

Competition Strategy between Proprietary and Open Source Software in the Presence of Software Piracy and Network Externalities

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บทความนี้เสนอแบบจำลองเพื่ออธิบายพฤติกรรมการแข่งขันและคุณภาพตลาดระหว่างซอฟต์แวร์ระบบปิดและระบบเปิด ซึ่งคำนึงถึงผลกระทบของสินค้าละเมิดลิขสิทธิ์และผลกระทบภายนอกของเครือข่าย (network externalities) ในตลาดซอฟต์แวร์ โดยประยุกต์ใช้ทฤษฎีความแตกต่างของสินค้าในแนวราบแบบ Hotelling (theory of horizontal product differentiation) ผลการศึกษาจากแบบจำลองนี้พบว่ากรณีที่บริษัทผู้ผลิตซอฟต์แวร์ระบบปิดยอมให้มีซอฟต์แวร์ละเมิดลิขสิทธิ์ในตลาดเป็นหนึ่งในกลยุทธ์ที่สำคัญซึ่งส่งผลกระทบต่อโครงสร้างการแข่งขันในตลาด ภายใต้ระบบซอฟต์แวร์ที่แตกต่างกัน การให้บริการบริการหลังการขายที่มากขึ้นเป็นกลยุทธ์หนึ่งซึ่งช่วยในการแข่งขันกับซอฟต์แวร์ระบบเปิดและขจัดสินค้าละเมิดลิขสิทธิ์ออกจากตลาด และผู้ผลิตซอฟต์แวร์ระบบเปิดจะดำรงอยู่ในตลาดได้ก็ต่อเมื่อสามารถสร้างฐานผู้ใช้ที่ใหญ่พอหรือเสนอสินค้าที่มีประโยชน์สูงต่อผู้บริโภค นอกจากนี้ยังพบว่าผู้ผลิตซอฟต์แวร์ระบบปิดมีแนวโน้มที่จะป้องกันการละเมิดลิขสิทธิ์ หากผลกระทบภายนอกของเครือข่ายมีผลเพียงเล็กน้อยต่อผู้บริโภค แต่หาก

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ผลกระทบดังกล่าวมีผลมากต่อผู้บริโภคผู้ผลิตซอฟต์แวร์ระบบปิดจะยอมให้มีการละเมิดลิขสิทธิ์ซอฟต์แวร์ เพื่อสร้างฐานผู้ใช้และใช้ประโยชน์จากผลกระทบภายนอกของเครือข่ายในภายหลัง

ABSTRACT

This study attempts to explore the price strategic competition between the proprietary and the open source software as well as the market outcomes in the presence of both software piracy and network externalities. The theory of horizontal product differentiation Hotelling-like framework is employed as a backbone of the model. The results suggest that software piracy is an important strategy for the commercial software firm affecting the structure of competition in the market. Under perfectly incompatible systems, giving more benefit to users in forms of after sale services is a useful strategy to beat both the open source software and the software piracy out of the software industry. However, open source software can survive in the market if its user base is large enough or it gives a significant benefit to its users. Finally, proprietary firms intend to protect their software against piracy if the degree of network externalities is weak. Otherwise, it will allow software piracy in order to increase its user base and exploit the network externalities later on.

1. INTRODUCTION

In recent decades, a notable rise in the role of information and the knowledge-based economy has brought the world to a new age economy. Advancements in digital technology and the Internet are the key engines igniting speed of information flow globally and at the same time, have driven down the costs of data transfer and telecommunication. Software is one of the digital network goods contributing considerably towards employment creation and economic growth. However, it is highly subject to a high level of software piracy due to its negligible cost of reproduction and characteristics of public good. Thus, it could be cheaply reproduced with an endless number of exact replicas. The most obvious and rather intuitive complaint of the software industry is that software piracy is a source of huge losses to the industry and jeopardizes all stages

of innovation. Considerable efforts devote to combat software piracy in the form of international treaties, such as the TRIPS Agreement and WIPO Copyright Treaty.

According to a report by the Business Software Alliance (BSA, 2007)¹, in 2006, the world software piracy rate was constant at 35 percent for three consecutive years implying a core and persistent piracy problem that is hard to solve. Statistically, software piracy varies tremendously from country to country, being noticeably high in developing countries. The rates were varying from 22 percent in Northern America to 68 percent in Central/Eastern Europe, and, by country, the software piracy rate ranged from 21 percent in U.S.A. to 95 percent in Armenia. From these statistics, the BSA eventually concludes that to promote economic growth, it is necessary to limit or eradicate piracy.

Fortunately, a rise of the upcoming issue of open source software, with its peculiar characteristics, is assisting people all over the world escape the crime of software piracy. Originating in 1998, open source software is generally recognized as the operating system GNU/Linux and its applications whose source code is made publicly available. Users can freely inspect this source code, extend it, modify it, and use it as a model for future software development. Therefore, not only it offers abundant useful software, but also gives users freedom and alternative ways out of being dominated by potential monopolists like Microsoft.

Evidently, the competition between proprietary and open source software seems to have become more and more intense over time. According to periodic surveys of web server sites by Netcraft, they indicated that the market share of publicly observable Web servers running Apache rose from 31 percent in April 1996 to 44 percent in June 1997 and to 58.86 percent in April 2007. The recent survey, as of April 2007, shows that the market share of Apache was followed by Microsoft with 31.13 percent, Sun with 1.68 percent and Lighttpd with 1.22 percent². Moreover,

¹ BSA. "2002, 2006 Global Software Piracy Report," A study conducted by International Planning and Research Corporation for the Business Software Alliance at <http://www.bsa.org>. For more detail on how the data collected please visit http://www.bsa.org/idcstudy/pdfs/White_Paper.pdf.

² For more detail, please see www.netcraft.com/survey.

in the operating system market, the survey indicates that approximately 50 percent of the computers running the web are MICROSOFT's Windows based followed by the open source Linux operating system with about 30 percent market share.³

In the research arena, many have documented reasons explaining where contributors find their incentives to develop and improve software which is usually available for free. Among possible answers are career concerns and peer recognition (Lakhani and von Hippel, 2003; Weber, 2000; Dalle and Jullien, 2001; Lerner and Tirole, 2002). Meanwhile some stress the idea that open source is a form of gift economy based on altruistic motives (Raymond, 1999; Weber, 2000; Edwards, 2001; Kelty, 2001). Other questions are interested in the analysis of open source software as a complex public good (Bessen, 2002); competition between open source software and proprietary software products (Dalle and Jullien, 2001; Casadesus-Masanell and Ghemawat, 2006); and the influence of public licenses on open source software development (Weber, 2000; Dalle and Jullien, 2001).

Unfortunately, there exist relatively few research papers documenting upon the strategic competition between proprietary software and open source software. According to my knowledge, none has put forth on firms' behavior in the presence of both software piracy and network externalities despite the fact that they coexist and increasingly fight each other in the software market. This analysis aims to contribute to fill up this gap. In particular, the study attempts to explore the price strategic competition between the proprietary and the open source software as well as the market outcomes in the presence of both software piracy and network externalities. In particular, what are competitive strategies introduced to the market by the open source software? Can a software company survive in facing competition from comparable open source software and software piracy? Moreover, what effects does software piracy have upon both closed source and open source software firms?

The theory of horizontal product differentiation in Hotelling-like framework is employed as a backbone of the model in which the effect of network externalities is taking into

³ Ibid., footnote 1.

consideration. Indeed, the study attempts to examine the role of software piracy on firms' price strategic behaviors providing perfectly incompatible products as well as an analysis of the equilibriums in which government takes action in performing the intensity in monitoring software piracy. The results suggest that software piracy is an important strategy for commercial software firm affecting the structure of competition in the market which goes against complaints made by the commercial firms. Besides, they could use non-price strategy such as after sale services as a tool in fighting both piracy and open source software. Finally, proprietary firms intend to protect their software against piracy if the degree of network externalities is weak. Otherwise, it will allow software piracy in order to increase its user base and exploit the network externalities thereafter.

This paper is divided into five sections. Section 2 provides related literature reviews on the competition between the proprietary and the open source software as well as the issues of software piracy. Section 3 is the theoretical model of the competition between the proprietary and the open source software given the existence of software piracy and network externalities. Section 4 gives analyses of the equilibriums and the comparative statics. Then conclusions and limitations for further studies would be provided in the last section.

2. LITERATURE REVIEWS

Since the early era, economic literature on open source has mainly focused on explaining its characteristics as well as elucidating contributors' incentives to participate in the projects they will not be paid for. Theoretically, the plausible explanations are wide ranged from altruism (Weber, 2000; and Kelty, 2001), intellectual challenges (Lakhani and Wolf, 2003), signaling incentives including career concerns and peer recognition (Raymond, 1999; Weber, 2000; Lerner and Tirole, 2001, 2002; Johnson, 2002; Lakhani and von Hippel, 2003), to political motivations (Bessen, 2002). Empirically, reputation is by no means the most prominent motive for the open source movements (Gosh et al., 2002; Hars and Ou, 2002; Lakhani and Wolf, 2003; Hertel et al., 2003).

Later on, the issues regarding quality of open source software and competition between proprietary and open source systems have been other prominent angles of interests (Kuan, 2001; Johnson, 2001; Dalle and Jullien, 2001; Ghosh et al., 2002; Lerner and Tirole, 2005; Bonaccorsi and Rossi, 2003, 2006; Gutsche, 2005; Casadesus-Masanell and Ghemawat, 2006; Economides and Katsamakos, 2006). However, few studies have been conducted, both theoretically and empirically, on competition between closed source and open source software. Indeed, our understanding on this particular issue is still an infant. Sometimes, it cannot easily be explained by the traditional economic ways of thinking.

Regarding strategic competition, the rivalry between open source and proprietary software is similar to the battle between profit-maximizing and non-profit firms. Open source software has already shown its ability to develop advanced products capable of competing with the proprietary software (von Krogh and von Hippel, 2003; Mustonen, 2003). The competitiveness of open source software depends on efficiency level of organization, compatibility with the existing proprietary software, and the distribution of adopter preferences (Dalle and Jullien, 2001). Mustonen (2003) and Lin (2006) emphasize that a low implementation cost of an open source application is crucial for survival in the rivalry against a proprietary application.

Buxmann (2001) suggests that to enter a market with a user base of a competitor's system, a penetration pricing strategy is always necessary when two software vendors selling incompatible products in network market. The higher the network effects, the more necessary are low pricing strategies. By employing the optimal two-sided pricing strategy toward platform and application market, Economides and Katsamakos (2006a) find that pricing strategy introduced by proprietary platform firm may be lower than its marginal cost in some cases. In addition, it is likely that the proprietary software would dominate the market even when open source software is adopted for free.

Bonaccorsi and Rossi (2003) and Katsamakos and Xin (2005) find that the strength of the network effect and the cost for the applications are crucial in making decision on adoption

patterns. The higher the IT capabilities of a firm, the more it adopts open source software. In the presence of network effects, both types of software always coexist in the market. These results are consistent with evidences from the IT press and are confirmed by the study of Bonaccorsi, Giannangeli, and Rossi (2006). They explore the entry strategies by collecting and analyzing survey data on 146 Italian software firms. They conclude that rather than being transferred from proprietary to open source software industry, the hybrid business model with the existence of the two software systems is likely to be permanent features of new industry. However, in the long run, if these two kinds of software are perfectly incompatible, only one technology will be survived (Bonaccorsi and Rossi, 2003).

Casadesus-Masanell and Ghemawat (2006) also give similar results employing a dynamic model of vertically differentiated mixed-duopoly with demand-side learning. They indicate that the existence of open source software leads to an intense price competition and possible a lower value of the proprietary software for users as fewer complementary products provided for the proprietary system. The main conclusion is that as long as the proprietary firm pricing decision is not myopic, the competition would lead to either the coexistence of the two systems or open source provider being driven out of the market. However, the proprietary firm could be defeated if government mandates toward the open source software or if the cost of open source software is significantly lower than that of the proprietary software.

So far, relatively few researches on strategic competition between the two paradigms have been documented. According to my knowledge, none has put forth on firms' behavior in the presence of both software piracy and network externalities despite the fact that they coexist and increasingly fight each other in the software market. Since software piracy is perfectly substitute for proprietary software, proprietary firms might allow it to exist in order to enlarge its user base and exploit network externalities in fighting against open source software – turns a foe into friend fighting against a greater opponent.

Shapiro and Varian (1998) outline this resolution as a unique business strategy available only in network industries. With the sizable user base, software piracy could give monopoly a

power to deter any potential competitor. Through network effect mechanism, the increase in the size of the network increases each consumer's willingness-to-pay for the good, thereby increasing firm's profits. These have been supported originally by Conner and Rumelt (1991), Takeyama (1994), Slive and Bernhardt (1998), and Shy and Thisse (1999).

Conner and Rumelt (1991) and Takeyama (1994) have proposed an optimal strategy for a monopolist software publisher who faces a linear demand of heterogeneous consumers and network externalities. They suggest that the policy of minimum or no protection might be optimal if the network externality is strong and if copyright protection downsizes the installed base rather than induces a sufficiently high proportion of pirates to buy. The similar results are found in Givon, Mahajan and Muller (1995), and Gayer and Shy (2003). These results also hold true in duopoly framework introduced by Shy and Thisse (1999).

Contrarily, Poddar (2002) and King and Lampe (2003) suggest that firms would be better off protecting their software even in the presence of network effects. Similarly, Belleflamme (2003) suggests that with a limited strength of the network effects, piracy always reduces profits contradicting to those of Takeyama (1994) and Gayer and Shy (2003). These lead to address a question of whether piracy actually favors or jeopardizes the proprietary firm. Moreover, how the presence of software piracy affects the proprietary firm in facing both the open source counterpart and network externalities. Therefore, in this study, both software piracy and network effects are incorporated to explore the strategic competition between closed source and open source software.

Closely similar to Shy and Thisse (1999), the theory of horizontal product differentiation Hotelling-like framework is selected as the backbone of the model to investigate firms' strategic behaviors. This approach seems fitting for three reasons. Firstly, it has already been successfully employed in modeling competition between software packages (Shy and Thisse, 1999; Katsamakos and Xin, 2005; Gutsche, 2005; Lin, 2006). Secondly, since these two platforms are different with respect to several product characteristics with similar quality, horizontal differentiation as in the Hotelling model seems to fit in naturally. Finally, since it is based on

explicit micro foundations, network externalities could be introduced into consumer utility to generate a consistent set of demands.

Though, there are also dissimilarities to that of Shy and Thisse (1999). In their model, they analyze the war between two profit maximizing firms competing in high-demand market and other two pirate firms competing in low-demand market. In our model, the competition in high-demand market is between proprietary and open source software while the rivalry in low-demand market is between illegal proprietary product and open source software. Thus, the model in this paper can be considered as an attempt to extend the literature on competition between the two paradigms by explicitly introducing both software piracy and network externalities into the model. Given particular environment, this study aims to find under which conditions that make these two systems of computer software coexist and further investigates the effects that software piracy have on firm's strategic interaction.

3. THE MODEL

The study employs a simple static duopoly where suppliers of differentiated products compete in price. The differentiation between products is captured by the standard Hotelling "linear city" model. The model also incorporates network externalities into consideration. With network externalities, the value of product increases in number of people joins the same or compatible product network. Thus, given each product characteristics and prices, each consumer would prefer a product more widely used.

At an equilibrium, the two-stage game played by firms and consumers is employed. In the first stage, firms simultaneously choose their software prices under a Bertrand strategic competition. In the second stage, given a pair of prices, potential software users make their decisions. The solution of the game will be made by backward induction. The general setting of the model and assumptions are as follows.

3.1 Software Providers

Consider a software industry consisting of two software packages located at endpoints of a unit line as in a standard Hotelling-like framework. For an analytical purpose, assume that the closed source software is located at position 0 whereas the open source software is located at position 1. The unit interval $[0,1]$ has been taken as a product space in order to capture the differences between the two software packages. Further assume that these two types of software are generally incompatible and each type of software has its own degree of network externality.

The open source software (O) is offered by a single independent vendor whereas the closed source software is supplied by a commercial firm (C) and a piracy (P). Since software is easily reproduced with negligible cost, the model assumes that production is costless for all suppliers. Moreover, further assume that firms are not capable of discriminating consumers according to types. The commercial firm sells legal closed source software with price $p_c \in [0, \infty)$, and provides its legal clients with after sale services $s \geq 0$. The pirate supplier offers perfectly substitutable and compatible with the commercial products with zero-price. Generally, the open source software is provided for free; however, operating business incurs some costs. Thus, the study assumes that the open source firm provides its software with delivery fee or service charge $p_o \in [0, \infty)$ to maximize total revenue of the firm.

3.2 Software Users

There is a continuum of types of the potential consumers who are heterogeneous in two main respects. Firstly, some consumers gain extra utility from the after sale services and supports provided by the commercial firm to its legal customers while the rest do not care for these services. Secondly, consumers have different taste parameters toward software packages. We assume that consumers determine their ideal product according to their level of programming skills – the more skill the user, the higher preference toward programming flexibility. Every consumer obtains a gross utility (v) from using software packages. Assume that it is sufficiently large enough so that every consumer always inelastically demands for a piece of software, either

by purchasing commercial closed source software for a price p_c , by pirating it for free, or by adopting open source software for a delivery fee p_o .

Typically, the model classifies consumers into two groups uniformly distributed on the product space with density of 1 per type. They are also characterized by a taste parameter x , i.e., $x \in [0,1]$, indicating each consumer's ideal product. Consumers with taste parameters close to 0 prefer closed source software while those whose taste parameters close to 1 prefer open source software. Formally, consumers are classified into high-type and low-type consumers as following.

(a) High-type consumers (H) always demands for legal products and cannot pirate any product due to prohibitively high cost of using illegal software. Cooperate users, organizations, and universities could be categorized into this group. Moreover, each high-type consumer gains an extra utility $s \geq 0$ from after sale services and supports such as training and updating provided by the commercial firm. In presence of network externalities, she/he is willing to pay more if others also have the same product. Thus, for each high-type consumer, his/her utility function indexed by $x \in [0,1]$ is given as follows.

$$U(x, H) = \begin{cases} v - x - p_c + s + \gamma_{cs} Q_{cs} & \text{if buy commercial software (C)} \\ v - (1-x) - p_o + b + \gamma_o Q_o & \text{if buy open source software (O)} \end{cases} \quad (3.1)$$

where γ_{cs} is the strength of network externality or the importance of network size to a closed source software user; $\gamma_{cs} \geq 0$,

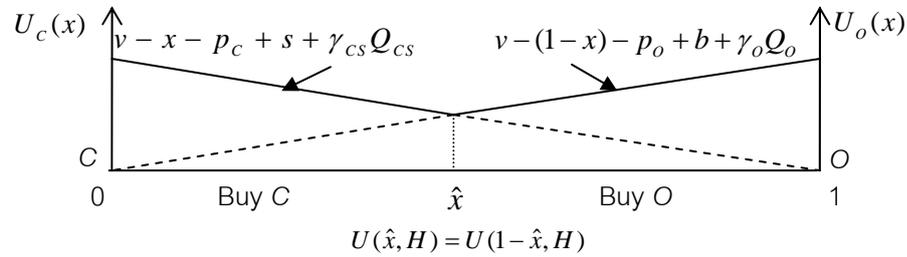
γ_o is the strength of network externality or the importance of network size to an open source software user; $\gamma_o \geq 0$,

Q_{cs} is total quantity demand for the closed source software from both the commercial software and the pirate software ($q_c + q_p$), and

Q_o is total quantity demand for the open source software from both high-type (q_{oh}) and low-type (q_{ol}) markets.

From equation (3.1), the market demand in the high-type market is obtained by calculating for a marginal consumer who is indifferent between buying the commercial software and the open source software as shown in Figure 3.1 below.

Figure 3.1
Linear City of High-Type Market



Formally, the marginal consumer \hat{x} can be derived by equating his/her utility from consuming these two products, i.e., $U_c(\hat{x}, H) = U_o(\hat{x}, H)$. We have:

$$\hat{x} = \frac{1}{2} + \left(\frac{s - p_c}{2} \right) - \left(\frac{b - p_o}{2} \right) + \frac{\gamma_{cs}}{2} Q_{cs} - \frac{\gamma_o}{2} Q_o$$

In this setting, the marginal consumer \hat{x} splits the continuum of consumers into two groups. All consumers with taste parameter $x \leq \hat{x}$ purchase the commercial software and those with taste parameter $x > \hat{x}$ purchase the open source software. Thus, demand for the commercial software is:

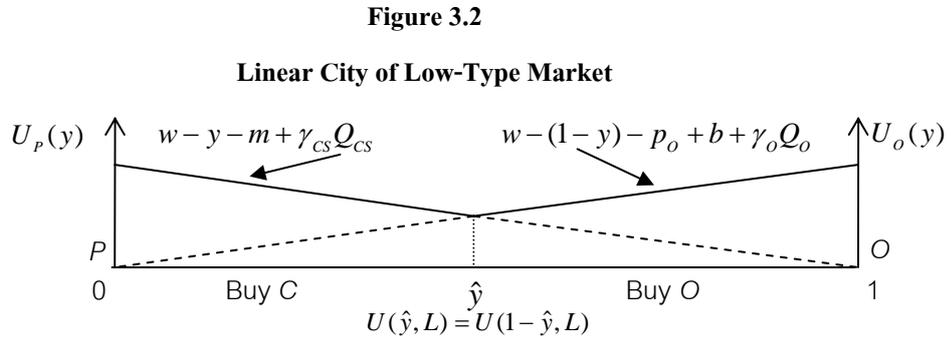
$$q_c = \frac{1}{2} + \left(\frac{s - p_c}{2} \right) - \left(\frac{b - p_o}{2} \right) + \frac{\gamma_{cs}}{2} Q_{cs} - \frac{\gamma_o}{2} Q_o \quad (3.2)$$

(a) Low-type consumers (L) refer to those having indifferent preference toward the after sale services. Hence, they put no value on the services and will not derive any additional utility from after sale services and support, i.e., $s = 0$. Besides, since this group of consumers represents a group of individual users, it is more difficult to monitor and charge penalties if they use pirate software. Then, the model simply assumes that the low-type consumers never buy legal closed source software if there is pirate software available in the market. However, by adopting pirate software, they will face the intensity of monitoring software piracy m which could be interpreted as irritation of being monitoring by the commercial firm. Thus, the utility function of

each low-type consumer and indexed by $y \in [0,1]$ is as follows. Note that index y is used instead of x for not being confused later on.

$$U(y, L) = \begin{cases} w - y - m + \gamma_{CS} Q_{CS} & \text{if use pirate software (P)} \\ w - (1 - y) - p_o + b + \gamma_o Q_o & \text{if use open source software (O)} \end{cases} \quad (3.3)$$

From the equation (3.3), the market demand in the low-type market is obtained by calculating for a marginal consumer who is indifferent between the pirate software and the open source software as shown in Figure 3.2 below.



By the same token as above, the marginal consumer \hat{y} can be derived by equating his/her utility from consuming these two products, i.e., $U_p(\hat{y}, L) = U_o(\hat{y}, L)$. Thus,

$$\hat{y} = \frac{1}{2} - \left(\frac{m}{2}\right) - \left(\frac{b - p_o}{2}\right) + \frac{\gamma_{CS}}{2} Q_{CS} - \frac{\gamma_o}{2} Q_o.$$

All consumers with taste parameter $y \leq \hat{y}$ use the unauthorized closed source software and those with taste parameter $y > \hat{y}$ adopt the open source software. Thus, demand for the pirate software is:

$$q_p = \frac{1}{2} - \left(\frac{m}{2}\right) - \left(\frac{b - p_o}{2}\right) + \frac{\gamma_{CS}}{2} Q_{CS} - \frac{\gamma_o}{2} Q_o \quad (3.4)$$

Since total demand for the closed source software is consist of the demand from both the commercial and the pirate products, i.e., $Q_{CS} = q_c + q_p$. From equation (3.2) and (3.4) and the assumption that software market is cover, i.e., $Q_{CS} + Q_o = 2$, we have:

$$Q_{cs} = \frac{(1-2\gamma_o)-(b-p_o)}{1-\gamma_{cs}-\gamma_o} + \left(\frac{s-p_c}{2(1-\gamma_{cs}-\gamma_o)} \right) - \left(\frac{m}{2(1-\gamma_{cs}-\gamma_o)} \right) \quad (3.5)$$

where $(\gamma_{cs} + \gamma_o) \neq 1$.

By the law of demand, a rise in price of the commercial software will cause demand for the closed source software to drop. The coefficient of p_c needs to be negative, thus, $1-\gamma_{cs}-\gamma_o$ must be positive in this case. Besides, by the definition of network externalities, both γ_{cs} and γ_o have to be non-negative. Thus, $(\gamma_{cs} + \gamma_o)$ must lie in the range of $[0,1)$.

Assumption 1: *The combined degree of network externalities parameter is in the range of $[0,1)$, i.e., $(\gamma_{cs} + \gamma_o) \in [0,1)$.*

Equation (3.5) shows that the demand for the closed source software depends positively on the value of after sale services (s) provided by commercial firm and the delivery fee (p_o) charged by the open source provider but negatively on the benefit of using the open source software (b) and the intensity level of monitoring pirate software (m).

Now, from equation (3.5) and the fact that $Q_o = 2 - Q_{cs}$, we could derive demand for the open source software as below. Given $(\gamma_{cs} + \gamma_o) \in [0,1)$,

$$Q_o = \frac{(1-2\gamma_{cs})+(b-p_o)}{1-\gamma_{cs}-\gamma_o} - \left(\frac{s-p_c}{2(1-\gamma_{cs}-\gamma_o)} \right) + \frac{m}{2(1-\gamma_{cs}-\gamma_o)} \quad (3.6)$$

The results show that the quantity demand for the open source software depends negatively on the value of after sale services to consumers s and its own price p_o but positively depends on the price of its rival p_c , the intensity level of monitoring pirate product m , and the benefit of using the open source software b .

Next, substitute equations (3.5) and (3.6) into the equation (3.2) for the demand for the commercial software, q_c . Given $(\gamma_{cs} + \gamma_o) \in [0,1)$, we have:

$$q_c = \frac{(1-2\gamma_o)}{2(1-\gamma_{cs}-\gamma_o)} + \frac{(s-p_c)(2-\gamma_{cs}-\gamma_o)}{4(1-\gamma_{cs}-\gamma_o)} - \left(\frac{m(\gamma_{cs}+\gamma_o)}{4(1-\gamma_{cs}-\gamma_o)} \right) - \left(\frac{b-p_o}{2(1-\gamma_{cs}-\gamma_o)} \right) \quad (3.7)$$

We could see that the demand for the legal software increases in value of the after sale services and the price of open source software but decreases in the price of commercial software and the benefit of using the open source software.

Surprisingly, an increase in the intensity level of monitoring pirate software will result in a lower demand for the commercial software through a mechanism of network externalities. As the intensity level of monitoring piracy rises, cost of using the pirate software goes up causing fewer demand for the pirate product and so do the total user base of the closed source software. Via network externalities, the user base of the closed source software gets smaller; the utility from using the commercial software becomes lesser leading to a drop in demand for the commercial software and a further decline in the user base of the closed source software.

Next, let calculate for demand for the pirate software q_p by substituting equations (3.5) and (3.6) into the equation (3.4). Given $(\gamma_{cs} + \gamma_o) \in [0, 1)$, we have:

$$q_p = \frac{(1-2\gamma_o)}{2(1-\gamma_{cs}-\gamma_o)} + \frac{(s-p_c)(\gamma_{cs}+\gamma_o)}{4(1-\gamma_{cs}-\gamma_o)} - \left(\frac{m(2-\gamma_{cs}-\gamma_o)}{4(1-\gamma_{cs}-\gamma_o)} \right) - \left(\frac{b-p_o}{2(1-\gamma_{cs}-\gamma_o)} \right) \quad (3.8)$$

That is the more the difference between the after sale services over the price of commercial software, $s - p_c$, the more is the demand for the illegal software. Intuitively, a rise of the differences makes demand for the commercial software grow in general and consequently, enlarges the user base of the closed source software. By means of network externalities, utility from employing the closed source software rises. Therefore, the higher the difference of $s - p_c$ in one market could result in a higher demand for compatible products in other markets. However, demand for the pirate software declines if the intensity level of monitoring piracy m increases or the benefit of using the open source software b rises.

3.3 The Equilibriums of the Model

Next, let consider strategic behaviors of the commercial and the open source providers. From the demand for the commercial software in the equation (3.7), the commercial firm sets its price to maximize the profits. The best response function of the commercial firm is:

$$p_c^* = \frac{1 - 2\gamma_o}{2 - \gamma_{cs} - \gamma_o} + \frac{s}{2} - \left(\frac{b - p_o}{2 - \gamma_{cs} - \gamma_o} \right) \quad (3.9)^4$$

Simultaneously, the open source provider will set its delivery fee p_o by maximizing its revenue. However, since we assume that the marginal cost of producing software is negligible and there is no fixed cost in the model, then from the equation (3.6), open source firm chooses p_o to maximize its total revenue $TR_o = p_o Q_o$. The best-response function of the open source firm is given by:

$$p_o^* = \left(\frac{1 - 2\gamma_{cs}}{2} \right) + \frac{b}{2} - \left(\frac{s - p_c}{4} \right) + \frac{m}{4} \quad (3.10)^5$$

Given $(\gamma_{cs} + \gamma_o) \in [0, 1)$, the equilibrium prices and intensity level of monitoring piracy are:

$$p_c^* = \frac{2(3 - 2\gamma_{cs} - 2\gamma_o) - 4\gamma_o + s(3 - 2\gamma_{cs} - 2\gamma_o) + m(1 - 2\gamma_{cs} - 2\gamma_o) - 2b}{7 - 4\gamma_{cs} - 4\gamma_o} \quad (3.11)$$

$$p_o^* = \frac{2(5 - 4\gamma_{cs} - 4\gamma_o) - 4\gamma_{cs}(3 - 2\gamma_{cs} - 2\gamma_o)}{2(7 - 4\gamma_{cs} - 4\gamma_o)} + \frac{-s(2 - \gamma_{cs} - \gamma_o) + m(4 - 3\gamma_{cs} - 3\gamma_o) + 2b(3 - 2\gamma_{cs} - 2\gamma_o)}{2(7 - 4\gamma_{cs} - 4\gamma_o)} \quad (3.12)$$

From equations (3.11) and (3.12), the condition in which all equilibriums to be positive is stated in Lemma 1.

Lemma 1: *Equilibrium prices of the commercial software and the open source software are positive if the benefits of using open source software are in the range of*

⁴ The second order condition also confirms the optimality as well as the stability of the equilibrium.

⁵ Ibid., footnote 4.

$$b \geq \frac{-2(5 - 4\gamma_{CS} - 4\gamma_o) + 4\gamma_{CS}(3 - 2\gamma_{CS} - 2\gamma_o) + s(2 - \gamma_{CS} - \gamma_o) - m(4 - 3\gamma_{CS} - 3\gamma_o)}{2(3 - 2\gamma_{CS} - 2\gamma_o)}$$

and $b \leq (3 - 2\gamma_{CS} - 2\gamma_o) - 2\gamma_o + \frac{s}{2}(3 - 2\gamma_{CS} - 2\gamma_o) + \frac{m}{2}(1 - 2\gamma_{CS} - 2\gamma_o)$, where $(\gamma_{CS} + \gamma_o) \in [0, 1]^6$.

Next, substitute the equilibriums in equations (3.11) and (3.12) into equations (3.5) to (3.8) for the equilibrium demand. Given $(\gamma_{CS} + \gamma_o) \in [0, 1]$, we have:

$$q_c^* = (2 - \gamma_{CS} - \gamma_o) \left[\frac{2(3 - 2\gamma_{CS} - 4\gamma_o) + s(3 - 2\gamma_{CS} - 2\gamma_o) + m(1 - 2\gamma_{CS} - 2\gamma_o) - 2b}{4(1 - \gamma_{CS} - \gamma_o)(7 - 4\gamma_{CS} - 4\gamma_o)} \right] \quad (3.13)$$

$$q_p^* = \frac{2(12 - 17\gamma_{CS} - 25\gamma_o) + 12(\gamma_{CS} + \gamma_o)(\gamma_{CS} + 2\gamma_o) - s(2 - 5\gamma_{CS} - 5\gamma_o) - 2s(\gamma_{CS} + \gamma_o)^2}{4(1 - \gamma_{CS} - \gamma_o)(7 - 4\gamma_{CS} - 4\gamma_o)} \quad (3.14)$$

$$- \frac{m(10 - 11\gamma_{CS} - 11\gamma_o) + 2m(\gamma_{CS} + \gamma_o)^2 + 2b(4 - 3\gamma_{CS} - 3\gamma_o)}{4(1 - \gamma_{CS} - \gamma_o)(7 - 4\gamma_{CS} - 4\gamma_o)}$$

$$Q_{CS}^* = \frac{6(3 - 4\gamma_{CS} - 4\gamma_o) + 8(\gamma_{CS} + \gamma_o)(\gamma_{CS} + 2\gamma_o) + s(2 - \gamma_{CS} - \gamma_o) - m(4 - 3\gamma_{CS} - 3\gamma_o) - 2b(3 - 2\gamma_{CS} - 2\gamma_o)}{2(1 - \gamma_{CS} - \gamma_o)(7 - 4\gamma_{CS} - 4\gamma_o)} \quad (3.15)$$

$$Q_o^* = \frac{2(5 - 4\gamma_{CS} - 4\gamma_o) + 4\gamma_{CS}(3 - 2\gamma_{CS} - 2\gamma_o) - s(2 - \gamma_{CS} - \gamma_o) + m(4 - 3\gamma_{CS} - 3\gamma_o) + 2b(3 - 2\gamma_{CS} - 2\gamma_o)}{2(1 - \gamma_{CS} - \gamma_o)(7 - 4\gamma_{CS} - 4\gamma_o)} \quad (3.16)$$

Hence, the optimal profit of commercial firm and the optimal revenue of open source providers are, respectively:

$$\Pi_c^* = (2 - \gamma_{CS} - \gamma_o) \left[\frac{(2(3 - 2\gamma_{CS} - 4\gamma_o) + s(3 - 2\gamma_{CS} - 2\gamma_o) + m(1 - 2\gamma_{CS} - 2\gamma_o) - 2b)^2}{4(1 - \gamma_{CS} - \gamma_o)(7 - 4\gamma_{CS} - 4\gamma_o)^2} \right] \quad (3.17)$$

$$TR_o^* = \frac{(2(5 - 4\gamma_{CS} - 4\gamma_o) - 4\gamma_{CS}(3 - 2\gamma_{CS} - 2\gamma_o) - s(2 - \gamma_{CS} - \gamma_o) + m(4 - 3\gamma_{CS} - 3\gamma_o) + 2b(3 - 2\gamma_{CS} - 2\gamma_o))^2}{4(1 - \gamma_{CS} - \gamma_o)(7 - 4\gamma_{CS} - 4\gamma_o)^2} \quad (3.18)$$

From equations (3.13) to (3.16), we need to ensure that all the equilibriums are positive. Thus, the conditions for all equilibrium demand to be positive are as shown in Lemma 2.

⁶ See Appendix A.

Lemma 2: *The equilibrium quantities of both the commercial software and the open source software are positive if the benefits of using open source software, is in the range of*

$$b \geq \frac{-2(5 - 4\gamma_{CS} - 4\gamma_o) + 4\gamma_{CS}(3 - 2\gamma_{CS} - 2\gamma_o) + s(2 - \gamma_{CS} - \gamma_o) - m(4 - 3\gamma_{CS} - 3\gamma_o)}{2(3 - 2\gamma_{CS} - 2\gamma_o)}$$

$$b \leq \frac{2(4 - 3\gamma_{CS} - 3\gamma_o)(3 - 2\gamma_{CS} - 2\gamma_o) - s(2 - \gamma_{CS} - \gamma_o)(1 - 2\gamma_{CS} - 2\gamma_o)}{2(4 - 3\gamma_{CS} - 3\gamma_o)}$$

and

$$\frac{m(10 - 11\gamma_{CS} - 11\gamma_o + 2(\gamma_{CS} + \gamma_o)^2)}{2(4 - 3\gamma_{CS} - 3\gamma_o)},$$

where $(\gamma_{CS} + \gamma_o) \in [0,1)$ ⁷.

Since the range of the benefits of using open source software b from Lemma 2 is narrower than that from Lemma 1, the range of b under consideration is as stated in Lemma 3 below.

Lemma 3: *equilibrium prices (p_c^*, p_o^*) and quantities (q_c^*, q_p^*, Q_o^*) are positive if the benefits of using open source software is in the range of*

$$b \geq \frac{-2(5 - 4\gamma_{CS} - 4\gamma_o) + 4\gamma_{CS}(3 - 2\gamma_{CS} - 2\gamma_o) + s(2 - \gamma_{CS} - \gamma_o) - m(4 - 3\gamma_{CS} - 3\gamma_o)}{2(3 - 2\gamma_{CS} - 2\gamma_o)}$$

$$b \leq \frac{2(4 - 3\gamma_{CS} - 3\gamma_o)(3 - 2\gamma_{CS} - 2\gamma_o) - s(2 - \gamma_{CS} - \gamma_o)(1 - 2\gamma_{CS} - 2\gamma_o)}{2(4 - 3\gamma_{CS} - 3\gamma_o)}$$

and

$$\frac{m(10 - 11\gamma_{CS} - 11\gamma_o + 2(\gamma_{CS} + \gamma_o)^2)}{2(4 - 3\gamma_{CS} - 3\gamma_o)},$$

where $(\gamma_{CS} + \gamma_o) \in [0,1)$.

⁷ See Appendix B.

4. COMPARATIVE STATICS ANALYSIS OF THE MODEL

Along with this section, the comparative statics analysis will be provided beginning with the effects of the value of after sale services, the benefits of using open source software, the intensity in monitoring software piracy and finally the strength of network externalities. Propositions of the model will be summarized along with this section.

4.1 The Effect of an Increase in the Value of after Sale Services

Given $(\gamma_{cs} + \gamma_o) \in [0,1)$ and the range of benefits in using the open source software stated in Lemma 3, from the equilibrium demand for the commercial software in equation (3.13), we have:

$$\frac{\partial q_C^*}{\partial s} = \frac{1}{24} \left[3 + \frac{1}{7 - 4\gamma_{cs} - 4\gamma_o} + \frac{2}{1 - \gamma_{cs} - \gamma_o} \right] > 0 \quad (4.1)$$

$$\frac{\partial q_{OH}^*}{\partial s} = -\frac{1}{24} \left[3 + \frac{1}{7 - 4\gamma_{cs} - 4\gamma_o} + \frac{2}{1 - \gamma_{cs} - \gamma_o} \right] < 0 \quad (4.2)^8$$

This indicates that given the network externalities level, as the after sale services becomes more valuable, demand for the commercial software rises which consequently causes demand for the open source software in the high-type market to fall. This is because as the value of after sale services becomes more valuable to the high-type consumers, the utility from consuming the commercial software rises. It leads to a higher demand for the commercial software as well as for the overall closed source software but to a lower demand for the open source software in the high-type market for a fixed q_p^* as shown.

$$\frac{\partial Q_{CS}^*}{\partial s} = \frac{2 - \gamma_{cs} - \gamma_o}{2(1 - \gamma_{cs} - \gamma_o)(7 - 4\gamma_{cs} - 4\gamma_o)} > 0 \quad (4.3)$$

⁸ $q_{OH} = 1 - q_C$

$$\frac{\partial Q_o^*}{\partial s} = \frac{-(2 - \gamma_{cs} - \gamma_o)}{2(1 - \gamma_{cs} - \gamma_o)(7 - 4\gamma_{cs} - 4\gamma_o)} < 0 \quad (4.4)$$

Through network effects, an increase in the user base of the closed source software then feeds back into the utility function of the high-type consumers and makes the commercial software becomes more attractive. Given delivery fee of the open source software, the commercial firm is now able to set a higher price than before as shown.

$$\frac{\partial p_c^*}{\partial s} = \frac{1}{2} - \frac{1}{7 - 4\gamma_{cs} - 4\gamma_o} > 0 \quad (4.5)$$

The best response for the open source software to a rise in price of and demand for the commercial software is to lower its delivery fee in order to regain its consumer base.

$$\frac{\partial p_o^*}{\partial s} = \frac{-(2 - \gamma_{cs} - \gamma_o)}{2(7 - 4\gamma_{cs} - 4\gamma_o)} < 0 \quad (4.6)$$

The interaction of the reaction functions of the commercial and the open source software from equations (3.9) and (3.10) will be introduced here to give an intuitive explanation for changes in the equilibrium prices with respect to the incremental increase in the value of after sale services as shown below.

Figure 4.1

Shifts in the Reaction Functions as the Value of after Sale Services Increases

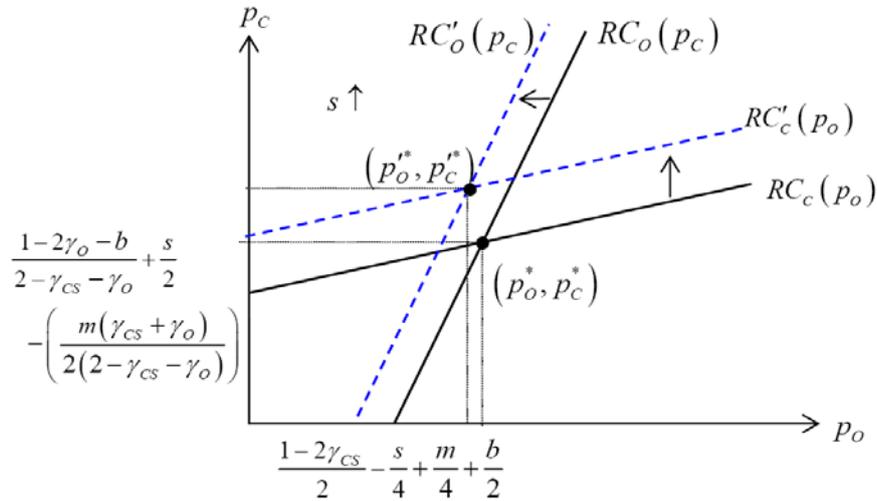


Figure 4.1 shows that given rise in the value of after sale services; the commercial firm has an incentive to raise its price corresponding to a greater demand for the product. Other things being constant, this shifts the commercial firm’s reaction function up. The commercial firm finds that it is better to set a higher price than it did previously for any given value of p_o .

However, the incremental increase in the value of after sale services directly and indirectly causes a fall in demand for the open source software. The open source firm’s best response is to lower its price. Thus, at a new equilibrium, as the value of after sale services increases, the price of commercial software rises while the price of open source software declines. The equilibrium prices change from (p_c^*, p_o^*) to (p_c^{r*}, p_o^{r*}) as shown in the figure above.

Interestingly, the effect of a small increase in the value of after sale services on the demand for the pirate software is unclear as summarized in the following proposition.

Proposition 1: *An increase in the value of after sale services provided by commercial firms could lead to a lower demand for the pirate software if the degree of combined network externalities $(\gamma_{CS} + \gamma_o)$ is weak or less than a half; otherwise, demand for the pirate software would rise.*

Proof: Considering the equilibrium demand for the pirate software in equation (3.17), we have:

$$\frac{\partial q_p^*}{\partial s} = \frac{-1}{24} \left[3 - \frac{2}{1 - \gamma_{CS} - \gamma_o} + \frac{5}{7 - 4\gamma_{CS} - 4\gamma_o} \right] \begin{cases} \leq 0 & \text{if } (\gamma_{CS} + \gamma_o) \in \left[0, \frac{1}{2}\right] \\ > 0 & \text{if } (\gamma_{CS} + \gamma_o) \in \left(\frac{1}{2}, 1\right) \end{cases} \quad (4.7)$$

$$\frac{\partial q_{OL}^*}{\partial s} = \frac{1}{24} \left[3 - \frac{2}{1 - \gamma_{CS} - \gamma_o} + \frac{5}{7 - 4\gamma_{CS} - 4\gamma_o} \right] \begin{cases} \geq 0 & \text{if } (\gamma_{CS} + \gamma_o) \in \left[0, \frac{1}{2}\right] \\ < 0 & \text{if } (\gamma_{CS} + \gamma_o) \in \left(\frac{1}{2}, 1\right) \end{cases} \quad (4.8)$$

The result indicates that demand for the pirate software could rise or fall as the high-type consumers put more value into the after sale services. Previously we know that as the after sale services become more valuable to consumers, demand for the commercial software and the total demand for the closed source software rise. Through the network effects, the larger user base of the closed source software causes the utility from using the pirate software and consequently demand for the pirate software to increase. However, it causes a drop in demand for the open source software. In order to maintain its position, the open source provider needs to lower its price as shown in Figure 4.1 to regain some of its customers in both markets which would partially dampen some of the demand for the pirate software.

In the case of weak network effects, the increase in demand for the pirate software via the effect of network externalities is not sufficiently large enough to compensate for a drop in demand for the pirate software as a result of a lower price for the open source software. In other words, the effect of network externalities is less than that of substitution effect in this case. Since the fall in demand for the pirate software is a secondary effect, in the end, the overall demand for the closed source software is still increasing as shown previously. Finally, the firm could also use the after sale services as a tool to fight software piracy in the case of weak network externalities as summarized in Proposition 1.

⁹ $q_{OL} = 1 - q_P$

However, in the case of strong network externalities, as the customer base for the closed source software enlarges from the incremental increase in the value of after sale services, the utility form using the closed source software increases significantly. In the low type market, demand for the pirate software increases drastically and more than compensates for a drop in demand for the pirate software as the open source software price becomes lower. Indeed, the result of network effects is more than that of substitution effect in this case. In the end, a rise in the value of after sale services in the high-type market also leads to a rise in demand for the pirate software in the low type market. These imply that, given that the closed source and the open source software are perfectly incompatible and there are strong network effects, a chance for the open source firm to survive in the market is slim if the commercial firm offers more services to its clients.

Finally, for the effect on the optimal profit of the commercial software firm and the total revenue of the open source firm, by considering the equations (3.17) and (3.18), respectively, we have:

$$\frac{\partial \Pi_c^*}{\partial s} = \frac{(2 - \gamma_{cs} - \gamma_o)(3 - 2\gamma_{cs} - 2\gamma_o)[2(3 - 2\gamma_{cs} - 2\gamma_o) - 4\gamma_o + s(3 - 2\gamma_{cs} - 2\gamma_o)]}{2(1 - \gamma_{cs} - \gamma_o)(7 - 4\gamma_{cs} - 4\gamma_o)^2} + \frac{(2 - \gamma_{cs} - \gamma_o)(3 - 2\gamma_{cs} - 2\gamma_o)[m(1 - 2\gamma_{cs} - 2\gamma_o) - 2b]}{2(1 - \gamma_{cs} - \gamma_o)(7 - 4\gamma_{cs} - 4\gamma_o)^2} > 0 \quad (4.9)$$

$$\frac{\partial TR_o^*}{\partial s} = \frac{-(2 - \gamma_{cs} - \gamma_o)[2(5 - 4\gamma_{cs} - 4\gamma_o) - 4\gamma_{cs}(3 - 2\gamma_{cs} - 2\gamma_o) - s(2 - \gamma_{cs} - \gamma_o)]}{(1 - \gamma_{cs} - \gamma_o)(7 - 4\gamma_{cs} - 4\gamma_o)^2} - \frac{(2 - \gamma_{cs} - \gamma_o)[m(4 - 3\gamma_{cs} - 3\gamma_o) + 2b(3 - 2\gamma_{cs} - 2\gamma_o)]}{(1 - \gamma_{cs} - \gamma_o)(7 - 4\gamma_{cs} - 4\gamma_o)^2} < 0 \quad (4.10)$$

The results show that the more the value of after sale services, the higher the profit to the firm but the fewer the total revenue to the open source shown above.

4.2 The Effects of an Increase in the Benefits of Using Open Source Software

For given $(\gamma_{cs} + \gamma_o) \in [0, 1)$ and the range of the benefits of using open source software stated in Lemma 3, we have, from the equation (3.16):

$$\frac{\partial Q_o^*}{\partial b} = \frac{3 - 2\gamma_{cs} - 2\gamma_o}{(1 - \gamma_{cs} - \gamma_o)(7 - 4\gamma_{cs} - 4\gamma_o)} > 0 \quad (4.11)$$

A rise in the benefits of using open source software causes the open source software to be more valuable. This will lead to a higher demand for the open source software. Through network effects, the result of an increase in the user base of the open source software will amplify the demand for the open source software. Now, for a given price of the commercial software, the open source firm is able to charge a higher delivery fee for the product as shown.

$$\frac{\partial p_o^*}{\partial b} = \frac{3 - 2\gamma_{cs} - 2\gamma_o}{7 - 4\gamma_{cs} - 4\gamma_o} > 0 \quad (4.12)$$

However, a rise in the benefits of using open source software will lower demand for the closed source software that is both demands for the commercial software and for the pirate software will drop regardless of the degree of network externalities as shown.

$$\frac{\partial q_c^*}{\partial b} = \frac{-(2 - \gamma_{cs} - \gamma_o)}{2(1 - \gamma_{cs} - \gamma_o)(7 - 4\gamma_{cs} - 4\gamma_o)} < 0 \quad (4.13)$$

$$\frac{\partial q_p^*}{\partial b} = \frac{-(4 - 3\gamma_{cs} - 3\gamma_o)}{2(1 - \gamma_{cs} - \gamma_o)(7 - 4\gamma_{cs} - 4\gamma_o)} < 0 \quad (4.14)$$

$$\frac{\partial Q_{cs}^*}{\partial b} = \frac{-(3 - 2\gamma_{cs} - 2\gamma_o)}{(1 - \gamma_{cs} - \gamma_o)(7 - 4\gamma_{cs} - 4\gamma_o)} < 0 \quad (4.15)$$

In order to survive in the market, the commercial firm needs to lower its price in response to a higher benefit from using the open source software to attract potential consumers as shown.

$$\frac{\partial p_c^*}{\partial b} = \frac{-2}{7 - 4\gamma_{cs} - 4\gamma_o} < 0 \quad (4.16)$$

The interaction of the two reaction functions of the commercial and the open source software from the equations (3.9) and (3.10) will be introduced to give an intuitive explanation for changes in the equilibrium prices with respect to the incremental increase in the benefits of using open source software as shown in Figure 4.2 below.

Figure 4.2

Shifts in the Reaction Functions as the Benefits of Using Open Source Software Increases

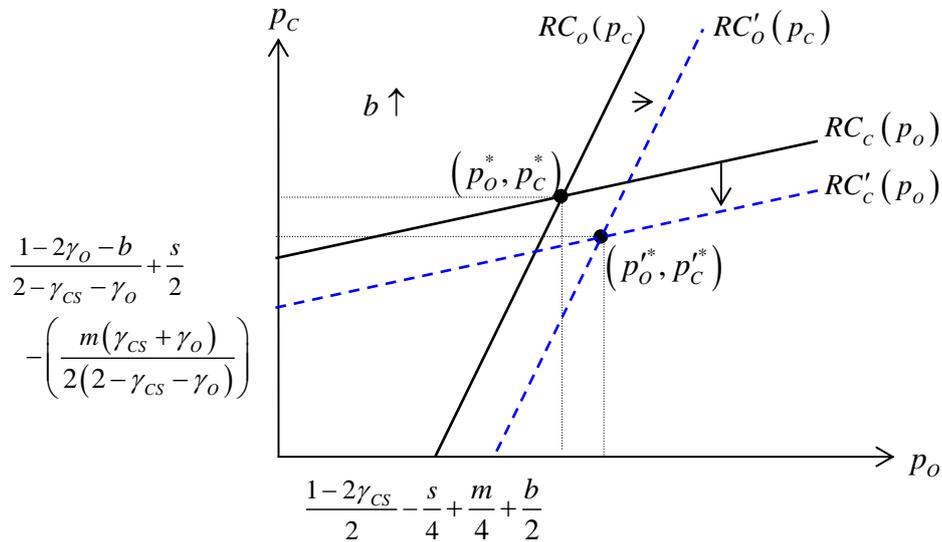


Figure 4.2 shows that given a rise in the benefits of using open source software, the open source provider has more incentive to raise its delivery fee due to a greater demand for the product. Other things being constant, this shifts the open source firm’s reaction function to the right. The provider finds it is better to set a higher price than it did previously for any given p_c .

Given the role of network externalities and a lower demand for the closed source software, the commercial firm realizes that its best response to the price of and demand for the open source software is to lower the price of commercial software leading the reaction function shifts downward. Thus, at new equilibrium prices, as the benefits of using open source software becomes greater to the consumer, the delivery fee of the open source software rises while the price of the commercial software declines, i.e., the equilibrium prices change from (p_c^*, p_o^*) to (p_c', p_o') as shown in the figure above.

4.3 The Effects of an Increase of the Intensity in Monitoring Software Piracy

Now, let investigate the effects of an increase in the intensity in monitoring piracy performed by a government on the equilibrium prices and quantities. For given

$(\gamma_{cs} + \gamma_o) \in [0,1)$ and the range of the benefits of using open source software as stated in Lemma 3, first we have:

$$\frac{\partial q_p^*}{\partial m} = \frac{-1}{24} \left[3 + \frac{2}{1 - \gamma_{cs} - \gamma_o} + \frac{25}{7 - 4\gamma_{cs} - 4\gamma_o} \right] < 0 \quad (4.17)$$

$$\frac{\partial q_{ol}^*}{\partial m} = \frac{1}{24} \left[3 + \frac{2}{1 - \gamma_{cs} - \gamma_o} + \frac{25}{7 - 4\gamma_{cs} - 4\gamma_o} \right] > 0 \quad (4.18)$$

Given the strength of network externalities, as the government increases the intensity in monitoring piracy, it causes more adverse effects to pirate users leading to a drop in demand for the pirate software. Thus, an increase in the intensity in monitoring leads to a higher demand for the open source software in the low-type market and the entire open source software, but a lower demand for the pirate software and the overall user base of the closed source software for a fixed q_c^* as follows.

$$\frac{\partial Q_{cs}^*}{\partial m} = \frac{-1}{6} \left[\frac{1}{1 - \gamma_{cs} - \gamma_o} + \frac{5}{7 - 4\gamma_{cs} - 4\gamma_o} \right] < 0 \quad (4.19)$$

$$\frac{\partial Q_o^*}{\partial m} = \frac{1}{6} \left[\frac{1}{1 - \gamma_{cs} - \gamma_o} + \frac{5}{7 - \gamma_{cs} - \gamma_o} \right] > 0 \quad (4.20)$$

Consequently, through the mechanism of network externalities, a fall in the client base of the closed source software would reduce the utility from using the commercial software. Then, demand for the commercial software falls. On the other hand, through the network effect, an increase in the clientele of the open source software causes the open source software to be more attractive. Given price of the commercial software, the open source firm is able to set a higher delivery fee and gain more revenue as follows.

$$\frac{\partial p_o^*}{\partial m} = \frac{3}{8} - \frac{5}{8(7 - 4\gamma_{cs} - 4\gamma_o)} > 0 \quad (4.21)$$

$$\begin{aligned}
\frac{\partial TR_o^*}{\partial m} &= \frac{2(3-2\gamma_{cs}-2\gamma_o)[2(5-4\gamma_{cs}-4\gamma_o)-4\gamma_{cs}(3-2\gamma_{cs}-2\gamma_o)]}{(1-\gamma_{cs}-\gamma_o)(7-4\gamma_{cs}-4\gamma_o)^2} \\
&\quad - \frac{2(3-2\gamma_{cs}-2\gamma_o)[s(2-\gamma_{cs}-\gamma_o)-m(4-3\gamma_{cs}-3\gamma_o)]}{(1-\gamma_{cs}-\gamma_o)(7-4\gamma_{cs}-4\gamma_o)^2} \\
&\quad + \frac{2(3-2\gamma_{cs}-2\gamma_o)[2b(3-2\gamma_{cs}-2\gamma_o)]}{(1-\gamma_{cs}-\gamma_o)(7-4\gamma_{cs}-4\gamma_o)^2} > 0
\end{aligned} \tag{4.22}$$

In response to the higher price of and demand for the open source software, the commercial firm could either increase or decrease its price with regard to the strength of the network externalities. The effects of an increase in the intensity in monitoring software piracy performed by the government on the equilibrium price of the commercial software are summarized in the following proposition.

Proposition 2: *An increase in the intensity in monitoring software piracy causes the price of the commercial software to rise if degree of the combined network externalities is weak or less than a half, otherwise, the commercial firm would lower its price.*

Proof: Consider equation (3.11), given all else constant, an increase in the intensity in monitoring software piracy affects the equilibrium price of the commercial software as stated below.

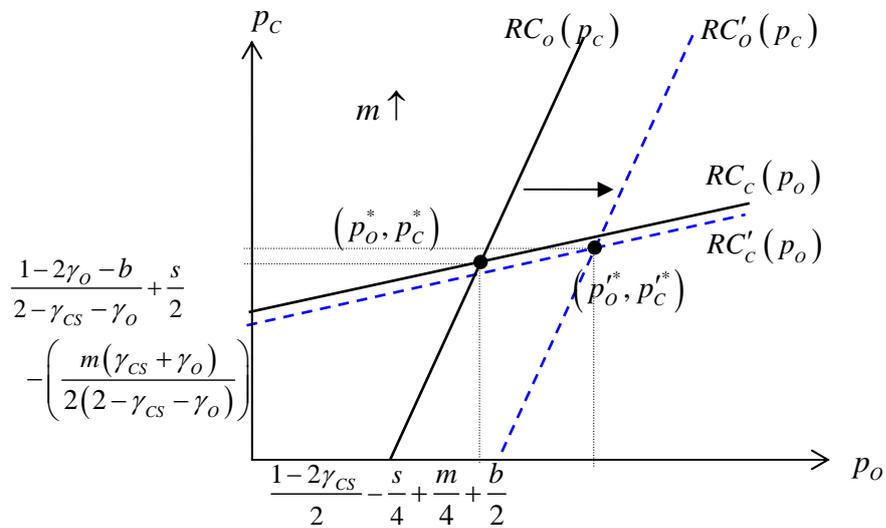
$$\frac{\partial p_c^*}{\partial m} = \frac{1}{2} - \frac{5}{2(7-4\gamma_{cs}-4\gamma_o)} \begin{cases} \geq 0 & \text{if } (\gamma_{cs} + \gamma_o) \in \left[0, \frac{1}{2}\right] \\ < 0 & \text{if } (\gamma_{cs} + \gamma_o) \in \left(\frac{1}{2}, 1\right) \end{cases} \tag{4.23}$$

Thus, an increase in the level of intensity in monitoring pirate software has an ambiguous affect on the optimal price of the commercial software depending upon the strength of combined network externalities. If the strength of the combined network effects is less than half, the commercial firm could set a higher price if the government monitors software piracy more. Instead, if degree of the combined network effects is greater than half, the commercial firm has to lower its price to fight the open source supplier. For intuitive purpose, shifts in the interactions of

the two reaction functions from equations (3.9) and (3.10) as a result of an increase in the intensity in monitoring pirate software are given for a better understanding of the mechanism behind the study are shown below.

Figure 4.3

**Shifts in the Reaction Functions as the Intensity in Monitoring Software Piracy Increases
in the Case of Weak Network Externalities**



(a) Case of weak network effects, i.e., $(\gamma_{cs} + \gamma_o) \in [0, 1/2)$

Generally, as the intensity in monitoring pirate software increases, some of the low type consumers switch to use the open source software due to higher cost of using the pirate software. As a result, demand for the open source software increases. Other things being constant, this shifts the open source firm’s reaction function to the right. Thus, given a rise in the intensity in monitoring piracy, the open source provider finds it is better off to set a higher price than it did previously for any given p_c .

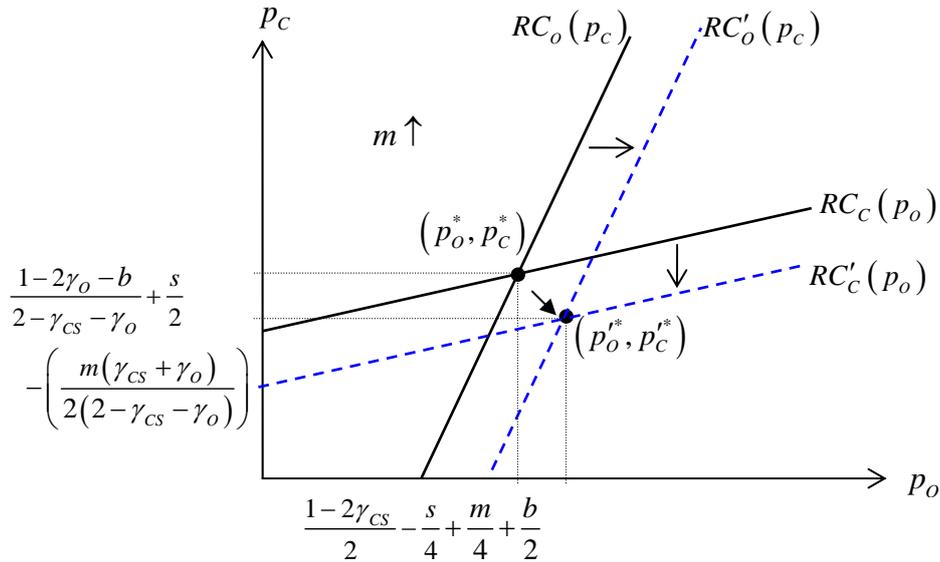
Through network externalities, as the intensity in monitoring piracy rises, demand for the pirate software declines leading to a lower user base of the closed source software as well as demand for the commercial software in the high-type market. As a result, the commercial firm

realizes that its best response to the price of and demand for the open source software is to lower its price. If the degree of network effects is weak or $(\gamma_{cs} + \gamma_o) \in [0, \frac{1}{2})$, it would be only a marginal drop in demand for the commercial software. There is no need for the commercial firm to push for a large drop in its price in order to regain its potential consumers in the high-type market. Thus, its reaction function slightly shifts downwards in this case.

Since a fall in the price of the commercial software is less than a rise in delivery price of the open source software, the best response of the commercial firm to a new equilibrium price of the open source software is to set a higher price than before. In this particular circumstance, the commercial firm realizes that the price competition from the commercial firm is now less intense and it then reacts by increasing its price as well. In other words, under weak network effects, the commercial firm is more willing to sacrifice some of the customer base of the closed source software and then cover that loss by increasing its price as well shown in Figure 4.3 above.

Figure 4.4

**Shifts in the Reaction Functions as the Intensity in Monitoring Software Piracy Increases
in the Case of Strong Network Externalities**



(b) Case of strong network effects, i.e., $(\gamma_{cs} + \gamma_o) \in (\frac{1}{2}, 1)$

As the government increases the intensity in monitoring pirate software, demand for the pirate software as well as the customer base of the closed source software decrease. Through strong network effects, it leads to a drastic decline in the demand for commercial software. As a result, the commercial firm needs to lower its price to maintain its customer base causing the reaction function to shift downward from RC_c to RC'_c as shown in Figure 4.4. On the other hand, the open source provider now realizes that for a rise in the intensity in monitoring, it is better for the firm to set a higher price than it did previously for any given p_c .

However, in the case of strong network effects, the noticeable drop in demand for the commercial software through the network effects cannot compensate for a drop in demand for the open source software due to an increase in the price of the open source software. In order to survive, the commercial firm needs to lower its price. Thus, in the end, as the government monitors the low-type market more frequently, it eventually leads to a new equilibrium where the

price of commercial software is set lower while the delivery fee of open source software is set higher than before as shown in the Figure 4.4 above.

Interestingly, the effect of an increase in the intensity in monitoring on the demand for commercial software is also unclear and depends upon the strength of network. The summary of the effects of an increase in the intensity in monitoring piracy by the government on the equilibrium demand for commercial software is shown in the following proposition.

Proposition 3: *An increase in the intensity in monitoring software piracy could lead to a higher demand for the commercial software if the degree of the combined network externalities is less than half; otherwise, the demand for the commercial software would fall.*

Proof: From the equilibrium demand for the commercial software in equation (3.13), we have:

$$\frac{\partial q_c^*}{\partial m} = \frac{1}{24} \left[3 - \frac{2}{1 - \gamma_{cs} - \gamma_o} + \frac{5}{7 - 4\gamma_{cs} - 4\gamma_o} \right] \begin{cases} \geq 0 & \text{if } (\gamma_{cs} + \gamma_o) \in \left[0, \frac{1}{2} \right] \\ < 0 & \text{if } (\gamma_{cs} + \gamma_o) \in \left(\frac{1}{2}, 1 \right) \end{cases} \quad (4.24)$$

The result emphasizes that there is no clear cut effect of an increase in the intensity in monitoring pirate software m on demand for the commercial software depending upon the degree of the combined network externalities $(\gamma_{cs} + \gamma_o)$. If it is less than a half or the strength of network externalities is weak, as m becomes higher, demand for the commercial software q_c^* rises and remains unchanged if the combined degree of network externalities equals a half. However, if $(\gamma_{cs} + \gamma_o) \in \left[0, \frac{1}{2} \right)$ or the strength of network externalities is strong, a higher m will cause demand for the commercial software q_c^* to fall.

Generally, an increase in the intensity in monitoring set by the government causes more adverse effects in using the pirate product leading to a drop in demand for the pirate software and consequently the total demand for the closed source software as respectively shown in equations (4.17) and (4.19). However, it causes a higher demand for the open source shown in

the equation (4.20). As a result, the open source provider will have an incentive to charge more for its delivery fee.

In the case of weak network effects, a decrease in the user base of the closed source software has only a marginal impact on demand for the commercial software. This combined with a drop in the price of commercial software and a rise in the delivery fee of open source software, thus, at a new equilibrium; demand for the commercial software rises. Thus, under this circumstance, limiting some levels of software piracy would have a favorable effect on the demand for the commercial software. Then, the commercial firm will have more incentive to complain of the damage caused by software piracy and ask the government to monitor it more frequently so that it can concentrate more on the high-type market. However, in the case of strong network effects, the decline in the clientele of the closed source software as the intensity in monitoring piracy increases causes the utility of using the commercial software to drop considerably. As the result, the demand for commercial software declines.

From all results of an increase in the intensity in monitoring piracy derived above, we can investigate the effect on the optimal profit of the commercial firm and total revenue of the open source provider which is summarized in following proposition.

Proposition 4: *An increase in the intensity in monitoring software piracy could lead to a higher profit of the commercial firm if degree of the combined network externalities is less than a half; otherwise, the profit would fall. However, it causes revenue of the open source to rise only.*

Proof: Consider the commercial firm's profit in the equation (3.17) and the open source provider's revenue in the equation (3.18), we have:

$$\frac{\partial \Pi_C^*}{\partial m} = \frac{(2 - \gamma_{CS} - \gamma_o)(1 - 2\gamma_{CS} - 2\gamma_o)[2(3 - 2\gamma_{CS} - 2\gamma_o) - 4\gamma_o + s(3 - 2\gamma_{CS} - 2\gamma_o)]}{2(1 - \gamma_{CS} - \gamma_o)(7 - 4\gamma_{CS} - 4\gamma_o)^2} + \frac{(2 - \gamma_{CS} - \gamma_o)(1 - 2\gamma_{CS} - 2\gamma_o)[m(1 - 2\gamma_{CS} - 2\gamma_o) - 2b]}{2(1 - \gamma_{CS} - \gamma_o)(7 - 4\gamma_{CS} - 4\gamma_o)^2}, \quad (4.25)$$

$$\text{where } \frac{\partial \Pi_c^*}{\partial m} = \begin{cases} \geq 0 & \text{if } (\gamma_{cs} + \gamma_o) \in [0, 1/2] \\ < 0 & \text{if } (\gamma_{cs} + \gamma_o) \in (1/2, 1) \end{cases}.$$

The result suggests that a rise in the intensity in monitoring pirate software has an ambiguous impact on the profit of the commercial firm depending on the degree of the combined network externalities. However, a rise in the intensity in monitoring piracy merely leads to higher revenue for the open source supplier. These factors confirm what we have observed regarding the price of and demand for the commercial software as stated in Proposition 2 and 3 above.

4.4 The Effect of an Increase in the Strength of Network Externalities

Now, let investigate the effects of the degree of network externalities on equilibrium prices, quantities and the optimal profit level. For given $(\gamma_{cs} + \gamma_o) \in [0, 1)$ and the range of the benefits of using open source software as stated in Lemma 3, we have the results on equilibrium prices summarized in Proposition 5.

Proposition 5: *The stronger the network externalities, the lower the equilibrium prices of both the commercial software and the open source software.*

Proof: Consider equations (3.11) and (3.12), respectively, for the effects of an incremental change in the degree of network externalities, we have:

$$\frac{\partial p_c^*}{\partial \gamma_{cs}} = \frac{-2(2 + 8\gamma_o + s + 5m + 4b)}{(7 - 4\gamma_{cs} - 4\gamma_o)^2} < 0 \quad (4.26)$$

$$\frac{\partial p_c^*}{\partial \gamma_o} = \frac{-2(16 - 8\gamma_{cs} + s + 5m + 4b)}{(7 - 4\gamma_{cs} - 4\gamma_o)^2} < 0 \quad (4.27)$$

$$\frac{\partial p_o^*}{\partial \gamma_{cs}} = \frac{-[4(25 - \gamma_{cs} - \gamma_o)(1 - \gamma_{cs} - \gamma_o) - 2\gamma_{cs} + 7(\gamma_{cs} + \gamma_o)^2 + s + 5m + 4b]}{2(7 - 4\gamma_{cs} - 4\gamma_o)^2} < 0 \quad (4.28)$$

$$\frac{\partial p_o^*}{\partial \gamma_o} = \frac{-(16 - 8\gamma_{cs} + s + 5m + 4b)}{2(7 - 4\gamma_{cs} - 4\gamma_o)^2} < 0 \quad (4.29)$$

As expected, the results show that an increase in the strength of network externalities causes both the commercial firm and the open source provider to have more incentives to capture as large a customer base as possible and exploit the network effects later on. To do so, undercutting prices against competitors will be a suitable strategy to employ. Thus equilibrium prices drop accordingly. These results confirmed what has been observed by previous studies (Katz and Shapiro, 1985; and Shy and Thisse, 1999)

The results emphasize that for other things being equal, as network effects become stronger, the equilibrium prices of the two differentiated software systems become smaller and close to zero if the combined network externalities approach unity. In other words, as network effects become stronger, price competition between these two paradigms becomes more intense which confirms what we observed previously.¹⁰

This fits to a stylized fact observed in the past few years as the open source software has the potential to create a local software industry and also lower the total cost of government information technology. In 2002, South Korea, Malaysia, Thailand, Indonesia, and Vietnam all embarked on projects to provide basic PCs equipped with open source software like Linux and OpenOffice to its citizens. It costs about \$250 per PC, a half price of a PC with the basic tools from Microsoft. In response to the low cost PCs for the masses campaigns in Asia, Microsoft rapidly responded with a special Thai language and Bahasa Malaysia versions of Windows and Office for \$40; and most recently an immediate 30% discount on its Tamil language version.

¹⁰For the effects of a change in degree of network externalities on the equilibrium quantities, the optimal profit of a commercial firm, and the revenue of the open source supplier are not able to give clear-cut directions in our analysis. Thus, we provide the detail of the results in Appendix D.

5. CONCLUSIONS AND LIMITATIONS

5.1 Conclusions

The major objective of this analysis is to study strategic competition between proprietary and open source software as well as the market outcomes in the presence of both software piracy and network externalities given product incompatibility. On the competition issue, under perfectly incompatible systems and non-price strategy, giving more benefit to users in forms of after sale services is a useful strategy to beat both the open source software and the software piracy out of the software industry. However, open source software can survive in the market if its user base is large enough or it gives a significant benefit to its users or just simply makes it more user-friendly.

For the level of protection, if strength of network effects is weak, a commercial firm should enforce copyright protection driving up price and profit, which states in Proposition 4. On the other hand, in the case of strong network externalities, it is benefit for firm to allow software piracy as one of the strategies firms pursues to increase their installed base. By allowing piracy, firm could certainly raise its customer base and win the market. This is because, through the effects of network externalities, firm could increase legal customers' willingness to pay. As the larger the number of users the greater is the value of product, thereby increasing profit for the firm selling the original. Thereafter, as market become more mature or being monopolized, firm could reap profit through protection. Finally, the model suggests that changes in level of the intensity of monitoring give the ambiguous results on demand for and price of the commercial software depending on strength of the network externalities. However, a rise in the intensity in monitoring piracy merely leads to higher revenue for the open source supplier.

5.2 Limitations

The model being studied here is far from being a complete analysis of a complex market like software. One of the reasons is that the model is in the form of a simple static model with no direct interaction between the commercial firm and the pirate supplier. Indeed, the model strictly

assumes that the high-type consumers never buy pirate software and the low-type consumers always prefer pirate software over the legal one. To solve this limitation, one could adopt a model of a circular city, so that firms can be kept in symmetric positions in which quality of all products is the same. Notice that it may come up with quite tedious results; one might need to pay great attention in setting assumptions in constructing the model.

Another resolution is employing the theory of vertical product differentiation. The reason is that even software could be copied with exact replicas; it incurs cost to pirate software in the forms of time consuming in downloading and hacking and having no documents or manuals, for instance. Likewise, not only provided for free, the open source software is handed users useful products. However, it does not come with documents and it also requires some advanced skills. In contrast, proprietary software is normally a user-friendly product coming with documents and supports. With all characteristics of each type of software, constructing model based on the vertical product differentiation might give more insight about the software industry.

In addition, the model gives the government authority in performing software piracy and leaves out investigation on the effect of after sale services as a non-price strategy of the commercial firm. By allowing them to be the firm's choice variables, we could derive a clearer picture for the analysis.¹¹ Other limitation of this article is that the model also ignores some other effects, such as switching costs and the lock-in effect for simplicity. This can be done for example by introducing more variables to the model, assigning higher degree of consumers' preference taste or higher cost of switching to employ other product, or relaxing some of the stringent assumptions of present study.

Finally, since software piracy is in some sense a dynamic phenomenon, it could be employed as strategic competition now and eliminated afterwards. By means of dynamics, the

¹¹ The extension of this article is done by allowing the intensity of monitoring piracy m to be the commercial firm's choice variable. Thereby, the firm is being able to conduct the copyright protection policy itself. The detail and results of the study is presented at the third National Conference of Economists at Kasetsart University.

model would capture more of the real world phenomena and gives more in-depth understanding of the software industry. Besides, it can be used to study the effects of switching costs and the lock-in customer at the same time as well as to investigate the issue of innovation and development.

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APPENDIX A
PROOF OF LEMMA 1

To find the condition, we first set the equilibriums p_C^* , m^* , and p_O^* to be positive and calculate for the range of b . From the equations (3.11) and (3.12), respectively, we have:

$$p_C^* \geq 0 \text{ if } b \leq (3 - 2\gamma_{CS} - 2\gamma_O) - 2\gamma_O + \frac{s}{2}(3 - 2\gamma_{CS} - 2\gamma_O) + \frac{m}{2}(1 - 2\gamma_{CS} - 2\gamma_O) \quad (\text{B.1})$$

$$p_O^* \geq 0 \text{ if } b \geq \frac{-2(5 - 4\gamma_{CS} - 4\gamma_O) + 4\gamma_{CS}(3 - 2\gamma_{CS} - 2\gamma_O) + s(2 - \gamma_{CS} - \gamma_O) - m(4 - 3\gamma_{CS} - 3\gamma_O)}{2(3 - 2\gamma_{CS} - 2\gamma_O)} \quad (\text{B.2})$$

Subtract the equation (B.1) by (B.2), we have:

$$\frac{(1 - \gamma_{CS} - \gamma_O)(7 - 4\gamma_{CS} - 4\gamma_O)(4 + s + m)}{2(3 - 2\gamma_{CS} - 2\gamma_O)} > 0 \quad \because (\gamma_{CS} + \gamma_O) \in [0, 1] \quad (\text{B.3})$$

Since the range of b from the equation (B.1) is greater than that of equation (B.2), we can conclude that given $(\gamma_{CS} + \gamma_O) \in [0, 1]$, the condition for $p_i^* \geq 0$, where $i = C, O$ is that the benefits of using the open source software b is in the range of

$$b \geq \frac{-2(5 - 4\gamma_{CS} - 4\gamma_O) + 4\gamma_{CS}(3 - 2\gamma_{CS} - 2\gamma_O) + s(2 - \gamma_{CS} - \gamma_O) - m(4 - 3\gamma_{CS} - 3\gamma_O)}{2(3 - 2\gamma_{CS} - 2\gamma_O)}$$

$$\text{and } b \leq (3 - 2\gamma_{CS} - 2\gamma_O) - 2\gamma_O + \frac{s}{2}(3 - 2\gamma_{CS} - 2\gamma_O) + \frac{m}{2}(1 - 2\gamma_{CS} - 2\gamma_O). \quad \text{Q.E.D.}$$

APPENDIX B

PROOF OF LEMMA 2

To find the condition in which the equilibrium quantities q_c^* , q_p^* , and Q_o^* are positive, the analysis first sets the equilibriums from equations (3.13), (3.14) and (3.16) to be positive and calculates the range of b for each equation. From the equation (3.13), (3.14) and (3.16), respectively, we have:

$$q_c^* \geq 0 \text{ if } b \leq (3 - 2\gamma_{cs} - 2\gamma_o) - 2\gamma_o + \frac{s}{2}(3 - 2\gamma_{cs} - 2\gamma_o) + \frac{m}{2}(1 - 2\gamma_{cs} - 2\gamma_o) \quad (C.1)$$

$$q_p^* \geq 0 \text{ if } b \leq \frac{2(4 - 3\gamma_{cs} - 3\gamma_o)(3 - 2\gamma_{cs} - 2\gamma_o) - s(2 - \gamma_{cs} - \gamma_o)(1 - 2\gamma_{cs} - 2\gamma_o)}{2(4 - 3\gamma_{cs} - 3\gamma_o)} - \frac{m(10 - 11\gamma_{cs} - 11\gamma_o + 2(\gamma_{cs} + \gamma_o)^2)}{2(4 - 3\gamma_{cs} - 3\gamma_o)} \quad (C.2)$$

$$Q_o^* \geq 0 \text{ if } b \geq \frac{-2(5 - 4\gamma_{cs} - 4\gamma_o) + 4\gamma_{cs}(3 - 2\gamma_{cs} - 2\gamma_o) + s(2 - \gamma_{cs} - \gamma_o) - m(4 - 3\gamma_{cs} - 3\gamma_o)}{2(3 - 2\gamma_{cs} - 2\gamma_o)} \quad (C.3)$$

Let us subtract the equation (C.1) by (C.2), we have:

$$\frac{(1 - \gamma_{cs} - \gamma_o)(7 - 4\gamma_{cs} - 4\gamma_o)(s + m)}{4 - 3\gamma_{cs} - 3\gamma_o} > 0 \quad \because (\gamma_{cs} + \gamma_o) \in [0, 1) \quad (C.4)$$

Then, subtracting the equation (C.2) by (C.3), we obtain:

$$\frac{(1 - \gamma_{cs} - \gamma_o)(7 - 4\gamma_{cs} - 4\gamma_o)(4(4 - 3\gamma_{cs} - 3\gamma_o) - (s + m)(2 - \gamma_{cs} - \gamma_o))}{2(3 - 2\gamma_{cs} - 2\gamma_o)(4 - 3\gamma_{cs} - 3\gamma_o)} > 0 \quad (C.5)$$

$$\because (\gamma_{cs} + \gamma_o) \in [0, 1)$$

Because the range of benefit of using the open source software b from the equation (C.1) is greater than those of equations (C.2) and (C.3) and the range of b from the equation (C.2) is also larger than that of equation (C.3). The study concludes that the condition for the equilibrium quantities being positive is that given $(\gamma_{cs} + \gamma_o) \in [0, 1)$, the benefit of using the open source software b is in the range of

$$b \geq \frac{-2(5-4\gamma_{cs}-4\gamma_o)+4\gamma_{cs}(3-2\gamma_{cs}-2\gamma_o)+s(2-\gamma_{cs}-\gamma_o)-m(4-3\gamma_{cs}-3\gamma_o)}{2(3-2\gamma_{cs}-2\gamma_o)}$$

$$\text{and } b \leq \frac{2(4-3\gamma_{cs}-3\gamma_o)(3-2\gamma_{cs}-2\gamma_o)-s(2-\gamma_{cs}-\gamma_o)(1-2\gamma_{cs}-2\gamma_o)}{2(4-3\gamma_{cs}-3\gamma_o)} - \frac{m(10-11\gamma_{cs}-11\gamma_o+2(\gamma_{cs}+\gamma_o)^2)}{2(4-3\gamma_{cs}-3\gamma_o)}$$

Q.E.D