

CHAPTER 1

INTRODUCTION

Let X be a nonempty set and let $T(X)$ denote the set of all transformations from X into itself. Then $T(X)$ is a semigroup under the composition of maps and it is called the full transformation semigroup on X .

Transformation semigroups play an important role in semigroup theory since it is well known that every semigroup is isomorphic to a subsemigroup of a suitable full transformation semigroup. Therefore, in a sense, in order to study semigroups it suffices to consider transformation semigroups.

It is known that $T(X)$ is a regular semigroup, that is, for every $\alpha \in T(X)$, $\alpha = \alpha\beta\alpha$ for some $\beta \in T(X)$.

In this thesis we study the subsemigroup of $T(X)$, namely $S(X, Y)$, define by

$$S(X, Y) = \{\alpha \in T(X) : Y\alpha \subseteq Y\}$$

where $\emptyset \neq Y \subseteq X$. Then $S(X, Y)$ is a semigroup of total transformations on X which leave a subset Y of X invariant. This semigroup was first introduced and studied by K.D. Magill Jr. [3] in 1966. In fact, if $Y = X$, then $S(X, Y) = T(X)$. So we may regard $S(X, Y)$ as a generalization of $T(X)$.

In 2005, S. Nenthein, P. Youngkhong and Y. Kemprasit [5] showed that $S(X, Y)$ is a regular semigroup if and only if $X = Y$ or Y contains exactly one element, and $\text{Reg } S(X, Y) = \{\alpha \in S(X, Y) : X\alpha \cap Y = Y\alpha\}$ is the set of all regular elements of $S(X, Y)$. Moreover, they count the number of regular elements of $S(X, Y)$ when X is a finite set.

In 2011, P. Honyam and J. Sanwong [4] described Mai Green's relations and ideals on $S(X, Y)$. The purposes of this thesis are:

- (1) To characterize left regular elements, right regular elements, intra-regular elements, unit regular elements and completely regular elements on $S(X, Y)$.

(2) To consider the relationships between left regular elements, right regular element, intra-regular elements, unit regular elements and completely regular elements.

(3) To count the numbers of left regular, right regular, intra-regular and unit regular elements of $S(X, Y)$ when X is a finite set.

This thesis comprises of four chapters. Chapter 1 is for the introduction. In Chapter 2, we list some well known results, definitions and notations that will be used throughout this thesis. In Chapter 3, we present the characterizations of left regular elements, right regular elements, intra-regular elements, unit regular elements and completely regular elements of $S(X, Y)$, and we consider the relationships between these elements. Moreover, we count the numbers of left regular, right regular, intra-regular and unit regular elements of $S(X, Y)$ when X is a finite set. Finally, Chapter 4 is for the conclusion.