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Competitions from the Product Line in a Consumer Search Model

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Abstract

This study aims to investigate the competitions from the product line when consumers search among two multiproduct firms for differentiated products. In the first model, a firm is created to be prominence that will be sampled first by all consumers. Firms compete in product line to insure certain utility to the consumers. Prominent firm earns higher profit, charges a lower price and provides longer product line. A counter intuitive result shows that the prominent firm's product line increases with the search cost. This model does not support the role of product line to enhance pricing power.

The model is further modified that consumers freely choose which the firm they will visit first. The product line serves the second role to create firm's saliency in consumer's memory. The firm with lower production cost can raise attention more easily and extends longer product lines. This salient firm maintains higher profit, and charges a higher price compared with its less-salient rival. With harder search, consumers face more brands with higher prices. In the social welfare perspective, we find that all social optimal product lines and prices are lower than those determined to maximize profits. Since firms need to gain the attentions, they over invest in product lines.

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บทคัดย่อ

งานวิจัยชิ้นนี้ศึกษาการแข่งขันในความหลากหลายของสินค้าเมื่อผู้บริโภคมีต้นทุนในการค้นหา (search cost) สินค้าแต่ละชิ้นมีคุณภาพเท่ากันแต่มีความแตกต่างกันในลักษณะของผลิตภัณฑ์ ขนาด สี หรือ รสชาติ สะท้อนความต้องการที่หลากหลายของผู้บริโภค ผู้ผลิตเสนอสินค้าที่หลากหลายเพื่อเพิ่มโอกาสให้ผู้บริโภคเลือกซื้อ แบบจำลองแรกหน่วยผลิตให้หน่วยผลิตหนึ่งมีความโดดเด่นมาก (Prominent Firm) หรือเป็นผู้ผลิตที่ชื่อเสียงมากจนผู้บริโภคจะมาเสาะหาสินค้าที่หน่วยผลิตนี้ก่อนเสมอหน่วยผลิตนี้จะตั้งราคาต่ำและมีความหลากหลาย (จำนวนสินค้า) ที่มากกว่า แต่ยังได้กำไรมากกว่าผู้ผลิตที่อยู่ในลำดับการค้นหาท้ายๆ (Non-prominent Firm) แบบจำลองนี้อธิบายได้ว่าทำไมผู้ผลิตพยามให้ผู้บริโภคพบเห็นสินค้าของตนก่อน ผลการศึกษาที่สำคัญประการหนึ่งคือความหลากหลายสินค้าของผู้ผลิตที่มีความโดดเด่นเพิ่มขึ้นตามต้นทุนการค้นหา แบบจำลองนี้ไม่สนับสนุนแนวคิดที่ว่าการมีผลิตภัณฑ์จำนวนมากๆ (ความหลากหลายมาก) จะช่วยเพิ่มอำนาจการตั้งราคา

แบบจำลองที่สองสมมติให้ผู้บริโภคสามารถเลือกลำดับการค้นหาได้ ผู้บริโภคมีแนวโน้มที่จะไปค้นหาสินค้าในผู้ผลิตที่มีความหลากหลายมากดังนั้นผู้ผลิตที่มีต้นทุนต่ำจะผลิตสินค้าที่หลากหลายได้มากกว่าและสร้างความโดดเด่นได้ดีกว่าผู้ผลิตรายนี้จะตั้งราคาขายสูงและได้กำไรมากกว่า เมื่อต้นทุนการค้นหาสูงขึ้นตลาดจะมีความหลากหลายและราคาสูงขึ้น แต่หากพิจารณาในแง่ของสวัสดิการสังคมพบว่าระดับของความหลากหลายและราคาของผู้ผลิตทุกรายจะสูงกว่าระดับที่ทำให้เกิดสวัสดิการสูงสุด สาเหตุหลักมาจากการที่หน่วยผลิตต้องการแข่งขันเพื่อสร้างความโดดเด่นจึงทำให้เกิดการสร้างความหลากหลายที่มากเกินไป

1. Introduction

Most of the firms do not produce only a single product. In recent years, the product extension has become one of the most favorite strategies (Draganska and Jain, 2006). A firm can sell various products which are different in favors, colors, or other attributes. There are many studies trying to explain why firms want to produce many products. The conventional wisdom informs that a firm extends the products in the horizontal direction to improve the ability to capture consumers' match values.[1] This will be very important when consumers have high heterogeneity in tastes. There are general acceptances in the marketing research that the product extension strategy allows firms to charge a higher price and a gross margin, increase a market share, soften price competition and increase profitability. However, by doing so, firms confront with more complicated production process. This causes firms to have increasing marginal cost for additional brands.

However, most studies in product line extension assume that consumers know the prices and characteristics of all products in the marketplace. Consumers meet products with costless search. Whereas some of the literature suggested that the consumers search can play the significant role in the markets. The consumer search can study why consumers shop around before buying. There are many industries in which buyer search is an important feature of the market interaction. For examples, women frequently shop around for shoes or a couple seriously looks for a house and does not hurry to make a decision.

One might wonder why the search cost still matters in the modern day, in which the internet supports the consumers to visit firm's website with almost costless. Rhodes (2011) shows that a prominent retailer earns significantly more profit than other firms, even when the cost of searching websites and comparing products are essentially zero. Consumers who intend to search will be imposed with search cost to get the information. With consumers' heterogonous tastes, consumers search for the match values and prices. If the search cost is high, they stop search sooner or, in a special case, they will not search at all.[2]

In the market which has no guidance, consumers may search randomly, and sellers equally share the market. However, firms have to concern their positions since being the first

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[1] Firms may compete through their product lines in two ways. (i) Vertical line extension involves various qualities which firms use price discrimination according to consumers' willingness to pay for quality. (ii) Horizontal line extension which all have the same price and quality but vary in other attributes.

[2] This special case is well-known as "Diamond Paradox" where the degree of product differentiation is very low (products are quite similar with each other.) and the search cost is in high level.

search gives their products more chance to be selected. In this sense, firms are willing to pay for their products to be prominence. The ways products are presented can influence consumer search orders. A firm can convince consumers to search its shop first in many ways such as advertising or producing various products. Without such a message, firms have equal chance to be sampled. For example, a shoe store provides various brands to gain the consumers' attentions. Consumers prefer visiting firms that provide more brands because of higher expected return. This is a reason why product extension has become favored strategy in recent years.

Some of the literature that has already done the study about the search model and how firms send message to convince consumers, such as Butters (1979) and Grossman and Shapiro (1984). However, these works employ Informative Advertising which provides information to eliminate search cost but does not influence consumers' search order. A few literature incorporate product line extension in the consumer search model. These works are Cachon et al. (2005) and Cachon et al. (2008). However, consumers randomly search. They do not study the role of product line in consumers' search order. To the best of my current knowledge, the study that incorporates product extension with the role of saliency in consumer search model has not yet been studied. In this study, I fill the gap of the existing literature in such a way that there will be a relationship between consumers' search order and a product extension strategy.

This study aims to analyze the circumstance that consumers do not know the prices and the match values of brands for them. Firms offer many brands to consumers. Analyzing impact of prominence on product lines and prices is the main objective. The study also investigates the competitions from product line when two firms are engaged in the battle for consumers' attention. Next, how firms' product lines and prices response to the changes in search cost is verified. Finally, we investigate the effect of the product line on price.

This study will focus on the duopoly competition with both symmetric and asymmetric technology. The product line in this study will include only horizontal line extension which all brands of both firms have the same quality but differ in other attributes. The first role of product line is to create firm saliency, that is, the prominence of brands in consumers' memories. The order in which firms are visited is influenced by their number of offered brands. The second role is to guarantee consumers' minimum utility.

2. Review of Related Literature

This article draws on the rich literature on consumer search. In particular, our model is related to the branch of the search literature concerned with product differentiation, where consumers must search both for price and match value. An early contribution is Weitzman (1979) proposes a sequential search strategy. An agent will terminate search whenever the sampled reward exceeds the reservation price. This reservation price depends on three properties of option i, the cost to open it, the time lag to learn the payoff and the distribution of the payoff. The model was later developed and applied to a market context by Wolinsky (1986). He claims that, with free entry condition and large number of firms, the search cost allows firms to maintain significant market power. Wolinsky's model is developed further by Anderson and Renault (1999) who discuss how equilibrium prices are affected by changes in the degree of product differentiation. The Diamond Paradox is limited in the market with homogeneous product. In search models with product differentiation, there are some consumers who are ill matched with their initial choice of supplier and then search further, so that the pro-competitive benefit of actual search is present. Thus, compared to the homogeneous product search model, models with product differentiation often better reflect consumer behavior in markets with non-standardized products.

The recent consumer search model focuses in biasing consumer search order. Armstrong et al. (2009) motivated by the reality that the consumers search order is not random but it is influenced by the way the options are presented. We use Armstrong et al. (2009) as the starting point for our article. The prominent firm sets lower price and maintains higher profit.[3] The prominent firm solely gains from its advantage of search order while the other non-prominent firms and consumers suffer from this condition. This study explains why firms try to be prominence.

Haan and Maranga-Gonzales (2011) generalizes Armstrong et al. (2009) by introducing advertising as the tool to create prominence. Unlike Grossman and Shapiro's model, advertising does not reduce search cost but rather affects a consumer's likelihood of sampling a firm. The firm that advertises more can attract more consumers. In symmetric equilibrium, all firms set the same price and advertise with the same intensity, and so no firm is more prominent than any other. So consumers end up searching randomly and advertising is pure waste. In asymmetric equilibrium, more efficient firm advertises more and sets a lower price. The industry profit can increase when the technology gap is larger. Such an effect that more prominent firms set lower prices is found in Bagwell and Ramey (1994), although for very different reasons. In their paper, firms are identical *ex ante* and attract consumers by means of advertising. Firms have economies of scale, so that a firm facing greater demand has a lower marginal cost. Consumers follow the rule of thumb whereby they buy from the firm which advertises most heavily. Because of economies of scale, this firm will have a lower price than its rivals. Thus, the consumer response to advertising is indeed rational even though advertising messages are not directly informative, and the more prominent firm sets a lower price.

The consumer search model also has some impact in multiproduct competition in which firms sell a set of brands which differ in various attributes except quality. Firms expect that more brands will increase the likelihood that one of the will be chosen. An active searching consumer will appreciate if firms provide various products because it gives higher expected benefits in a visit. This presents in Cachon et al. (2008) that consumers randomly search among the infinite number of firm. Contrast with the general wisdom that the firms

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[3]This condition does not hold in the case of infinite number of firms. Prices of prominent firm and non-prominent firm are equal if n goes to infinity.

price, number of product, and profit all positively relate to search cost, they find an opposite results that easier search raises numbers of brand and does not necessary reduce profit. An easier search brings new consumers, and so the competition in the assortment becomes more profitable. Draganska and Jain (2005) consider product assortment as a competitive tool in the U.S. yogurts industry. A result shows that firms extend assortments because they can charge higher prices. Furthermore, firms would prefer to commit to an assortment if this were possible that means firms are forced to take excessive variety competition. This result is drawn without the effects of search cost, the firm's saliency or both. If consumers lack information about the characteristics and prices of brands in the market, the study should investigate the impact of search cost and firms saliency in the consumer's memory.

3. Model

This study focuses on the product line as a competition tool of two multiproduct firms. The extended product line occurs when a firm has various brands. This can be appeared through different color combinations, product sizes and different brand uses but does not in quality. All firms compete in product line first to become saliency, that is, the prominence of a firm in consumers' memories. Moreover, product line also guarantee the minimum utility to the consumers.

The consumers are assumed to be heterogeneity in tastes and have imperfect information about the match values of the available brands in the market place. To gain that information, they must sequentially search. In each search, consumers will be imposed by an explicit search cost. Once they visit a firm, they know the highest utility, and decide whether to terminate or continue search.

All firms simultaneously choose number of brands (how much the minimum utility it guarantees to the consumers) and a price for those brands. The assumption that firm sets the same price for all brands is very valid when consumers values in qualities more than products' attributes.[4] Firms have asymmetric production technology. The variable costs are normalized to zero while the marginal cost to extend the product line is positive. The longer product line, the higher production cost at increasing rate.

3.1 Information

The main concern of this study is in consumers' information. Here it is assumed that initially a consumer knows the number of the available brands, but she does not exactly know the prices and her highest match values y_j of both firms' brands.

A consumer, however, can gather information by sequentially search among firms. I make the following assumptions concerning consumers' search:

1. At a cost s per firm a consumer can sample firm's brands and finds out the price and her maximum value in the shop.

2. The consumer's search is without replacement and with costless recall. That is, each time the consumer incurs the costs she will learns about a set of different brands, and she can proceed to purchase any one of the brands she has already sampled without incurring additional search costs.

3.2 Consumer Search

The number of consumer in this market is normalized to 1, which are uniformly distributed along the characteristic line [0,1]. A consumer intends to buy a single brand from the firm that gives them a sufficient high utility. The maximum utility received from consuming her most favorite brand from firm *i* is y_i. Consumers learn about y_i only upon visiting firm *i*.

The value of y_i is assumed to be sufficiently high such that the consumers will always buy a single unit of brand. The standard optimal stopping rule is employed with the form

$$s = \int_{x_i}^{1} (y_i - x_i) dF(y_i, r_i) \qquad (3.1)$$

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[4] With this condition, Draganska and Jain (2006) confirm that setting the different prices according to products' qualities and the same price in the same product line (same quality) are the corrected pricing strategy.

The reservation price x_i is decreasing in search cost per firm s while increasing in firm's product line r_i. This implies consumers are more likely to terminate search sooner and may accept the payoff that far from their preferences when search cost is in high level. In contrast, as more brands are offered by firms, consumers will expect better payoff in the next search, therefore they become more choosy.

Noting that if there is no search cost $s \approx 0$, $x_i \approx 1$. The consumers will search until they meet a brand which gives the maximum utility. This can be perceived as a special case.

Given the finite options available, one might suppose that an optimal search strategy might exhibits (i) the consumer becomes less choosy as the number of remaining brands shrinks, or (ii) when there are fewer firms, a consumer is less choosy. However, Wolinsky (1986) shows that the optimal search rule is stationary when consumers can costless go back to the earlier sampled brands. If both firms are expected to offer the price *p*,

Claim:

1) If $p_i > x_i$, a consumer should not participate in the market.

2) If $p_i \leq x_i$, a consumer should stop searching when she finds a brand with $y_i - p_i > x_j - p_j$; if no such brand is eventually found in both shops, she goes back to buy the brand with the highest y_i, provided $y_i > p_i$. If all y_i are below p_i, then the consumer buys nothing.

3.3 Product Line

Firms' production technologies are differed.[5] Without loss generality, firm 1 has more efficient production technology which is captured by a parameter $\mu_1 \in [a,1)$, where *a* is the lower bound that firm 1 has very high efficiency so that becomes the pure monopoly. The marginal costs for firm *I* is defined as follow:

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[5] In the symmetric case, the game in (3.2) can be considered as "Prisoners' dilemma". Firms equally advertise and also equally share pool of consumers who sample them first. However, in order to achieve this result, all firms are better if they do not advertise at all.

$$c_{i,r_i} = \frac{r_i}{1-r_i} \cdot \mu_i .$$

3.3.1 Attention

The first propose of firms to engage in a product line battle is to lure consumers to their shops. In particular, a consumer is more likely to go to a firm *i* if she has observed the longer product line of that firm. Intuitively, the longer product line, the more likely is that consumers remember it.

The probability that the consumer will recall firm *j* in her next search is given by

$$\frac{r_i}{\sum_{i=1}^{2} r_i} . \qquad (3.2)$$

This modeling of the recall probability captures the consumers' memorability of competing firms. A firm that has zero product line will be visited last. If no firm has positive product line, the consumer visits firms randomly.

One can think of brand of a firm as a ball this firm puts in an urn. Each firm can put as many balls as it can. Whenever the consumer needs a product, she draws one ball from the urn and visits the corresponding firm. In the first visit, if a consumer does not satisfy with those products she already sampled, she will search another one firm without drawing a ball.

3.3.2 Guarantee Utility

$F(y_i, r_i)$ is the CDF of the maximum utility offered by firm *i* which is assumed to be a standard uniform distribution function.

$$F(y_i, r_i) = \frac{y_i - r_i}{1 - r_i} \qquad , r_i \in [0,1], \ and \ \ y_i \in [r_i, 1]$$

3.4 Firm

Assume that, upon visiting a firm, a consumer learns its type. Let $\Omega \in \{1, 2, 12, 21\}$ denote which firms a particular consumer visits, and in what order. Thus $\Omega = 12$ implies that the consumer has first visited firm 1, and then firm 2. Let q_i^{Ω} denote total demand for firm i from such consumers. Thus q_1^{12} denotes demand for firm 1 from consumers that visit firm 1 and 2 in that order, while q_1^{1} denotes demand for firm 1 from consumers that only visit firm 1.

The probability that consumers sample firm *i* in the first search is λ_i which depends on its level of product line relative to the rival firm *j*, $\lambda_i = \dfrac{r}{r + r_j}$. A consumer immediately discovers her utilities attached to each brands and their single price *p*. Since she can accurately expect that another firm will charge p_j, a brand in firm *i* will be chosen if $\Pr\left(y_i - p > x_j - p_j\right) = 1 - F(y_i, r)$. Here, $x_j - p_j$ is the reservation surplus when a consumer deals with this firm. Thus, firm's "fresh demand" will be

$$q_i^i(p) = \lambda_i \left[1 - \frac{x_j - p_j + p - r}{1 - r}\right]. \tag{3.3}$$

The second part of demand arises when a consumer visit firm *j* first but does not decide to buy any brand. The term $h_i = \dfrac{x_i - p_i + p_j - r_j}{1 - r_j}$ reflects a fraction of consumers who do not satisfy utility offered by firm *j*. She will then search firm *i* and may buy from firm *i* if

$$\Pr(y_j - p_j < r_j - p_j < x_i - p) \text{ and } \Pr(y_i - p > y_j - p_j) \text{ and } \Pr(y_i > p).$$

Therefore, firm's "weak demand" q_i^{ji}, is defined as follow

$$q_i^{ji}(p) = \lambda_j \left[h_i \cdot \left(1 - \frac{x_i - p + p_i - r}{1 - r}\right) + \int_{p}^{x_i + p - p_i} \frac{y_i - p + p_j - r_j}{1 - r_j} \cdot \frac{1}{1 - r} dy_i \right] \tag{3.4}$$

When a consumer paid visits firm *i* first and then firm *j* but she still cannot find any brand that give a utility exceed her reservation threshold. She will buy from firm *i* if it gives higher utility than firm *j*. Formally, $\Pr(y_i - p < x_j - p_j)$ and $\Pr(y_j - p_j < y_i - p)$ and $\Pr(y_i > p)$ is the probability that a consumer will buy from firm *i*

$$\Pr\left(y_j - p_j < y_i - p < x_j - p_j\right) = \int_{p}^{p+x_j-p_j} F(y_i - p + p_j - r_j, r_j) \cdot f(y_i, r) dy_i$$

With costless recall assumption, this consumer can choose a brand that she already explored without any cost. This faction is "returning demand" and the probability of this event is

$$q_i^{ij}(p) = \lambda_i \cdot \int_{p}^{p+x_j-p_j} \frac{y_i - p + p_j - r_j}{1-r_j} \cdot \frac{1}{1-r} dy_i \tag{3.5}$$

The demand of firm i comprises with three parts. The first part is fresh demand q_i^i, the second part is weak demand q_i^{ji} and the third part is returning demand q_i^{ij}.

$$q_i = q_i^i + q_i^{ji} + q_i^{ij} \tag{3.6}$$

Engaging in product line battle, all firms are imposed with the production costs $c_i(r_i)$. When this firm charges *p*, the profit will be

$$\pi_i(p) = pq_i - c_i(r_i) \tag{3.7}$$

Firm *i* maximizes profit by choosing a price p_i, and a level of product line r_i, given the level of search cost and strategies of firm *j*.

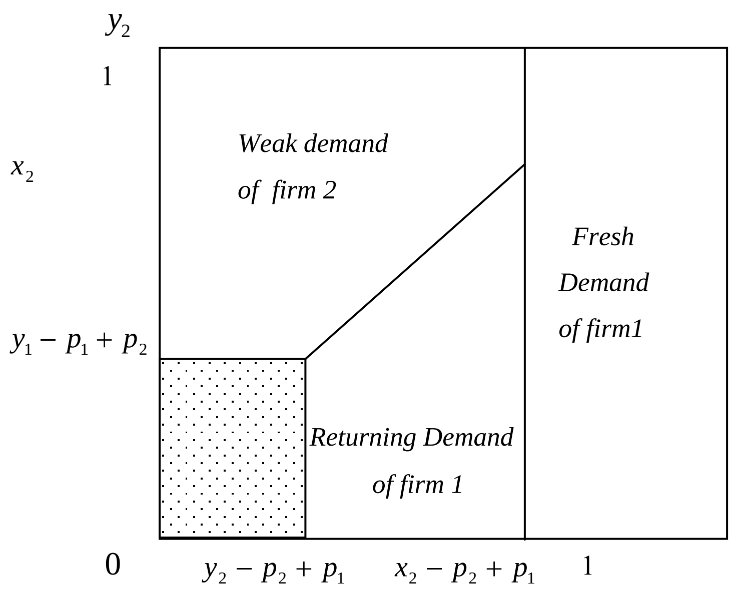
4. Prominence and Product Line

This chapter limits the product line to be a guarantee tool. It can be treated as a general case of search model in Armstrong et al. (2009) in which firms can produce more than one brand. Firms compete to extend the product lines to capture consumers' tastes. Studying the impacts of the prominent position in product lines, and prices are the main objectives.

The prominent search model assumes all consumers search the prominent firm first. We can explain the idea of firm's saliency in consumers' memory as followed. The prominent firm will always be search first by all consumers. The saliency can lure the larger pool of consumers than the less-saliency. The non-prominence won't be sampled first by any consumer. In this chapter, a firm is assumed to be prominence while another firm acts as a non-prominent firm.

Figure 4.1

The consumer demand



The pattern of consumer demand is depicted in figure 4.1 that exhibits the characteristic space $[0,1]^2$. The realized highest match values, y_2, for a non-prominent firm is in the vertical axis and the highest value y_1, of prominent firm is in horizontal axis. A consumer's preference for each brand is unique and represented by a point in the space.

The product line is a strategic substitute. Firm's extension has positive impact on its demand but negative on its rival's demand. Higher search cost induces consumers to terminate search sooner causing more consumers buy in fresh demand of the prominent firm. Then, fewer search non-prominent shop, and hence reduces the weak demand.

4.1 Prominent Firm

Since all consumers search firm 1 first, they search in the market when they expect the positive surpluses $p \le x_1(r_1, s)$, where *p* is a price charged by prominent firm and x_1 is a reservation price defined by

$$s = \int_{x_1(r_1,s)}^{1} \left(y_1 - x_1(r_1, s)\right) dF(y_1, r_1) \tag{4.1}$$

An active consumer pays a search cost to visit firm 1. She immediately discover highest values of the brands $y_1 \in [r_1, 1]$. Then she must decide whether buying a brand from this firm or start next search. To make this decision, she adopts the reservation price x_2 which is defined by[6]

$$s = \int_{x_2(r_2,s)}^{1} \left(y_2 - x_2(r_2, s)\right) dF(y_2, r_2). \tag{4.2}$$

Consumers buy from firm 1 if $\Pr(y_1 - p > x_2 - p_2) = 1 - F(y_1, r)$. Otherwise, she will continue search. The firm's "fresh demand" is

$$q_1^1(p, r) = 1 - \frac{(x_2 - p_2 + p - r)}{1 - r}. \tag{4.3}$$

If a consumer sampling all firms but she finds out that the net surpluses of all brands is still less than $x_2 - p_2$, $\Pr(y_1 - p < x_2 - p_2)$ and $\Pr(y_2 - p_2 < x_2 - p_2)$,she may choose a brand of prominent firm if it gives higher utility. The returning demand of prominent firm is described as

$$q_1^{12}(p) = \int_{p}^{p + x_2 - p_2} \frac{\left(y_1 - p + p_2 - r_2\right)}{1 - r_2} \cdot \frac{1}{1 - r} dy_1 \tag{4.4}$$

Interestingly, this returning demand is independent with its price. To clarify this event, reducing a firm's price has two opposite effects on its returning demand:

(i) Negative effect; more consumers are satisfied with prominent firm's options at the first visit (fresh demand increases) and so fewer consumers go on to sample all firms, and

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[6] These two reservation prices x_1 and x_2 are equal if both firms produce the equal number of brands.

(ii) Positive effect; the prominent firm's share of those consumers who sample all firms is increased.

With a uniform distribution of match values, the negative effect will completely offset the positive effect. Therefore, reducing price has no effect to returning demand.

The prominent firm's demand comprises with fresh demand and the returning demand.[7]

$$q_1(p) = q_1^1(p) + q_1^{12}(p) \tag{4.5}$$

The production cost $c(r)$ is strictly convex in r, so the marginal cost is $c_r = \frac{r}{1-r} > 0$. When prominent firm charges p, the profit is

$$\pi_1(p) = p \cdot q_1(p) - c(r) \tag{4.6}$$

Each firm maximizes profit by setting a price p_1 and chooses a product line r_1 given a level of search cost. The two FOCs of price $\pi_{1,p} = 0$ and product line $\pi_{1,r} = 0$ are rewritten in equation (4.7) and equation (4.8) respectively.

$$p_1 = \frac{1 - x_2 + p_2}{2} + \frac{1}{2} \cdot \int_{p_2}^{x_2} \frac{y_1 - r_2}{1 - r_2} dy_1 \tag{4.7}$$

$$r_1 = p_1 q_1 \tag{4.8}$$

4.2 Non-prominent Firm

If consumers do not buy in firm 1 $\Pr\left(y_1 - p_1 < x_2 - p_2\right)$, they search to non-prominent firm. When she visit, she discovers the highest utility y_2 which is realization of a random variable with distribution $F(y_2, r_2) = \frac{y_2 - r_2}{1 - r_2}$.

The probability that she will terminate search and buy from current firm is $\Pr\left(y_1 - p_1 < x_2 - p_2 < y_2 - p\right)$ and $\Pr\left(y_1 - p_1 < y_2 - p < x_2 - p_2\right)$. Non-prominent

[7] Prominent firm is visited first. Then it has no weak demand.

firm charges a price *p* which is set to allow small deviation from p_2. Its weak demand is going to be

$$q_2^{12}(p) = h_2\left[1 - \frac{x_2 - p_2 + p - r}{1-r}\right] + \int_{p}^{p+x_2-p_2} \frac{(y_2 - p + p_1 - r_1)}{1-r_1} \cdot \frac{1}{1-r} dy_2 \qquad (4.9)$$

, where $h_2 = \frac{x_2 - \Delta - r_1}{1-r_1}$ is the probability that consumers visit this firm.

Non-prominent firm's demand comprises only with weak demand. The technology to produce product line is symmetric with prominent firm. When this firm charges price p , its profit is

$$\pi_2(p) = p \cdot q_2(p,r) - c(r) \qquad (4.10)$$

A firm chooses a price and product line to maximize profit. The FOCs is going to be $\pi_{2,p} = 0$ and $\pi_{2,r} = 0$. Then, substituting $p = p_2$, and $r = r_2$ to receive the best response function of price and product line of non-prominent firm in equation (4.11) and equation (4.12), respectively.

$$p_2 = 1 - x_2 + \int_{p_2}^{x_2} \frac{y_2 - \Delta - r_1}{x_2 - \Delta - r_1} dy_2 \qquad (4.11)$$

$$r_2 = p_2 q_2 \qquad (4.12)$$

Lemma 1: *There exists price equilibrium. Under the uniform distribution, within the square* $[0, x_2]^2$ *expression (4.7) and (4.11) have a unique solution, which satisfies* $(p_1^*, p_2^*) \in \left[1 - x_2, \frac{1}{2}\right]^2$.

Proof. See Appendix.

Lemma 2: *There exists product line equilibrium. Under the uniform distribution, within the square* $[0,1]^2$ *expression (4.8) and (4.12) have a unique solution, which satisfies* $\left(r_1^*, r_2^*\right) \in \left[0, \frac{1}{2}\right]^2$.

Proof. See Appendix.

4.3 The Impact of Prominence

Here we present results describing the impact of making a firm prominent on market outcomes, specifically, product lines and prices. (Proofs of each result are presented in the Appendices.) The first question is how making a firm prominent influences the equilibrium product lines and prices:

Proposition 3: *When a firm is made prominent, the prominent firm produces a longer product line and charges a lower price than non-prominent firm. Formally,*

$$r_1^* \geq r_2^* \quad and \quad p_1^* \leq p_2^*$$

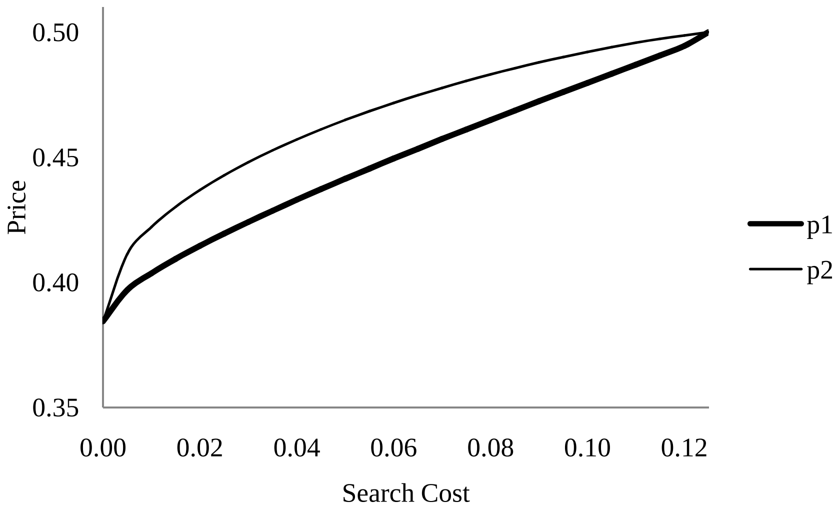
, where the inequality of product line is strict if $s > 0$*, and the inequality of prices is strict if* $0 < s < 0.125$.

Proof. See Appendix.

The intuition for this result is as follows: if p_1 and p_2 are not too far apart from each other, then the prominent firm's demand consists more of fresh demand while non-prominent firm's demand depends solely of weak demand. Because fresh demand is more price sensitive than weak demand, the prominent firm faces more elastic demand than tits non-prominent rival. The price elasticity of demand for each firm creates the different price equilibrium. This result is in line with the study of Armstrong et al.(2009) that claims the prominent firm charges a lower price.

Figure 4.2

Price and Search Cost (i)



It is useful to consider two polar cases. when $x_2 \approx 1$ (i.e., $s = 0$), consumers sample all firms before they purchase, and so prominence has no impact and all prices converge to the full-information price p_F. At the other extreme, when $x_2 = 1/2$, all prices converge to the same price 1/2 , the monopoly price. Here, the high search cost makes a consumer willing to buy whenever she finds a brand which yields her positive surplus, and so prominent firm acts as a monopolist. Thus, the price difference caused by prominence becomes negligible when the search cost is too high or too low, and it is most pronounced when the search cost is at an intermediate level. The Figure 4.2 describes the relationship between the three equilibrium prices when s varies from 0 to 0.125.

Proposition 4: *When a firm is made prominent and all firms share the same technology, all prices increase with the search cost.*

Proof. See Appendix.

The figure 4.2 exhibits both prices strictly increase with the search costs. The changes in the search costs have both direct and indirect effects on the equilibrium prices. The direct effects are drawn by firm's demand and its sensitivity on prices. The indirect ones are from its product lines and its rival's strategies.

Higher search costs induce consumers to be less choosy and thus more of them buy in prominent firm. Therefore, this direct effect allows prominent firm to raise the prices. Even though, the search costs increase the prominent firm's product lines ($r^*_{1,s} > 0$ in the Proposition 4), the prominent prices surprisingly are not affected by changes in its product lines ($b_{1,r^*_1} = 0$).[8]

It seem that higher search costs and then lower product lines $r^*_{2,s} < 0$ cause less consumers search and buy the non-prominent brands. The non-prominent firm is supposed to reduce the prices. However, its consumers reveal that they are less sensitive on prices. Consequently, the non-prominent firm can raises the prices for them.

Charging higher prices for higher search costs is reinforced by the strategic responses. Because both firms intend to raise prices, it in turn drives up the rival prices.

Proposition 5: *When a firm is made prominent, (i) the prominent firm's product lines increase with search cost and non-prominent firm's product lines decrease with search cost. (ii) Total product lines increase with the search costs.*

Proof. See Appendix.

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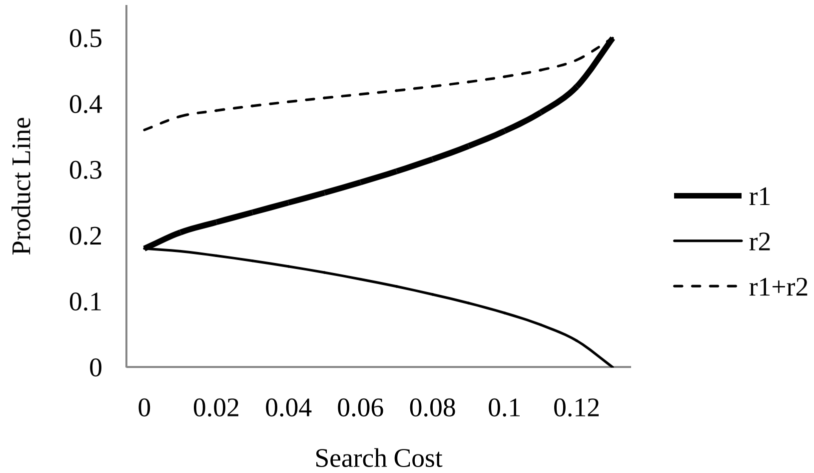
[8] To clarify this indirect effect, increases in the product lines have two completely opposite forces. The consumers are more likely to pick brands in the prominent firm but if the firm tries to increase the prices, it faces to lose all new demand.

Figure 4.3

Product Line and Search Cost (i)



Why the prominent firm provides longer product lines can be explained by its demand. Intuitively, how long the product line a firm provides is depend on its prices and demand. Despite of lower prices that make prominent product lines seem to be less profitable, the prominent firm gains a big advantage from high demand. With higher search cost, the gain in prominent firm's demand and the loss in non-prominent firm's demand make $r_1^* > r_2^*$.

The result in (i) seems counter intuitive. Since higher search costs give prominent firm more market power and the current brands can ensure higher demand, one might wonder why this firm guarantees higher utility. This study finds out that, by doing so, the firm can raise the profits.

When the search costs increase, the consumers do not want to search many times. The prominent firm's brands are more attractive to the consumers. A consumer can buy a prominent brand even if it provides low utility. Consequently, more consumers buy in prominent shop, and fewer of them search and buy in non-prominent shop. It is more profitable to prominent firm if there are more brands in the shop. On the contrary, non-prominent firm's brands are seemed to be less attractive because the consumers must pay higher cost to visit the shop. Thus, it is better to reduce the production cost by reducing its product lines.

The result in (ii) is opposed with Cachon et al. (2008) studying the product line in the random search model and claiming the negative relationship, or easier search costs give more total brands. This study gives the opposite result based on the advantage of prominent firm's demand.

Higher search cost means the consumers are more likely to buy in prominent shop. That is higher marginal revenue for product line. Hence, adding a brand increases profit a lot. The increase of all prices also makes brands more profitable. It makes brands more profitable and higher the rival's price strategically increase demand for firm's brands.

In the Figure 4.3, a dash line represents the total brands which increase with cost. This shape means to the consumers that they can expect more brands from the market when the search cost is high. Furthermore, since each brand has equally chance to be picked, it is easier for consumers to find their match values as the search cost increases.

Proposition 6: *When a firm is made prominent, (i) the prominent firm earns more than non-prominent firm. (ii) The prominent firm's profit increases with search cost while non-prominent firm's profit decreases with search cost.*

Proof. See Appendix.

The result in (i) is not surprising. For instance, the prominent firm could choose to set non-prominent firm's equilibrium price, in which case it still makes more profit than its rival because it has greater demand. But it can do still better than this by choosing a lower price than its rival.

This result is also drawn from the impact of prominence on the non-prominent firms' profit. Specifically, the impact is determined due to two effects: (i) the non-prominent firm suffers from being pushed further back in each consumer's search order and (ii) from the lower price offered by the prominent firm.

Since firms' profit are strongly correlated with the level of their product lines, the firm that produce longer product line will earn more than firm that produce shorter product line. In the Proposition 4, we know that prominent firm's product line increases with search cost while non-prominent firm reacts in the opposite direction. Therefore, the differences of firms' profits are greater in responses to higher search costs.

We end this chapter by pointing out the implications of these results. When the consumers are imposed with search costs, firms want to become a prominence. Even though they must discount prices and produce more varieties that can be recognized as lower incomes and generating more costs but the prominent position gives a big advantage in high demand. This leads to a surprising result that providing many brands does not improve the pricing powers. More brands increases demand but if firms try to charge higher prices, the same amount will leave the prominent firm or more of them will not visit non-prominent firm.

5. Competitions for Attention and Utility Guarantee

The analysis in the previous chapter is on a firm is created to be prominence. Many results are no longer true if we allow consumers to freely range their search orders. In such a case, the portions of consumers that choose to visit a firm depend on how many brands it provides. A firm which has longer product line is more likely to gain a larger pool of consumers in the first visit. The central question in this chapter is firm which attract more consumers in the first visit charge higher or lower price. More specifically, is longer product line correlated with higher prices, or lower ones? How are firms' profits affected by firms' asymmetry?

There are alternative ways to introduce asymmetries across firms. To focus on a case where the asymmetry in equilibrium prices stems exclusively from differences in product line levels, we assume that firms differ in the costs they have to incur to undertake a product line campaign. Technically, we write the marginal cost of firm i, $c_i = \frac{r_i}{1-r_i} \cdot \mu_i$.[9] Even in such a simple setting it is difficult to derive analytical results, so we will partly have to resort to a numerical analysis.

One complication has to do with consumer search behavior after out-of-equilibrium moves. Suppose that firms charge different prices in equilibrium. Let us assume consumers know the equilibrium prices but do not know which firm has which price. Suppose now that a consumer observes an out-of-equilibrium price at her first visit. Her decision whether to continue searching will then be affected by whether she interprets this out-of-equilibrium price as coming from the low-price firm or from the high-price firm.

There are various ways to circumvent this complication. The simplest is to assume that, upon visiting a firm, a consumer can learn its type.[10] We compute our equilibrium under this assumption. Another possibility is to specify a set of beliefs after disequilibrium moves that sustain a given equilibrium.

Firm *i* has three types of demand, fresh demand q_i^i, weak demand q_i^{ji} and returning demand q_i^{ij}. When firm *i* charges p, its profit is

$$\pi_i(p) = pq_i - c_i(r_i) \quad (5.1)$$

Taking the FOCs with respect to own advertising intensity and price, imposing $p = p_i$ and $r = r_i$, and doing so for $i = 1, 2$ and $j \neq i$ yields four nonlinear equalities that can be solved to find equilibrium product lines and prices. From these FOCs,

$$p_i = q_i \frac{1-r_i}{\lambda_i + h_i \lambda_j} \quad (5.2)$$

---

[9] Notice that in this case, if we chose marginal costs to be different, price variation due to marginal cost differences would be augmented by price variation due to different product line.

[10] For example, from observing the lay-out and the colors in the store, she may realize that she has actually seen more brands from the other store and hence this store must be the one with the more costly technology.

$$r_i = \frac{p_i}{\mu_i}\left[q_i\left(\frac{r_i^2 + r_j}{r_i + r_j^2}\right) - q_i^{ji}\left(\frac{1-r_i}{r_i}\right)\right] \tag{5.3}$$

, where $\mu_1 = [a,1)$, and $\mu_2 = 1$.

Lemma 7: *There exists price equilibrium. Under the uniformly distributed of match values, within the range* $[0, x_i]$ *, the pair of expression (5.2) have a unique solution, and this solution satisfies* $p_i^* \in \left[1 - x_i, \frac{1}{2}\right]$.

Proof. See Appendix.

Lemma 8: *There exists product line equilibrium. Under the uniformly distributed of match values, within the range* $[0,1]$*, the pair of expression (5.3) have a unique solution, and this solution satisfies* $r_i^* \in \left[(1-x_i)^2, p_i\right]$.

Proof. See Appendix.

5.1 Asymmetric Production Technology

Proposition 10. *With two firms, a uniform distribution of matching values and asymmetric production technologies, we have that the firm that provide longer product line sets a lower price:* $r_i^* > r_j^*$ *necessarily implies* $p_i^* > p_j^*$.

Proof. See Appendix.

Figure 5.1

Product Line and Firm Asymmetry

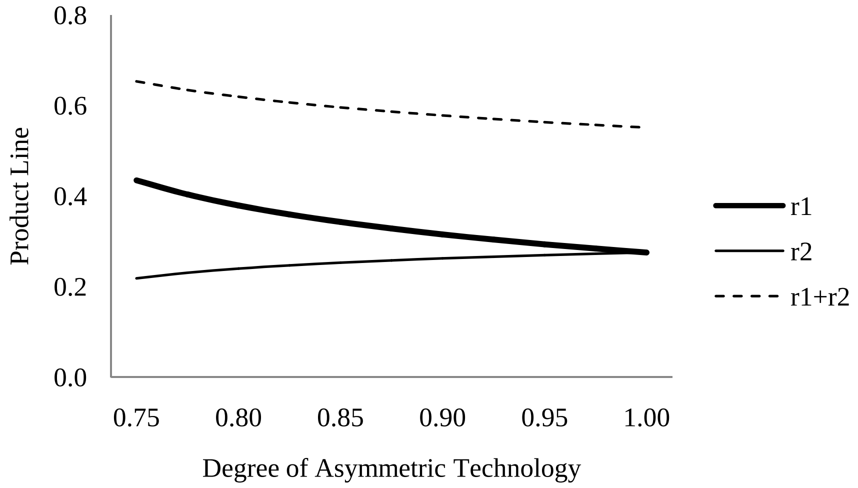
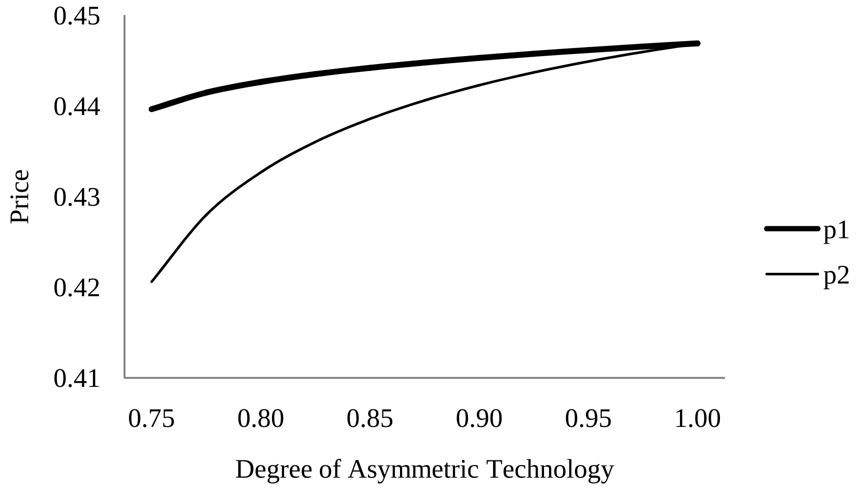


Figure 5.2

Price and Firm Asymmetry



This is a surprising result. The previous literatures, including our result in the Chapter 4, suggest that a firm with higher degree of prominence will discount.[11] Oppose to

[11] See Armstrong et al. (2009), and Haan and Moraga-Gonzalez (2011).

those studies, we find that the product line gives pricing power. Even though a firm that extends product line more intensively attracts a broader range of consumers, those are more likely to buy in this particular firm. Hence longer product line increases price. By choosing to continue search, consumers must sure enough that they can explore a high surplus in the second shop. The less-salient firm that relies more on the second search will discount to attract the consumers. For example, when the tourists was shopping, they know a little which shop provides highest value. Then they look for the shop that has plenty of products that they can expect high utilities and more likely to buy. Thus salient shop charges higher price than that presents in the late search.

To see which firm produces longer product line, we need to put additional structure on the model. Assume that production technologies are strictly convex, so $c_{i,r_i} = \frac{r_i}{1-r_i}\mu_i$, with $c_{1,r_1} < c_{2,r_2}$.[12] Then:

Proposition 10. *With two firms, a uniform distribution of matching values and convex asymmetric production technologies, in equilibrium, the more production-efficient firm will produce longer product line.*

Proof. See Appendix.

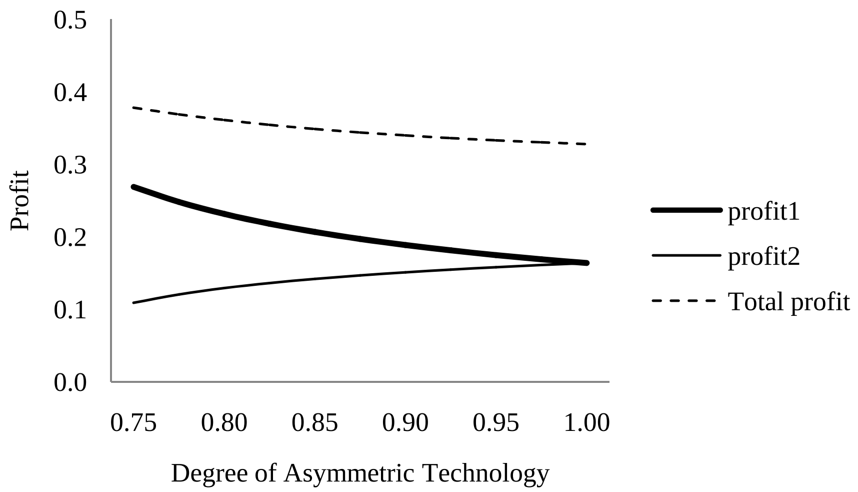
To perform comparative statics, we have to resort to a numerical analysis. We again assume convex production technologies. Without loss of generality, we assume that firm 1 has a more efficient technology, and normalize c_{2,r_2} to $\frac{r_2}{1-r_2}$, so $\frac{r_1}{1-r_1}\cdot\mu_1 = c_{1,r_1} < c_{2,r_2}$. We know that this implies that $r_1^* \geq r_2^*$ and $p_1^* \geq p_2^*$. In Figure 5.1 and 5.2, we depict equilibrium product lines and prices as a function of asymmetric cost. The horizontal axis now reflects the extent of asymmetry between production technologies: as closer to 1, production technologies become more symmetric.

[12] With convex production costs, it guaranteed that our profit functions are globally well-behaved.

An increase in the asymmetry in firm production efficiency has the following effects. The prices of both firms decrease due to higher shares on fresh demand of both firms. The price gaps increase because the differences on product lines are larger. The product line level of the cheapest firm increases, that of the most expensive firm decreases, while average product line levels also increase.

Figure 5.3

Profit and Firm Asymmetry



To fully appreciate the effect of a change in μ_1 on profits, we have to resort to numerical analysis. In figure 5.3, we depict the components of equilibrium profits of firm1 and firm2 and industry profit as a function of μ_1, for the case that $s = 0.08$. For different level of *s*, the picture looks qualitatively the same. Firm1's profits decrease as the technologies are more symmetric, while firm2's profits increase, but less so. The industry profits thus increase as firm1's production cost falls.

5.2 The Impact of Search Cost

To verify how firms react to changes in search cost, we employ the numerical analysis. The asymmetric technology parameter μ_1 is restricted to 0.8 and allows search cost to vary. The finding results are presented below.

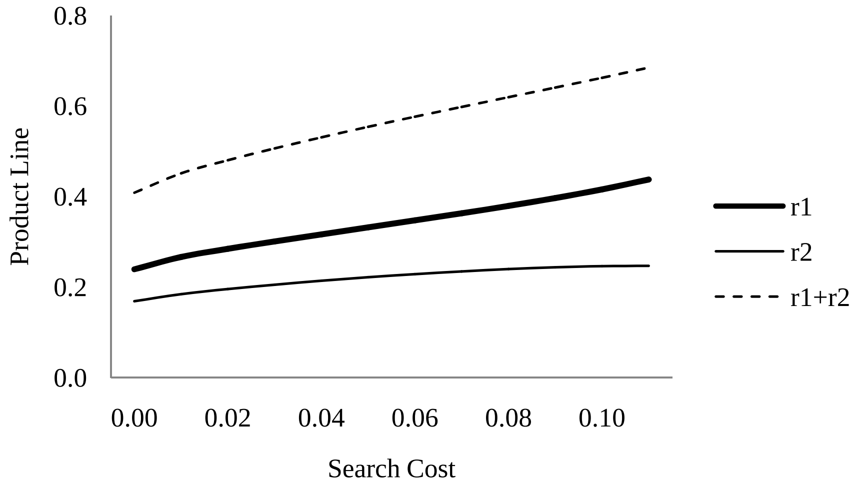
Proposition 11. *With asymmetric technology, (i) the salient firm's product line increases with search cost, and (ii) less-salient firm's product line both increases and decreases with search cost.*

Proof. See Appendix.

Simulations show that the comparative statics with respect to search costs are qualitatively unaffected by the asymmetry of technologies. The increases of the equilibrium product lines level of firm1 are stronger than firm2, $r^*_{1,s} > r^*_{2,s}$ caused by two effects, attention and guarantee effect. We will describe how the guarantee effect dominates the attention one.

Figure 5.4

Product Line and Search Cost (ii)



For given the technology asymmetry μ_1, total product lines are increasing with search costs because high search costs give a big reward to the firm that being the first station. Consequently, the competition in product line is more aggressive. This motivation is greater in firm2 because it will gain high benefit from luring firm1's larger consumer pool. Higher search costs also mean that firm's products are more attractive in the first search, hence it guarantee higher utilities. The firm1 that able to convince more consumers to visit the shop will receive higher benefits because its additional brand brings greater demand.

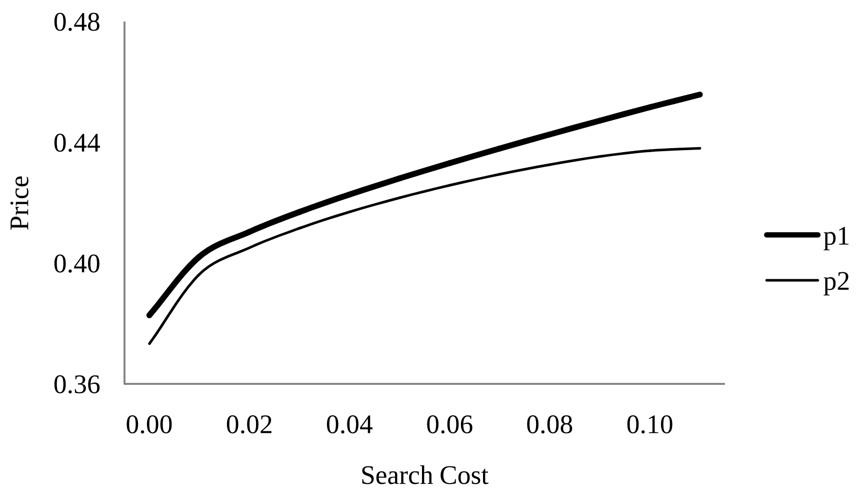
The guarantee effect dominates the attention effect and hence the impacts of search costs is stronger in firm1 that relies more on the guarantee effect, $e_{1,s} > e_{2,s}$. The intuitive is described as follow. By extending longer product lines, Firm1 may lure additional fewer consumers from firm2 (given the constant r_2^*) but it can well convince these consumers to buy. On the contrary, firm 2 can steal consumers from firm1's larger consumers base (given the constant r_1^*) but these new consumers are less likely to buy than their friends who were lured in firm1. Therefore, the firm1 efficiently improves its demand by extending the product lines.

$$r_{i,s}^* = e_{i,s} + \left[e_{i,r_j^*} \cdot r_{j,s}^* \right] + \left[e_{i,p_i^*} \cdot p_{i,s}^* \right] + \left[e_{i,p_j^*} \cdot p_{j,s}^* \right] \tag{5.4}$$

The expression (5.4) describes how the firm's product lines react with search costs. The core effect $e_{i,s}$ is just described above and another three terms in the square blanket are indirect effects. The first indirect effect $e_{i,r_j^*} \cdot r_{j,s}^*$ is the strategic reaction from the rival's product lines and this term can explain why r_2^* decreases when the search costs are very high.

Figure 5.5

Price and Search Cost (ii)



We now turn to the relationship of price and search cost. Prices of both firms increase with search cost. Intuitively, the higher search cost makes consumers less choosy and gives firms higher market power. The salient firm gains more pricing power due to its large consumer base. The salient firm's demand increases with the search cost, while less salient firm's demand decrease. Therefore, the pricing power increases more in salient firm.

The profit of firm1, the most efficient firm, monotonically increases in *s*. while the opposite direction is exhibited in firm2. This coincides with the firms' demands. Since, more consumers first visit firm 1, it also retains more consumers when the search cost increases. For firm2, it gains more consumers in fresh demand but it losses more proportionately in weak and returning demand. Another reason is the higher prices of both firms due to search cost. In the Figure 5.5, Firm 1 charges higher price more aggressively than firm2. Hence, firm1's profit increases with search cost.

5.3 Optimal Social Level of Prices and Product Line

One question we have not yet answered is, in the welfare point of view, whether the market serves the appropriate level of prices and product lines. This section discusses the prices and product lines firms should operate to maximize social welfare. One might suggest that the firm's competition to maximize profit could lead to overinvests in the product lines since the firm want to attract consumers. The low production means that it will loss consumers to the rival. Can the policymaker restrict this kind of competition to improve the overall social welfare? Should the firms' prices be reduced to make the consumers better? The following result will give the answer to these questions.

We first describe the formulation of consumer surplus. The total consumer surplus equals

$$CS = \lambda_1\left(CS_1^1 + CS_1^{12} + CS_2^{12}\right) + \lambda_2\left(CS_2^2 + CS_2^{21} + CS_1^{21}\right) + CS_{\varnothing}^{ij}$$

The surplus for consumers who first visit and buy in firm *i* equals

$$CS_i^i = \int_{r_j}^{1}\left(\int_{x_j - p_j^* + p_i^*}^{1} (y_i - p_i^* - s)\, f(y_i)dy_i\right) f(y_j)dy_j$$

The surplus for consumers who first visit firm *i* and buy in firm *i* only after they visit both firms equals

$$CS_i^{ij} = \int_{p_i^*}^{x_j - p_j^* + p_i^*} \left(\int_{r_j}^{y_i + p_j^* - p_i^*} (y_i - p_i^* - 2s) f(y_j) dy_j \right) f(y_i) dy_i$$

The consumers who first visit firm *i* but continue search and buy in firm *j* have the consumer surplus equals

$$CS_j^{ij} = \int_{x_j}^{1} \left(\int_{r_i}^{x_j - p_j^* + p_i^*} (y_j - p_j^* - 2s) f(y_i) dy_i \right) f(y_j) dy_j$$
$$+ \int_{p_j^*}^{x_j} \left(\int_{r_i}^{y_j - p_j^* + p_i^*} (y_j - p_j^* - 2s) f(y_i) dy_i \right) f(y_j) dy_j$$

The consumer surplus for some consumers who do not buy at all equals

$$CS_{\varnothing}^{ij} = \frac{-2sp_i^* p_j^*}{(1-r_i)(1-r_j)}$$

and total welfare is $WF = \pi_1 + \pi_2 + CS$ as usual.

Next, the objective of the policy maker is to choose social prices p_i^s and social product lines r_i^s. Firms maximize social welfare by choosing the prices and product lines simultaneously. Specifically, we differentiate the social welfare function with respect to prices and product lines

$$\frac{dWF}{dp_i} = \pi_{i,p_i} + \pi_{j,p_i} + CS_{p_i} = 0 \tag{5.4}$$

$$\frac{dWF}{dr_i} = \pi_{i,r_i} + \pi_{j,r_i} + CS_{r_i} = 0 \tag{5.5}$$

Therefore, the prices p_1^s, p_2^s and product lines r_1^s, r_2^s that maximize the social welfare are

$$p_i^s = \frac{q_i + p_j q_{j,p_i} + CS_{p_i}}{q_{i,p_i}} \tag{5.6}$$

$$r_i^s = \left(p_i q_{i,r_i} + p_j q_{j,r_i} + CS_{r_i} \right) \frac{(1-r_i)}{\mu_i} \tag{5.7}$$

Proposition 12. *When firms maximize social welfare and make the all decisions simultaneously, the social optimal prices and product lines are lower than those determined to maximize profit.*

Proof. See Appendix.

It should be noted that these equilibria are still asymmetric among the firms caused solely by asymmetric technology μ_i. When firm maximizes social welfare, it must concern the rival's profit and the consumer surplus in addition to its own profit. We explore that the profit-seeking prices are too high as well as the product lines. Reducing prices make the social better off because it improves consumer welfare. However, this heavily deteriorates firms' revenues. To protect the losses, firms must cut the production costs by reducing the product lines. In the consumers' point of view, they are better off by trading the product lines with lower prices. The firms definitely suffer but this worth for better social welfare. If the policymaker can intervene, the market prices and product lines should be reduced in order to achieve the optimal social welfare.

Why the profit seeking decision fails to achieve optimal social welfare can be explained in the nature of the competition for attention. Since the probability that consumers first visit firm depends on its product line relatives to the rival's one, the firm is in duced to invest more than its counterpart. With the asymmetric technology, highly efficient firm invests more and lures proportionately more consumers. To achieve the same proportion, both firms can reduce product lines but this does not happen in the free market because the cooperation is failed by the incentive for deviation. When the policymaker is in charge, it can prevent such deviation leading to higher efficient level of product lines.

The soften competition in product line means firms insure lower utility to consumers. This can make the consumers more choosy and thus generate more search costs. However, the policy also offers the lower prices that preferable to all consumers. Even if the consumers become more choosy and more of them search two times but they are still better off due to these lower prices.

6. Conclusions

This research has examined the implications of biasing each consumer's search order with firm's competition in product lines. We model this idea in the framework of search with differentiated products. The first model assumes consumers encounter prominent shop first. In an environment without technology differences, we find that the prominent firm produces more brands and charge a lower price than non-prominent firm. Making a firm prominent will typically increase prominent firm's profit, but lower non-prominent firm's profit. This explains why firms want to be prominent. For example, when using a search engine online, people might first click on links displayed at the top of the page. Then they find plenty of products with discounted prices. The firm tends to be largest and the most profitable.

We also find that prominent firm's product line increases with search cost. In this situation, prominence guides consumers toward firm which give expected higher utility. Moreover, high search cost facilitates market to provide plenty of brands.

In a richer environment, firms engage in a battle for attention in an attempt to being visited early by consumers. Since consumers are more likely to expose to the firm that provides more brands, more of them visits the shop for which they see many brands earlier than another shop, and, hence, because search costs are non-negligible, they are also more likely to buy from such shops. This difference has an implication on the relationship between prices and product line outlays. In this a framework, product line is not a winner-takes-all contest: after a consumer has visited a firm, she may still decide to go to a different one if she does not sufficiently like the brands of the current particular firm.

Through investments in product line, a firm can achieve a salient place in consumer memories. Consumers will then visit this firm sooner than the rival firm. The firm with the better technology becomes salient. The core finding is that firms with more efficient technologies has longer product line, charge higher prices and obtain greater profits than less efficient rival. Making firms more asymmetric technology increases the gaps in price, product line, and profit. The only result that differs from the first model is that the firm with higher product line charge higher price. This is drawn from (i) the degree of prominence is narrowed because firms share consumers in the first search, and (ii) the less-salient firm relies on fresh demand that is the most sensitive on price.

One can reconsider the case of tourists as we already described. When the tourists meet two shops, more of them search the shop that has longer product line because there's higher expected utilities. The most efficient shop is also the bigger one and charges higher price that resulted by the difference of product lines.

When firms engage in attention battles, all product lines are overinvested and all prices are overcharge. It is not surprising since firm's demand depends crucially on its product lines. This facilitates firms to harshly compete in product line, and thus reduces the profit of each other. The over production however seems to benefit overall consumers because of higher utility guarantees. To determine the social optimal decisions, there's tradeoff between loss of profits and gain in utility. When the firms compete to maximize social welfare, all product lines and prices should be reduced. The production costs decrease as well as lower utility guarantees. However, firms' profits significantly decrease due to the lower prices, and these new prices make the consumers better off.

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APPENDIX

Proof of Lemma 1. To check that the pricing policies are optimal that make both firms have the highest profits, we need to ensure that firms have no profitable deviation. The demand function needs to be modified if a firm deviates too high price, and the profit function may no longer be globally concave. Nevertheless, the prices defined in FOCs are the equilibrium price even if we take this into account.

All demand function are legitimate only when the deviation price *p* is not too high. If $p > 1 - x_2 + p_2$, then the fresh demand vanishes and the total demand becomes

$$q_1(p) = \int_p^1 \frac{(y_1 - p + p_2 - r_2)}{1 - r_2} \cdot \frac{1}{1 - r_1} dy_1 ,$$

$$q_2(p) = \int_p^1 \frac{(y_2 - p + p_1 - r_1)}{1 - r_1} \cdot \frac{1}{1 - r_2} dy_2 ,$$

which is no longer independent of *p*. Therefore, a firm's profit has a kink at $p > 1 - x_2 + p_2$ and may fail to globally concave.

For a pair of deviation prices $(p_1, p_2) \in [1 - x_2 + p_2, 1]^2$,

$$\pi_1(p) = p \cdot \int_{p_2}^{1-p+p_2} \frac{(y_1 - r_2)}{1 - r_2} \cdot \frac{1}{1 - r_1} dy_1 - c(r_1)$$

$$\pi_2(p) = p \cdot \int_{p_1}^{1-p+p_1} \frac{(y_2 - r_1)}{1 - r_1} \cdot \frac{1}{1 - r_2} dy_2 - c(r_2)$$

If we can show that these profit function are decreasing on this price interval, then we have done.

Because both $\frac{y_1 - r_2}{(1 - r_2)(1 - r_1)}$, $\frac{y_2 - r_1}{(1 - r_1)(1 - r_2)}$ are log concave, the integration term are log concave in *p*. So $\pi_1(p)$ and $\pi_2(p)$ are log concave (so quasi concave). One can check that $\pi_1^/(p_1) < 0$ and $\pi_{2,p_2}^/ < 0$ so that all firms have no intention to deviate from the best response function.

To show the existence, assume $x_2 \geq \frac{1}{2}$. Fix $p_1 \in [0, x_2]$, and consider equation (4.11). Then $p_2 = 1 - x_2 + t_2$. Here t_2 is a decreasing function of p_2 for $p_2 \in [0, x_2]$ because the integrand $\frac{y_2 - \Delta - r_1}{x_2 - \Delta - r_1}$ is positive and decrease with p_2. If $p_2 = 1 - x_2$, $p_2 < 1 - x_2 + t_2$, but when $p_2 = \frac{1}{2}$, then $p_2 > 1 - x_2 + t_2$.

Therefore, for $p_1 \in [0, x_2]$, (4.11) has a unique solution for $p_2 \in [0, x_2]$, say $p_2 = b_2(p_1)$, and $b_2(p_1) \in \left[1 - x_2, \frac{1}{2}\right]$.

Next from (4.7), we have that $b_1^/(p_2) = \frac{1}{2}\left[1 - \frac{p_2 - r_2}{1 - r_2}\right] \in \left[0, \frac{1}{2}\right]$, ands how $b_1(p_/) \in \left[1 - x_2, \frac{1}{2}\right]$, when $p_2 \in \left[1 - x_2, \frac{1}{2}\right]$.If $p_1 = 1 - x_2$, $p_1 < \frac{1 - x_2 + p_2}{2} + t_1$ but when $p_1 = \frac{1}{2}$, $p_1 > \frac{1 - x_2 + p_2}{2} + t_1$.

A solution to the pair of equation (4.7) and (4.11) involves $p_1^* = b_1(b_2(p_1^*))$, and by a fixed point argument, there exists such a $p_1^* \in \left[1 - x_2, \frac{1}{2}\right]$. Because $p_2^* = b_2(p_1^*) \in \left[1 - x_2, \frac{1}{2}\right]$, the pair of FOCs has at least one solution $(p_1^*, p_2^*) \in \left[1 - x_2, \frac{1}{2}\right]^2$.

Next consider the uniqueness. Substituting $r_1 = e_1(p_2)$, $r_2 = e_2(p_2)$, $p_1 = b_1(p_2)$ into (4.11), we have

$$p_2^* = 1 - x_2\left(e_2(p_2^*)\right) + \int_{p_2^*}^{x_2(e_2(p_2^*))} \frac{y_2 - p_2^* + b_1(p_2^*) - e_1(p_2^*)}{x_2 - p_2^* + b_1(p_2^*) - e_1(p_2^*)} dy_2 . \qquad \text{(A.1)}$$

Because $b_{1,p_2} \in \left[0, \frac{1}{2}\right]$, $b_1(p_2^*) - p_2^*$ decreases with p_2^*. Moreover, $e_{1,p_2^*} > 0$ and e_2 is independent of p_2^*. These imply the RHS of (A.1) decreases with p_2^*. Therefore, the solution is unique. ***Q.E.D.***

Proof of Lemma 2. To check whether the profit functions are globally concave with the product lines, we assume the condition of marginal costs

$$c_{r_i} = \frac{r_i}{1-r_i}. \tag{A.2}$$

This cost function ensures the globally concave of π_i in $r_i \in [0,1]$ and ensure $r_1^* < 1$ (because of $p_i^* \in \left(1-x_2, \frac{1}{2}\right)$ and $q_1 \in [0,1]$). For most firms' FOCs, $\pi_{r_i} = p_i q_{i,r_i} - c_{r_i}$, where $q_{i,r_i} = \frac{q_i}{1-r_i}$,

$$r_i = q_i p_i$$

Then, the SOCs, $\pi_{r_i,r_i} = p_i q_{r_i,r_i} - c_{r_i,r_i} < 0$, where $q_{r_i,r_i} = 2\frac{q_i\,(r_i)}{(1-r_i)^2}$

$$2q_i p_i < 1 \tag{A.3}$$

First, we fix $r_2 \in [0,1], (p_1, p_2) \in \left[1-x_2, \frac{1}{2}\right]^2$, and consider the equation (4.8). Here q_1 is a increasing function of r_1 for $r_1 \in [0,1]$. If $r_1 = \frac{1}{2}, r_1 > p_1 q_1$ but when $r_1 = 0$, we have $r_1 < p_1 q_1$.Therefore, (4.8) has a unique solution for $r_1 \in [0,1]$, say $r_1 = e_1(r_2)$, and $e_1(r_2) \in \left[0, \frac{1}{2}\right]$.

Next from (4.12), one can check that $e_{2,r_1} < 0$. We show that $e_2(r_1) \in \left[0, \frac{1}{2}\right]$, when $r_1 \in \left[0, \frac{1}{2}\right]$. (i) If $r_2 = \frac{1}{2}$, $r_2 > p_2 q_2$. (ii) If $r_2 = 0$, $r_2 \leq p_2 q_2$.

A solution to the pair of equation (4.8) and (4.12) involves $r_1^* = e_1(e_2(r_1^*))$, and by a fixed point argument, there exists such a $r_1^* \in \left[0, \frac{1}{2}\right]$. Because $r_2^* = e_2(r_1^*) \in \left[0, \frac{1}{2}\right]$, the pair of FOCs has at least one solution $(r_1^*, r_2^*) \in \left[0, \frac{1}{2}\right]^2$.

Next consider the uniqueness. Substituting $r_2 = e_2(r_1)$, $p_1 = b_1(r_1)$, $p_2 = b_2(r_1)$ into (4.8),

$$r_1^* = b_1(r_1^*) \cdot q_1\left(r_1^*, e_2(r_1^*), b_1(r_1^*), b_2(r_1^*)\right). \tag{A.4}$$

Since $q_{1,r_1} > 0$, it is necessary to show that e_2, b_1, and b_2 are all decrease with r_1. First the b_1 is independent of r_1. One can see that $e_{1,b_2} \cdot b_{2,r_1^*} < 0$ (because $b_{2,r_1^*} < 0$ and $e_{1,b_2} > 0$). Therefore, the solution is unique. ***Q.E.D.***

Proof of Proposition 3. Here we show $p_1^* \le p_2^*$. Recall that

We define $\Delta = p_2 - p_1 = \frac{1}{2}\left[\int_{p_2}^{x_2} \frac{y_2 - \Delta - r_1}{x_2 - \Delta - r_1} dy_2 - \int_{p_2}^{x_2} \frac{y_1 - r_2}{1 - r_2} dy_1\right]$.From $\frac{x_2 - \Delta - r_1}{1 - r_1} < 1$, then

$$\Delta > \frac{1}{2}\left[\int_{p_2}^{x_2} \frac{y_2 - \Delta - r_1}{1 - r_1} dy_2 - \int_{p_2}^{x_2} \frac{y_1 - r_2}{1 - r_2} dy_1\right] \tag{A.5}$$

Because (A.5) states the LHS > RHS and the RHS has the same sign as $p_1 - p_2$, implies the term $p_1 - p_2$ must be negative.

Next consider $r_1^* \ge r_2^*$. The product lines equilibrium is $r_i^* = p_i q_i$. When $s \approx 0$, then $q_1^* = q_2^*$, $p_1^* = p_2^*$, and hence $r_1^* = r_2^* = r_F$. In the Proposition 5, we show that $r_{1,s}^* > 0$ and $r_{2,s}^* < 0$. Therefore, we claim $r_1^* > r_F > r_2^*$ for $s \in (0, 0.125]$. ***Q.E.D.***

Proof of Proposition 4.(i) $p_{1,s}^* > 0$. Write

$$p_{1,s}^* = \left(q_{1,s}^* + q_{1,p_2^*}^* \cdot p_{2,s}^* + q_{1,r_2^*}^* \cdot r_{2,s}^*\right)(1 - r_1) \tag{A.6}$$

First, $q^*_{1,s} = -\frac{1-x_2}{1-r_2} \cdot \frac{x_{2,s}}{1-r_1} > 0$. Next $q^*_{1,p_2} = \frac{1-p_2}{1-r_2} \cdot \frac{1}{1-r_1} > 0$. Finally, $q^*_{1,r_2^*} \cdot r^*_{2,s} = \left[-\frac{(1-x_2)\cdot x_{2,r_2}}{(1-r_1)(1-r_2)} - \int_{p_2}^{x_2} \frac{1-y_1}{\left((1-r_1)(1-r_2)\right)^2} dy_1 \right] \cdot r^*_{2,s} > 0$

(ii) $p^*_{2,s} > 0$. Write

$$p^*_{2,s} = b_{2,s} + b_{2,p_1^*} \cdot p^*_{1,s} + b_{2,r_1^*} \cdot r^*_{1,s} + b_{2,r_2^*} \cdot r^*_{2,s} \qquad \text{(A.7)}$$

First $b_{2,s} = -x_{2,s} \cdot \int_{p_2^*}^{x_2} \frac{y_2 - p_2^* + p_1^* - r_1}{\left(x_2 - p_2^* + p_1^* - r_1\right)^2} dy_2 > 0$. Next,

$b_{2,p_1^*} \cdot p^*_{1,s} + b_{2,r_1^*} \cdot r^*_{1,s} = \left(p^*_{1,s} - r^*_{1,s}\right) \int_{p_2^*}^{x_2} \frac{x_2 - y_2}{\left(x_2 - p_2^* + p_1^* - r_1\right)^2} dy_2 > 0$ because $p^*_{1,s} > r^*_{1,s} > 0$.

Finally, $x_{2,r_2} > 0$ and $r^*_{2,s} < 0$ make $b_{2,r_2^*} \cdot r^*_{2,s} = -x_{2,r_2} \int_{p_2^*}^{x_2} \frac{y_2 - p_2^* + p_1^* - r_1}{\left(x_2 - p_2^* + p_1^* - r_1\right)^2} dy_2 \cdot r^*_{2,s} > 0$.

Q.E.D.

Proof of Proposition 5.(i) $r^*_{1,s} > 0$. Write

$$r^*_{1,s} = \underset{(+)}{e_{1,s}} + \left[\underset{(-)}{e_{1,r_2^*}} \cdot \underset{(-)}{r^*_{2,s}}\right] + \left[\underset{(0)}{e_{1,p_1^*}} \cdot \underset{(+)}{p^*_{1,s}}\right] + \left[\underset{(+)}{e_{1,p_2^*}} \cdot \underset{(+)}{p^*_{2,s}}\right] \qquad \text{(A.8)}$$

, where $e_{1,s} = p_1^* \cdot q_{1,s}$, $e_{1,r_2} = q_1 p^*_{1,r_2} + p_1^* \cdot q_{1,r_2}$, $e_{1,p_1} = q_1 + p_1^* \cdot q_{1,p_1}$, and $e_{1,p_2} = q_1 p^*_{1,p_2} + p_1^* \cdot q_{1,p_2}$.

First $q_{1,s} = -\frac{1-x_2}{1-r_2} \cdot \frac{x_{2,s}}{1-r_1} > 0$ makes $e_{1,s} > 0$. Next $p^*_{1,r_2} < 0$ and $q_{1,r_2} < 0$ make $e_{1,r_2} < 0$. Finally, the $p^*_{1,p_2} > 0$ and $q_{1,p_2} > 0$ cause $e_{1,p_2} > 0$.

(ii) $r^*_{2,s} < 0$. Write

$$r^*_{2,s} = \underset{(-)}{e_{2,s}} + \left[\underset{(-)}{e_{2,r_1^*}} \cdot \underset{(+)}{r^*_{1,s}}\right] + \left[\underset{(+)}{e_{2,p_1^*}} \cdot \underset{(+)}{p^*_{1,s}}\right] + \left[\underset{(-)}{e_{2,p_2^*}} \cdot \underset{(+)}{p^*_{2,s}}\right] \qquad \text{(A.9)}$$

, where $e_{2,s} = p_2^* \cdot q_{2,s}$, $e_{2,r_1} = q_2 p^*_{2,r_1} + p_2^* \cdot q_{2,r_1}$, $e_{2,p_2} = q_2 + p_2^* \cdot q_{2,p_2}$, and $e_{2,p_1} = q_2 p^*_{2,p_1} + p_2^* \cdot q_{2,p_1}$.

First $q_{2,s} = \frac{1-x_2}{(1-r_1)(1-r_2)} \cdot x_{2,s} < 0$ makes $e_{2,s} < 0$. Next $p_2^* = -\frac{q_2}{q_{2,p}}$, and $q_{2,p_2} < q_{2,p} < 0$ causes $e_{2,p_2^*} < 0$. The $p_{2,r_1}^* < 0$ and $q_{2,r_1} < 0$ cause $e_{2,r_1} < 0$. Finally, the only positive force $e_{2,p_1} > 0$ (because $p_{2,p_1}^* > 0$ and $q_{2,p_1} > 0$). However, it is numerically smaller than other negative effects. ***Q.E.D.***

Proof of Proposition 6.(i) Write π_1 and π_2 for the respective equilibrium profit of the prominent firm, and of non-prominent firm. Then

$$\pi_1 > p_2\left(1-\frac{x_2-r_1}{1-r_1}+\int_{p_2}^{x_2}\frac{y_1-r_2}{1-r_2}\cdot\frac{1}{1-r_1}dy_1\right)$$

$$> p_2\left[h_2\left(1-\frac{x_2-r_2}{1-r_2}\right)+\int_{p_2}^{x_2}\frac{y_2-\Delta-r_1}{1-r_1}\cdot\frac{1}{1-r_2}dy_2\right]=\pi_2 .$$

The first inequality holds because the prominent firm makes less profit if it deviates from p_1 to p_2 . The second inequality holds because $h_2 < 1$, $r_1 > r_2$, and $\int_{p_2}^{x_2}\frac{y_1-r_2}{1-r_2}\cdot\frac{1}{1-r_1}dy_1 > \int_{p_2}^{x_2}\frac{y_2-\Delta-r_1}{1-r_1}\cdot\frac{1}{1-r_2}dy_2$.

(ii) In the equilibrium, the firm's product line equals to its own revenue, $p_i q_i = r_i$. The total cost can be derived by integrating marginal cost from 0 to r_i , $c_i = \int_0^{r_i} c_{i,r_i} dr_i$. Therefore, we can write equilibrium profits as

$$\pi_i = r_i - \int_0^{r_i} c_{i,r_i} dr_i = r_i - \left(\log\frac{1}{1-r_i} - r_i\right).$$

The firm's profit increases with its product line, $\pi_{i,r_i} = 2 - \frac{1}{1-r_i} > 0$ because $2 > \frac{1}{1-r_i}$ caused by $r_i < \frac{1}{2}$. In the Proposition 4, we show $r_{1,s}^* > 0$, while $r_{2,s}^* < 0$. Then, $\pi_{1,s} = \pi_{1,r_1^*} \cdot r_{1,s}^* > 0$, and $\pi_{2,s} = \pi_{2,r_2^*} \cdot r_{2,s}^* < 0$. ***Q.E.D.***

Proof of Lemma 7. By assumption, $x_i \geq \frac{1}{2}$. Fix $p_j \in [0, x_j]$, and $r_i < p_i$, consider equation (5.2). The RHS is non-negative and it can increase or decrease with p_i depends on of λ_i (because of both q_i and h_i decrease with p_i). (i) If $p_i = 1 - x_i$, $p_i < \frac{q_i(1-r_i)}{\lambda_i(1-h_i)-h_i}$. (ii) If $p_i = \frac{1}{2}$, $p_i > \frac{q_i(1-r_i)}{\lambda_i(1-h_i)-h_i}$. Therefore, expression (5.2) has a unique solution for $p_i \in [0, x_i]$, say $p_i = b_i(p_j)$, and $b_i(p_j) \in \left[1 - x_i, \frac{1}{2}\right]$.

A solution to the pair of equation (5.2) involves $p_i^* = b_i(b_j(p_i^*))$, and by a fixed point argument, there exists such a $p_i^* \in \left(1 - x_j, \frac{1}{2}\right)$. Because $p_j^* = b_j(p_i^*) \in \left(1 - x_i, \frac{1}{2}\right)$, the pair of FOCs has at least one solution $p_i^* \in \left(1 - x_j, \frac{1}{2}\right)$.

Next consider the uniqueness. Substituting $r_i = e_i(p_i)$, $r_j = e_j(p_i)$, $p_j = b_j(p_i)$ into $b_i(r_i, r_j, p_j)$, we have

$$p_i^* = q_i^*\left(e_i(p_i^*), e_j(p_i^*), b_j(p_i^*)\right) \cdot \frac{1 - e_i(p_i^*)}{\lambda_i\left(e_i(p_i^*), e_j(p_i^*)\right) \cdot (1 - h_i) + h_i} \qquad \text{(A.18)}$$

The RHS of (A.18) is positive. One can check that $b_{i,p_i^*} < 0$. Because $b_{i,e_i} < 0$ and $e_{i,p_i^*} > 0$, $b_{i,e_i} \cdot e_{i,p_i^*} < 0$. There is a definite positive values of $b_{i,b_j} \cdot b_{j,p_i^*}$. The term $b_{i,e_j} \cdot e_{j,p_i^*}$ can be positive or negative. ***Q.E.D.***

Proof of Lemma 8. By assumption, $x_i \geq \frac{1}{2}$. Fix $r_j \in [0,1]$, $p_i \in \left(1 - x_i, \frac{1}{2}\right)$, and consider equation (5.3). For $r_i \in [0,1]$, the RHS is an increasing function of r_i,

$$e_{i,r_i} = \frac{p_i}{\mu_i}\left\{\left(\frac{r_i^2 + r_j}{r_i + r_j^2}\right) q_{i,r_i} + q_i\left(\frac{r_i^2 + 2r_i r_j^2 - r_j}{r_i^2 + 2r_i r_j^2 - r_j^4}\right) - \left(\frac{1 - r_i}{r_i}\right) q_{i,r_i}^{ji} - \frac{q_i^{ji}}{r_i^2}\right\} > 0.$$

The values of e_{i,r_i} can be greater than 1 if the term in curly blanket is greater than $\frac{p_i}{\mu_i}$. Thus, to bring the equilibrium in range, we add the assumption on *a* ,the lower bound of μ_1. The *a* cannot have very low value, or the technologies of firms do not much far apart. (i)

If $r_i = \frac{1}{2}$, $\frac{1}{2} > \frac{p_i}{\mu_i}\left[q_i\left(\frac{r_i^2 + r_j}{r_i + r_j^2}\right) - q_i^{ji}\left(\frac{1-r_i}{r_i}\right)\right]$. (ii) If $r_i = 1 - x_i$, $r_i < \frac{p_i}{\mu_i}\left[q_i\left(\frac{r_i^2 + r_j}{r_i + r_j^2}\right) - q_i^{ji}\left(\frac{1-r_i}{r_i}\right)\right]$.

A solution to the pair of equation (5.3) involves $r_i^* = e_i\left(e_j(r_i^*)\right)$, and by a fixed point argument, there exists such a $r_i^* \in \left[1 - x_i, \frac{1}{2}\right]$. Because $r_j^* = e_j(r_i^*) \in \left[1 - x_j, \frac{1}{2}\right]$, the pair of FOCs has at least one solution $(r_i^*, r_j^*) \in \left[1 - x_i, \frac{1}{2}\right]^2$.

Next consider the uniqueness. Substituting $r_j = e_j(r_i)$, $p_i = b_i(r_i)$, $p_j = b_j(r_i)$ into $e_i(r_j, p_i, p_j)$, we have

$$r_i^* = \frac{b_i(r_i^*)}{\mu_i}\left[\begin{array}{c} q_i\left(e_j(r_i^*), b_i(r_i^*), b_j(r_i^*)\right)\left(\dfrac{r_i^{*2} + e_j(r_i^*)}{r_i^* + e_j(r_i^*)^2}\right) \\ - q_i^{ji}\left(e_j(r_i^*), b_i(r_i^*), b_j(r_i^*)\right)\left(\dfrac{1 - r_i^*}{r_i^*}\right) \end{array}\right] \tag{A.19}$$

Since the RHS of (A.19) has positive values, and also increases with r_i^*, its slope must be less than one. Formally,

$$r^*_{i,r_i^*} = e_{i,r_i^*} + e_{i,e_j} \cdot e_{j,r_i^*} + e_{i,b_i} \cdot b_{i,r_i^*} + e_{i,b_j} \cdot b_{j,r_i^*} < 1 \tag{A.20}$$

All of four terms are positive. These make the proof of the uniqueness in (A.20) harder. Then, we employ the numerical values to solve for r_i^*, p_i^*. Next, they are assigned to the RHS of (A.20)at all levels of *s*and $\mu_1 \in [0.75, 1)$. The result confirms the unique solution ***Q.E.D.***

Proof of Proposition 9. Define the probability that firm i is visited first as λ_i. Thus $\lambda_i = \frac{r_i}{r_i + r_j}$. After setting $p = p_i$, the FOC in prices can be written as

$$v_1(\lambda_1, p_1, p_2) = q_1 + p_1 q_{1,p} = 0$$

$$= \lambda_1 \left[1 - \frac{x_2 - p_2 + 2p_1 - r_1}{1 - r_1} + \int_{p_2}^{x_2} \frac{y_1 - r_2}{1 - r_2} \frac{1}{1 - r_1} dy_1 \right]$$

$$+ (1 - \lambda_1) \left[\frac{x_1 + p_2 - p_1 - r_2}{1 - r_2} \left[1 - \frac{x_1 - r_1 + p_1}{1 - r_1} \right] + \int_{p_1}^{x_1} \frac{y_1 + p_2 - p_1 - r_2}{1 - r_2} \frac{1}{1 - r_1} dy_1 \right]$$

The FOC for the less-salient firm yields a similar condition:

$$v_2(1 - \lambda_1, p_2, p_1) = q_2 + p_2 q_{2,p} = 0$$

$$= (1 - \lambda_1) \left[1 - \frac{x_1 - p_1 + 2p_2 - r_2}{1 - r_2} + \int_{p_1}^{x_1} \frac{y_2 - r_1}{1 - r_1} \frac{1}{1 - r_2} dy_2 \right]$$

$$+ \lambda_1 \left[\frac{x_2 + p_1 - p_2 - r_1}{1 - r_1} \left[1 - \frac{x_2 - r_2 + p_2}{1 - r_2} \right] + \int_{p_2}^{x_2} \frac{y_2 + p_1 - p_2 - r_1}{1 - r_1} \frac{1}{1 - r_2} dy_2 \right]$$

This implies the equilibrium also requires $v_1(\lambda_1, p_1^*, p_2^*) = v_2(1 - \lambda_1, p_2^*, p_1^*)$. Rewriting to move the term λ_1 to the LHS, we have

$$2\lambda_1 \left\{ \begin{array}{l} 2 - \frac{x_2 - p_2 + 2p_1 - r_1}{1 - r_1} - \frac{x_1 - p_1 + 2p_2 - r_2}{1 - r_2} + \int_{p_2}^{x_2} \frac{y_1 - r_2}{1 - r_2} \frac{1}{1 - r_1} dy_1 + \int_{p_1}^{x_1} \frac{y_2 - r_1}{1 - r_1} \frac{1}{1 - r_2} dy_2 \\ - \frac{x_2 + p_1 - p_2 - r_1}{1 - r_1} \left[1 - \frac{x_2 - r_2 + p_2}{1 - r_2} \right] - \int_{p_2}^{x_2} \frac{y_2 + p_1 - p_2 - r_1}{1 - r_1} \frac{1}{1 - r_2} dy_2 \\ - \frac{x_1 + p_2 - p_1 - r_2}{1 - r_2} \left[1 - \frac{x_1 - r_1 + p_1}{1 - r_1} \right] - \int_{p_1}^{x_1} \frac{y_1 + p_2 - p_1 - r_2}{1 - r_2} \frac{1}{1 - r_1} dy_1 \end{array} \right\}$$

$$= 2 \left\{ \begin{array}{l} 1 - \frac{x_1 - p_1 + 2p_2 - r_2}{1 - r_2} + \int_{p_1}^{x_1} \frac{y_2 - r_1}{1 - r_1} \frac{1}{1 - r_2} dy_2 \\ - \frac{x_1 + p_2 - p_1 - r_2}{1 - r_2} \left[1 - \frac{x_1 - r_1 + p_1}{1 - r_1} \right] - \int_{p_1}^{x_1} \frac{y_1 + p_2 - p_1 - r_2}{1 - r_2} \frac{1}{1 - r_1} dy_1 \end{array} \right\}$$

Let simply denote *B* for RHS and *A* for the curly term in LHS and we will get $2\lambda_1 = \frac{B}{A}$. Because $r_1 > r_2$ means $2\lambda_1 > 1$, the term $\frac{B}{A} > 1$ is also hold. Next, we show that, with $p_2 > p_1$, the following inequality cannot be satisfied.

$$B > A \tag{A.21}$$

(i) When $p_1 = p_2$, the expression (A.21)cannot be satisfied. Define $p_1 = p_2 = p_0$. Form (A.21), substituting p_0 into *A* and *B* and rearrange the terms, we have $B > A$ in the following expression

$$p_0\left(r_1 - r_2\right) > x_1\left(2 - x_1 - 3p_0\right) - x_2\left(2 - x_2 - 3p_0\right) \\ + r_1\left(2 - p_0 - 2x_2\right) - r_2\left(2 - p_0 - 2x_1\right)$$

Because $\frac{x_1}{x_2} > \frac{\left(2 - x_2 - 3p_0\right)}{\left(2 - x_1 - 3p_0\right)} > 1$ and $\frac{r_1}{r_2} > 1 > \frac{\left(2 - 2p_0 - 2x_1\right)}{\left(2 - 2p_0 - 2x_2\right)}$, the expression cannot be satisfied.

(ii) When $p_1 < p_2$, the expression (A.21) cannot be satisfied. Rearrange the expression (A.22) to get $B = 0$. Then,

$$A = -p_1\left(5 - \frac{x_1}{2} - \frac{7x_2}{2} - 2p_1\right) + p_2\left(5 - \frac{x_2}{2} - \frac{7x_1}{2} - 2p_2\right) \\ + x_1\left(2 - x_1\right) - x_2\left(2 - x_2\right) + r_1\left(2 - 2p_2 - 2x_2\right) - r_2\left(2 - 2p_1 - 2x_1\right)$$

Because of $\frac{p_2}{p_1} > \frac{\left(5 - \frac{x_1}{2} - \frac{7x_2}{2} - 2p_1\right)}{\left(5 - \frac{x_2}{2} - \frac{7x_1}{2} - 2p_2\right)} > 1$, $\frac{x_1}{x_2} > \frac{\left(2 - x_2\right)}{\left(2 - x_1\right)} > 1$ and $\frac{r_1}{r_2} > 1 > \frac{\left(2 - 2p_1 - 2x_1\right)}{\left(2 - 2p_2 - 2x_2\right)}$, the term *A* is definite positive. The inequality $0 = B > A$ cannot be satisfied. ***Q.E.D.***

Proof of Proposition 10. The best response function of in the product line level is

$$\mu_i = \frac{p_i}{r_i}\left[q_i\left(\frac{r_i^2 + r_j}{r_i + r_j^2}\right) - q_i^{ji}\left(\frac{1 - r_i}{r_i}\right)\right]. \tag{A.22}$$

The firm 1 is created to has more efficient firm, so $\mu_1 < \mu_2 = 1$. We then require that the RHS of (A.22) for firm 1 is smaller than that for firm 2. It can be shown that the inequality simplifies to

$$\frac{p_1}{r_1} < \frac{p_2}{r_2} \tag{A.23}$$

Suppose that $r_1 < r_2$. This, by Proposition 10, necessary implies $p_1 < p_2$, hence $p_1 r_2 > p_2 r_1$. But this contradicts with (A.23), thus Proposition 11 is established. ***Q.E.D.***

Proof of Proposition 11. To verify $r_{i,s}^* > 0$, we generate

$$r_{i,s}^* = e_{i,s} + \left[e_{i,r_j^*} \cdot r_{j,s}^* \right] + \left[e_{i,p_i^*} \cdot p_{i,s}^* \right] + \left[e_{i,p_j^*} \cdot p_{j,s}^* \right] \tag{A.24}$$

First we show $e_{1,s} > 0 > e_{2,s}$ by

$$e_{i,s} = \frac{p_i}{\mu_i} \left[q_{i,s} + \lambda_j \left(\frac{1 - r_i^*}{\sum r_i^*} \right) \left(- \frac{(1 - x_j) x_{j,s} - (1 - x_i) x_{i,s}}{(1 - r_j)(1 - r_i)} \right) \right] \tag{A.25}$$

Because $q_{i,s} = \lambda_i \left(\frac{1}{1 - r_i} \right) - \lambda_j \left(\frac{1}{1 - r_j} \right)$, $q_{1,s} > 0 > q_{2,s}$. The last term in square blanket is positive for firm1 but negative for firm 2 because $x_1 > x_2$, and $x_{2,s} < x_{1,s} < 0$.

Next, we show $e_{i,r_j^*} < 0$ and $e_{2,r_1} < e_{1,r_2} < 0$.

$$e_{i,r_j} = \frac{p_i}{\mu_i} \left[\begin{array}{c} q_{i,r_j} + (1 - r_i^*) \left(\frac{r_i^2 - r_j^2}{\left(\sum r_i^* \right)^4} \right) \left(FD_i + RD_i - WD_i \right) \\ + \left(\frac{1 - r_i^*}{\sum r_i^*} \right) \lambda_j \left(FD_{i,r_j} + RD_{i,r_j} - WD_{i,r_j} \right) \end{array} \right], \tag{A.26}$$

where $q_{i,r_j} = -\frac{q_i - WD_i}{\sum r_i} + \lambda_i \left(FD_{i,r_j} + RD_{i,r_j} \right) + \lambda_j \left(WD_{i,r_j} \right) < 0$, $FD_{i,r_j} = -\frac{x_{j,r_j}}{1 - r_i}$,

$WD_{i,r_j} = -\frac{1 - x_i}{1 - r_j} \cdot \frac{h_{i,r_j}}{1 - r_i} + \int_{p_i}^{x_i} \frac{y_i - p_i + p_j - 1}{(1 - r_j)^2} \cdot \frac{1}{1 - r_i} dy_i$, and

$RD_{i,r_j} = \frac{x_j - r_j}{1 - r_j} \cdot \frac{x_{j,r_j}}{1 - r_i} + \int_{p_j}^{x_j} \frac{y_i - 1}{(1 - r_j)^2} \cdot \frac{1}{1 - r_i} dy_i$.

The effects come from guarantee effect (the first term in the square blanket) and attention effect (the last two terms in the square blanket).The guarantee effect exhibits $q_{2,r_1} < q_{1,r_2} < 0$. The attention effect is also negative for both firms and the impact for firm 2 is stronger (firm 2 has less positive second term and more negative third term).

For $e_{i,p_i^*} < 0$ and $e_{2,p_2^*} < e_{1,p_1^*} < 0$, write

$$e_{i,p_i} = \frac{1}{\mu_i}\left[q_i + \left(\frac{1-r_i^*}{\sum r_i^*}\right)\lambda_j\left(FD_i + RD_i - WD_i\right)\right] + \frac{p_i}{\mu_i}\left[q_{i,p_i} + \left(\frac{1-r_i^*}{\sum r_i^*}\right)\lambda_j\left(\frac{-p_i + p_j}{(1-r_j)(1-r_i)}\right)\right], \tag{A.27}$$

where

$$q_{i,p_i} = -\frac{\left(\lambda_i + \lambda_j h_i\right)}{1-r_i} - \frac{\lambda_j(1-x_i)}{(1-r_i)(1-r_j)} < 0 .$$

Because of $p_i = q_i \dfrac{1-r_i}{\lambda_i + \lambda_j h_i}$, the expression (A.28) can be rewritten as

$$e_{i,p_i} = \frac{\lambda_j}{\mu_i}\left[\left(\frac{1-r_i^*}{\sum r_i^*}\right)\left(FD_i + RD_i - WD_i\right) - \frac{p_i}{(1-r_i)(1-r_j)}\left(1 - x_i + \frac{(1-r_i^*)}{\sum r_i^*}\cdot(p_j - p_i)\right)\right].$$

The guarantee effect is positive, $\left(\dfrac{1-r_i^*}{\sum r_i^*}\right)\lambda_j\left(FD_i + RD_i - WD_i\right) > 0$, and strong in firm 1. The attention effect is negative for firm 2 but positive for firm 1 because $p_1 > p_2$.

The last effect is $e_{i,p_j^*} > 0$ and $e_{1,p_2^*} > e_{2,p_1^*} > 0$

$$e_{i,p_j} = \frac{p_i}{\mu_i}\left[q_{i,p_j} + \lambda_j\left(\frac{1-r_i^*}{\sum r_i^*}\right)\left(\frac{-p_j + p_i}{(1-r_i)(1-r_j)}\right)\right], \tag{A.29}$$

where $q_{i,p_j} = \dfrac{\lambda_i(1-p_j)}{(1-r_i)(1-r_j)} + \dfrac{\lambda_j(1-p_i)}{(1-r_i)(1-r_j)}$.

The guarantee effect $q_{i,p_j} > 0$. Furthermore, $q_{1,p_2} > q_{2,p_1}$ because $\lambda_1 > \lambda_2$ and $p_1 > p_2$. The attention effect (the last term) is positive for firm1 and negative for firm 2 because $p_1 > p_2$. Finally, $e_{1,p_2^*} > e_{2,p_1^*} > 0$ since $p_1 > p_2$ and $\mu_1 < \mu_2 = 1$. ***Q.E.D.***

Proof of Proposition 12. (i) Social prices are lower than the market determined prices, $p_i^* > p_i^s$.

Recall the price when firm *i* maximizes profit

$$p_i^* = \frac{q_i}{q_{i,p_i}} \tag{A.30}$$

We show that, $p_i^* - p_i^s > 0$. From (A.30)and (5.6), we have

$$-p_j q_{j,p_i} - CS_{p_i} > 0 \tag{A.31}$$

Substituting q_{j,p_i} and CS_{p_i} in (A.31) and rearranging the term to

$$\lambda_i\left(\frac{1}{2} - p_i + \frac{p_j^2}{2} + s\right) + \lambda_j\left(\frac{1}{2} - p_i + \frac{p_i^2}{2} + \frac{(x_i - x_j)^2}{2} + s\right) > 0$$

(ii) Social product lines are lower than the market determined, $r_i^* > r_i^s$.Recall the product line when firm *i* maximizes profit

$$r_i^* = \left(p_i q_{i,r_i}\right)\frac{(1-r_i)}{\mu_i} \tag{A.32}$$

We show that, $r_i^* - r_i^s > 0$. From (5.7) and (A.32),

$$\left\{\begin{array}{l} -\lambda_{i,r_i}\left(p_j q_{j,\lambda_i} - p_j q_{j,\lambda_j} + CS_{\lambda_i} - CS_{\lambda_j}\right) \\ \qquad -x_{i,r_i}\left(p_j q_{j,x_i} + CS_{x_i}\right) - p_j q_{j,F(y_i)} \cdot F(y_i)_{r_i} - CS_{f(y_i)} \cdot f(y_i)_{r_i} \\ \qquad\qquad + CS^{ij}_{\varnothing,r_i} \end{array}\right\} > 0$$

, where $x_{i,r_i} = \dfrac{s}{1-x_i}$, $q_{j,x_i} = -\lambda_j\left(\dfrac{1-x_i}{(1-r_j)(1-r_i)}\right)$,

$CS_{x_i} = -\lambda_j\left(\dfrac{s}{1-r_j} - \dfrac{(x_i-1)^2}{2(1-r_j)(1-r_i)}\right) = 0$,

$CS_{f(y_i)} \cdot f(y_i)_{r_i} = \dfrac{\lambda_i\left(CS_i^i + CS_i^{ij}\right) + \lambda_j CS_i^{ji}}{1-r_i}$, and

$q_{j,F(y_i)} \cdot F(y_i)_{r_i} = \dfrac{\lambda_j}{1-r_i}\left[\dfrac{x_i - p_i}{1-r_j} - \dfrac{q_j^{ji}}{\lambda_j}\right] + \dfrac{\lambda_i}{1-r_i}\left[\dfrac{1-p_i}{1-r_j} - \dfrac{q_j^{ij}}{\lambda_i}\right]$.

Rearranging the term to get

$$-\frac{\lambda_j}{r_i + r_j}\left(p_j \frac{q_j^{ij}}{\lambda_i} - p_j \frac{q_j^j + q_j^{ji}}{\lambda_j} + CS_i^i + CS_i^{ij} + CS_j^{ij} - CS_j^j - CS_j^{ji} - CS_i^{ji}\right)$$
$$-\frac{p_j}{1-r_i}\left\{\lambda_j\left[\frac{x_i - p_i + s}{1-r_j} - \frac{q_j^{ji}}{\lambda_j}\right] + \lambda_i\left[\frac{1-p_i}{1-r_j} - \frac{q_j^{ij}}{\lambda_i}\right]\right\}$$
$$-\frac{\lambda_i\left(CS_i^i + CS_i^{ij}\right) + \lambda_j CS_i^{ji}}{1-r_i} + \frac{CS_\varnothing^{ij}}{1-r_i}$$

which is positive definite. ***Q.E.D.***