Appendix A

Analytical treatments for the results shown in Table 3.1

Generally, a sensitivity of y to a variation of x is given by S_x^y = [∂y/∂x][x/y)] where y is a parameter of interest and x is a parameter of variation. For the Techniques 1 and 2, it can be seen from Equations 3.9 that $y = \omega_{HQ}$, $x = C_1, V_T, I_2$ and β, and therefore

$$S_{C_1}^{\omega_{HQ}} = \frac{C_1}{\omega_{HQ}}\frac{\delta\omega_{HQ}}{\delta C_1} = \frac{C_1}{\omega_{HQ}}\frac{\delta\left(\frac{I_2}{4C_1V_T}\sqrt{\frac{\beta}{\beta+1}}\right)}{\delta C_1} = \frac{C_1\left(\frac{I_2}{4V_T}\sqrt{\frac{\beta}{\beta+1}}\right)}{\omega_{HQ}}\frac{\delta C_1^{-1}}{\delta C_1}$$

$$= \frac{C_1^2\omega_{HQ}}{\omega_{HQ}}\frac{\delta C_1^{-1}}{\delta C_1} = C_1^2\frac{\delta C_1^{-1}}{\delta C_1} = -1.0$$

$$S_{V_T}^{\omega_{HQ}} = \frac{V_T}{\omega_{HQ}}\frac{\delta\omega_{HQ}}{\delta V_T} = \frac{V_T}{\omega_{HQ}}\frac{\delta\left(\frac{I_2}{4C_1V_T}\sqrt{\frac{\beta}{\beta+1}}\right)}{\delta V_T} = \frac{V_T\left(\frac{I_2}{4C_1}\sqrt{\frac{\beta}{\beta+1}}\right)}{\omega_{HQ}}\frac{\delta V_T^{-1}}{\delta V_T}$$

$$= \frac{V_T^2\omega_{HQ}}{\omega_{HQ}}\frac{\delta V_T^{-1}}{\delta V_T} = V_T^2\frac{\delta V_T^{-1}}{\delta V_T} = -1.0$$

$$S_{I_2}^{\omega_{HQ}} = \frac{I_2}{\omega_{HQ}}\frac{\delta\omega_{HQ}}{\delta I_2} = \frac{I_2}{\omega_{HQ}}\frac{\delta\left(\frac{I_2}{4C_1V_T}\sqrt{\frac{\beta}{\beta+1}}\right)}{\delta I_2} = \frac{I_2\left(\frac{1}{4C_1V_T}\sqrt{\frac{\beta}{\beta+1}}\right)}{\omega_{HQ}}\frac{\delta I_2}{\delta I_2}$$

$$= \frac{\omega_{HQ}}{\omega_{HQ}}\frac{\delta I_2}{\delta I_2} = 1.0$$

$$S_{\beta}^{\omega_{HQ}} = \frac{\beta}{\omega_{HQ}}\frac{\delta\omega_{HQ}}{\delta\beta} = \frac{\beta}{\omega_{HQ}}\frac{\delta\left(\frac{I_2}{4C_1V_T}\sqrt{\frac{\beta}{\beta+1}}\right)}{\delta\beta} = \frac{\beta\left(\frac{1}{2}\cdot\frac{I_2}{4C_1V_T}\sqrt{\frac{\beta+1}{\beta}}\right)}{\omega_{HQ}}\frac{\delta\left(\frac{\beta}{\beta+1}\right)}{\delta\beta}$$

$$= \frac{(\beta+1)\omega_{HQ}}{2\omega_{HQ}}\left[\frac{(\beta+1)\frac{\delta\beta}{\delta\beta} - \beta\frac{\delta(\beta+1)}{\delta\beta}}{(\beta+1)^2}\right] = \frac{(\beta+1)}{2}\left[\frac{\beta+1-\beta}{(\beta+1)^2}\right] = \frac{1}{2[\beta+1]}$$

For the Techniques 1, it can be seen from Equations 3.10 that $y = Q_{HQ1}$, $x = \beta$, and

therefore $$S_{\beta}^{Q_{HQ1}} = \frac{\beta}{Q_{HQ1}} \frac{\delta \beta}{\delta \beta} = \frac{Q_{HQ1}}{Q_{HQ1}} = 1$$

For the Techniques 2, it can be seen from Equations 3.13 that $y = Q_{HQ2}$, $x = R_C, V_T$, and I_3, and therefore

$$S_{R_C}^{Q_{HQ2}} = \frac{R_C}{Q_{HQ2}} \frac{\delta Q_{HQ2}}{\delta R_C} = \frac{R_C}{Q_{HQ2}} \frac{\delta\left(\dfrac{1/2}{1-\dfrac{R_C I_3}{V_T}}\right)}{\delta R_C} = -\frac{4R_C\left(\dfrac{1/2}{1-\dfrac{R_C I_3}{V_T}}\right)^2}{2Q_{HQ2}} \frac{\delta\left(1-\dfrac{R_C I_3}{V_T}\right)}{\delta R_C}$$

$$= -\frac{2R_C\left(Q_{HQ2}\right)^2}{Q_{HQ2}} \frac{\delta\left(-\dfrac{R_C I_3}{V_T}\right)}{\delta R_C} = 2\frac{R_C I_3}{V_T} Q_{HQ2} = 2Q_{HQ2} \quad \text{where } \frac{R_C I_3}{V_T} = 1 \text{ for High - Q}$$

$$S_{V_T}^{Q_{HQ2}} = \frac{V_T}{Q_{HQ2}} \frac{\delta Q_{HQ2}}{\delta V_T} = \frac{V_T}{Q_{HQ2}} \frac{\delta\left(\dfrac{1/2}{1-\dfrac{R_C I_3}{V_T}}\right)}{\delta V_T} = -\frac{4V_T\left(\dfrac{1/2}{1-\dfrac{R_C I_3}{V_T}}\right)^2}{2Q_{HQ2}} \frac{\delta\left(1-\dfrac{R_C I_3}{V_T}\right)}{\delta V_T}$$

$$= -\frac{2V_T\left(Q_{HQ2}\right)^2}{Q_{HQ2}} \frac{\delta\left(-\dfrac{R_C I_3}{V_T}\right)}{\delta V_T} = -2\frac{R_C I_3}{V_T} Q_{HQ2} = -2Q_{HQ2} \quad \text{where } \frac{R_C I_3}{V_T} = 1 \text{ for High - Q}$$

$$S_{I_3}^{Q_{HQ2}} = \frac{I_3}{Q_{HQ2}} \frac{\delta Q_{HQ2}}{\delta I_3} = \frac{I_3}{Q_{HQ2}} \frac{\delta\left(\dfrac{1/2}{1-\dfrac{R_C I_3}{V_T}}\right)}{\delta I_3} = -\frac{4I_3\left(\dfrac{1/2}{1-\dfrac{R_C I_3}{V_T}}\right)^2}{2Q_{HQ2}} \frac{\delta\left(1-\dfrac{R_C I_3}{V_T}\right)}{\delta I_3}$$

$$= -\frac{2I_3\left(Q_{HQ2}\right)^2}{Q_{HQ2}} \frac{\delta\left(-\dfrac{R_C I_3}{V_T}\right)}{\delta I_3} = 2\frac{R_C I_3}{V_T} Q_{HQ2} = 2Q_{HQ2} \quad \text{where } \frac{R_C I_3}{V_T} = 1 \text{ for High - Q}$$