is employed to compute the desired controller. The complete system dynamics are obtained under the MATLAB environment. The time domain simulations are also carried out to investigate the damping performance of the designed controllers in the system. The proposed controller performance is compared with the uncontrolled system and the output feedback H∞ controller. The model parameters employ the following nominal values (Rubio-Massegu et al., 2013)

$$m_s=504.5\text{ kg},\ m_u=62\text{ kg},\ k_s=13100\text{ N/m},$$
$$k_u=252000\text{ N/m},\ c_s=400\text{ Ns/m},$$

and the particular values of the weighting coefficients: $\lambda_1=8, \lambda_2=10,\ \lambda_3=0.0015.$ Moreover, for the proposed controller, we assign all closed-loop poles in D(100,50) and set $\|T_{zw}(s)\|_\infty \le 32.$

6.1. Time response to a bump disturbance

To illustrate the performance achieved by the proposed controller, we consider the case of an isolated bump in an otherwise smooth road surface. We present the time response of the quarter-case suspension system to a road disturbance. The corresponding isolated bump is provided in the form of:

$$z_r(t)=\begin{cases}\dfrac{A}{2}\left(1-\cos\left(\dfrac{2\pi V}{L}t\right)\right), & 0\le t\le \dfrac{L}{V}\\ 0 & ,\ t>\dfrac{L}{V}\end{cases} \qquad (24)$$

where A and L denote the bump height and length, respectively; and V is the vehicle velocity. The parameter values are used throughout the numerical simulation are as follows: $A=0.1\text{ m}, L=5\text{ m}, V=12.5\text{ m/s}$ (Rubio-Massegu et al., 2013).

For this road disturbance, the magnitudes of the developed controller and the existing controllers used as performance criteria are computed. In Figure 3, the solid line exhibits the bump response where the proposed method and the dashed lines are the output feedback H∞ method. It can be seen that the oscillations in bump responses are sluggishly damped by the uncontrolled system (passive suspension) shown in dash-dotted line. In comparison with the uncontrolled system, it is easy to observe that the active controller (the proposed controller and the output feedback H_∞ controller) improves significantly, that is, better transient dynamic performance on ride comfort (lower peak and shorter setting time in the sprung mass acceleration), suspension deflection, and road holding ability. Moreover, the developed control law practically offers the level of vibrational response mitigation more than the output feedback H∞ control law. Comparing with the output feedback H∞ controller, it is obvious that transient responses of the proposed controller, in particular $\ddot{z}_s(t), z_s(t)-z_u(t), z_u - z_r(t)$ do decay faster and exhibit smaller overshoot along with shorter settling time. Figure 4 illustrates that the closed-loop poles of the presented control law are placed in the desired region D(100,50). But the closed-loop poles of uncontrolled systems and the output feedback H_∞ control are nearer the imaginary axis than that of the proposed controller. As known well that the more all closed-loop pole locations are pushed toward the left-half plane, the more the settling time and overshoot decreases. Therefore, this is why transient responses of the proposed strategy clearly outperform both the uncontrolled system and the output feedback H∞ controller.

The simulation results supported that the presented control law exhibits a remarkably improved performance in terms of time responses comparing with those from the uncontrolled system and the output feedback H_∞ controller. Clearly, the developed scheme not only achieves two performance requirements but also offers the best transient dynamic properties as resulted in faster transient responses of the closed-loop systems under a road disturbance.

7. Conclusion

In this paper, the design combination of an output feedback H_∞ control with circular pole constraints can effectively minimize the H∞ norm and can appropriately assign the closed-loop poles into the desired stable region. The resulting controller is also effectively used to attenuate a road disturbance and to improve transient dynamic performances in terms of lower peak, shorter setting time, etc. The numerical simulations show the effectiveness of the proposed scheme as the transient response is improved and better than the passive system and the output feedback H_∞ controller. Future studies will be devoted to the extension of this strategy to different mathematical complexities including input and output constraints, input delay (Li, Liu, Hand, & Hilton, 2013; Li, Jing, & Karimi, 2014), actuator dynamics (Chen & Guo, 2004; Chen, Shiu, & Hsieh, 2011) and road excitation model. Besides, the extension of this approach on the controller design that includes the effects of uncertain and unknown parameters deserves further study as well.

8. References

Chen, H., & Scherer, C. W. (2004). An LMI based model predictive control scheme with guaranteed H∞ performance and its application to active suspension. Proceedings of the American Control Conference. Boston, MA, USA. pp. 1487-1492 vol. 2.

Chen, H., & Guo, K.-H. (2005). Constrained H∞ control of active suspensions: an LMI approach.*IEEE Transactions on Control Systems Technology*, *13*(3), 412-421. DOI: 10.1109/TCST.2004.841661

Chen, B.-C., Shiu, Y.-H., & Hsieh, F.-C. (2011). Sliding-mode control for semi-active suspension with actuator dynamics. *Vehicle System Dynamics, 49*(1-2), 277-290.. DOI: 10.1080/00423111003602376

Chilali, M., & Gahinet, P. (1996). H∞ design with pole placement constraints. *IEEE Transactions on Automatic Control, 41*(3), 358-367. DOI: 10.1109/9.486637

Chu, J.-H. (1991). Pole-assignment robustness in a specified disk. *System & Control Letters, 16*(1), 41-44. DOI: 10.1016/0167-6911(91)90027-C

Du, H., & Zhang, N. (2009). Fuzzy control for nonlinear uncertain electrohydraulic active suspension with input constraint. *IEEE Transactions on Fuzzy Systems, 17*(2), 343-356. DOI: 10.1109/TFUZZ.2008.2011814

Fialho, I., & Balas, G. J. (2002). Road adaptive active suspension design using linear parameter-varying gain- scheduling. *IEEE Transactions on Control Systems Technology, 10*(1), 43-54. DOI: 10.1109/87.974337

Garcia, G. & Bernussou, J. (1995). Pole assignment for uncertain systems in a specified disk by state feedback. *IEEE Transactions on Automatic Control, 40*(1), 184-190. DOI: 10.1109/9.362872

Garcia, G. (1997). Quadratic guaranteed cost and disc pole location control for discrete-time uncertain systems. *IEE Proceedings - Control Theory Applications, 144*(6), 545-548. DOI: 10.1049/ip-cta:19971542

Haddad, W. M., & Bernstein, D. S. (1992). Controller design with regional pole constraints. *IEEE Transactions on Automatic Control, 37*(1), 54-69. DOI: 10.1109/9.109638

Hrovat, D. (1997). Survey of advanced suspension developments and related optimal control applications. *Automatica, 33*(10), 1781-1817. DOI: 10.1016/S0005-1098(97)00101-5

Kanchanaharuthai, A., & Ngamsom, P. (2005). Robust H∞ load-frequency control for interconnected power systems with D-stability constraints via LMI approach. Proceedings of the 2005 American Control Conference, pp. 4387-4392 Vol.6. DOI: 10.1109/ACC.2005.1470670

Koch, G., & Kloiber, T. (2014). Driving state adaptive control of an active vehicle suspension system. *IEEE Transactions on Control Systems Technology*, *22*(1), 44-57. DOI: 10.1109/TCST.2013.2240455

Li, H., Liu, H., Hand, S. & Hilton, C. (2013). Design of robust H∞ controller for a half-vehicle active suspension with input delay. *International Journal of Systems Science*, *44*(4), 625-640. DOI: 10.1080/002077221.2011.617895

Li, H., Jing, X., & Karimi, H. R. (2014). Output-feedback-based H∞ control for vehicle suspension systems with control delay. *IEEE Transactions on Industrial electronics*, *61*(1), 436-446. DOI: 10.1109/TIE.2013.2242418

Prabakar, R. S., Sujatha, C., & Narayanan, S. (2013). Response of a quarter car model with optimal magnetorheological damper parameters. *Journal of Sound and Vibration, 332*(9), 2191-2206. DOI:10.1016//j.jsv.2012.08.021

Robio-Masegu, J., Palocios-Quinonero, F., Rossell, J. M., & Karimi, H. R. (2013). Static output-feedback control for vehicle suspension: a single-step linear matrix inequality approach. Mathematical Problems in Engineering, 2013, Article ID 907056, 12 pages.

Scherer, C., Gahinet, P., & Chilali, M. (1997). Multi-objective output feedback control via LMI approach optimization. *IEEE Transactions on Automatic Control, 42*(7), 896-911. DOI: 10.1109/9.599969

Wang, Z. (1998). Robust H∞ state feedback control with regional pole constraints: an algebraic Riccati equation approach. *Journal of Dynamic Systems, Measurement, and Control, 120*(2), 289-292. DOI: 10.1115/1.2802422

Wang, Z., Zeng, H., Ho, D. W. C, & Unbehauen, H. (2002). Multi-objective control of a four-link flexible manipulator: a robust H∞ approach. *IEEE Transactions on Control Systems Technology, 10*(6), 866-875. DOI: 10.1109/TCST.2002.804132

Xiao, L., & Zhu, Y. (2014). Passivity-based integral sliding mode active suspension control. In *Proceedings of the 19th International Federation of Automatic Control World Congress*, Cape Town, South Africa. August 24-29, 2014, pp. 5205-5210.

Yedavalli, R. K., & Liu, Y. 1995). H∞ control with regional stability constraints. *Automatica, 31*(4), 611-615. DOI: 10.1016/0005-1098(95)98491-N

Yu, L., Chen, G., & Nan, Y. (2002). Design of output feedback guaranteed cost controller with disk closed-loop pole constraints for uncertain discrete-time system. Proc. 15th Triennial World Congress.

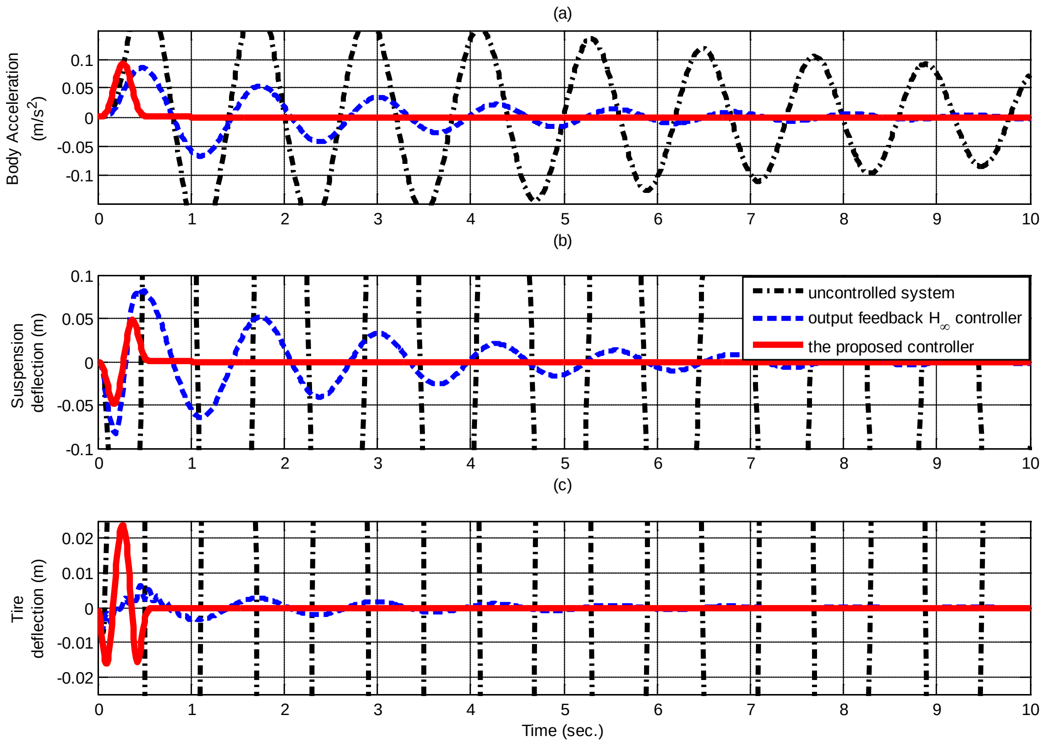

Figure 3 Bump responses: Time histories of body acceleration, suspension deflection, and tire deflection

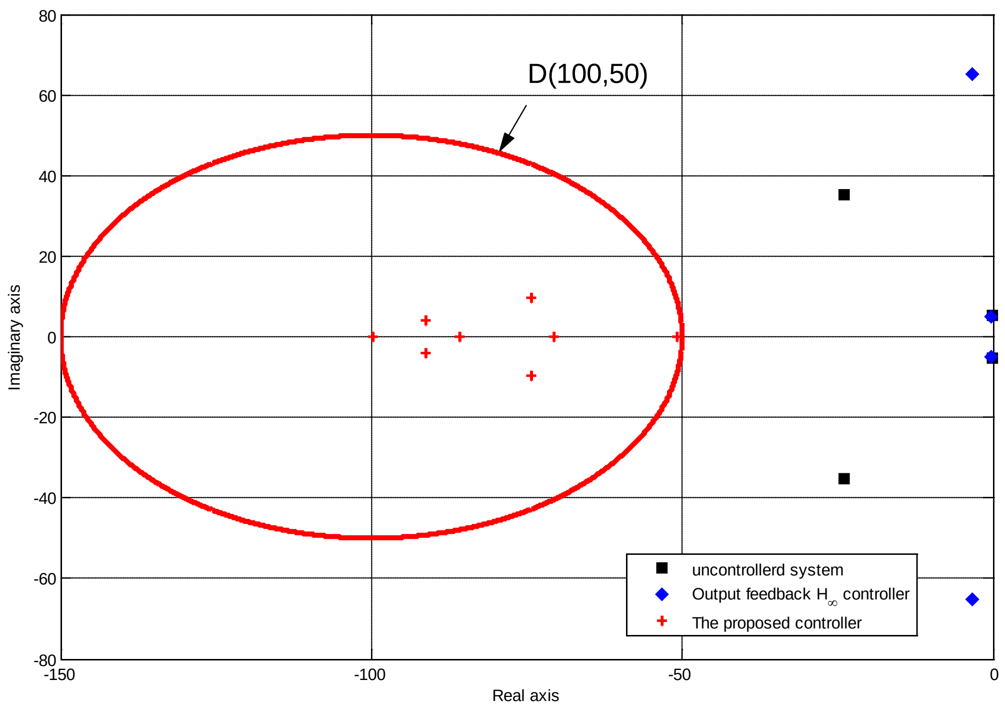

Figure 4 Pole location