Stress Testing Framework Based on Market Risk Models:

Analyses on Foreign Exchange and Stock Markets in Thailand

Comprehensive Exam

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Introduction

- Financial institutions are required by regulators to have sufficient capital to cover probable losses.
- Several studies have evaluated about proper risk models to be used for valuing risk and which is the most accurate.
- While Value-at-Risk (VaR) seems to be a standard tool for risk management, it fails to capture extreme events which are unlikely but possible to occur. A stress test is then developed to overcome the problem.
- While VaR's objective is to quantify potential losses under normal market condition, a stress testing's objective is to evaluate extreme losses that rarely but possible to occur.

Literature Review

- Assumption that return distribution is normally distributed is proven to be invalid in several studies.
  - The distribution in fact exhibits fat tails. That is extreme movement is more likely to happen rather than suggested by the normal distribution.
- Alternative distributions, e.g., historical simulation, student's *t* distribution, etc., have been suggested for risk modeling. However, each has its own merits and drawbacks, and does not suits for all financial markets or countries.
- Although the Extreme Value Theory (EVT) is well accepted that it is superior to traditional VaR, it has been criticized for its complexity with only slightly improved performance.

Literature Review

- Stress testing is not associated with a probability statement like a VaR and as a result difficult for interpretation and implementation.
- Subjective ‘what if’ scenarios have become key inputs for calculating losses when performing stress test.
- Since the method is highly subjective, several scenarios might be unreasonable or even be ignored resulting in inappropriate losses and could lead to improper policy

Objective

- This study will investigate a stress testing methodology based on market risk model developed by Alexander and Sheedy (2008) in the context of Thai financial markets.
- The objective of this study is to identify the most suitable risk models for conducting a stress test on Thai financial markets.

Benefits and Limitations

- **This methodology is superior to the traditional method in that**
  - able to link stress tests to a targeted probability
  - provides greater statistical reliability
  - able to examine market response following a shock event
- **However, this methodology is not perfect.**
  - Vulnerable to model risk

Data

Foreing Exchange Market

- Daily exchange rate of Thai Baht in terms of US dollar (THB/USD)
- Period: July 2, 1997 to December 30, 2008 (2,823 observations)
- Source: Datastream (BOT simple averages of closing rates quoted by commercial banks)

Stock Market

- Daily closing SET50 index
- Period: August 16, 1995 to December 30, 2008 (3,283 observations)
- Source: SET Smart

Methodology

Step 1 — VaR and ETL Estimations

Step 2 — Risk Model Selection (Backtesting)

Step 3 — Stress Testing

Step 4 — Model Evaluation

Methodology:
VaR and ETL Estimations

- **There are 4 risk models in this study.**
  - Unconditional Risk Model
    - Unconditional Normal Risk Model
  - Conditional Risk Model
    - Conditional Normal Risk Model
    - Conditional Student's *t* Risk Model
    - Conditional Empirical Risk Model
- **Rolling estimation window of 250 days**
- **Estimate $VaR_{99\%,1\text{-}day}$**

Unconditional Coverage (Kupiec, 1995)

$$LR = \frac{\pi_{\exp}^{n_1}(1-\pi_{\exp})^{n_0}}{\pi_{obs}^{n_1}(1-\pi_{obs})^{n_0}} \qquad \text{where} \qquad -2\ln LR \sim \chi_1^2$$

The null hypothesis is:

H_0: the actual number of violations is equal to the expected number of violations

where

π_{exp}	is the expected proportion of returns that lie in the prescribed interval of the distribution
π_{obs}	is the observed proportion of returns that lie in the prescribed interval of the distribution
n_1	is the number of returns that lie inside the interval (the number of violations)
n_0	is the number of returns that lie outside the interval (the number of good returns).

Conditional Coverage (Christoffersen,1998)

$$LR = \frac{\pi_{\exp}^{n_1}(1-\pi_{\exp})^{n_0}}{\pi_{01}^{n_{01}}(1-\pi_{01})^{n_{00}}\pi_{11}^{n_{11}}(1-\pi_{11})^{n_{10}}} \quad \text{where} \quad -2\ln LR \sim \chi_1^2$$

The null hypothesis is:

H_0: Violations are spread evenly over time

where

n_1	is the number of returns that lie inside the interval (the number of violations)
n_0	is the number of returns that lie outside the interval (the number of good returns).
n_{10}	is the number of times a violation is followed by a good return
n_{11}	is the number of times a violation is followed by another violation
n_{01}	is the number of times a good return is followed by a violation and
n_{00}	is the number of times a good return is followed by another good return.
π_{exp}	is the expected proportion of returns that lie in the prescribed interval of the distribution
π_{01}	is the proportion of exceedances, give that the last return was a good return
π_{11}	is the proportion of exceedances, given that the last return was an exceedance.

ETL Test (McNeil and Frey, 2000)

$$r = \frac{(R_{t+h} - ETL_{h,\alpha})}{\hat{\sigma}_{t+h}} \qquad \text{if} \quad R_{t+h} < -VaR_{h,\alpha} \qquad \text{and}$$

$$r = 0 \qquad \text{otherwise}$$

The distribution of the test statistic is $t = \dfrac{\bar{r}}{\hat{\sigma}_r}$

The null hypothesis is:

H_0: The standardized exceedance residuals have zero mean, or equivalently ETL does not consistently understate the true potential for losses beyond the VaR.

Methodology:
Stress Testing

- **Identify Initial Shock**
  - The model based stress test interprets α as the probability of a market shock.
  - The size of shock is $-VaR_{1,\alpha}$ for a long position, and is $VaR_{1,(1-\alpha)}$ for a short position.
- **Evaluate After-Shock Effect**
  - We will investigate return movement in total horizon of h days
  - Initial shock occurs at time T and portfolio returns are assessed for h-1 days after the shock.
  - Portfolio is assumed to be held constant along the stress test duration.

Methodology:
Stress Testing

Evaluate After-Shock Effect (Con't)

- Standard deviation is set equal to long-term value at the first day
- For the subsequent days, innovation drawn from the distribution of initial shock is used to determined variance.

$$\hat{\sigma}^2_{T+i+1} = \hat{\gamma}_1 + \hat{\gamma}_2 \varepsilon^2_{T+i} + \hat{\gamma}_3 \overline{\sigma}^2_{T+i} \qquad \text{for } i = 1, \ldots, h-1$$

- Monte Carlo simulation is used to simulate possible return after initial shock.

Methodology:
Model Evaluation

- Comparing stress test results by risk model
  - the risk model that can provide the stress loss beyond the worst historical loss should be preferred
- Comparing traditional stress tests with model based stress tests
  - compare stress test results of unconditional normal model with the other conditional risk models
- Comparing stress loss with VaR-based regulatory capital
  - whether the capital requirement estimates could sufficiently cover the loss generated from model based stress test
- Model-based stress test overtime
  - assess the stability of the model-based stress test over time

Empirical Results: Risk Model Backtesting

This table presents backtesting result of VaR and ETL calculated at 99% confident level over 1- day horizon. We backtest VaR and ETL calculated for 3 data set which are SET50 index, long USD/ short THB, and short USD/ long THB. VaR and ETL are estimated using estimation window of 250 days. The sample is rolled over daily to keep estimation window constant. For the unconditional normal model, 1-day VaR and ETL are forecasted directly from the recent estimation window. VAR and ETL for the conditional risk models, on the other hand, are estimated by simulating 10,000 paths of possible returns accordingly to the assumed return distribution of each risk model.

1. The unconditional coverage VaR tests a null hypothesis that the actual number of violations is equal to the expected number of violations.
2. The conditional coverage VaR tests a null hypothesis that violations are spread evenly over time.
3. The ETL backtest tests a null hypothesis that the ETL does not consistently understate the true potential for losses beyond the VaR.
4. Hypothesis is tested at 95% confident level meaning that a null hypothesis is rejected if p-value is less than 5%.

SET50 index

Innovation Process	Unconditional Coverage VaR[1] *p*-value[4]	Conditional Coverage VaR[2] *p*-value	ETL[3]
Unconditional Model	0.29%	0.00%	1.35%
Conditional Model			
Normal	6.43%	5.29%	5.18%
Students' *t*	31.40%	20.44%	26.78%
Empirical	9.21%	0.32%	8.08%

Long USD/ Short THB

Innovation Process	Unconditional Coverage VaR *p*-value	Conditional Coverage VaR *p*-value	ETL
Unconditional Model	0.53%	0.04%	0.00%
Conditional Model			
Normal	0.01%	0.01%	0.01%
Students' *t*	23.06%	13.20%	48.09%
Empirical	3.59%	2.90%	63.88%

Long THB/ Short USD

Innovation Process	Unconditional Coverage VaR *p*-value	Conditional Coverage VaR *p*-value	ETL
Unconditional Model	16.69%	0.02%	0.00%
Conditional Model			
Normal	0.89%	0.42%	3.65%
Students' *t*	52.41%	29.64%	22.98%
Empirical	23.06%	13.20%	27.24%

Empirical Results: Stress Test Results

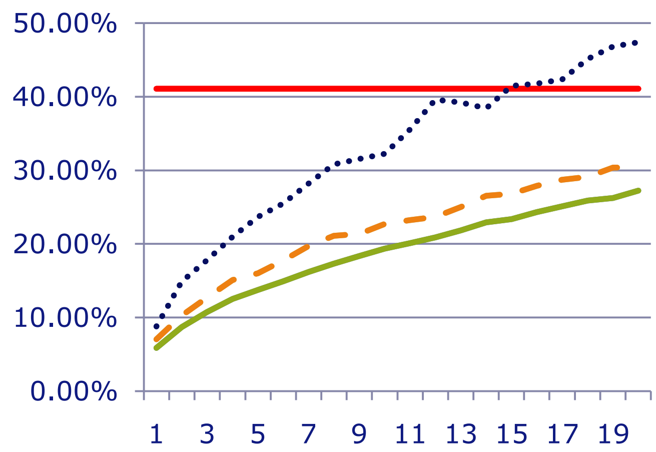
The initial shock assumed to occur on the first day. For the conditional model, this will affect variance of the following day and hence the returns. The forecasted returns are simulated with 10,000 paths and are aggregated along the holding period of each path to estimate the stress loss at 99% confidence level.

1. Long-term volatility estimate is sample standard deviation of all daily log returns, expressed on a per annum basis
2. VaR-based regulatory capital is calculated using $3VaR_{0.01,10\text{-day}}$
3. Stress loss refers to -1 times the stress test outcome expressed as a percentage of initial portfolio value. Initial shock is set at α. Portfolio is assumed to be held constantly during the period of h-day.

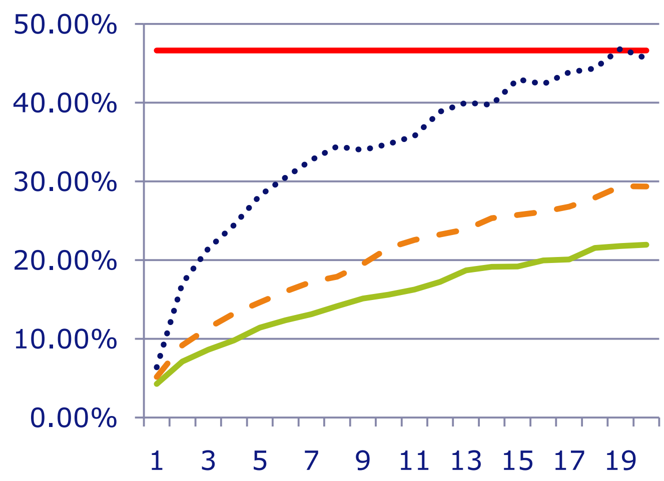
	Unconditional	Conditional	Conditional Students' t	Conditional Empirical
SET50				
The worst historical loss over 3 days = 20.62%, over 10 days = 30.13%				
Long term volatility estimate[1]	32.78%	32.78%	21.25%	21.25%
VaR-based regulatory capital[2]	45.75%	41.09%	44.18%	46.63%
99% Stress loss[3]:				
h=3, α=0.01	8.28%	10.73%	7.99%	8.61%
h=3, α=0.005	9.37%	12.66%	10.63%	11.37%
h=3, α=0.001	10.98%	17.81%	22.83%	21.48%
h=10, α=0.01	15.37%	19.34%	14.16%	15.63%
h=10, α=0.005	17.51%	22.68%	19.41%	21.54%
h=10, α=0.001	21.22%	32.26%	38.24%	34.76%
Long USD/ Short THB				
The worst historical loss over 3 days = 12.64%, over 10 days = 20.38%				
Long term volatility estimate	10.08%	10.08%	3.39%	3.39%
VaR-based regulatory capital	14.08%	9.91%	10.15%	11.21%
99% Stress loss:				
h=3, α=0.01	2.54%	3.89%	2.39%	3.24%
h=3, α=0.005	2.87%	4.57%	4.35%	4.92%
h=3, α=0.001	3.42%	7.07%	17.95%	11.38%
h=10, α=0.01	4.54%	7.39%	4.44%	5.19%
h=10, α=0.005	5.33%	8.89%	8.04%	7.59%
h=10, α=0.001	6.19%	14.17%	34.64%	16.36%
Short USD/ Long USD				
The worst historical loss over 3 days = 10.17%, over 10 days = 17.66%				
Long term volatility estimate	10.08%	10.08%	3.39%	3.39%
VaR-based regulatory capital	14.08%	10.06%	10.08%	11.66%
99% Stress loss:				
h=3, α=0.01	2.59%	3.91%	2.34%	3.47%
h=3, α=0.005	2.91%	4.71%	4.21%	5.46%
h=3, α=0.001	3.43%	7.10%	18.13%	12.52%
h=10, α=0.01	4.81%	7.60%	4.32%	5.35%
h=10, α=0.005	5.18%	9.44%	7.73%	7.60%
h=10, α=0.001	5.98%	13.73%	34.07%	14.61%

Empirical Results:
Regulatory Capital and Stress test of SET50 Index

Conditional Normal

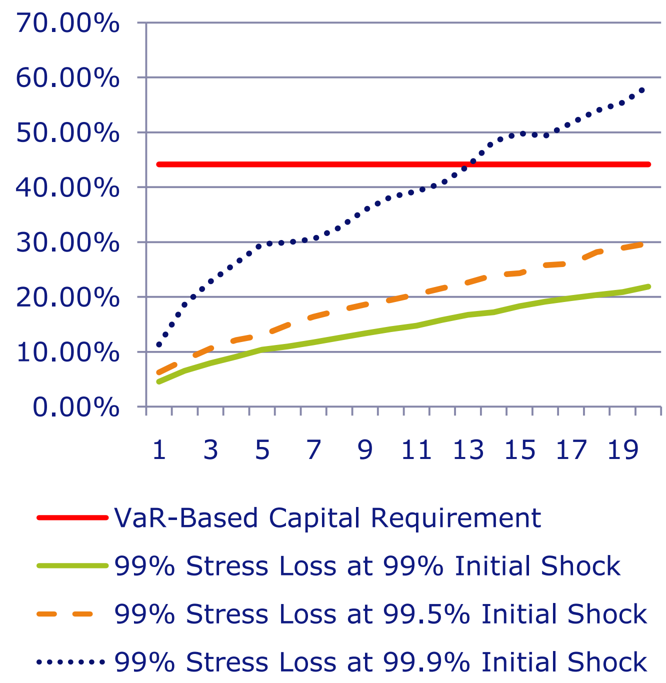


Conditional Empirical



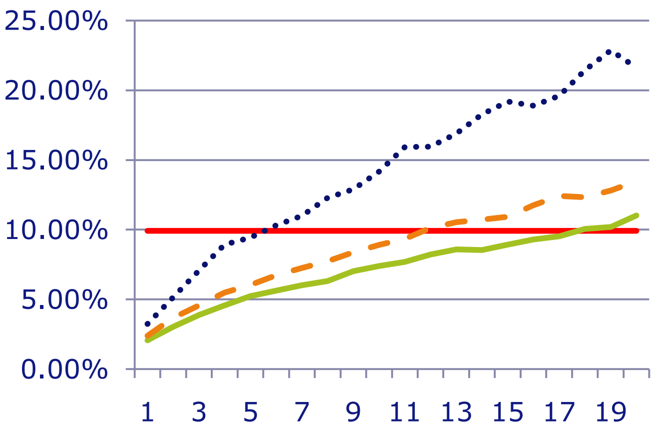
The figures depict comparison of Basel I regulatory capital calculated according to Alexander and Sheedy (2008) as $3VaR_{0.01,10\text{-day}}$, represented by a horizontal solid line, and 99% confident stress losses for various initial shocks. The horizontal axis represents holing period in days, and the vertical axis represents percentage of portfolio value.

Conditional Student's t

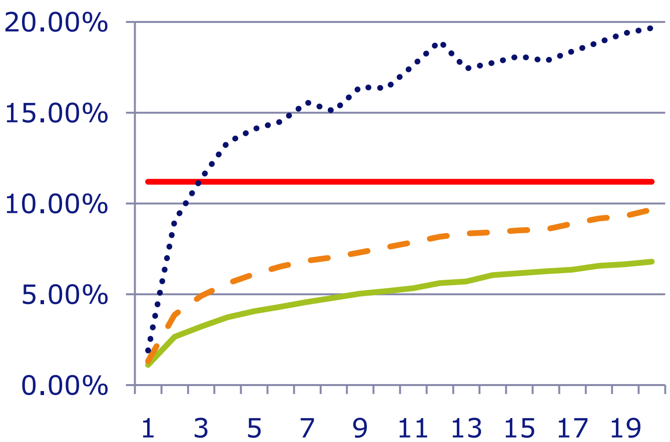


Empirical Results:
Regulatory Capital and Stress test of Long USD/ Short THB

- **Conditional Normal**



- **Conditional Empirical**



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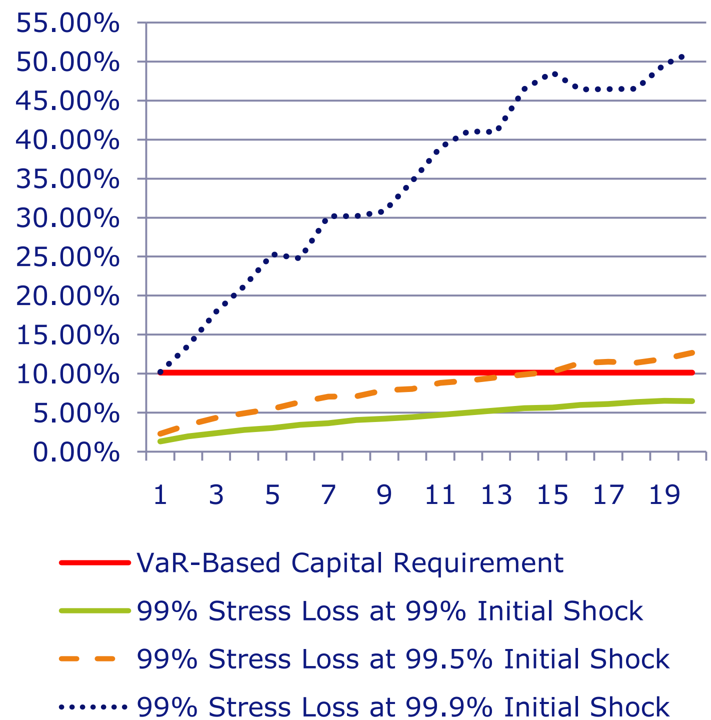
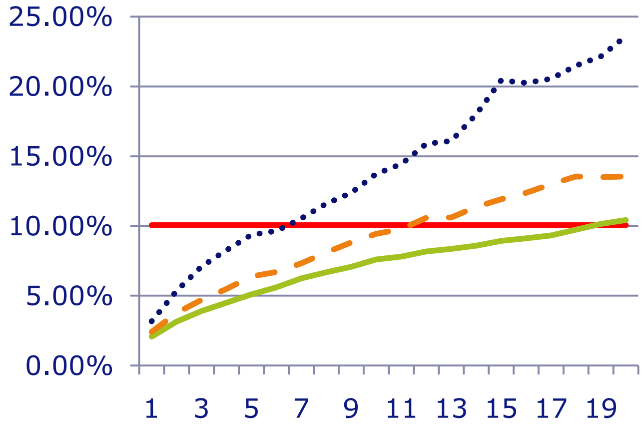
- **Conditional Student's t**



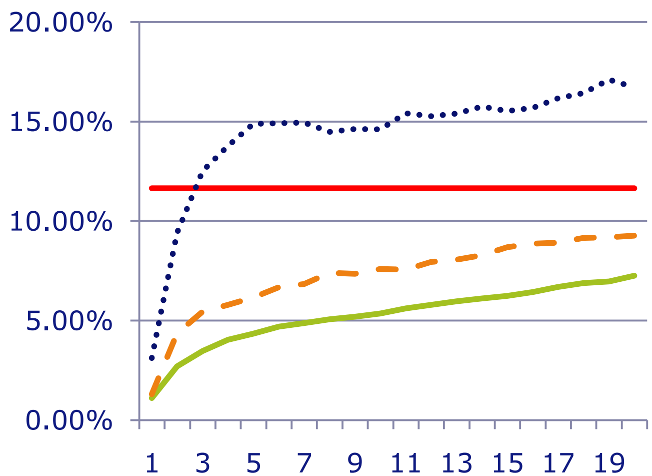
Empirical Results:

Regulatory Capital and Stress test of Short USD/ Long THB

Conditional Normal

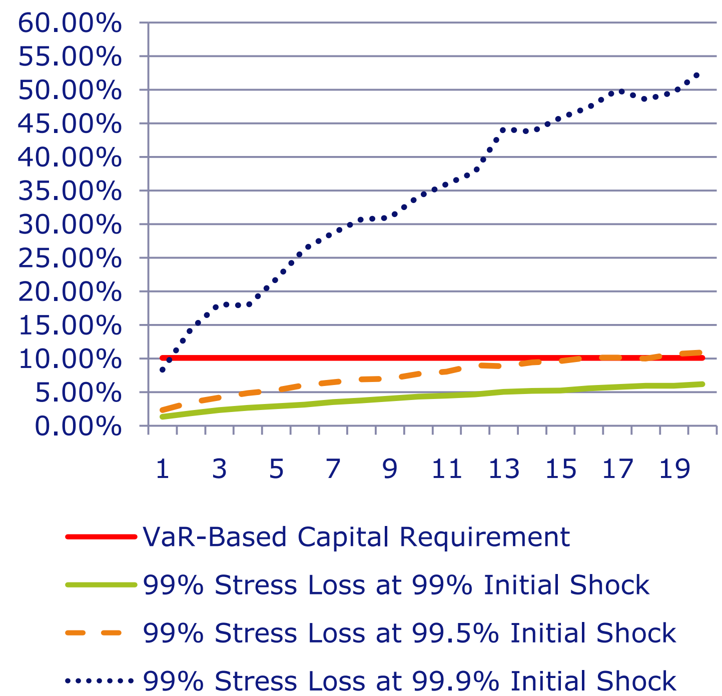


Conditional Empirical



The figures depict comparison of Basel I regulatory capital calculated according to Alexander and Sheedy (2008) as $3VaR_{0.01,10\text{-day}}$, represented by a horizontal solid line, and 99% confident stress losses for various initial shocks. The horizontal axis represents holing period in days, and the vertical axis represents percentage of portfolio value.

Conditional Student's t

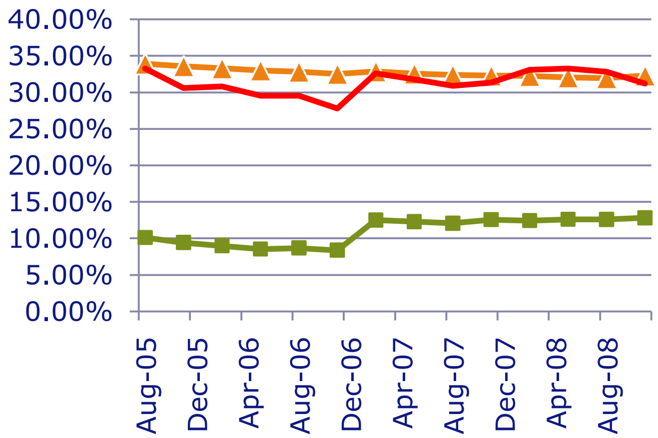


Empirical Results:

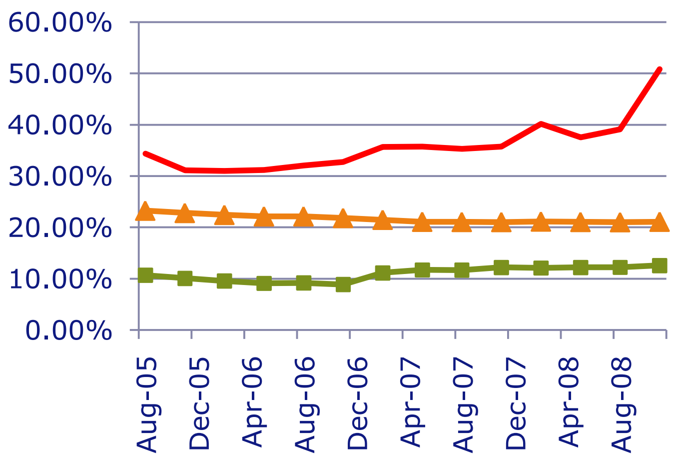
Stress Test Over Time: SET50 Index

We use 10 years of data, i.e., August 1995 to August 2005 to estimate long term volatility, the reaction term of the GARCH equation (γ_2), and other necessary parameters of each model. The stress loss is estimated quarterly using all available data since the first data to the estimation point. The horizontal axis represents quarter of estimation, and the vertical axis represents percentage of portfolio value. The initial shock is set at $\alpha=0.001$

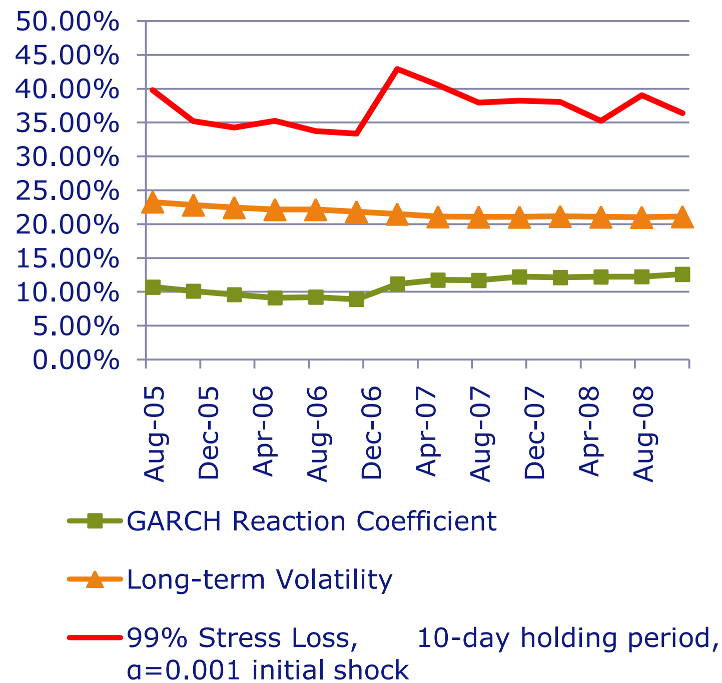
- Conditional Normal



- Conditional Empirical



- Conditional Student's t

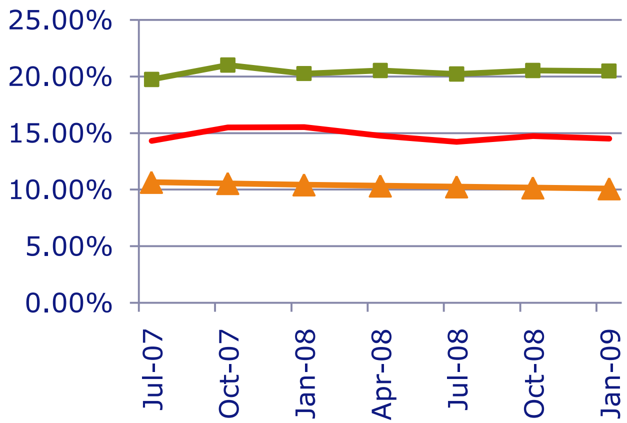


Empirical Results:

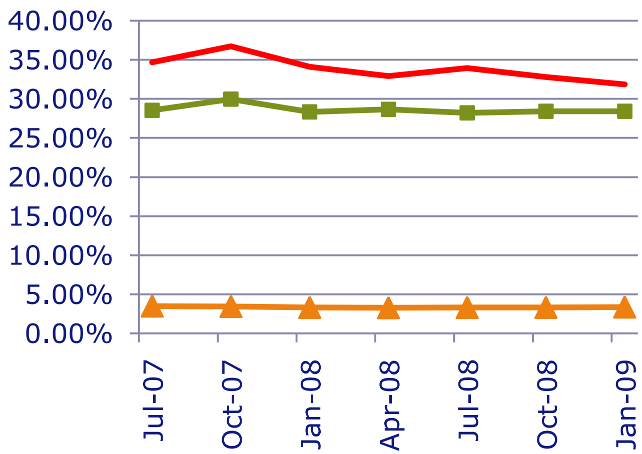
Stress Test Over Time: Long USD/ Short THB

We use 10 years of data, i.e., July 1997 to July 2007 to estimate long term volatility, the reaction term of the GARCH equation (γ_2), and other necessary parameters of each model. The stress loss is estimated quarterly using all available data since the first data to the estimation point. The horizontal axis represents quarter of estimation, and the vertical axis represents percentage of portfolio value. The initial shock is set at $\alpha=0.001$

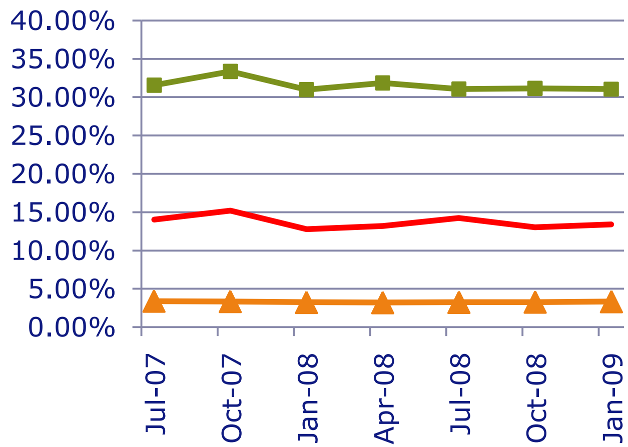
- ### Conditional Normal



- ### Conditional Empirical



- ### Conditional Student's t



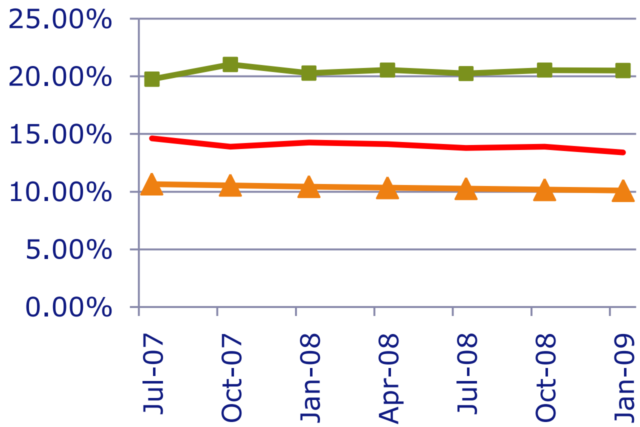
GARCH Reaction Coefficient

Long-term Volatility

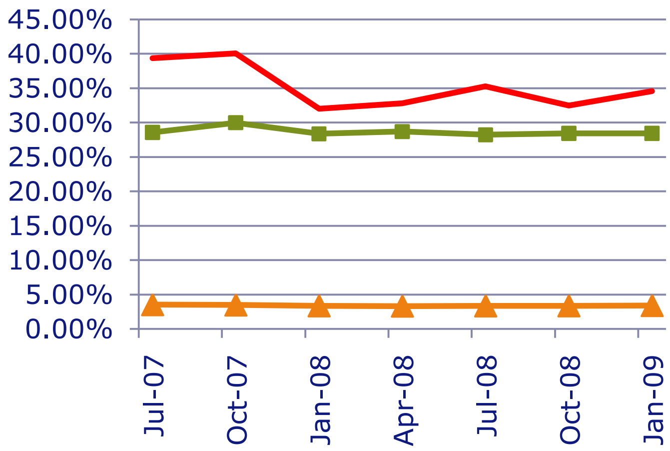
99% Stress Loss, 10-day holding period, α=0.001 initial shock

Empirical Results:
Stress Test Over Time: Short USD/ Long THB

- **Conditional Normal**

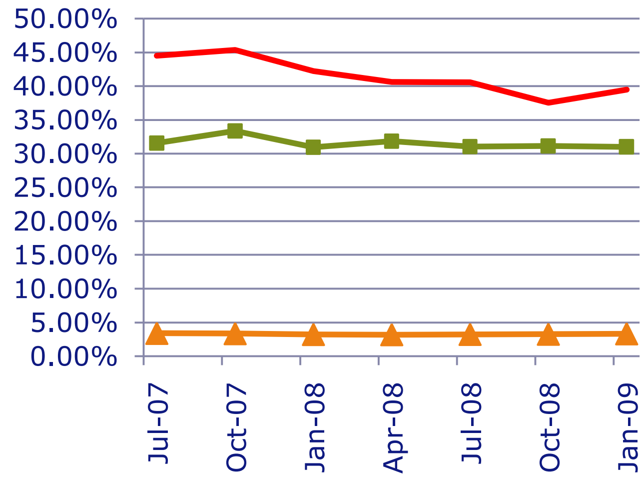


- **Conditional Empirical**



We use 10 years of data, i.e., July 1997 to July 2007 to estimate long term volatility, the reaction term of the GARCH equation (γ_2), and other necessary parameters of each model. The stress loss is estimated quarterly using all available data since the first data to the estimation point. The horizontal axis represents quarter of estimation, and the vertical axis represents percentage of portfolio value. The initial shock is set at α=0.001

- **Conditional Student's t**



GARCH Reaction Coefficient

Long-term Volatility

99% Stress Loss, 10-day holding period, α=0.001 initial shock

Conclusions

- The unconditional risk model is inferior to other conditional risk models.
- Among conditional risk models, the conditional Student's t risk model seems to be preferred than the others.
- While the conditional Student's t risk model provides the best estimate of stress loss among other risk models, the stress losses are much larger than the VaR-based regulatory capital. This may weaken the use of stress testing framework based on market risk model with Thai financial data.
- Therefore, the model based stress test for SET50 index and foreign currencies would be more appropriate to use if we have a longer series of data. In addition, adjusting for VaR-based regulatory capital is also required.

Thank You !